NSWCCD-65-TR–2004/16 Design Equations and Criteria of Orthotropic Composite Panels

Naval Surface Warfare Center
Carderock Division
West Bethesda, MD 20817-5700

NSWCCD-65-TR–2004/16 June 2004

Survivability, Structures, and Materials Department

Technical Report

Design Equations and Criteria of Orthotropic Composite Panels

by

Brian J. Jones

DEPARTMENT OF THE NAVY

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2. Comments or questions may be referred to Mr. Brian J. Jones, Code 655; telephone (301) 227-4130; e-mail, JonesBJ@nswccd.navy.mil.

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The U.S. Navy is currently pursuing lightweight material options for shipboard structural applications, including hull, topside and mast structures. It is necessary to develop preliminary design criteria and methods to assess structures before the development of detailed finite element models. The design equations and criteria presented in this report are for simply-supported orthotropic plates typically used in composite ship structures. The design equations presented provide orthotropic plate solutions for principal use in code development. In addition, the method could extend to overall ship structural design, for scantling design, or for input into an overall design program. This document provides design equations and criteria for simply-supported orthotropic plates in the context of the design of composite hull and topside structures. It identifies ship structural loads, structural design criteria, and design equations for solid, sandwich and hat-stiffened panels, as well as girders and frames.

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1. Introduction

The U.S. Navy is currently pursuing lightweight material options for shipboard structural applications, including hull, topside and mast structures. It is necessary to develop preliminary design criteria and methods to assess structures before the development of detailed finite element models.

The design equations and criteria presented in this report are for simply-supported orthotropic plates typically used in composite ship structures. The design equations presented provide orthotropic plate solutions for principal use in code development. In addition, the method could extend to overall ship structural design, for scantling design, or for input into an overall design program.

This document provides design equations and criteria for simply-supported orthotropic plates in the context of the design of composite hull and topside structures. It identifies ship structural loads, structural design criteria, and design equations for solid, sandwich and hat-stiffened panels, as well as girders and frames.

2. Ship Structural Design Loads

Loads on the hull structure define the basic requirements for any ship structure. The hull loads are related to geometry, size, speed, and operational and combat environments of the ship. The determination of the loads is a crucial part of the composite structural design process.

Within the ship structure, there are a variety of loads which are experienced by the various parts. Because of the variety of loads acting on the hull, bulkheads and decks, it is important to first identify, then define each load and conditions in which they can occur. Only when there is an understanding of the frequency can we understand when and how the loads may be combined.

From the structural design manual for surface ships [1], loads are grouped into four categories, basic loads, sea environment, operational environment, and combat environment; see Figure 1. Table 1 shows ship structural members and pertinent loads and their combination for analysis.

Loads for naval ship design are discussed in the structural design manual for surface ships [1] and also by, a topside design guide [2]. These documents provide much of the background information on which this report is based. Recently, classification societies, Det Norske Veratis (DNV) and American Bureau of Shipping (ABS), have added more specific information about basic, sea environment, and operational environment loads of naval ships and of ships with non-traditional hull forms [3,4,5].

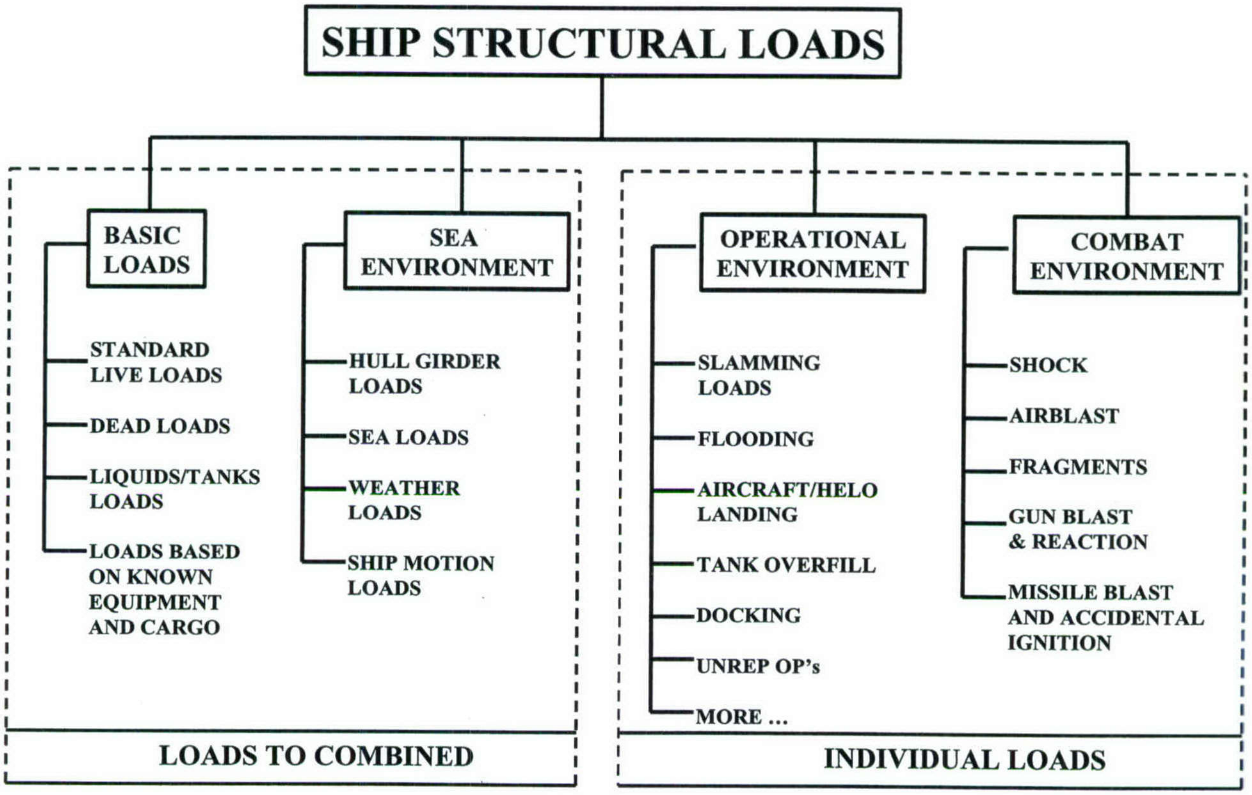


Figure 1. Categories of Ship Structural Loads

Table 1. Ship Structures Load Application Chart

	Ship Component	Loads to be Combined							Load to be Applied Independently									
		Primary	Hydrostatic	Tank Loads	Dead Loads	Live Load	Weather	Stowage	Wave Slap	Tank Overflow	Flooding	Gun and Missile Blast	Interior Pressure	Dry-Docking	Operating	Aircraft/Helo Landing	Nuclear Air Blast	Shock
1	**Shell & Frame**																	
A	Midship	X	X	X	X	-	-	-	-	X	-	-	-	X	X	-	X	-
B	Forward	X	X	X	X	-	-	-	X	X	-	-	-	X	X	-	X	-
C	Aft	X	X	X	X	-	-	-	-	X	-	-	-	X	X	-	X	-
D	Sponson Shell	-	X	-	X	-	X	-	X	-	-	-	-	-	X	-	X	-
E	Web Frame	-	X	X	X	-	X	X	X	X	X	X	-	X	X	-	X	-
2	**Bulkhead**																	
A	Longitudinal	X	-	X	X	-	-	-	-	X	X	-	X	-	-	-	-	-
B	Transverse	-	-	X	X	-	-	-	-	X	X	-	X	-	-	-	X	-
C	Bends	-	-	-	X	X	-	X	-	-	-	-	-	-	-	X	X	-
D	Misc.	-	-	X	X	-	-	-	-	X	X	-	X	-	-	-	-	-
3	**Decks**																	
A	Interior	X	-	X	X	X	-	X	-	X	X	-	X	-	X	-	-	-
B	Weather	X	X	-	X	X	X	X	X	X	-	X	X	-	X	X	X	-
C	Platforms	-	-	X	X	X	-	X	-	X	X	-	X	-	-	-	-	-
4	**Stanchion**	**-**	**X**	**X**	**X**	**X**	**X**	**X**	**-**	**X**	**X**	**X**	**X**	**-**	**X**	**X**	**X**	**-**
5	**Superstructures**																	
A	Long	X	X	-	X	X	X	X	-	-	-	X	X	-	X	X	X	-
B	Short	-	X	-	X	X	X	X	-	-	-	X	X	-	X	-	X	-
C	Deckhouse	-	X	-	X	X	X	X	-	-	-	X	X	-	X	-	X	-
6	**Foundations**																	
A	Critical Machinery/Equipment	-	-	-	X		-	-	-	-	-	-	-	-	X	-	-	X
B	Secondary Machinery/Equipment	-	-	-	X		-	-	-	-	-	-	-	-	X	-	-	-
C	Exposed Locations	-	-	-	X		X	-	X	-	-	X	-	-	X	-	X	-

3. Composite Failure Modes

The Definition of Failure

The following excerpts are from *Ship Structural Design Concepts*, SSDC-P1, Ship Structure Committee [6].

Damage

A structure is damaged if its original form has changed in a way which is detrimental to its future performance, even though there may be no immediate loss of function. Examples of damage include excessive permanent deformations or the appearance of cracks due to fatigue or local brittleness. In such cases the structure may still be able to sustain its design loads, but because of the possible adverse effects on performance or appearance, and hence on the confidence of operators and users, repairs should be effected as soon as convenient.

Collapse

This occurs when a structure is damaged so badly that it can no longer fulfill its function. This loss of function may be gradual, as in the case of a lengthening fatigue crack or spreading plasticity; or sudden, as when the failure occurs through plastic instability or through propagation of a brittle crack. In all cases the collapse load may be defined as the minimum load which will cause this loss of function.

The strength of laminates cannot, in general, be predicted reliably from their constituent properties; it is therefore usually evaluated by reference to test data. Theoretical models, supported by microscopic examination, have, however, provided insight into the mechanics of laminate failure. For composite materials, internal material failure frequently occurs before any macroscopic change in the composite is observed. Examples of internal failures that may lead to catastrophic failure of the composite structure include [7]:

1) Microcracking of the matrix,
2) Separation of the fibers from the matrix (debonding),
3) Failure or rupture of individual fibers, and
4) Separation of individual lamina from each other (delamination).

For composite sandwich laminates, failure modes include:

5) Above faiures for skin of sandwich,
6) Core failure due to yielding, cracking or ultimate strength, and
7) Skin/core delamination.

These failures modes can cause damage which may require composite repair; others may lead to overall structural failure or collapse. In the case of preliminary design, these failure modes can affect structural design criteria. Four classes of design criteria must be met for a feasible design:

- Deflection requirement,
- Strength limits,
- Buckling, and
- Natural frequency.

Properties normally required for design purposes and failure evaluation/prediction are tensile, compressive, flexural, in-plane and interlaminar shear strengths. Material modulus data are needed to determine deflection, buckling strength and natural frequencies. Material strength data are needed for comparison with strength allowables.

4. Overall Ship Design Process

The structural design process is initiated by assuming a set of ship hull cross-sectional scantlings for the midship plating. For the first cycle of the design, the contributions of longitudinal stiffeners to the overall hull sectional modulus may be neglected. Using the longitudinal hull girder loads, the induced primary stresses in the deck and hull bottom are calculated [8,9,10,11,12]. With these primary in-plane stresses and assumed transverse frame spacing and plate thickness, the assumed scantlings are checked for buckling under in-plane loads and ultimate strength under combined in-plane and lateral loadings. The calculated stress results and design criteria are used to modify the assumed set of cross-sectional scantlings and the iterative processes continues until an optimum set of scantlings is obtained.

4.1. Ship Cross-Section Analysis

Hull girder analysis requires that deck and bottom stress and overall ship deflection is checked. Using a maximum bending at mid-ship, the induced stress in the deck and bottom is given by:

$$\sigma_{Deck} = \frac{Mc_D}{I} \le \frac{\sigma_{ult}}{F.O.S.}$$
$$\sigma_{Keel} = \frac{Mc_B}{I} \le \frac{\sigma_{ult}}{F.O.S.}$$
$$where: \sigma_{ult} = \min(\sigma_{Compression}, \sigma_{Tension})$$

In addition to stress, hull girder deflection must be checked. Assume that the ship's weight is evenly distributed between perpendiculars and that the moment of inertia throughout the length of the hull is equal to the value at midships. The evenly distributed weight required to give a bending moment for the case of simply-supported ends gives [13]:

$$W = \frac{8M}{L_P}$$

The beam deflection can be calculated from the following:

$$\delta = \frac{5WL_P^{\ 3}}{384EI} \leq 1/200$$

The hull girder deflection is required to be less than 1/200.

4.2. Composite Panel Analysis

From the ship cross-sectional analysis mentioned above, in-plane axial forces and in-plane bending moments are determined. With knowledge of the out-of-plane load requirements, the ship structure can be segmented into panels for design analysis. Necessary equations and design criteria will be discussed in subsequent sections.

The design equations are split into three categories: solid laminate panels, sandwich panels, hat-stiffened-panels. Four classes of design criteria must be met for a feasible design:

- Deflection requirement,
- Strength limits,
- Buckling, and
- Natural frequency.

5. Design Criteria (Limit States)

5.1 Deflection Requirements

Structural stiffness is a major driver in the design of composite structures due to the low composite modulus. Deflection limits can be absolute limits or limits due to imposed design criteria. Absolute limits represent physical constraints based on tolerances or interference with adjacent structures or equipment. Limits due to design criteria aid in avoidance of other requirements such as crew habitability/visibility, motions (velocity or acceleration) or natural frequency.

The absolute deflection limits must be set on a case-by-case basis and are typically *not* load case dependent. Specified design criteria limits have values which could be as much as 1/25 and as little as 1/1000 in some civil engineering applications [14].

Deflection limits used in the past for composite structural design are:

L/200	internal (walking) decks panels
L/200	internal (walking) decks frame
L/100	internal (non-walking) decks panels
L/100	internal (non-walking) decks frames
L/200	hull exterior frames
L/150	hull exterior deck panels
L/150	hull exterior panels (elsewhere)
L/100	topside exterior panels and frame
L/50	bulkhead to bulkhead panels

5.2 Strength Limits

Using current methodology, material strengths are determined via stress. Historically, material knockdown factors that reduce ultimate strength values to account for laminate service conditions have also been derived from stress-based testing and calculation.

To determine failure with analytical or numerical (FE) results, maximum stress theory is most-commonly used. In comparison with other failure theories, maximum stress theory is conservative and well understood. Jenkins [15] stated that failure occurs when one or all of the orthotropic stress values exceed their maximum limits as obtained in uni-axial tension, compression or pure shear stress test, when material is tested to failure. For plane stress, this is:

$$\sigma_{11} = X$$
$$\sigma_{22} = Y$$
$$\sigma_{12} = S$$

5.3 Buckling Criteria

Ship structures generally have: [16]

Plate Buckling-Critical Member	4
Plate Buckling-Non-Critical Member	2
Stanchion and Stiffener Buckling	4

5.4 Natural Frequency Criteria

From the DDX deckhouse program, the approach to account for vibration at preliminary design level is to determine the panel's natural frequency. The local response of individual panels (panel natural frequencies) will be above 125% of the ship's blade rate values [17].

Where: Shaft Rate (Hz) = Shaft RPM / 60

Blade Rate (Hz) = # of Blades × Shaft RPM / 60

5.5 Design Allowables and Factors of Safety

5.5.1 Traditional Factor of Safety Approach

Typically, a traditional single factor of safety design has been used. Design data sheets for hull structures have used a single factor of safety from the ultimate strength [18].

stiffener or stanchion	4
static loading	4
long-term loading (creep)	4
fatigue	6
air blast load (one-time-load)	1.25
repeated impact load	8

5.5.2 Design Allowable Method

Design allowable stress levels should account for all applicable fabrication and loading condition uncertainties. These uncertainties include, but are not limited to the following:

1. End-use/In-service environment,
2. Fatigue loadings,
3. Sustained loadings,
4. Impact loadings,
5. Form and shape factors,
6. Laminate thickness effects,
7. Manufacturing variables,
8. Residual stresses and strains, and
9. Corrosion effects.

When accounting for these effects by testing or other methods, one may establish design allowables by first determining material allowables, and then applying a reduced factor of safety. The material allowables account for the anticipated in-service, worst-case environmental conditions, for example, elevated temperature, moisture, processing variability (B-basis statistical determinations). When material allowables are known, then the design allowables are determined as follows:

Using the material knockdown factors combined with a load uncertainty, the factor of safety can be determined.

$$\text{F.S.} = \frac{\prod_{i=1}^{N} \phi_i}{\prod_{j=1}^{M} K_j}$$

K = material knockdown factors

φ = load uncertainty factors

6. Panel Design Equations

6.1 Solid Laminate Panel Design

Consider a rectangular plate of length, a, width, b, and thickness, t shown in Figure 2. For marine construction, the rectangular panel loads can be simplified into an average edge force N_x, or N_y combined with an in-plane bending moment and in-plane shear, N_{xy}. In addition, the panel may be subjected to a uniform pressure, P, on the entire panel. Using classical plate theory for thin plates, Z-stress = 0 (no transverse shear deformation), the governing equation for displacement, w, of the plate in the Z direction is

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=F(x,y)$$

where D_{ij}'s are the flexural rigidities of the plate given by:

$$D_{11}=\frac{E_1 t^3}{12(1-\upsilon_{12}\upsilon_{21})},D_{22}=\frac{E_2 t^3}{12(1-\upsilon_{12}\upsilon_{21})}D_{12}=\frac{\upsilon_{21}E_1 t^3}{12(1-\upsilon_{12}\upsilon_{21})},D_{66}=\frac{G_{12}t^3}{12}$$

With these terms, the governing equation can be rewritten as:

$$\frac{\partial^4 w}{\partial x^4}+2\eta\lambda^{\frac{1}{2}}\frac{\partial^4 w}{\partial x^2\partial y^2}+\lambda\frac{\partial^4 w}{\partial y^4}=F(x,y)$$

where:

$$\lambda=\frac{D_{22}}{D_{11}},\qquad \eta=\frac{D_{12}+2D_{66}}{\sqrt{D_{11}D_{22}}}$$

The non-dimensional adjusted plate aspect ratio is given by:

$$R=\frac{\lambda^{1/4}a}{b}$$

Plate equations are valid for symmetric laminates (with no bending twist coupling, $B_{ij}=0$). For further discussion on plate equations, go to the detailed progression in Reference [19]. Additionally, in laminated parts, the flexural rigidities, D_{ij}'s, can be determined through classical laminate theory see Appendix A.

6.1.1 Deflection of Solid Laminate Panel

The deflection of a composite panel is important in composite design due to the low modulus of laminates. Because the modulus ranges from 1 to 10 Msi, deflection limits can govern design. For a simply-support panel, the deflection solution for a uniform pressure is given by the Navier solution [20]. This solution converges rapidly for deflection within 20 to 40 terms of the summation.

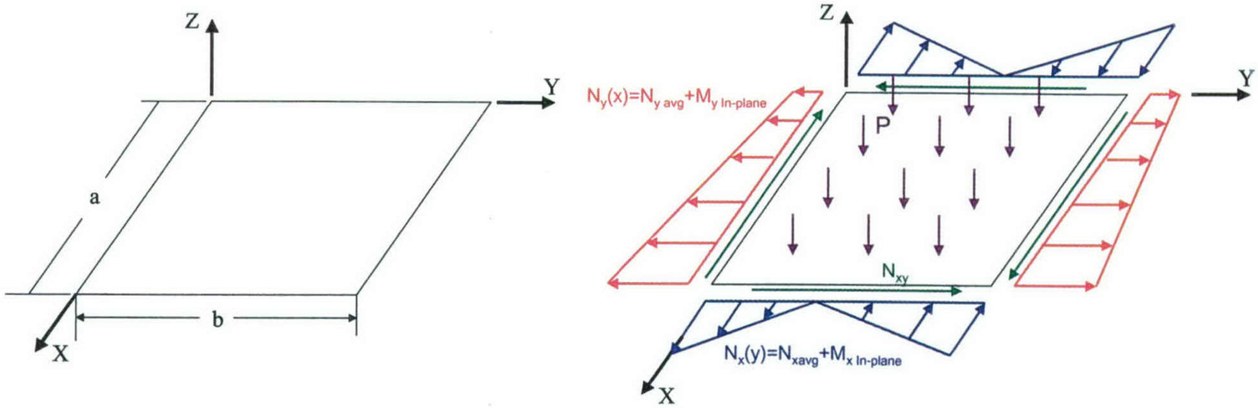


Figure 2. Generalized Rectangular Plate with Typical Panel Loads

$$\delta_{ss}(x,y)=\frac{16P}{\pi^6}\sum_{n=1,3,5\ldots}\sum_{m=1,3,5\ldots}\frac{1}{mn\overline{D}}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

where:

$$\overline{D}=D_{11}\left(\frac{m}{a}\right)^4+2(D_{12}+2D_{66})\left(\frac{mn}{ab}\right)^2+D_{22}\left(\frac{n}{b}\right)^4$$

For most panels in marine construction, the simply-support deflection is overly conservative. For this reason, the clamped-clamped rectangular panel is calculated [19]. The maximum deflection of a clamped-clamped rectangular panel is:

if $-1<\eta<(3.5)^{\wedge 0.5}$

$$\delta_{clamped}=\frac{Pb^4}{384D_{11}}\left[1-\frac{2\lambda_1\cosh\frac{\lambda_1 R}{2}\sin\frac{\lambda_2 R}{2}+2\lambda_2\sinh\frac{\lambda_1 R}{2}\cos\frac{\lambda_2 R}{2}}{\lambda_1\sinh\lambda_2 R+\lambda_2\sinh\lambda_1 R}\right]$$

$$\lambda_1=\sqrt{6\left(\sqrt{3.5}+\eta\right)},\lambda_2=\sqrt{6\left(\sqrt{3.5}-\eta\right)}$$

if $\eta>(3.5)^{\wedge 0.5}$

$$\delta_{clamped}=\frac{Pb^4}{384D_{11}}\left[1-\frac{\lambda_1\sinh\frac{\lambda_1 R}{2}-\lambda_2\sinh\frac{\lambda_2 R}{2}}{\lambda_1\sinh\frac{\lambda_1 R}{2}\cosh\frac{\lambda_2 R}{2}-\lambda_2\sinh\frac{\lambda_1 R}{2}\cosh\frac{\lambda_2 R}{2}}\right]$$

$$\lambda_1 = 2\sqrt{3\left(\eta + \sqrt{\eta^2 - 3.5}\right)}, \lambda_2 = 2\sqrt{3\left(\eta - \sqrt{\eta^2 - 3.5}\right)}$$

The maximum deflection of the simply-supported and clamped-clamped panel are combined using a weighted average W to estimated the panel deflection. W ranges between 0 and 1 and is typically taken to be 0.75.

$$\delta = \frac{W\delta_{ss} + (1-W)\delta_{Clamped}}{\left(1 - \frac{N_X^{Compression}}{N_{X_{CR}}}\right)\left(1 - \frac{N_Y^{Compression}}{N_{Y_{CR}}}\right)}$$

6.1.2 Stress of Solid Laminate Panel

For preliminary design and assessment, stresses in the simply-supported rectangular panel offer a conservative estimate of stress in the panel. Stress components can be determined through the extension of the Navier solution [20]. The Navier solution for a rectangular panel with a uniform pressure gives the following moment and shear resultants due to bending:

$$M_x(x,y) = \frac{16P}{\pi^4} \sum_{n=1,3,5\ldots} \sum_{m=1,3,5\ldots} \frac{1}{mn\overline{D}} \left[D_{11}\left(\frac{m}{a}\right)^2 + D_{12}\left(\frac{n}{b}\right)^2 \right] \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$M_y(x,y) = \frac{16P}{\pi^4} \sum_{n=1,3,5\ldots} \sum_{m=1,3,5\ldots} \frac{1}{mn\overline{D}} \left[D_{12}\left(\frac{m}{a}\right)^2 + D_{22}\left(\frac{n}{b}\right)^2 \right] \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$M_{xy}(x,y) = \frac{-32P}{\pi^4} \sum_{n=1,3,5\ldots} \sum_{m=1,3,5\ldots} \frac{D_{66}}{mn\overline{D}} \left(\frac{m}{a}\right)\left(\frac{n}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

$$Q_x(x,y) = \frac{16P}{\pi^2} \sum_{n=1,3,5\ldots} \sum_{m=1,3,5\ldots} \frac{1}{mn\overline{D}} \left[D_{11}\left(\frac{m}{a}\right)^3 + (D_{12} + 2D_{66})\left(\frac{m}{a}\right)\left(\frac{n}{b}\right)^2 \right] \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$Q_y(x,y) = \frac{16P}{\pi^2} \sum_{n=1,3,5\ldots} \sum_{m=1,3,5\ldots} \frac{1}{mn\overline{D}} \left[D_{22}\left(\frac{n}{b}\right)^3 + (D_{12} + 2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right) \right] \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

The maximum stresses for a solid laminate panel are:

$$\sigma_X^{Bending} = \frac{6M_x\left(\frac{a}{2},\frac{b}{2}\right)}{t^2}$$

$$\sigma_Y^{Bending} = \frac{6M_y\left(\frac{a}{2},\frac{b}{2}\right)}{t^2}$$

$$\sigma_{xy}^{Bending} = \frac{6M_{xy}(0,0)}{t^2}$$

$$\tau_{xz}^{Bending} = \frac{3Q_x(0,b/2)}{2t}$$

$$\tau_{yz}^{Bending} = \frac{3Q_y(a/2,0)}{2t}$$

In addition to the bending stress due to out-of-plane pressure, membrane components can be determined from in-plane loading.

$$\sigma_x^{Membrane} = \frac{N_X}{t}$$

$$\sigma_y^{Membrane} = \frac{N_Y}{t}$$

$$\sigma_{xy}^{Membrane} = \frac{N_{xy}}{t}$$

6.1.3 Buckling of Solid Laminate Panel

Solid laminate panel buckling can arise from any loads shown in Figure 2. Here we will show the solutions for a simply-support panel under N_x, N_y, N_{xy} and in-plane bending moments. There is little information on resulting buckling due to combined loaded cases. However, it should be noted that all components of the loads are typically not substantial at the same time. These equations will form the basis for an interaction equation to be used to assess combinations of loading.

In-plane buckling of a simply supported panel due to a uniform edge load for both the X-face and Y-face are given below [19].

X-Edge Load:

$$N_{X_{cr}} = \frac{\pi^2 \sqrt{D_{11} D_{22}}}{b^2} \left(\frac{R_x^{\ 2}}{m^2} + \frac{m^2}{R_x^{\ 2}} + 2\eta \right)$$

$$where:$$

$$R_x = \frac{\left(\frac{D_{22}}{D_{11}} \right)^{1/4} a}{b}$$

Y-Edge Load:

$$N_{Y_{cr}} = \frac{\pi^2 \sqrt{D_{11} D_{22}}}{a^2} \left(\frac{R_y^{\ 2}}{m^2} + \frac{m^2}{R_y^{\ 2}} + 2\eta \right)$$

$$where:$$

$$R_y = \frac{\left(\frac{D_{11}}{D_{22}} \right)^{1/4} b}{a}$$

The panel will buckle in *m* half waves if

$$\sqrt{m(m-1)} \le R_i \le \sqrt{m(m+1)}, m \ge 1$$

The critical in-plane bending moment along each edge of a simply-supported panel is given by [21]:

X-Moment

$$M_{X_{Cr}}^{In-plane} = \frac{\pi^2 \sqrt{D_{11} D_{22}}}{b^2} \left(0.047 \pi^2 b^2 \sqrt{\alpha_x \beta_x} \right)$$

$$where:$$

$$\alpha_x = \left(\frac{R_x^{\ 2}}{k^2} + \frac{k^2}{R_x^{\ 2}} + 2\eta \right)$$

$$\beta_x = \left(\frac{16 R_x^{\ 2}}{k^2} + \frac{k^2}{R_x^{\ 2}} + 8\eta \right)$$

Y-Moment

$$M_{Y_{Cr}}^{In-plane} = \frac{\pi^2\sqrt{D_{11}D_{22}}}{a^2}\left(0.047\pi^2 a^2\sqrt{\alpha_y\beta_y}\right)$$

where:

$$\alpha_y = \left(\frac{R_y^{\ 2}}{k^2} + \frac{k^2}{R_y^{\ 2}} + 2\eta\right)$$

$$\beta_y = \left(\frac{16R_y^{\ 2}}{k^2} + \frac{k^2}{R_y^{\ 2}} + 8\eta\right)$$

the panel buckles in k m-half waves when:

$$\begin{aligned} &k = 1 \ \textit{if} \ \ 0 \le R_i < 1 \\ &k = 2 \ \textit{if} \ \ 1 < R_i < 1.7346 \\ &k = 3 \ \textit{if} \ \ 1.7346 < R_i < 2.4513 \\ &k = 4 \ \textit{if} \ \ 2.4513 < R_i < 3.1637 \\ &k = 5 \ \textit{if} \ \ 3.1637 < R_i < 3.8741 \end{aligned}$$

if R_i is greater than 3.8741, these critical in-plane bending moment can be estimated by:

$$M_{x_{Cr}}^{In-plane} = \frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2}\left(18.24\frac{b^2}{\pi^2}(1.25+\eta)\right)$$

$$M_{y_{Cr}}^{In-plane} = \frac{\pi^2\sqrt{D_{11}D_{22}}}{a^2}\left(18.24\frac{a^2}{\pi^2}(1.25+\eta)\right)$$

For in-plane shear, there are limited solutions for rectangular plate buckling. Here the plate is assumed to be infinitely long, for which R_x approaches infinity. The critical in-plane shear for this case is given by [21]:

$$if\eta \le 1$$

$$N_{xy_{cr}} = \frac{4\left(D_{11}D_{22}^{\ 3}\right)^{1/4}}{b^2}(8.125+5.05\eta)$$

$$if\eta \ge 1$$

$$N_{xy_{cr}} = \frac{4\left(D_{11}D_{22}^{\ 3}\right)^{1/4}}{b^2}\sqrt{\eta}\left(11.7+\frac{0.532}{\eta}+\frac{0.938}{\eta^2}\right)$$

6.1.4 Natural Frequency of Solid Laminate Panel

In addition to deflection concerns, the low modulus of composites can make the natural frequency of the panel a concern. The general form of the panel natural frequency is given [22]:

$$\omega = \frac{1}{a^2\sqrt{\rho}}\sqrt{D_{11}\alpha_1{}^4 + 2(D_{12} + 2D_{66})R^2\alpha_2{}^2 + D_{22}R^4\alpha_3{}^4}$$

where:

ρ = rotational inertia

for a simply support panel the fundamental natural frequency is given by the following constants

$$\alpha_1 = \pi$$
$$\alpha_2 = \pi^4$$
$$\alpha_3 = \pi$$

for a clamped-clamped panel, the fundamental natural frequency is given by the following constants

$$\alpha_1 = 4.730$$
$$\alpha_2 = 151.3$$
$$\alpha_3 = 4.730$$

The true natural frequency can be estimated by a weighting function given by following:

$$\omega = W\omega_{ss} + (1 - W)\omega_{Clamped}$$

Typically W is taken to be 0.25.

6.1.5 Design Criteria for Solid Laminate Panel

The following 11 criteria, arising from the above equations, ***must*** be satisfied.

1. $\dfrac{\delta}{ad_{LIMIT}} \le 1$ Panel Deflection Criteria with respect to Length, a

2. $\dfrac{\delta}{bd_{LIMIT}} \le 1$ Panel Deflection Criteria with respect to Width, b

3. $\sigma_{xM}^C + \dfrac{\sigma_{xB}^C}{\left(1 - \dfrac{N_X}{N_{X_{Cr}}}\right)} \le \dfrac{S_{XC}}{F.S.}$ Laminate Compressive X - Stress

4. $\sigma_{xM}^T + \sigma_{xB}^T \le \dfrac{S_{XT}}{F.S.}$ Laminate Tensile X - Stress

$$5.\sigma_{yM}^{C} + \frac{\sigma_{yB}^{C}}{\left(1 - \dfrac{N_Y}{N_{Y_{Cr}}}\right)} \leq \frac{S_{YC}}{F.S.} \qquad \text{Laminate Compressive Y - Stress}$$

$$6.\sigma_{yM}^{T} + \sigma_{yB}^{T} \leq \frac{S_{YT}}{F.S.} \qquad \text{Laminate Tensile Y - Stress}$$

$$7.\sigma_{xyM} + \sigma_{xyB} \leq \frac{S_{XY}}{F.S.} \qquad \text{Laminate In - plane Shear Stress}$$

$$8.\tau_{xz} \leq \frac{S_{XZ}}{F.S.} \qquad \text{Laminate Through - Thickness Shear Stress, X}$$

$$9.\tau_{yz} \leq \frac{S_{YZ}}{F.S.} \qquad \text{Laminate Through - Thickness Shear Stress, Y}$$

$$10.\frac{N_X}{N_{X_{Cr}}} + \frac{N_Y}{N_{Y_{Cr}}} + \left(\frac{N_{XY}}{N_{XY_{Cr}}}\right)^2 + \left(\frac{M_X^{In-Plane}}{M_{X_{CR}}^{In-Plane}}\right)^2 + \left(\frac{M_Y^{In-Plane}}{M_{Y_{CR}}^{In-Plane}}\right)^2 \leq \frac{1}{F.S.B.} \qquad \text{Buckling Interaction}$$

$$11.\frac{\omega_{LIMIT}}{\omega} \leq 1 \qquad \text{Natural Frequency Limit}$$

where:

d_{LIMIT}	=	deflection limit for the panel
S_i	=	stress allowable for each component of stress
$F.S.$	=	Factor of safety for stress
$F.S.B.$	=	Factor of safety for buckling
ω_{LIMIT}	=	minimum natural frequency of the panel

6.2. Sandwich Panel Design

The design of sandwich panels in this discussion is limited to equal face sheet thickness and similar material (see Figure 3), where D_{ij}'s are flexural rigidities of the plate given by [23]:

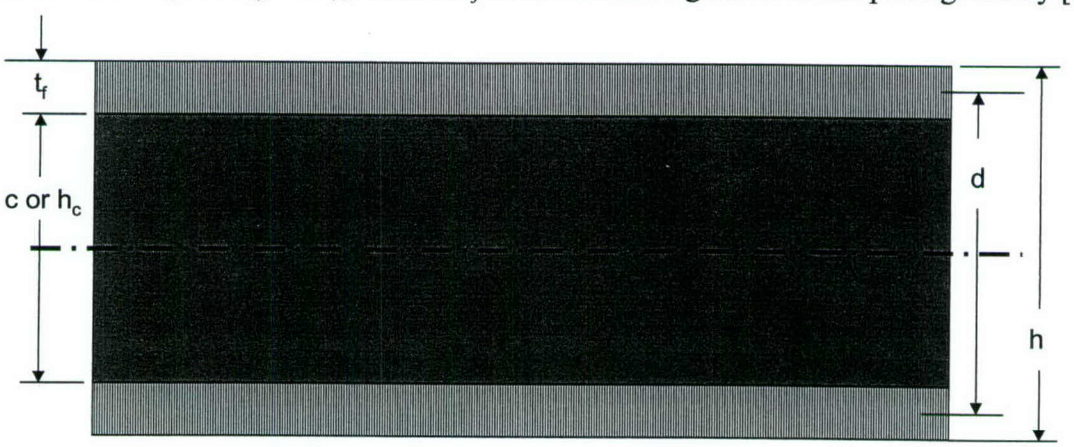


Figure 3. Sandwich Panel Dimensions

$$D_{11}=\frac{E_1^F h^3}{12\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}\left[1-\left(\frac{c}{h}\right)^3+\frac{\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}{\left(1-\upsilon_{12}^C\upsilon_{21}^C\right)}\left(\frac{E_1^C}{E_1^F}\right)\left(\frac{c}{h}\right)^3\right],$$

$$D_{22}=\frac{E_2^F h^3}{12\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}\left[1-\left(\frac{c}{h}\right)^3+\frac{\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}{\left(1-\upsilon_{12}^C\upsilon_{21}^C\right)}\left(\frac{E_2^C}{E_2^F}\right)\left(\frac{c}{h}\right)^3\right],$$

$$D_{12}=\frac{\upsilon_{21}^F E_1^F h^3}{12\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}\left[1-\left(\frac{c}{h}\right)^3+\frac{\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}{\left(1-\upsilon_{12}^C\upsilon_{21}^C\right)}\left(\frac{\nu_{21}^C}{\nu_{21}^F}\right)\left(\frac{E_2^C}{E_2^F}\right)\left(\frac{c}{h}\right)^3\right],$$

$$D_{66}=\frac{G_{12}^F h^3}{12\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}\left[1-\left(\frac{c}{h}\right)^3+\left(\frac{G_{12}^C}{G_{12}^F}\right)\left(\frac{c}{h}\right)^3\right]$$

Additional terms are needed to represent transverse shear stiffness of the sandwich laminate. These terms are given by:

$$A_{55}=\frac{G_{13}^C(h+c)^2}{4c}$$

$$A_{44}=\frac{G_{23}^C(h+c)^2}{4c}$$

6.2.1 Deflection of Sandwich Laminate Panel

Due to transverse shear deformation of the core, a sandwich panel will deflect more than estimated by the classical plate theory. The revised Navier solution for a simply-supported sandwich panel is given by Dobyns [24]. This solution is general and can apply to sandwich or solid laminate plates where:

$$\textit{Displacement}:$$

$$\delta(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}C_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$\textit{Rotations}:$$

$$\overline{\alpha}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$\overline{\beta}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$\textit{Load}(\textit{Generalized}):$$

$$p(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

where:

$$q_{mn} = \frac{4p}{mn\pi^2}(1-\cos m\pi)(1-\cos n\pi)$$

A_{mn}, B_{mn}, and C_{mn} can be determined by substitutions into the governing equations resulting in the following solution:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \end{Bmatrix}$$

where:

$$\begin{aligned}
L_{11} &= D_{11}\lambda_m{}^2 + D_{66}\lambda_n{}^2 + \kappa A_{55} \\
L_{12} &= (D_{12} + D_{66})\lambda_m \lambda_n \\
L_{13} &= \kappa A_{55}\lambda_m \\
L_{22} &= D_{22}\lambda_n{}^2 + D_{66}\lambda_m{}^2 + \kappa A_{44} \\
L_{23} &= \kappa A_{44}\lambda_n \\
L_{33} &= \kappa A_{55}\lambda_m{}^2 + \kappa A_{44}\lambda_n{}^2 \\
\lambda_m &= \frac{m\pi}{a} \\
\lambda_n &= \frac{n\pi}{b} \\
\kappa &= \pi^2/12
\end{aligned}$$

The A_{mn}, B_{mn}, and C_{mn} can be solved:

$$\begin{aligned}
A_{mn} &= \frac{(L_{12}L_{23} - L_{22}L_{13})q_{mn}}{\det(L)} \\
B_{mn} &= \frac{(L_{12}L_{13} - L_{11}L_{23})q_{mn}}{\det(L)} \\
C_{mn} &= \frac{(L_{11}L_{22} - L_{12}L_{12})q_{mn}}{\det(L)}
\end{aligned}$$

As in the solid panel case, the simply-support solution is overly conservative. For this reason, the clamped-clamped rectangular panel is calculated from the work by Roberts and Bao [23].

The maximum deflection of a clamped-clamped rectangular panel is:

$$\delta_{clamped} = \frac{12Pb^4\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}{c^3\sqrt{E_{11}^F E_{22}^F}\pi^4}\left(\frac{a}{b}\right)^2 \frac{1+\varphi}{\left[\left(\frac{h}{c}\right)-1+2\left(\frac{t}{c}\right)^3\varphi\right]R}$$

$$where:$$

$$\varphi = \frac{FR}{H}$$

$$H = G_{23}^C + G_{13}^C\left(\frac{a}{b}\right)^2$$

$$R = 3s^2 + \frac{3}{s^2} + 2\eta, \qquad \eta = \frac{D_{12}+2D_{66}}{\sqrt{D_{11}D_{22}}}$$

$$s = \left(\frac{E_{11}^F}{E_{22}^F}\right)^{1/4}\frac{a}{b}$$

$$F = \frac{2\pi^2 ct\sqrt{E_{11}^F E_{22}^F}}{b^2\left(1-\upsilon_{12}^F\upsilon_{21}^F\right)}$$

These terms are combined using a weighted average, W, to estimate the true panel deflection. The weighting function, W, ranges between 0 and 1. Typically W is taken to be 0.25.

$$\delta = \frac{W\delta_{ss} + (1-W)\delta_{Clamped}}{\left(1-\frac{N_X^{Compression}}{N_{X_{CR}}}\right)\left(1-\frac{N_Y^{Compression}}{N_{Y_{CR}}}\right)}$$

6.2.2 Stress of Sandwich Laminate Panel

For panel stresses the Navier Solution for a simply-supported panel under uniform pressure with transverse shear deformation was used to determine M_x, M_y, M_{xy}. In addition, the shear resultants can be found. This is important because in a sandwich panel the core (only) is assumed to resist shear. Using the solutions for displacement and rotations, the stress couples, M_x, M_y, and M_{xy} and shear resultants Q_x and Q_y can be determined.

$$M_x = -D_{11}\frac{\partial\overline{\alpha}}{\partial x} - D_{12}\frac{\partial\overline{\beta}}{\partial y}$$

$$M_x = -D_{11}\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} - A_{mn}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right] - D_{12}\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} - B_{mn}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right]$$

$$M_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[D_{11}A_{mn}\left(\frac{m\pi}{a}\right) + D_{12}B_{mn}\left(\frac{n\pi}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$M_y = -D_{12}\frac{\partial\overline{\alpha}}{\partial x} - D_{22}\frac{\partial\overline{\beta}}{\partial y}$$

$$M_y = -D_{12}\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} - A_{mn}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right] - D_{22}\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} - B_{mn}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right]$$

$$M_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[D_{12}A_{mn}\left(\frac{m\pi}{a}\right) + D_{22}B_{mn}\left(\frac{n\pi}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$M_{xy} = -2D_{66}\left[\frac{1}{2}\left(\frac{\partial\overline{\alpha}}{\partial y} + \frac{\partial\overline{\beta}}{\partial x}\right)\right] = -D_{66}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[A_{mn}\left(\frac{n\pi}{b}\right) + B_{mn}\left(\frac{m\pi}{a}\right)\right]\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$Q_x = 2A_{55}\left(\overline{\alpha} + \frac{\partial w}{\partial x}\right) = 2A_{55}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[A_{mn} + C_{mn}\left(\frac{m\pi}{a}\right)\right]\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$Q_y = 2A_{44}\left(\overline{\beta} + \frac{\partial w}{\partial y}\right) = 2A_{44}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[B_{mn} + C_{mn}\left(\frac{n\pi}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

The resulting stresses in the face sheet and core shear are:

$$\sigma_X^{Bending} = \frac{M_x\left(\frac{a}{2},\frac{b}{2}\right)}{t_f d}$$

$$\sigma_Y^{Bending} = \frac{M_y\left(\frac{a}{2},\frac{b}{2}\right)}{t_f d}$$

$$\sigma_{xy}^{Bending} = \frac{M_{xy}(0,0)}{t_f d}$$

$$\tau_{xz}^{Core} = \frac{Q_x(0,b/2)}{d}$$

$$\tau_{yz}^{Core} = \frac{Q_y(a/2,0)}{d}$$

$$\text{where: } d = c + t_f$$

In addition to the bending stress due to out-of-plane pressure, membrane components can be determined from in-plane loading.

$$\sigma_x^{Membrane} = \frac{N_X}{2t_f}$$

$$\sigma_y^{Membrane} = \frac{N_Y}{2t_f}$$

$$\sigma_{xy}^{Membrane} = \frac{N_{xy}}{2t_f}$$

6.2.3 Buckling of Sandwich Laminate Panel

There are several modes of buckling for a solid-core sandwich panel. These modes include face-wrinkling instability, core shear instability, or overall buckling. The critical stress for buckling is discussed by Vinson [25] and given below. Overall buckling is given by:

$$N_{X_{CR}}^{O.B.} = \frac{2t_f \pi^2}{4\left(1-\nu_{12}^F \nu_{21}^F\right)} \sqrt{E_{11}^F E_{22}^F} \frac{h_c^{\,2}}{b^2} K \quad \text{for } V_x \le k_1 B_1 r$$

$$K = \frac{B_1 C_1 + 2B_2 C_2 + \dfrac{C_3}{B_1} + A\left(\dfrac{V_y}{C_4} + V_x\right)}{1 + \left(B_1 C_1 + B_3 C_2\right)\dfrac{V_y}{C_4} + \left(\dfrac{C_3}{B_1} + B_3 C_2\right) V_x + \dfrac{V_x V_y A}{C_4}}$$

where:

$$A = C_1 C_3 - B_2 C_2^{\,2} + B_3 C_2 \left(B_1 C_1 + 2B_2 C_2 + \frac{C_3}{B_1}\right)$$

$$B_1 = \sqrt{\frac{D_{22}}{D_{11}}}, B_3 = \frac{D_{12}}{\sqrt{D_{11} D_{22}}}, B_2 = 2B_3 + B_1 \nu_{12}^F, \quad r = \frac{G_{13}^C}{G_{23}^C}$$

$$V_x = \frac{\pi^2}{b^2} \frac{\sqrt{D_{11} D_{22}}}{D_{13}}, \quad V_y = \frac{\pi^2}{b^2} \frac{\sqrt{D_{11} D_{22}}}{D_{23}}$$

for a simply-supported panel

$$C_1 = C_4 = \frac{a^2}{b^2}, C_2 = 1, C_3 = \frac{b^2}{a^2}, \; k_1 = 1$$

When the condition for overall buckling is satisfied, the panel may still buckle due to core shear instability given by:

$$\sigma_{X_{CR}}^{C.S.} = \frac{G_{13}^C h_c}{2t_f}$$

Other modes of core shear crimping or instability are given by:

$$\sigma_{Y_{CR}}^{C.S.} = \frac{G_{23}^C h_c}{2t_f}$$

$$\sigma_{XY_{CR}}^{C.S.} = \frac{\sqrt{G_{13}^C G_{23}^C}\, h_c}{2t_f}$$

Face wrinkling instability causes a short-wave buckling of the sandwich skins, primarily when core modulus is low or skins are thin. The critical in-plane edge load to cause face wrinkling is estimated by two methods. The first by Vinson [25]:

$$\sigma_{X_{CR}}^{F.W.1} = \left[\frac{2}{3} \frac{t_f}{h_c} \frac{E_3^C \sqrt{E_{11}^F E_{22}^F}}{\left(1 - \upsilon_{12}^F \upsilon_{21}^F\right)} \right]^{1/2}$$

A second estimate of critical in-plane edge load to cause face wrinkling is given by Hoff and Mauter [26]:

$$\sigma_{X_{CR}}^{F.W.2} = c\left(E_{11}^F E_3^C G_{13}^C\right)^{1/3}$$
$$where \quad c = 0.5$$

Buckling will occur at the minimum edge load as calculated above. For the assessment of buckling in the Y direction, terms can be rearranged with proper directionality. To assess shear face wrinkling, stress is given by the lower of the following [7]:

$$\sigma_{XY_{CR}}^{F.W} = c \frac{\left(E_{45}^F E_3^C G_{13}^C\right)^{1/3}}{\sqrt{3}}$$
$$\sigma_{XY_{CR}}^{F.W} = c \frac{\left(E_{45}^F E_3^C G_{23}^C\right)^{1/3}}{\sqrt{3}}$$

$$where \quad c = 0.44$$

$$E_{45}^F = \frac{1}{4E_{11}^F} + \left(\frac{1}{4G_{12}^F} - \frac{\nu_{12}}{2E_{11}^F}\right) + \frac{1}{4E_{22}^F}$$

The in-plane moment and shear critical loads equations can be used with proper flexural rigidities for the sandwich panel.

6.2.4 Natural Frequency of Sandwich Laminate Panel

Additionally, the natural frequency of the sandwich panel can be approximated with equations used in Section 6.1.4 with proper flexural rigidities for the sandwich panel. Note in this case, transverse shear deformation consideration is not used.

6.2.5 Design Criteria for Sandwich Laminate Panel

The 14 design constraints which must be satisfied for a feasible design are shown below. Note that core shear stresses become a critical component in the design.

$1.\dfrac{\delta}{ad_{LIMIT}} \le 1$	Panel Deflection Criteria with respect to Length, a
$2.\dfrac{\delta}{bd_{LIMIT}} \le 1$	Panel Deflection Criteria with respect to Width, b
$3.\sigma_{xM}^{C} + \dfrac{\sigma_{xB}^{C}}{\left(1-\dfrac{N_X}{N_{X_{Cr}}}\right)} = \sigma_x^C \le \dfrac{S_{XC}}{F.S.}$	Facesheet Compressive X - Stress
$4.\sigma_{xM}^{T} + \sigma_{xB}^{T} = \sigma_x^T \le \dfrac{S_{XT}}{F.S.}$	Facesheet Tensile X - Stress
$5.\sigma_{yM}^{C} + \dfrac{\sigma_{yB}^{C}}{\left(1-\dfrac{N_Y}{N_{Y_{Cr}}}\right)} = \sigma_y^C \le \dfrac{S_{YC}}{F.S.}$	Facesheet Compressive Y - Stress
$6.\sigma_{yM}^{T} + \sigma_{yB}^{T} = \sigma_y^T \le \dfrac{S_{YT}}{F.S.}$	Facesheet Tensile Y - Stress
$7.\sigma_{xyM} + \sigma_{xyB} = \sigma_{xy} \le \dfrac{S_{XY}}{F.S.}$	Facesheet In - plane Shear Stress
$8.\tau_{xz} \le \dfrac{S_{XZ}^{Core}}{F.S.}$	Core Shear Stress, X - direction
$9.\tau_{yz} \le \dfrac{S_{YZ}^{Core}}{F.S.}$	Core Shear Stress, Y - direction
$10.\tau_{yz} \le \dfrac{S_{Z}^{Core}}{F.S.}$	Core Compressive Stress, Z - direction
$11.\dfrac{N_X}{N_{X_{Cr}}} + \dfrac{N_Y}{N_{Y_{Cr}}} + \left(\dfrac{N_{XY}}{N_{XY_{Cr}}}\right)^2 + \left(\dfrac{M_X^{In-Plane}}{M_{X_{CR}}^{In-Plane}}\right)^2 + \left(\dfrac{M_Y^{In-Plane}}{M_{Y_{CR}}^{In-Plane}}\right)^2 \le \dfrac{1}{F.S.B.}$	Global Buckling Interaction
$12.\dfrac{\sigma_X}{\sigma_{X_{Cr}}^{C.S.}} + \dfrac{\sigma_Y}{\sigma_{Y_{Cr}}^{C.S.}} + \left(\dfrac{\sigma_{XY}}{\sigma_{XY_{Cr}}^{C.S.}}\right)^2 \le \dfrac{1}{F.S.L.B.}$	Local Buckling, Core Shear Instabilty Interaction
$13.\dfrac{\sigma_X}{\sigma_{X_{Cr}}^{F.W.}} + \dfrac{\sigma_Y}{\sigma_{Y_{Cr}}^{F.W.}} + \left(\dfrac{\sigma_{XY}}{\sigma_{XY_{Cr}}^{F.W.}}\right)^2 \le \dfrac{1}{F.S.L.B.}$	Local Buckling, Face Wrinkling Interaction
$14.\dfrac{\omega_{LIMIT}}{\omega} \le 1$	Natural Frequency Limit

6.3. Hat-Stiffened Panel Design

Consider a panel with several hat-stiffeners attached; see Figure 4. In this case, three separate analysis checks must be done. First, the unsupported panel, which is now length *a*, width $s\text{-}b_w$, can be designed using the procedures discussed in Section 6.1 or 6.2. Second, the plate-stiffener combination must be checked. Finally, the overall panel length *a* width *b* with effective stiffness properties must assessed.

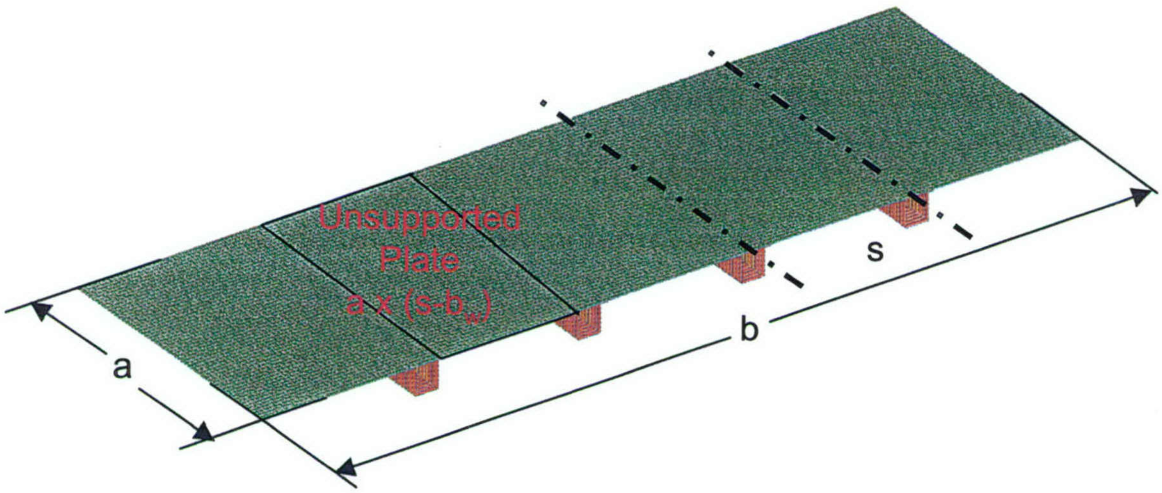


Figure 4. Hat-Stiffened Panel Dimensions

6.3.1 Stiffener or Girder/Frame Design

Figure 5 shows hat stiffener geometry. For analysis of hat-stiffened structure, the core is assumed to be ineffective. When calculating the characteristics of the plate-hat-stiffener combination, the effective width of the beam is given by ABS Rules [4]:

$$b_{eff} = \text{the lesser of} : \left(18t + b_w, s\right)$$

For a sandwich panel, the equivalent thickness *t* must be calculated to have the same inertia of sandwich construction. For a sandwich panel with equal skin thickness and similar material, the equivalent thickness equals:

$$t_{eq} = \left(h^3 \left[1 - \left(\frac{c}{h}\right)^3 + \frac{\left(1 - \upsilon_{12}^F \upsilon_{21}^F\right)}{\left(1 - \upsilon_{12}^C \upsilon_{21}^C\right)} \left(\frac{E_1^C}{E_1^F}\right) \left(\frac{c}{h}\right)^3 \right] \right)^{1/3}$$

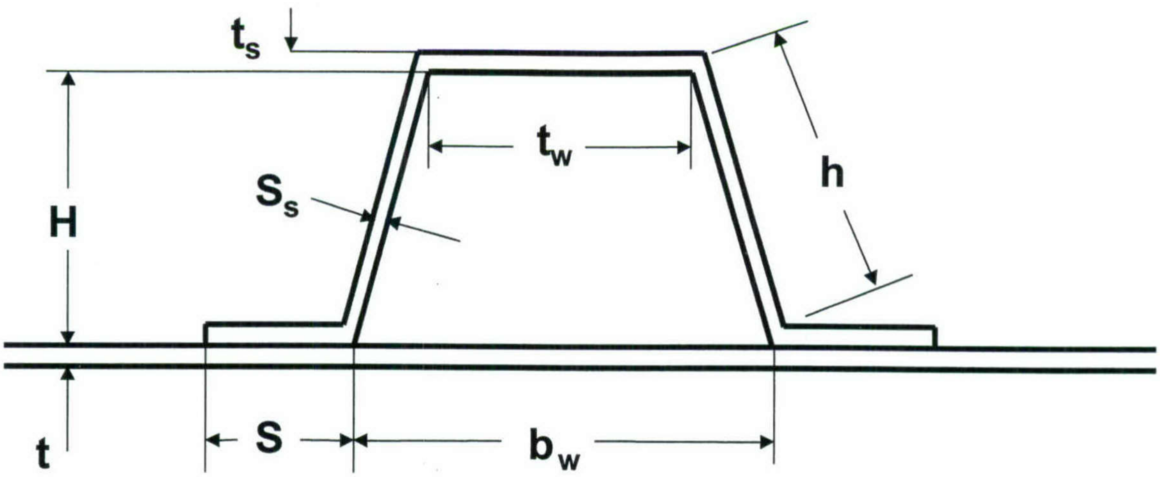


Figure 5. Hat-Stiffener Dimensions

Composite hat-stiffeners typically have lay-ups which vary between the webs and cap. Also, composite hat-stiffeners have different composite materials in stiffener caps. To represent a composite hat-stiffened beam, a designer must:

Develop an effective axial stiffness: $(EA)_{eff} = \sum_{i=1}^{N} E_i A_i$

Develop an effective bending stiffness: $(EI_y)_{eff} = \sum_{i=1}^{N} E_i A_i z_i^2 + E_i I_{yi}$

From this, the area A and moment of inertia of the plate-stiffener combination I can be determined. See Appendix B for an example.

For beam bending due to out-of-plane load, the beam is assumed to have a uniform load w (lb/in). The resulting shear and moments will cause maximum tensile or compressive stress values either at the mid-span or ends of the beam, shown Figure 6.

The bending moments and shear forces are defined as [1]:

$$M_e = \text{end bending moment} = \frac{wL^2}{f_1}$$

$$M_m = \text{midspan bending moment} = \frac{wL^2}{f_2}$$

$$V = \text{end shear force} = \frac{wL}{f_3}$$

where:

w	=	normal load in lb/in
L	=	span of plate-stiffener combination
f_1, f_2, f_3	=	response factors

For hydrostatic, dead, damage or tank overflow load of plate-stiffener combination (that is, load that is uniform over a large area):

$f_1 = 12.0$

$f_2 = 24.0$

$f_3 = 2.0$

For hydrostatic, dead, damage or tank overflow load of plate-frame stiffener combination (that is, load that is not uniform over a large area):

$f_1 = 10.0$

$f_2 = 20.0$

$f_3 = 1.67$

For live loads and dynamic loads (slamming) of plate-frame or plate-stiffener combination (that is, load that is intermittent from frame to frame):

$f_1 = 9.46$

$f_2 = 12.90$

$f_3 = 1.76$

From the moments, stress in the plate-stiffener combination can be determined:

$$\sigma^T_{end} = \frac{M_e c_{plate} E_{plate}}{(EI)_{eff}}$$ tensile stress in the plate at the edge

$$\sigma^T_{midspan} = \frac{M_m c_{flange} E_{flange}}{(EI)_{eff}}$$ tensile stress in the flange at the midspan

$$\sigma^C_{end} = \frac{M_e c_{flange} E_{flange}}{(EI)_{eff}}$$ compressive stress in the flange at the edge

$$\sigma^C_{midspan} = \frac{M_m c_{plate} E_{plate}}{(EI)_{eff}}$$ compressive stress in the plate at the midspan

$$\sigma_{xy} = \frac{V}{A_s}$$ average shear stress at the edge

where: A_s = shear area ($2S_sH$) for a hat - stiffener

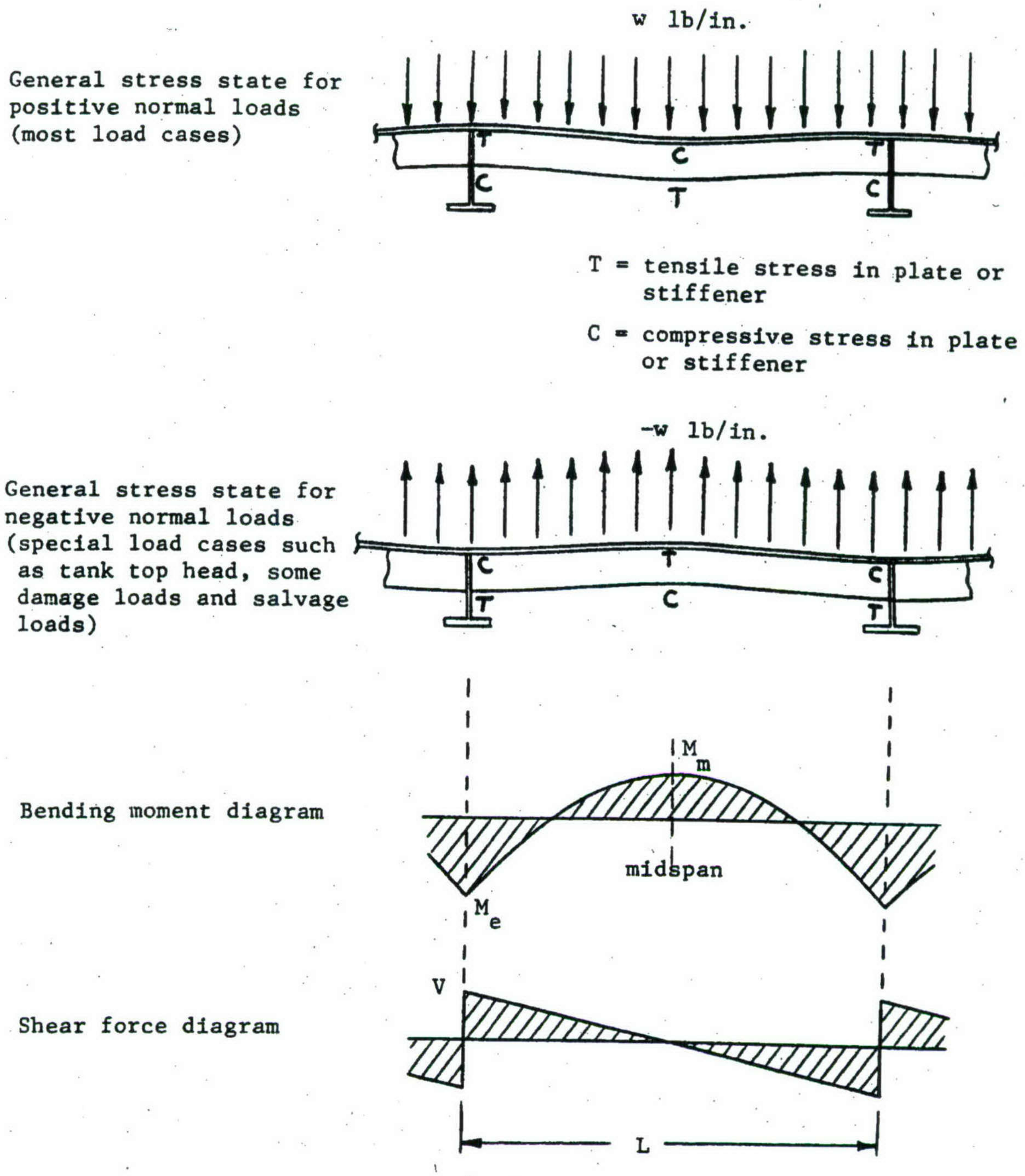


Figure 6. General Shear and Moments

The column buckling of the plate-stiffener combination with uniform cross-section is given by Reference [1]:

$$P_{cr} = \frac{\pi^2 E}{\left(K_4 \frac{L}{r}\right)^2}$$

$where:$

$r = \sqrt{\frac{I}{A}}$ =radius of gyration

E = modulus parallel to the column direction

For pinned ends $K_4 = 1.0$

For fixed ends $K_4 = 0.5$

For fixed - pinned $K_4 = 0.75$

For fixed - free $K_4 = 2.0$

The defection of the plate-stiffener combination is defined by Navy standards [1]:

$$\delta_B = \frac{0.004 w L^4}{EI}$$

For interaction with an in-plane (column) loads, the deflection is increased via:

$$\delta = \frac{\delta_B}{\left(1 - \frac{P}{P_{cr}}\right)}$$

In addition to overall buckling, web buckling should be checked. This can be assessed using the equations for solid laminate (the hat stiffener core is taken to be non-structural) for edge compression and shear, respectively.

6.3.2 Overall Stiffened Panel Design

To this point, the hat-stiffened panel has been examined for the unsupported panel and the plate-stiffener combination. Now the entire panel with length *a* and width *b* must be examined for defection criteria, natural frequency, and overall buckling. This can be done by developing flexural rigidities for the entire hat-stiffened panel. This work was done Bao, *et al* [19]:

$$D_{11} = \frac{EI_s}{b_{eff}},$$

$$D_{22} = \frac{E_2 t^3}{12(1-\upsilon_{12}\upsilon_{21})}\left(\frac{a}{a-2b_w}\right)$$

$$D_{12} = \frac{\upsilon_{21}E_1 t^3}{12(1-\upsilon_{12}\upsilon_{21})},$$

$$D_{66} = \frac{G_{12}t^3}{12} + \frac{D_{Tx}}{2s}$$

$$where:$$

$$D_{Tx} = \frac{4G_{12}A_m{}^2}{\oint \frac{ds}{t}} = \text{torsional rigidity of the stiffener}$$

$$A_m = \left[\frac{(b_w + t_w)}{2}\right]H = \text{area enclosed by the centerlines of the elements forming the closed section of the stiffener}$$

$$\oint \frac{ds}{t} = \frac{b_w}{t} + 2\frac{h}{S_s} + \frac{t_w}{(S_s t_s)} = \text{integration with respect to the distance s, around the perimter}$$

The response of this panel can be calculated via solid laminate equations in the Section 6.1.

Longitudinally framed structures are also subject to buckling failure; see Figure 7 and Figure 8. A suitable formula for critical buckling stress σ_{ycr} (simply supported ends) for longitudinally framed composite structures is given as [2]:

$$\sigma_{ycr} = \frac{\frac{\pi^2 EI}{AL^2}}{1 + \frac{\pi^2 EI}{L^2 GA_S}}$$

where:

L	=	longitudinal panel span,
EI	=	the flexural rigidity of a longitudinal with assumed effective plate width,
A	=	total cross-sectional area of the longitudinal, including effective width of plate,
GA_S	=	shear rigidity with A_S = the area of the stiffener webs.

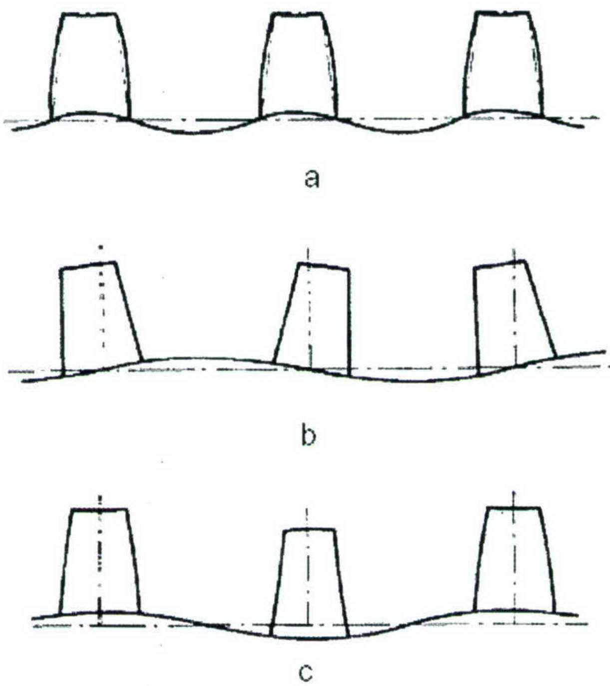
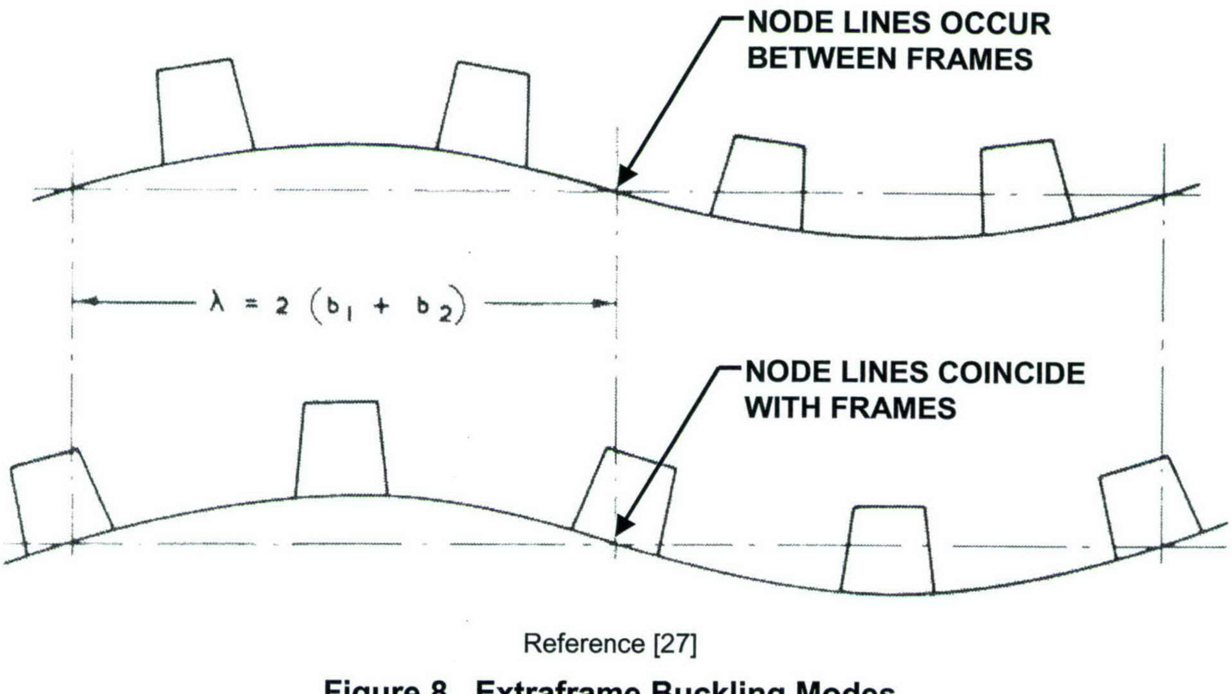


Figure 7. Interframe Buckling Modes



Reference [27]

Figure 8. Extraframe Buckling Modes

6.3.3 Design Criteria for Hat-Stiffened Panels

For the unsupported plate between the stiffeners

$1.\dfrac{\delta}{ad_{LIMIT}} \leq 1$ Plate Deflection Criteria with respect to Length, a

$2.\dfrac{\delta}{sd_{LIMIT}} \leq 1$ Plate Deflection Criteria with respect to Spacing, s

$3.\sigma^C_{xM} + \dfrac{\sigma^C_{xB}}{\left(1-\dfrac{N_X}{N_{X_{Cr}}}\right)} \leq \dfrac{S_{XC}}{F.S.}$ Compressive X - Stress

$4.\sigma^T_{xM} + \sigma^T_{xB} \leq \dfrac{S_{XT}}{F.S.}$ Tensile X - Stress

$5.\sigma^C_{yM} + \dfrac{\sigma^C_{yB}}{\left(1-\dfrac{N_Y}{N_{Y_{Cr}}}\right)} \leq \dfrac{S_{YC}}{F.S.}$ Compressive Y - Stress

$6.\sigma^T_{yM} + \sigma^T_{yB} \leq \dfrac{S_{YT}}{F.S.}$ Tensile Y - Stress

$7.\sigma_{xyM} + \sigma_{xyB} \leq \dfrac{S_{XY}}{F.S.}$ In - plane Shear Stress

$8.\tau_{xz} \leq \dfrac{S_{XZ}}{F.S.}$ Shear Stress, X

$9.\tau_{yz} \leq \dfrac{S_{YZ}}{F.S.}$ Shear Stress, Y

$10.\dfrac{N_X}{N_{X_{Cr}}} + \dfrac{N_Y}{N_{Y_{Cr}}} + \left(\dfrac{N_{XY}}{N_{XY_{Cr}}}\right)^2 + \left(\dfrac{M_X^{In-Plane}}{M_{X_{CR}}^{In-Plane}}\right)^2 + \left(\dfrac{M_Y^{In-Plane}}{M_{Y_{CR}}^{In-Plane}}\right)^2 \leq \dfrac{1}{F.S.B.}$ Buckling Interaction

$11.\dfrac{\omega_{LIMIT}}{\omega} \leq 1$ Natural Frequency Limit

For Plate - Stiffener Combination

$12.\dfrac{\delta}{ad_{LIMIT}} \leq 1$ Beam Deflection Criteria with respect to Length, a

$13.\dfrac{E^{flange}P}{(EA)_{eff}} + \dfrac{\sigma^C_{End}}{\left(1-\dfrac{P_C}{P_{cr}}\right)} \leq \dfrac{S_{XC}}{F.S.}$ End Compressive Stress

$14.\dfrac{E^{plate}P}{(EA)_{eff}} + \sigma^T_{End} \leq \dfrac{S_{XT}}{F.S.}$ End Tensile Stress

$15.\dfrac{E^{plate}P}{(EA)_{eff}} + \dfrac{\sigma^C_{Midspan}}{\left(1-\dfrac{P_C}{P_{cr}}\right)} \leq \dfrac{S_{XC}}{F.S.}$ Midspan Compressive Stress

$16.\dfrac{E^{flange}P}{(EA)_{eff}} + \sigma^T_{MisSpan} \leq \dfrac{S_{XT}}{F.S.}$ Midspan Tensile Stress

$17.\sigma_{xy} \leq \dfrac{S_{XY}}{F.S.}$ Web Shear Stress

$18.\dfrac{P}{P_{Cr}} \leq \dfrac{1}{F.S.B.}$ Buckling

$19.\dfrac{N_X}{N_{X_{Cr}}} + \left(\dfrac{N_{XY}}{N_{XY_{Cr}}}\right)^2 \leq \dfrac{1}{F.S.B.}$ Web Buckling Interaction

For Overall Panel Response

20. $\frac{\delta}{ad_{LIMIT}} \leq 1$ Plate Deflection Criteria with respect to Length, *a*

21. $\frac{\delta}{bd_{LIMIT}} \leq 1$ Plate Deflection Criteria with respect to Spacing, *b*

22. $\frac{N_X}{N_{X_{Cr}}} + \frac{N_Y}{N_{Y_{Cr}}} + \left(\frac{N_{XY}}{N_{XY_{Cr}}}\right)^2 + \left(\frac{M_X^{In-Plane}}{M_{X_{CR}}^{In-Plane}}\right)^2 + \left(\frac{M_Y^{In-Plane}}{M_{Y_{CR}}^{In-Plane}}\right)^2 \leq \frac{1}{F.S.B.}$ Buckling Interaction

23. $\frac{\omega_{LIMIT}}{\omega} \leq 1$ Natural Frequency Limit

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Appendix A
Classical Laminate Theory (CLT):

In Section 6 of this report, preliminary design equations are presented with smeared composite laminates properties. In most Navy projects, smeared laminates representing common structural configurations, that is, quasi-isotropic, have been tested. In this case, a designer can now input a set of modulii and strength properties for a single plate thickness.

However, depending on the material test data developed through the material test program, the use of material properties in laminated plates can be determined. In some cases, lamina (ply) properties are generated (warps-parallel or uni-directional). In this case, the user can define a stacking sequence for construction which provides individual ply information in the material set (lamina elastic constants), thickness, and angular orientation; see Figure A-1. Micro-mechanics analysis can be used to predict the behavior of the laminate. Equivalent properties can be determined using:

- Classical Laminate Theory, CLT, to Predict Equivalent Stiffness Characteristics, First-Ply Strength Note: CLT is valid for symmetric laminates (with no bending twist coupling, $B_{ij} = 0$)
- CLT with Ply-Discount Method to Predict Equivalent Stiffness Characteristics and Ultimate Strength Predictions
- Micro-Mechanics Analysis with Progressive Failure/Damage Analysis (Currently there are no standard methods for using these models.)
- Classical Laminated Plate Theory covered in detail in Reference 25.

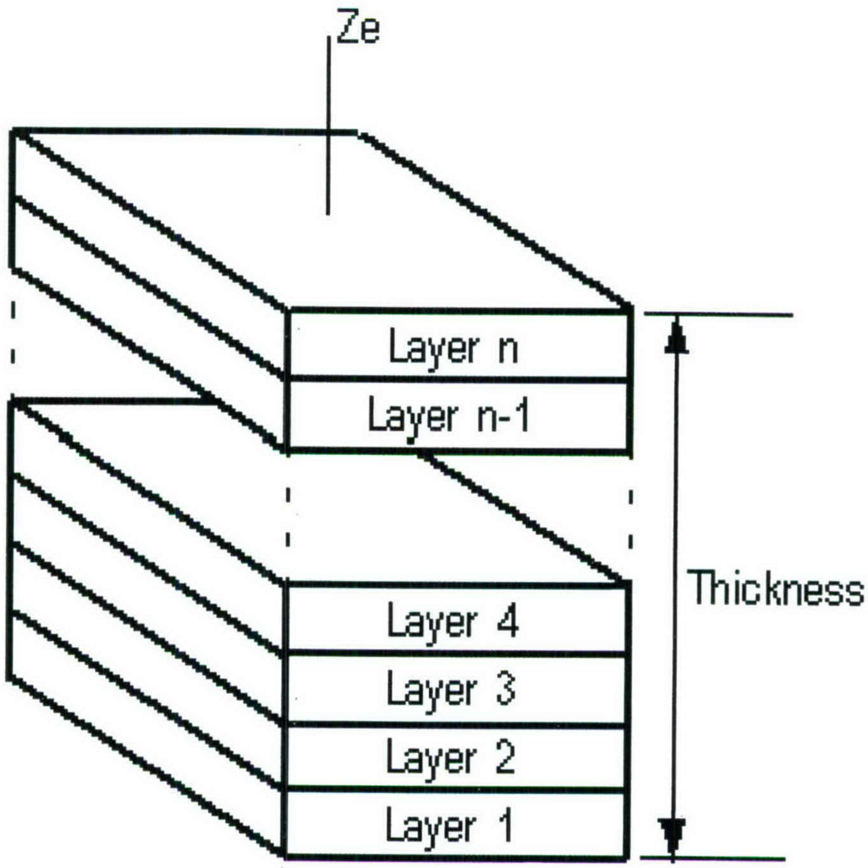
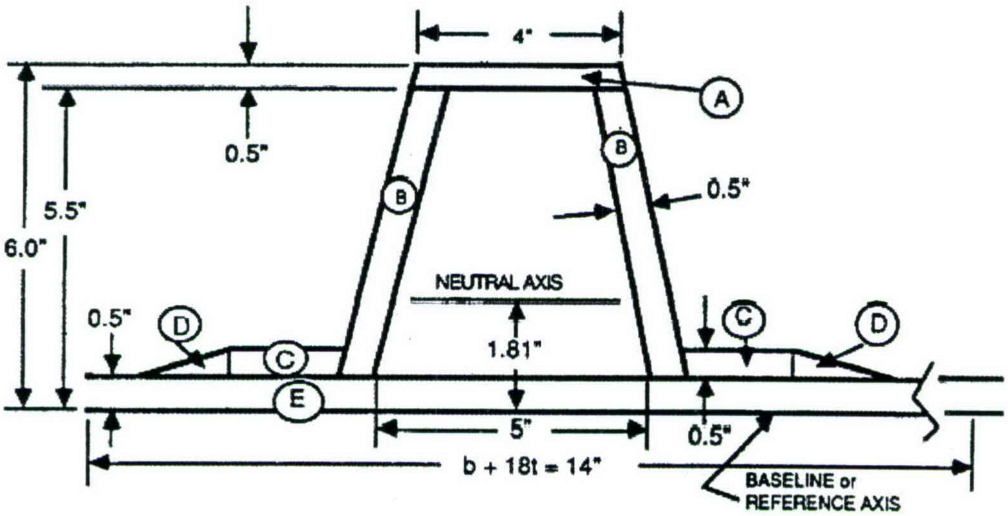


Figure A-1. Laminated Plate Assembly

Appendix B
Sample Area and Moment of Inertia Calculation

SECTION MODULUS and MOMENT of INERTIA CALCULATION GUIDE

Figure A-1



ITEM	b	h	A=bxh	d	Ad	Ad^2	i_o
A	4.00	0.50	2.00	5.75	11.50	66.13	0.04
B	0.50	5.10	2.55	3.00	7.65	23.95	5.31
B	0.50	5.10	2.55	3.00	7.65	23.95	5.31
C	2.00	0.50	1.00	0.75	0.75	0.56	0.02
C	2.00	0.50	1.00	0.75	0.75	0.56	0.02
D	14.00	0.50	7.00	0.25	1.75	0.44	0.15
E	3.00	0.50	0.75	0.67	0.50	0.33	0.01
			16.85		30.55	115.92	10.86

$d_{NA} = \Sigma Ad / \Sigma A = 30.55/16.85 = 1.81$ in.

$I_{NA} = \Sigma i_o + \Sigma Ad^2 - [A(d^2)]$

$= 10.86 + 115.92 - [16.85 \times (1.81)^2] = 71.58 \text{ in}^4$

$SM_{top} = I/d_{NA\ top} = 71.58/4.19 = 30.26 \text{ in}^3$

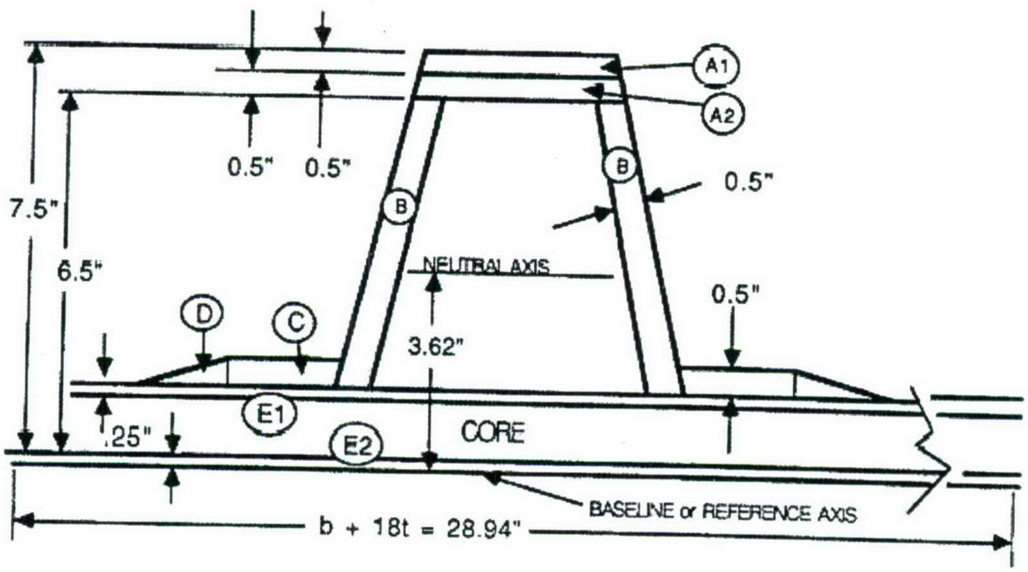
$SM_{bottom} = I/d_{NA\ bottom} = 71.58/1.81 = 39.55 \text{ in}^3$

Reference [28]

Figure B-1. Sample Area and Moment of Inertia Calculation Solid Hat-Stiffener

Consider the use of a 3 inch wide layer of kevlar in the top flange of the stiffener, in a polyester resin, with a 1 inch cored shell.

Figure A-2.



ITEM	b	h	A=bxh	d	Ad	Ad^2	i_o
A1	3.70	0.50	3.29*	7.25	52.56	381.08	0.039
A2	3.80	0.50	1.90	6.75	12.83	86.57	0.040
B	0.50	5.00	2.50	4.00	10.00	40.00	5.208
B	0.50	5.00	2.50	4.00	10.00	40.00	5.208
C	2.00	0.50	1.00	1.75	1.75	3.06	0.021
C	2.00	0.50	1.00	1.75	0.75	0.56	0.021
D	3.00	0.50	0.75	0.67	0.50	0.33	0.01
E1	28.94	0.25	7.23	1.375	9.95	13.68	0.038
E2	28.94	0.25	7.23	0.125	0.90	0.11	0.038
			27.40		99.24	565.39	10.620

$d_{NA} = \Sigma\, Ad / \Sigma\, A = 99.24/27.40 = 3.62$ in.

$I_{NA} = \Sigma\, i_o + \Sigma\, Ad^2 - [A(d^2)]$

$= 10.62 + 565.39 - [27.40 \times (3.62)^2] = 216.95 \text{ in}^4$

$SM_{top} = I/d_{NA\ top} = 216.95/3.88 = 55.92 \text{ in}^3$

$SM_{bottom} = I/d_{NA\ bottom} = 216.95/3.62 = 59.93 \text{ in}^3$

Reference [28]

Figure B-2. Sample Area and Moment of Inertia Calculation Sandwich Hat-Stiffener [28]