

Design of Nonlinear Autopilots for High Angle of Attack Missiles

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Abstract

Two nonlinear autopilot design approaches for a tail-controlled high angle of attack air-to-air missile are described. The research employs a highly nonlinear, time varying pitch plane rigid-body dynamical model of a short range missile. Feedback linearization technique together with linear control theory are then used for autopilot design. In order to manage the difficulties associated with “zerodynamics” that arise in tail controlled missiles, two distinct approaches for approximate feedback linearization are advanced. The first approach imposes a time-scale structure in the closed-loop dynamics, while the second technique redefines the output. Performance of these autopilots are illustrated in a nonlinear simulation.

Introduction

Recent aeronautical research has identified high angle of attack maneuverability as an important requirement in future combat aircraft [1 - 3]. Recently, the X-31 program at NASA Dryden Flight Research Center investigated high angle of attack maneuvers through an ambitious flight test-based program [4]. These developments have major implications on missile technologies. Superior maneuverability of high angle of attack aircraft will demand significantly

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better agility from tactical missiles. In air-to-air missions, missile launch at arbitrary aircraft attitudes will require the missile airframe to be able to maneuver effectively through the high angle of attack regime to ensure successful target intercept.

Operating a missile in the high angle of attack regime can lead to several new difficulties. The first of these is that the missile dynamics can undergo significant changes during the flight. In fixed-wing aircraft, operation at elevated angle of attack is known to lead to new dynamic modes such as roll divergence, nose slice, and wing rock [5 -10]. High angle of attack missile dynamic modes are yet to be similarly characterized. Additionally, conventional aerodynamic surfaces may no longer be effective at extreme angles of attack, and alternate control effectors such as a reaction jets will have to be employed to maintain control over the missile.

Designing a satisfactory autopilot and guidance system for such missiles will require the use of multiple control strategies and techniques. Various approaches to the control of high angle of attack flight vehicles have been discussed in the recent literature [11 - 16]. Most of this research focuses on high performance aircraft. References 15 and 16 discuss some of the issues that arise while designing unconventional flight control systems for missiles. However, the development of nonlinear flight control systems has not been addressed to any significant degree.

This paper presents the results of an exploratory study to examine autopilot design methods for high angle of attack missile flight. Some of the guidance related issues are addressed in a companion paper [17]. Admittedly, the missile and subsystem models used in the present study are simpler than the real situation. However, this preliminary study adequately demonstrates the performance potential of high angle of attack missiles, and exposes the underlying issues that require further investigation.



Planar Rigid Body Missile Dynamics

The pitch-plane rigid-body model of the missile used in the present study incorporates two translational degrees of freedom and one rotational degree of freedom. Figure 1 illustrates the missile coordinate systems and the variables of interest.

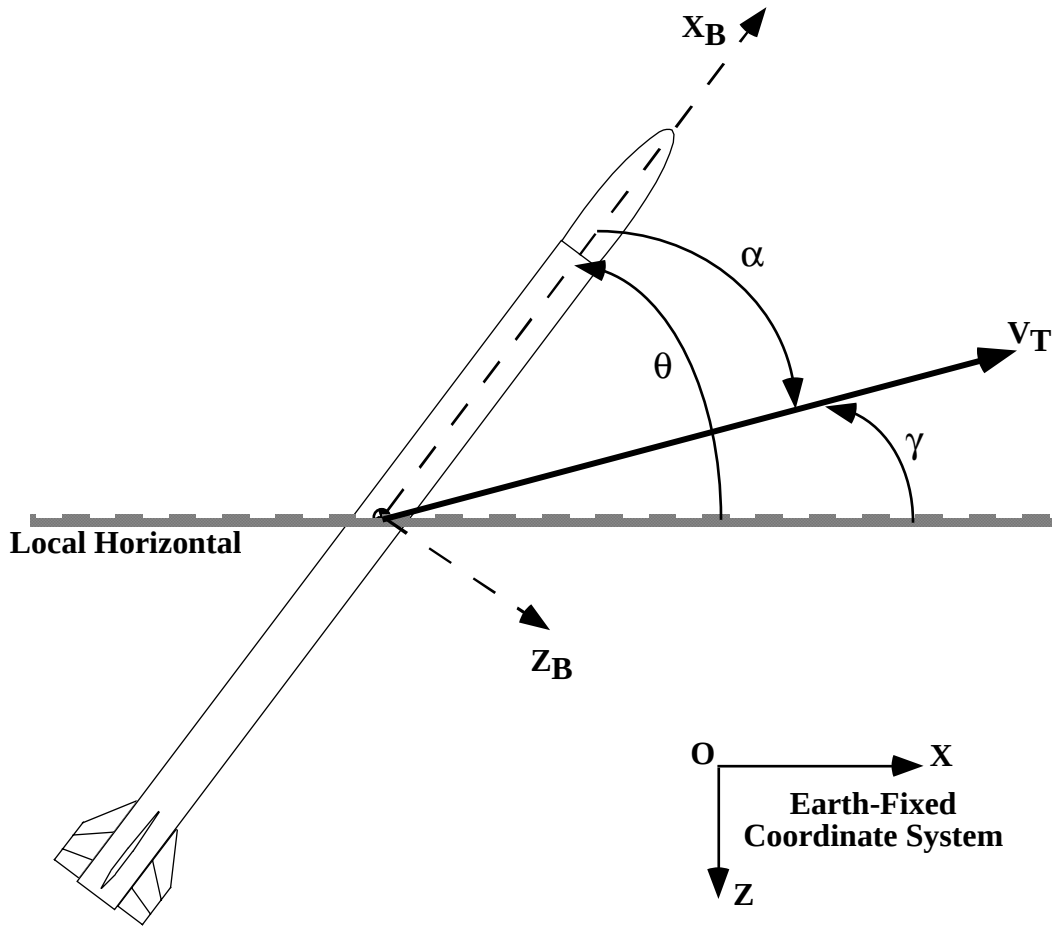


Fig. 1. Missile Pitch Plane Coordinate System

The missile equations of motion are expressed in terms of two coordinate systems. The coordinate system $X - Z$ is used to define the position and attitude of the missile with respect to an earth fixed reference, while the body fixed coordinate system $X_B - Z_B$ is used to define the missile velocity components and body rates. Aerodynamic forces and moments are also expressed in the body frame. The Z -axis in the earth-fixed coordinate system points in the



direction of the local gravity vector. The missile equations of motion in the pitch plane are given by the following six nonlinear differential equations:

$$\begin{aligned} \dot{u} &= [T - F_A]/m - g \sin \theta - q w \\ \dot{w} &= q u + g \cos \theta - [F_N + F_r]/m \\ \dot{q} &= \frac{M - l_r F_r}{I_{yy}}, \quad \dot{\theta} = q \\ z &= V_T \sin (\theta - \alpha), \quad \dot{x} = V_T \cos (\theta - \alpha) \end{aligned}$$

These equations assume a flat, non-rotating earth with a quiescent atmosphere. The variable u is the missile velocity component along the X_B body axis, w is the velocity component along the Z_B body axis, T is the missile thrust, F_A is the axial force, F_N is the normal force, m is the vehicle mass, g the acceleration due to gravity, θ the pitch attitude, q the pitch rate, F_r is the force generated by the reaction jets, and l_r is the reaction jet lever arm. M is the aerodynamic pitching moment, I_{yy} is the pitch moment of inertia, $-z$ is the altitude, and x is the down range. The variable γ shown in Figure 1 is the flight path angle.

The form of the missile aerodynamic forces and moments, and the definition of other related variables are given by the following expressions:

$$\begin{aligned} F_A &= C_A(\text{Mach}, \alpha, \delta) \bar{q} s \\ F_N &= C_N(\text{Mach}, \alpha, \delta) \bar{q} s \\ M &= C_m(\text{Mach}, \alpha, \delta) \bar{q} s l_{\text{ref}} \end{aligned}$$

C_A is the axial force coefficient, C_N is the normal force coefficient, and C_m is the pitching moment coefficient, all given as functions of Mach number, angle of attack α and aerodynamic

surface deflection δ . The variable s is the reference area and l_{ref} is the reference length. Mach number, dynamic pressure $\bar{q}$, total velocity V_T , and angle of attack α are defined as:

$$\text{Mach} = V_T/a, \quad \bar{q} = \frac{1}{2} \rho V_T^2, \quad V_T = \sqrt{u^2 + w^2}, \quad \alpha = \tan^{-1}[w/u]$$

The variable 'a' is the speed of sound and ρ is the atmospheric density given as a function of altitude. The aerodynamic surface actuator and reaction jet actuator dynamics are modeled as:

$$\dot{\delta} = \tau_f(\delta_c - \delta), \quad \dot{F}_r = \tau_r(F_{rc} - F_r)$$

The quantities with subscript 'c' are the commanded values of the actuator, and τ_f, τ_r are the actuator time constants.

The missile acceleration components measured by the on-board accelerometers are defined as:

$$a_x = -\frac{(T - C_A \bar{q} s)}{m}, \quad a_z = -\frac{(C_N \bar{q} s + F_R)}{m}$$

Pitch plane high angle of attack missile autopilot is required to track a normal acceleration command generated by the guidance law [17]. The normal acceleration is given by the expression:

$$a_n = a_z \cos \alpha - a_x \sin \alpha$$

In order to include the actuator dynamics in the analysis, it is sometimes convenient to differentiate the pitch rate equation once with respect to time, to yield:

$$\dot{q} = \frac{1}{I_{yy}} [\bar{q} s l_{\text{ref}} (C_{m\alpha} \dot{\alpha} + C_{m\delta} \dot{\delta}) - l_r \dot{F}_r]$$

The derivatives of the aerodynamic surface deflection and reaction jet thrust can be eliminated from this expression using actuator dynamics. This results in:

$$\dot{q} = \frac{1}{I_{yy}} [\bar{q} s l_{\text{ref}} (C_{m\alpha} \dot{\alpha} - C_{m\delta} \tau_f \delta) + l_r \tau_r F_r] + \left(\frac{1}{I_{yy}} \bar{q} s l_{\text{ref}} C_{m\delta} \tau_f \right) \delta - \left(\frac{1}{I_{yy}} l_r \tau_r \right) F_{rc}$$

The angle of attack rate can be computed from measured normal acceleration, pitch rate and the airspeed as:

$$\dot{\alpha} = \frac{a_n}{\sqrt{u^2 + w^2}} + q$$

The pitch plane missile model discussed in this section forms the basis for the development of nonlinear autopilots.

Autopilot Design

Target intercept under arbitrary launch conditions is the primary objective of high angle of attack missiles. Based on the missile and target states, the guidance law generates steering commands to orient the missile velocity vector in the direction of the target. The missile velocity vector can be oriented in a desired direction by applying a force normal to the direction of the instantaneous velocity vector. At low angles of attack, the velocity vector can be oriented by using the aerodynamic force normal to the missile body as is done in conventional missile configurations. However, in high angle of attack missiles, a force normal to the velocity vector will include both the aerodynamic normal force and a component of the main motor thrust. Note that at low angles of attack, the acceleration along the Z_B axis adequately approximates the acceleration normal to the velocity vector.

The autopilot has the responsibility for tracking the normal acceleration commands generated by the guidance law while stabilizing the missile airframe. As mentioned in the section on vehicle modeling, the autopilot uses the aerodynamic surface actuators and reaction jets to carry out its functions. Aerodynamic surfaces are used for high speed/low angle of attack flight, while a combination of reaction jets and aerodynamic surfaces may be employed in the high angle of attack regime. Depending upon the nature of autopilot logic, reaction jet operation may or may not require a pulse modulator.



The airframe stabilization and acceleration command tracking functions of the autopilot can be accomplished by formulating the control problem in terms of the missile pitch rate q , and either the transverse velocity w or the angle of attack α . Consequently, the missile longitudinal velocity, attitude, down range and altitude are ignorable variables. It will be subsequently seen that although the longitudinal velocity and altitude do not explicitly appear in the autopilot logic, they would be required for computing the linearizing transformations.

Autopilots can be designed based on any of the several control design techniques available in the literature. Several variations of linear multivariable control theory [18 - 21], feedback linearization [14, 22 - 26], sliding mode control [27 - 28], Lyapunov theory [29], and bang-bang control [30] can all be used to synthesize missile autopilots.

The development of two nonlinear autopilots based on feedback linearization methods will be discussed in this paper. In this design technique, a portion of the missile model is used to transform the plant into a linear, time-invariant form. The control laws are then designed using the feedback linearized model. The nonlinear transformation, together with the control law forms the autopilot logic. However, since the present missile configuration is tail controlled, it has a strong non-minimum phase behavior with respect to the normal acceleration. Consequently, a straightforward application of the feedback linearization technique will not be successful, because the unstable zerodynamics would not assure the internal stability of the system [22, 27, 28].

Two approaches have been proposed to overcome this difficulty. First of these is the time-scale separation of the system dynamics into slow and fast modes [14, 25, 26]. The second approach is to redefine the system outputs to suppress the zerodynamics [27, 28]. Both these methods will be discussed in the following sections.



Two Time-Scale Nonlinear Autopilot Design

The first step in the two time-scale design process is to split the system dynamics into time scales based upon the notion of slow and fast dynamic modes. Note that even if a clear separation between the modes are not present in the open-loop dynamics, mode separation can be enforced during control system design. For the present case, the actuator dynamics, together with the pitch rate dynamics are included in the fast time-scale. The normal acceleration dynamics is considered to be the slow time scale mode.

A differential equation describing the normal acceleration rate can be obtained by differentiating the expression for normal acceleration to yield:

$$\dot{a}_n = \left[\frac{C_{N\alpha} \bar{q} s \cos\alpha}{m} + a_z \sin\alpha + a_x \cos\alpha \right] \left(\frac{a_n}{\sqrt{u^2 + w^2}} + q \right)$$

Derivation of this equation assumed that the acceleration due to gravity is much smaller than the normal acceleration commands, and that the rate of change of dynamic pressure does not influence the normal acceleration rate. Further, it assumes that the force contribution due to the aerodynamic surface deflection is small. The primary reason to make these assumptions is to simplify the development.

Assuming that the pitch rate dynamics is much faster than the normal acceleration dynamics, the above equation can be used to generate commanded pitch rate in response to a commanded normal acceleration. This can be accomplished by first defining the right hand side of the normal acceleration equation to be a pseudo control variable P to yield:

$$\dot{a}_n = P$$

Since the system is now in a linear, time-invariant form, a proportional plus integral control law can be designed to track normal acceleration commands a_{nc} . The proportional plus integral control law can then be equated to the right hand side of the $\dot{a}_n$ expression to yield:



$$q_c = -\frac{a_n}{\sqrt{u^2 + w^2}} + \frac{K_{anP}(a_{nc} - a_n) + K_{anI} \int_0^t (a_{nc} - a_n) dt}{\left[\frac{C_{N\alpha} \bar{q} s \cos \alpha}{m} + a_z \sin \alpha + a_x \cos \alpha \right]}$$

K_{anP} is the proportional gain and K_{anI} is the integral gain. Since the slow time-scale dynamics is of first-order, a proportional plus integral control law is necessary to ensure zero steady state error. The feedback gains are chosen to yield slow time scale undamped natural frequency of 0.5 Hz and a damping ratio of 1.

The fast time-scale dynamics has the responsibility for tracking the commanded pitch rate q_c . Fast time-scale control law can be designed using the expression for the rate of change of angular acceleration:

$$\dot{q} = \frac{1}{I_{yy}} [\bar{q} s l_{ref}(C_{m\alpha} \dot{\alpha} - C_{m\delta} \tau_f \delta) + l_r \tau_r F_r] + \left(\frac{1}{I_{yy}} \bar{q} s l_{ref} C_{m\delta} \tau_f \right) \delta - \left(\frac{1}{I_{yy}} l_r \tau_r \right) F_{rc}$$

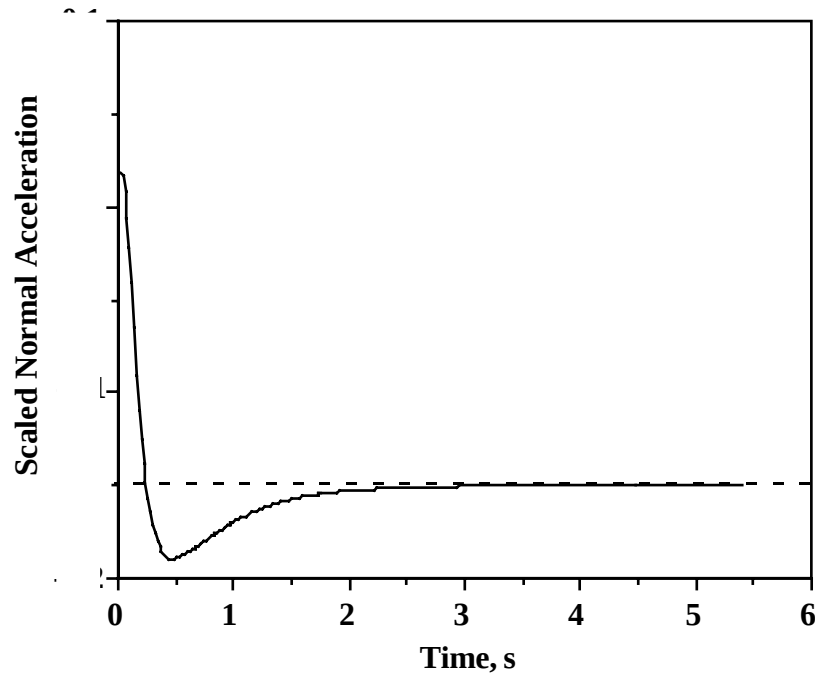
The right hand side of this differential equation can be denoted by a pseudo control variable for nonlinear control law design. The resulting system will be linear, time invariant and of second-order. A proportional plus derivative control law can be used to track the pitch rate commands generated by the slow normal acceleration dynamics. Proceeding as before, the nonlinear control law for the aerodynamic surface actuator in the fast time-scale is given by:

$$\delta_c = \frac{K_{qp}(q_c - q) - K_{qd} \dot{q} - \frac{1}{I_{yy}} [\bar{q} s l_{ref}(C_{m\alpha} \dot{\alpha} - C_{m\delta} \tau_f \delta)]}{\left(\frac{1}{I_{yy}} \bar{q} s l_{ref} C_{m\delta} \tau_f \right)}$$

The fast time-scale feedback gains K_{qp} and K_{qd} are chosen to yield an undamped natural frequency of 10 Hz and unity damping ratio.

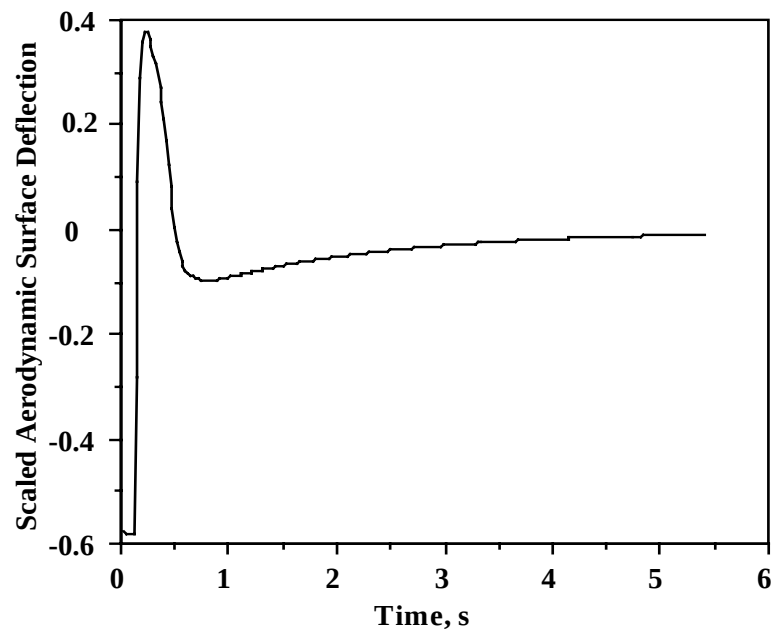
The two time-scale control law is next evaluated in a nonlinear missile simulation. The step response of the two time-scale nonlinear autopilot is given in Figure 2. It can be observed that the system has a relatively fast rise time, moderate overshoot and no steady state error.





**Fig. 2. Two Time-Scale Nonlinear Autopilot Response
to a Step Normal Acceleration Command**

The aerodynamic surface deflection history given in Figure 3 reveals the nonlinear nature of the missile dynamics.



**Fig. 3. Aerodynamic Surface Deflection Required to Track Step Normal Acceleration
Command
(Two Time-Scale Nonlinear Autopilot)**



Additional simulations at different flight conditions within the flight envelope have shown that the two time-scale nonlinear autopilot consistently delivers uniform performance. Thus, the nonlinear autopilot provides the same level of performance at all Mach numbers and altitudes. Additionally, since no gain scheduling is required to implement the control law, tremendous savings in the design effort can be realized. Finally, since the system structure after time-scale separation is of low-order, the autopilot design can be accomplished using rather elementary control system design tools. Note that one can also employ modern robust control methods in the design process, if desired.

Nonlinear Autopilot Design Using Modified Output

As indicated at the beginning of this section, direct feedback linearization of a tail controlled missile dynamics will result in marginally stable or unstable zerodynamics [22, 27, 28], which will adversely impact the closed loop system performance. Time scale separation is one of the approaches to circumvent this difficulty. Another approach is via the redefinition of the system output [27, 28].

In this approach, one attempts to find a new output variable that behaves like the desired output in steady state, but which limits or suppresses any unstable zerodynamics arising out of the feedback linearization process. The transient response of the resulting system may not exactly be what the user desires, but it will ameliorate the difficulties with zerodynamics.

In the present case, instead of attempting to track the normal acceleration a_n , consider the problem of tracking a new variable a_n^* defined as:

$$a_n^* \equiv a_n + b \dot{q} + c \dot{\alpha}$$

The variables b and c will be selected as a part of the design process. In steady state, it can be observed that $a_n^* = a_n$, since both the vehicle pitch acceleration and the angle of attack rate goes



to zero. However, during the transients, a_n^* will include the pitch rate dynamics in addition to the normal acceleration dynamics. Including these variables in the output will ensure that the normal acceleration as well as the pitching moments will remain bounded during transient maneuvers. However, in steady state, these quantities will vanish from the output. The variables b and c can be chosen to provide adequate damping for angle of attack and pitch rate.

The system can next be expressed in terms of the new output variable. Noting that the angle of attack rate without including the effect of gravitational acceleration is given by:

$$\dot{\alpha} = \frac{a_n}{\sqrt{u^2 + w^2}} + q$$

one may substitute for $\dot{\alpha}$ in the expression for a_n^* . The resulting expression may be differentiated with respect to time. After eliminating the terms $\dot{a}_n$ and $\dot{q}$ on the right hand side one has:

$$\begin{aligned} \dot{a}_n^* = & - \left[1 + \frac{c}{\sqrt{u^2 + w^2}} \right] \left[\frac{C_{N\alpha} \bar{q} s \cos \alpha}{m} + a_z \sin \alpha + a_x \cos \alpha \right] \left(\frac{a_n}{\sqrt{u^2 + w^2}} + q \right) \\ & + \frac{b}{I_{yy}} \left[\bar{q} s l_{ref} (C_{m\alpha} \dot{\alpha} - C_{m\delta} \tau_f \delta) + I_r \tau_r F_r \right] + \left(\frac{b}{I_{yy}} \bar{q} s l_{ref} C_{m\delta} \tau_f \right) \delta_c - \left(\frac{b}{I_{yy}} I_r \tau_r \right) F_{rc} + cq \end{aligned}$$

Note that the force contribution by the aerodynamic control surfaces are neglected in the above derivation.

As in the previous section, the right hand side of this expression can next be considered to be a pseudo-control variable P , and a proportional plus integral control law can be designed. The feedback gains are chosen to yield an undamped natural frequency of 0.5 Hz and a unity damping ratio. Inverse transformation will then yield a nonlinear control law for aerodynamic surface deflection as:

$$\delta_c = \frac{K_{anP} (a_{nc}^* - a_n^*) + K_{anI} \int_0^t (a_{nc}^* - a_n^*) dt - cq + a_1 + a_2}{\left(\frac{b}{I_{yy}} \bar{q} s l_{ref} C_{m\delta} \tau_f \right)}$$

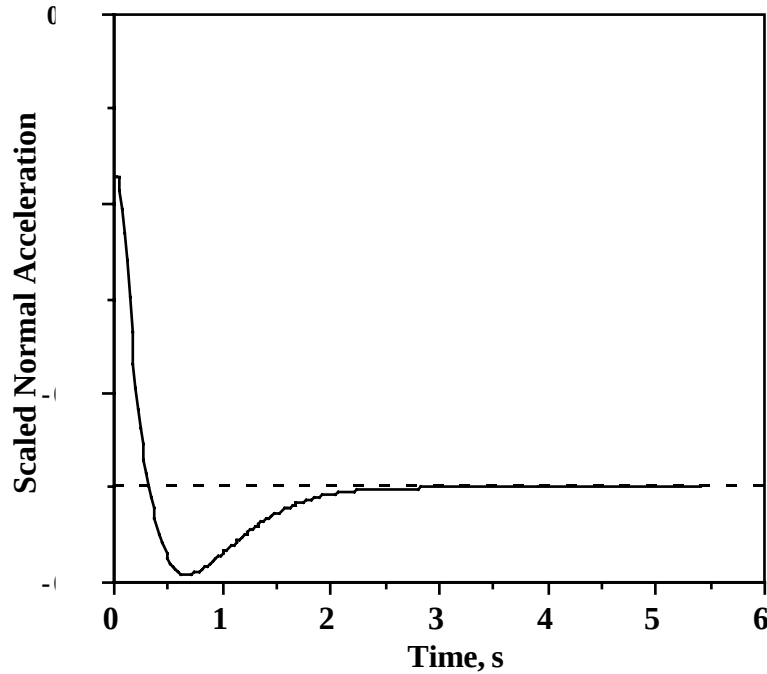


where:

$$a_1 = \left[1 + \frac{c}{\sqrt{u^2 + w^2}} \right] \left[\frac{C_{N\alpha} \tau s \cos \alpha}{m} + a_z \sin \alpha + a_x \cos \alpha \right] \left(\frac{a_n}{\sqrt{u^2 + w^2}} + q \right)$$

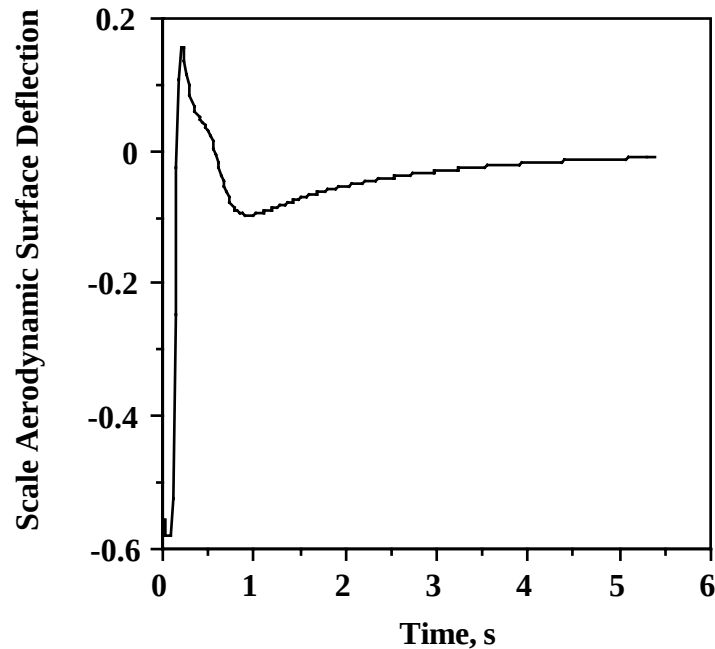
$$a_2 = - \frac{b}{I_{yy}} [\tau s l_{ref} (C_{m\alpha} \dot{\alpha} - C_{m\delta} \tau_f \delta)]$$

The variables b and c are chosen to give a well damped, second-order response to the pitch rate dynamics. The response of the nonlinear autopilot to a step normal acceleration command is shown in Figure 4. Good command tracking characteristics can be observed from this figure. The aerodynamic surface deflection corresponding to the step response is given in Figure 5. The nonlinearities in the missile dynamic model can be discerned from the aerodynamic surface deflection history.



**Fig. 4. Response of the Modified Output Nonlinear Controller
to a Step Normal Acceleration Command**





**Fig. 5. Aerodynamic Surface Deflection Required to Track a Step Normal Acceleration Command
(Modified Output Nonlinear Controller)**

As with the nonlinear two time scale autopilot, the nonlinear autopilot with modified output provides uniform response over the entire flight envelope. However, the advantage in using the second nonlinear control formulation when compared with the first is that explicit time scale separation assumption need not be invoked to synthesize the autopilot. The disadvantage is that the desired output has to be modified. Consequently, the system transient response cannot be precisely controlled.

Conclusions

This paper presented an initial research effort on nonlinear control design methods for synthesizing agile autopilots for high angle of attack missiles. Starting from a pitch-plane missile model assembled using wind tunnel data, two different autopilot logics were discussed. Nonlinear autopilot logics exploited feedback linearization and time-scale separation. The performance of these autopilots were demonstrated in nonlinear simulations. Due to the wide variation in the flight



conditions encountered by the high angle of attack missile, linear autopilots require gain scheduling with respect to Mach number, altitude and angle of attack. Although powerful multivariable techniques are available for design, gain scheduling can consume an enormous amount of effort. On the other hand, nonlinear autopilots require significantly higher initial analysis effort, but are considerably easier to design than linear-gain-scheduled controllers. If sufficient computational resources are available on-board, nonlinear autopilots are viable candidates for implementation in high angle of attack missiles.

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