

A Finite Element Method for the Electromagnetic Characterization of Quasi-Magnetostatic Problems Found in UXO Detection and Discrimination

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Abstract: A Finite Element Method (FEM) is presented to solve for the scattered magnetic vector potential from arbitrarily polarized targets typical of those encountered in Unexploded Ordnance (UXO) detection. Most previously developed numerical methods only solved problems in which axial symmetry could be exploited. This, of course, severely limits the capabilities and utility of such methods. The method presented in this paper however seeks to overcome these limitations by allowing not only arbitrarily polarized targets but also allowing the size and shape of the target to be arbitrary as well. We show also the capabilities of the method for extending work in measurement technique and discrimination algorithm development.

Keywords: Unexploded ordnance, finite element method, remote sensing

1. Introduction

Detection and removal of buried unexploded ordnance (UXO) has become a center of attention by humanitarian groups and governments throughout the world. Several detection methods including electromagnetic induction techniques (EMI) are in use and undergoing constant improvement. Currently, there is a significant need to develop measurement and signal processing techniques which discriminate between typical UXO objects and so-called “clutter” items. The future deployment of an accurate UXO detection and discrimination system will depend heavily on the measurement and discrimination capabilities of that system. To date, numerous measurement techniques have been developed as well as several discrimination schemes (although none are accurate enough for permanent real-world use) [1-3]. To aid in the development of these measurement and discrimination techniques, it is beneficial to have a numerical method which can accurately predict the response for an arbitrarily polarized target of any shape. Traditionally, objects with analytical response solutions (e.g., spheres, infinite cylinders, etc.) have been used to gauge the accuracy of a measurement technique [4-6]. The development of a robust numerical method would allow data verification for more realistic geometries and polarizations. However, the utility of such a numerical scheme is not limited purely to verification of measurements.

Previously, the majority of numerical codes developed to study the UXO problem have been based on integral equation formulations (Method of Moments) with a body-of-revolution (BOR) assumption [3,7-8]. BOR codes however, as their name implies, can only solve axisymmetric problems. One problem of extreme interest in the UXO community is the response of a finite cylinder with uniform transverse excitation or, more generally, arbitrary polarization. BOR codes are completely useless for this type of analysis since the symmetry of the situation has been destroyed. A Finite Element Method is presented in the remainder of this paper which can accurately solve for the response of an arbitrarily polarized (excited) target as well as perform many other types of analyses. Following a brief overview of the formulation, results will be presented in order to further discuss the capabilities of the method.

An FEM method such as the one proposed here can quickly establish the upper and lower frequency bounds useful for discrimination purposes beyond what is currently available from measurement techniques. The very-low frequency response of targets may in fact be a key component in development of more accurate

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discrimination algorithms. It is well-known that to reach the low-frequency asymptotic behavior of the in-phase response for targets of appreciable size, the measurements must be taken at frequencies near DC. Knowledge of the lower bound of useful frequencies would allow researchers to more efficiently focus their development of new measurement and discrimination techniques. Results of a study of this nature are presented in a later section.

Additionally, a numerical method provides a way to perform fast parametric and sensitivity analyses which may also be important for discrimination purposes. Many realistic targets are found with driving bands at different positions. One useful analysis, which is presented later, is that of the variation in response due to changes in driving band position. Also, the sensitivity of the response to variation in constitutive parameters may yield important information as to the range of these parameters over which an object may be accurately discriminated. An analysis of the response sensitivity to changes in relative permeability as well as conductivity will be presented.

The organization of the remainder of the paper is as follows. In Section 2, a formulation of the FEM method will be presented. In Section 3, results will be presented for a variety of situations applicable to current research in UXO detection/discrimination.

2. Formulation

The formulation of finite element problems is well-documented [9]. Therefore, only the information pertinent to FEM solution of UXO problems will be discussed here. As is well-noted in the literature, the UXO problem may be characterized by the electromagnetic induction (EMI) technique [1-5,7-8]. In this low frequency range, the displacement current in the target may be assumed negligible with respect to the conduction current. Therefore, Maxwell's equations reduce to what is known as the quasi-magnetostatic form

$$\begin{aligned}\nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \times \vec{H} &= \sigma\vec{E}\end{aligned}\tag{1}$$

where μ is the permeability of the target/medium, ω is the angular frequency, and σ is the conductivity of the target/medium. Taking the magnetic flux density to be the curl of the magnetic vector potential, the electromagnetic diffusion equation can be written in the frequency domain as

$$\nabla \times \mu^{-1} \nabla \times \vec{A} = j\omega\sigma\vec{A}\tag{2}$$

where A is the magnetic vector potential. This form of the diffusion equation is slightly different than that found in analytic formulations. The presence of the double curl operation rather than the Laplacian preserves the possibility of a spatially varying permeability thereby increasing the robustness of the method. Of course, in this form, no assumptions have been made about the spatial variation of the conductivity, and it can, in general, be allowed to vary.

As was mentioned previously, this method can evaluate the response of arbitrarily polarized targets. The density of the discretization is a function of the wavelength within the target and can be varied appropriately for the given frequency range. One weakness of the method similar to the MoM codes previously developed is the dramatic increase in the number of unknowns required to achieve a solution within the metallic region. Even at relatively low frequencies, wavelengths can easily be on the order of a millimeter. This is of course much smaller than the feature size of typical UXO targets which can be on the order of a meter. However, using modern computing resources and novel solvers, realistic target geometries can be solved without becoming computationally prohibitive. For general orientations/excitations, quadratic tetrahedral elements are used to discretize the solution domain. When BOR symmetry can be exploited, quadratic triangular elements are employed. An example of a

BOR geometry to be implemented using FEM is shown in Figure 2. Unlike the Method of Moments (MoM), the entire solution space must be discretized when using FEM. Therefore, some consideration must be taken when truncating the domain outside of the target. Since a general target will have a dipole response that is dominant some distance from the target, the boundary of the domain is established such that the $1/r^3$ variation of the dipole response is approximately zero. This assumption necessarily introduces some small error into the method. However, with proper placement of the boundary, this error can be made negligible.

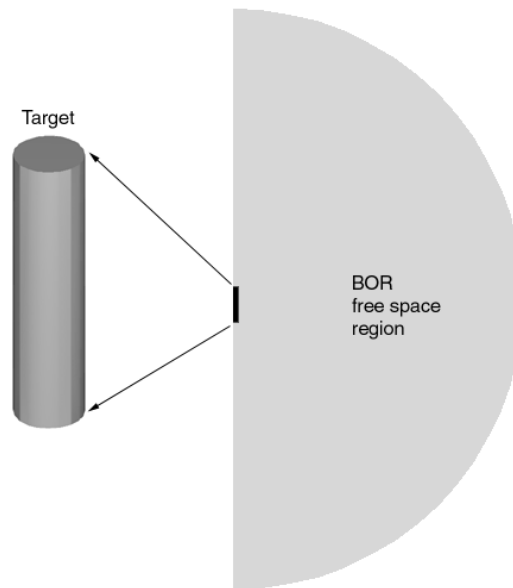


Fig. 1. A finite cylinder with BOR symmetry placed in an axisymmetric FEM domain.

3. Results & Discussion

We wish to compare the response of finite cylinders for the FEM method and an EMI measurement technique [10]. The FEM cases presented here assume BOR symmetry to reduce the computational complexity. The cylinders are placed in a uniform, axially-directed magnetic field. This assumption should be valid provided that the transmit coils of the measurement system produce a reasonably uniform field in the vicinity of the cylinder. Unlike the measurement system which measures the flux through the receive coil, the FEM measures the scattered magnetic vector potential some distance away from the target. At some reasonable distance from the target, the dipole contribution of the multipole expansion is the dominant term in the response. With this in mind, the dipole moment can be easily calculated from the magnetic vector potential with a minimum of additional error. Another possibility for comparison is to simply take the vertical magnetic field some distance away from the target. This quantity however is, unlike the dipole moment, spatially dependent and involves taking numerical derivatives. Therefore, this method was viewed as a less attractive option although it will show the same trends in the response. Since the dipole moment is spatially independent, it is within a multiplicative constant of the true response function ($X+jY$) discussed in the literature. Similarly, it is assumed that the quantity measured using the EMI technique discussed in [10] is also within a multiplicative constant (plus some frequency-dependent systematic error) of the response function. Assuming that the error in the FEM results is negligible, a least squared error minimization algorithm is used to effectively normalize the measured data with respect to the FEM results. The resulting data can now be compared and the discrepancies more easily noted.

Figure 2 is a comparison of FEM and measured data [10] for an axially polarized steel cylinder (2 in. diameter, 8 in. length) with a relative permeability of 70 and conductivity of $5.82E6$. As can be seen from the figure, the results are quite good with some discrepancy in the high frequency range. The measured data obviously contains some error in the high frequency region since the quadrature data above 35kHz is negative.

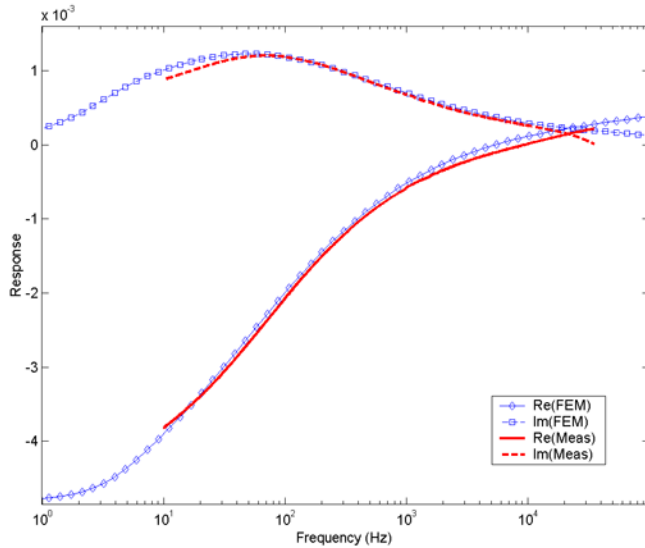


Fig. 2. EMI response of an axially polarized steel cylinder.

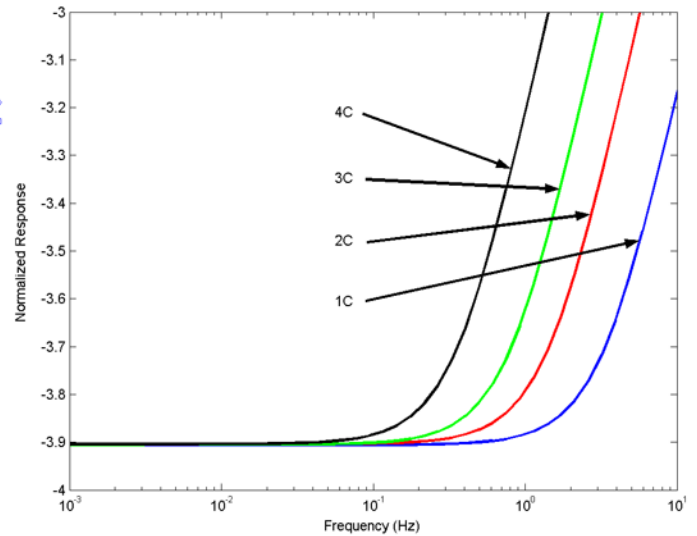


Fig. 3. Normalized low frequency asymptotic behavior of in-phase response for 4 cylinders of varying size.

A. Low frequency response

As mentioned previously, the FEM may be used to establish a lower frequency bound for discrimination purposes. Figures 3 & 4 show the low frequency asymptotes of the in-phase target response for four solid cylinders and 3 cylinders of equal dimensions but different thicknesses, respectively. The dimensions of these cylinders may be found in Table 1. In Figure 3, we see that the real part does not reach its asymptotic value until approximately 10 mHz for the largest cylinder which has dimensions characteristic of large UXO items. As the cylinder size decreases, the asymptotic limit of the response is reached at higher frequencies. Similarly, for the cylinders of different thickness (see Figure 4), a decrease in cylinder thickness corresponds to an increase in the frequency at which the asymptotic limit is reached. This information as well as many other response characteristics which have been addressed in the literature [4] may be very useful for discrimination of targets.

Designation	Diameter	Length	Wall Thickness
1C	2 in.	8 in.	Solid
2C	3 in.	12 in.	Solid
3A	4 in.	16 in.	0.25 in.
3B	4 in.	16 in.	0.5 in.
3C	4 in.	16 in.	Solid
4C	6 in.	24 in.	Solid

Table 1. Dimensions of steel cylinders used for FEM experiments and measurements found in [10].

B. Sensitivity Analysis

One of the primary weaknesses of many discrimination algorithms is the need for a large pool of training data. To develop this database as well as provide a greater pool of information from which to derive more intelligent algorithms, parametric analyses and sensitivity analyses must be performed. Since the FEM has been shown to be a viable tool for very low frequencies (0 Hz – 30 Hz) as well as the more common EMI frequency range (30 Hz – 50 kHz), this may be a more efficient means of gathering accurate data for algorithm development than measurements. The first example of such an application is an analysis of the sensitivity of the response to changes in the relative permeability of a target. In this study, the 1C cylinder with axial polarization was

simulated with relative permeabilities of $\pm 10\%$ and $\pm 20\%$ of the nominal value (for the all cylinders $\mu_{\text{nominal}} = 70$). As can be seen from Figure 5, this change caused only a slight frequency shift as well as a change in low frequency signal amplitude. A similar study was conducted for the conductivity (nominal conductivity of $5.82\text{E}6$). The results of which are shown in Figure 6. In this case, only a slight logarithmic frequency shift in the response occurred.

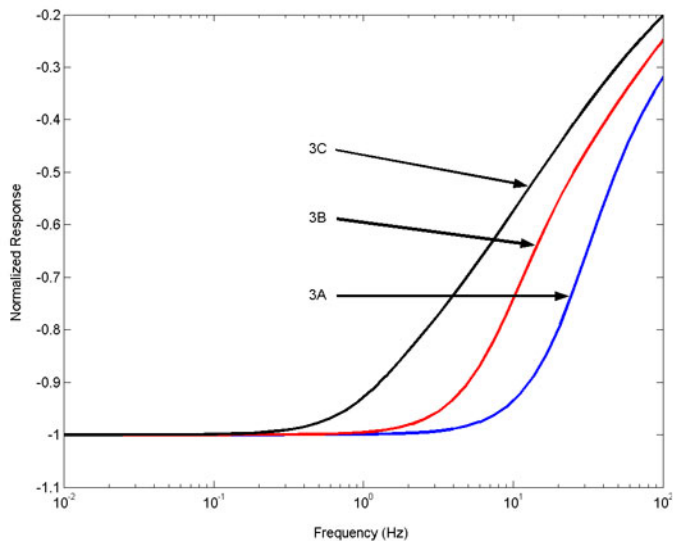


Fig. 4. Normalized low frequency asymptotic behavior of in-phase response for 3 cylinders of varying thickness.

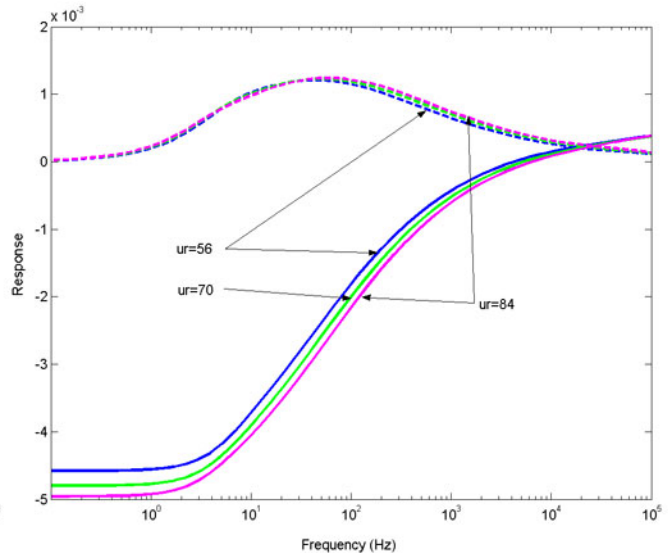


Fig. 5. Sensitivity analysis of different relative permeability values for the same cylinder size.

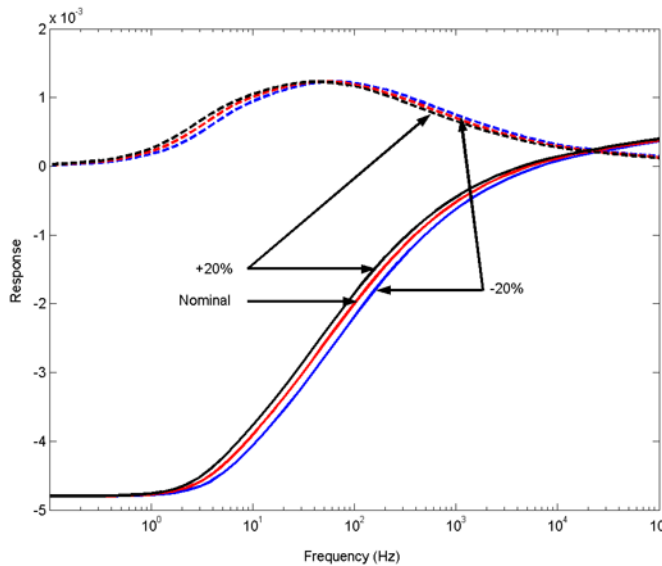


Fig. 6. Sensitivity analysis of different conductivity values for the same cylinder size.

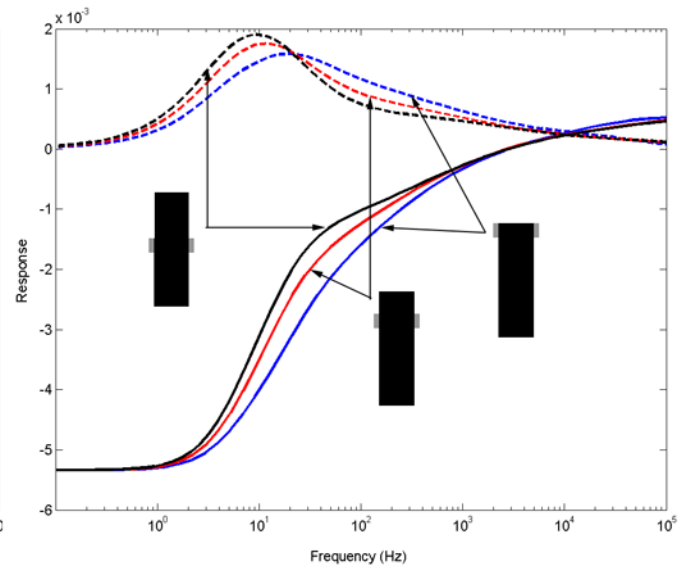


Fig. 7. Comparison of responses for driving bands at different positions on the 1C cylinder.

C. Effects of Driving Bands

A metallic driving band is often encountered on realistic UXO items. These driving bands can have a drastic effect on the response of a target. A parametric analysis is performed in which the response is simulated for driving bands at different positions on the 1C cylinder. Due to symmetry, the band need only be simulated for

positions on one half of the cylinder. Figure 7 shows the response of the axially polarized 1C cylinder with a driving band placed at different locations ranging from the center of the cylinder to the top of the cylinder. Clearly, as the driving band nears one end of the cylinder, the response begins to “decouple” into the sum of the individual object responses. This type of analysis will be imperative for future work in discrimination.

4. Conclusions

A Finite Element Method has been presented which can perform many types of analyses beneficial to the development and improvement of measurement and discrimination techniques needed for UXO detection and removal. One benefit of this method over previous methods is its ability to obtain a response from arbitrarily polarized targets. Additionally, as was presented, the FEM is capable of a variety of parametric and sensitivity studies which will be pertinent to development of discrimination algorithms. Future work in this area will include improvements on the speed and accuracy of simulations of arbitrarily polarized targets.

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