

TECHNICAL RESEARCH REPORT

A Multilevel ON/OFF Model for Multifractal Internet Traffic

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**CSHCN TR 2002-5
(ISR TR 2002-9)**



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Report Documentation Page				Form Approved OMB No. 0704-0188	
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1. REPORT DATE 2002		2. REPORT TYPE		3. DATES COVERED -	
4. TITLE AND SUBTITLE A Multilevel ON/OFF Model for Multifractal Internet Traffic				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Defense Advanced Research Projects Agency, 3701 North Fairfax Drive, Arlington, VA, 22203-1714				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT see report					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES 10	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

A Multilevel ON/OFF Model for Multifractal Internet Traffic

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Abstract

In this paper, an open-loop multilevel ON/OFF model is proposed to capture the multifractal behavior of the HTTP traffic on the Internet. It is assumed that the life time of a TCP session and the active time of a burst within a TCP session have a heavy-tail type distribution. The aggregate traffic of this model is shown to be multifractal. We analyze its second and higher order statistics by the wavelet analysis and develop a simple method to estimate the model parameters from a real Internet trace. We show that real and synthesized traffic produce the same Logscale Diagram with accuracy, for proper selection of the model parameter. Finally, we compare using the NS-2 simulator the queueing behavior of FIFO queues fed by real and synthetic traffic demands.

I. INTRODUCTION

Recent studies [3] [4] on Internet traffic have shown that the aggregate traffic driven by TCP based protocols such as HTTP is not only monofractal (self-similar) but also multifractal. The wavelet analysis demonstrates that the traffic is monofractal on large time scales ($> 1sec$), which is mainly due to the heavy-tailed distribution of file sizes on the Internet. However, the traffic behavior on small time scales is much more complicated and has been shown to be multifractal. This multifractal behavior is mainly due to the protocol dynamics such as TCP flow control, network congestion, packet loss and packet retransmission. Taqqu and Willinger [2] explained the monofractal behavior on large time scales by aggregating a large number of independent ON/OFF type traffic with ON and/or OFF duration which have heavy-tailed distribution. The ON duration is corresponding to the total transmission time of a file. They proved that the aggregate traffic converges to the well-known fractional Brownian motion asymptotically in the limit of many flows. They also found a simple relationship between the shape parameter of heavy-tailed distribution and the *Hurst* parameter of self-similarity. However, their single level ON/OFF model can not explain the multifractal behavior on small time scales with its constant rate assumption in the ON duration. We seek a more precise model which can capture the traffic behavior in all regions and at the same time provide a physical interpretation of the model via natural network mechanisms. We are also interested in the impact of protocol dynamics on the network performance and control. Some important parameters such as round-trip time and active time of burst will be studied and discussed. Furthermore, based on these parameters estimated from a real trace, we propose an open-loop traffic model with a multilevel ON/OFF structure for HTTP traffic on the Internet. The proposed model can capture the traffic behavior within a wide range of time scales and offers useful physical information of the effects on protocol parameters. By employing this model, we can facilitate the consideration of the optimal control problem in network management and estimate its performance.

Research partially supported by DARPA contract No. N66001-00-C-8063.

II. MULTILEVEL ON/OFF MODEL FOR ONE TCP SESSION OF HTTP TRAFFIC

By using the discrete wavelet analysis[5], the Logscale Diagram of a typical HTTP traffic in Fig.1 shows that the logarithm of the energy of the wavelet detail coefficient $\log_2 E[d_{j,k}^2]$ is a linear function at large time scales. However, the traffic behavior at small time scales ($\leq 1\text{sec.}$) is more complicated and regarded to be multifractal. The single ON/OFF model is unable to explain this multifractal behavior on small time scales by its constant packet rate assumption. For reliable communication, TCP has a well-known congestion control mechanism. After sending out a batch of packets (burst), the sender will stop and wait until receiving acknowledgment from the receiver. In order to avoid congestion, the burst size (the number of packets in one burst) is controlled by the current TCP sliding window. The time interval for waiting the ACK is almost equal to the network round-trip time (RTT). This stop and wait behavior of HTTP connection can also be modeled by a second level ON/OFF process. Since most objects on the web pages are small graphic or texture files, the transfer of these files is usually finished in its slow start phase. However, when the web objects are large enough, the TCP will finish the slow start phase and enter to its congestion avoidance phase. In this case, the TCP session will have a very long active period and this behavior will be modeled in this paper by a heavy-tailed distribution of active time. We propose a two level ON/OFF model for one TCP session

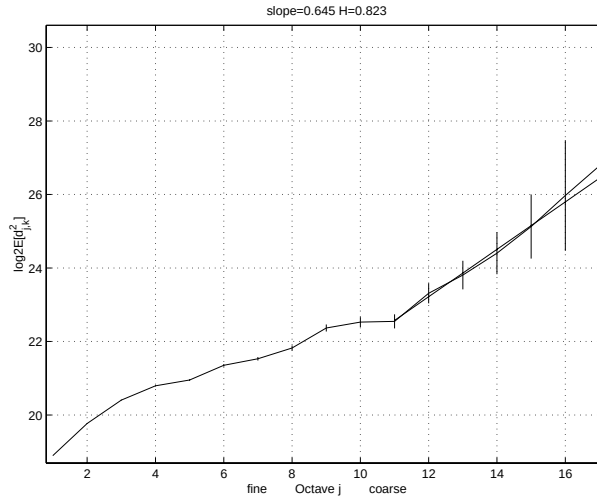


Fig. 1. Logscale Diagram of Real Trace

as shown in Fig.2. The first level is an ON/OFF process which models the life time of one TCP session (T_{11}) and the OFF time (T_{10}) between two TCP sessions. In order to capture the behavior of the TCP mechanism inside the duration of T_{11} , there is another ON/OFF process, which mimics the active time (duration of a burst T_{21}) and the inactive time (T_{20}) between two successive bursts. The packet rate B in the active time is assumed to be a constant. The T_{11} and T_{21} have the Pareto Type I distribution with Parameters (K_{11}, α_{11}) and (K_{21}, α_{21}) respectively. i.e

$$Pr[T > t] = \begin{cases} (K/t)^\alpha & , \text{ if } t \geq K \\ 1 & , \text{ if } 0 < t < K \end{cases}$$

The OFF time T_{10} and T_{20} are chosen to be Exponential random variables with mean $1/\lambda_{10}$ and $1/\lambda_{20}$ respectively. All these random variables are statistically independent of each other.

$$\begin{aligned} T_{10} &:= r.v. \text{ Exp}(1/\lambda_{10}) \\ T_{11} &:= r.v. \text{ Pareto}(K_{11}, \alpha_{11}) \\ T_{20} &:= r.v. \text{ Exp}(1/\lambda_{20}) \\ T_{21} &:= r.v. \text{ Pareto}(K_{21}, \alpha_{21}) \\ B &:= \text{Data rate within the active period} \\ N &:= \text{Number of connections} \end{aligned}$$

Similar to the original single level ON/OFF model [2], when a file is ready to be transferred, the user connection

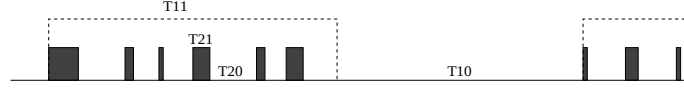


Fig. 2. Traffic model for one TCP session

begins a TCP session with duration T_{11} and then returns to the OFF state when transmission is completed. Some empirical studies have shown that the mean OFF time is 30 to 60 times greater than the mean ON time on the Internet. Note that the network round-trip time is assumed to be the sum of T_{21} and T_{20} . For the HTTP traffic on the Internet, T_{21} is usually far less than T_{20} .

III. PARAMETER ESTIMATION AND MODEL FITTING

In order to match the second order statistical properties of the real HTTP traffic, we have to properly estimate the model parameters from the real trace. Veitch and Abry [5] provided an asymptotically unbiased and efficient estimator for the slope m of the Logscale diagram within a certain region (j_1, j_2) . The shape parameters α_{11} and α_{21} can be estimated by the relation in [2]:

$$\alpha := 2 - m.$$

However, this slope estimator is semiparametric and depends on the selection of (j_1, j_2) . We need to determine this linear region before estimating the slope. Since the traffic is monofractal on large time scales and the Logscale Diagram is also linear in this region, the selection of j_1 and j_2 within this region will not affect the result of α_{11} . For the selection of the linear region at small time scales, we use an Exponential ON/OFF process to verify our selection, with the details of this technique provided in [7]. Fig.1 and Fig.3 show the slope estimated from the Logscale Diagram via these algorithms.

The parameter K_{11} is chosen to match the mean TCP session ON time of the real HTTP trace. The mean of TCP session ON time (T_m) is easily obtained by estimating the time interval between the SYN packet and FIN packet within the same session. From the definition of Pareto distribution, we have

$$K_{11} := T_m \frac{a_{11} - 1}{a_{11}}.$$

Unlike the estimation of mean session ON time, there is no packet in the real trace to indicate the beginning or the end of each burst. The parameter K_{21} of T_{21} is estimated by the normalized autocorrelation function $a(t)$ of the real traffic for values of t very close to zero. Let X_i be the number of bytes of the real trace transmitted in the interval $[(i - 1/2)\Delta, (i + 1/2)\Delta)$, where Δ is the minimum time resolution, chosen to be 1msec in this paper. Let $W(t)$ be the stationary ON/OFF process of the second level with the magnitude B . The autocorrelation function is

$$\begin{aligned} A(t) &:= E[W(0)W(t)] \\ &= B^2 Pr[W(t) = 1 | W(0) = 1] Pr[W(0) = 1]. \end{aligned}$$

Letting $\pi_{11}(t) := Pr[W(t) = 1 | W(0) = 1]$, we have

$$A(t) = B^2 \frac{ET_{21}}{ET_{21} + ET_{20}} \pi_{11}(t).$$

The normalized autocorrelation function is

$$a(t) := \frac{A(t)}{A(0)} = \pi_{11}(t).$$

According to renewal theory in [2] [8], the renewal equation of $\pi_{11}(t)$ is

$$\pi_{11}(t) = G_{1c}(t) + \int_0^t F_{1c}(t-u) dH_{12}(u)$$

where $G_{1c}(t) = Pr[\text{residual life of the first ON interval} > t \mid \text{at time 0 is ON}]$. Since we are interested in the behavior of $a(t)$ around t close to zero, we have the approximation

$$a(t) = \pi_{11}(t) \approx G_{1c}(t).$$

Since the active time T_{21} has a *Pareto* distribution, the complement CDF of the residual life of the first ON period is

$$\begin{aligned} G_{1c}(t) &= \frac{1}{ET_{21}} \int_t^\infty \left(\frac{K_{21}}{u}\right)^{\alpha_{21}} du \\ &= \frac{K^{\alpha_{21}-1}}{\alpha_{21}} t^{-\alpha_{21}+1}. \end{aligned}$$

By estimating the normalized autocorrelation function at the first lag from the trace, the estimator of K_{21} is

$$\begin{aligned} \hat{a}(\Delta) &:= \frac{\sum_{i=1}^M X_i X_{i+1}}{\sum_{i=1}^M X_i^2} \\ K_{21} &:= \Delta(\alpha_{21} \hat{a}(\Delta))^{1/(\alpha_{21}-1)}. \end{aligned}$$

To estimate the parameter $1/\lambda_{20}$, or equivalently the mean inactive period, we need to measure the network round-trip time from the trace. It can be extracted from the real trace by the duration between the SYN packet and the SYN-ACK packet at the beginning of each TCP session. In our model, the mean round-trip time R_m is equal to the sum of the mean active time and the mean inactive time, i.e.:

$$1/\lambda_{20} = R_m - \frac{\alpha_{21} K_{21}}{\alpha_{21} - 1}.$$

The parameter B is the constant data rate in the active period T_{21} . The mean rate M_t and the variance V_t of a connection are:

$$\begin{aligned} M_t &= B R_1 R_2 \\ V_t &= B^2 R_1 R_2 (1 - R_1 R_2) \end{aligned}$$

where $R_1 := \frac{ET_{11}}{ET_{11}+ET_{10}}$ and $R_2 := \frac{ET_{21}}{ET_{21}+ET_{20}}$. Assuming $R_1 R_2 \ll 1$, we have $V_t \approx B^2 R_1 R_2$. With the independence assumption of the connections, B is obtained by the Fano factor [1] of the real trace.

$$B \approx \frac{V_t}{M_t} = \frac{N V_t}{N M_t} = \frac{Var(X_i)}{Mean(X_i)}$$

The number of connections N and the ratio R_1 are chosen by matching the mean rate (time average) of the real trace, over all intervals using

$$Mean(X_i) = N R_1 R_2 B.$$

We have one degree of freedom to choose N and R_1 . In order to satisfy the assumption of $R_1 R_2 \ll 1$, we can pick a large integer for N and then the mean off time $1/\lambda_{10}$ is determined by R_1 at the same time. Since T_{10} is an Exponential random variable and $ET_{10} \gg ET_{11}$, the starting time of each TCP session can be approximated by a Poisson process.

IV. COMPARISON USING WAVELET ANALYSIS

We compare our synthesized traffic with the real HTTP traffic, which was collected from a gateway of the DirecPC¹ system on Oct 13 1999 17:00-18:00. We extracted all the HTTP traffic by the sender's port number. The

¹ DirecPC is a product of Hughes Network System

mean round-trip time and the mean TCP session time are $0.130sec$ and $4.896sec$ respectively. It has the mean rate $513.98Bytes/msec$ and the variance $9.8990e05Bytes^2/msec$. The normalized autocorrelation function $a(1msec)$ is 0.3519 . The shape parameters α_{11} and α_{21} are estimated by the slopes of the Logscale Diagram at the small scale and large scale regions shown in Figure 1 and 3. The following table provides the corresponding parameters of this model.

Para.	K_{11}	α_{11}	$1/\lambda_{10}$	B
Value	1.27sec	1.35	167.55sec	1926B/ms
Para.	K_{21}	α_{21}	$1/\lambda_{20}$	N
Value	0.54ms	1.77	128.75ms	1000

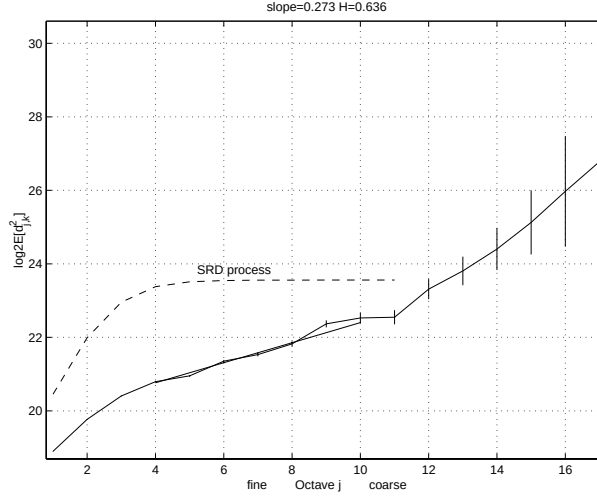


Fig. 3. Logscale Diagram of Real Trace and estimated slope in region (4,10)

A. Second Order Statistics

We apply the discrete wavelet transform to analyze the real and synthesized traffic. The mathematical properties of wavelet transform and its advantages are beyond the scope of this paper [6] [3][4]. The second order analysis of the traffic is obtained by studying the detail process of wavelet transforms $d_{j,k}$. As mentioned before, the X_i , $i = 1, 2, \dots$ is the time series of total transmitted bytes during the interval $[(i - 1/2)\Delta, (i + 1/2)\Delta]$. In order to avoid the estimation error from the deterministic trend, the mother wavelet of the discrete wavelet decomposition is chosen to be the *Daubechies* wavelet with 5 vanishing moments. The Logscale Diagram in Fig.4 is the energy of the detail process $\log_2 E[d_{j,k}^2]$ v.s. the octave j of the real traffic and synthesized traffic. It shows that the second order statistics of these two traffic traces have almost the same values on every scale. This matching implies their similar autocorrelation structure in time. There is a breaking point around the scale $j = 11$ ($2^{11}\Delta \approx 2sec$) related to the minimum value of T_{21} ($=K_{21}$ in the model). When the observing time scale is less than K_{21} , the behavior of traffic is dominated by the second level ON/OFF process or equivalently by the TCP congestion control mechanism. This figure shows that TCP dynamics of HTTP traffic can be modeled well in second order behavior by a simple open-loop ON/OFF process. On the right-hand side of the breaking point, the behaviors of the real and synthesized traces are both monofractal with the same Hurst parameter ($H \approx 0.823$). Note that the slope m and the *Hurst* parameter [1] have the relation $m = 2H - 1$.

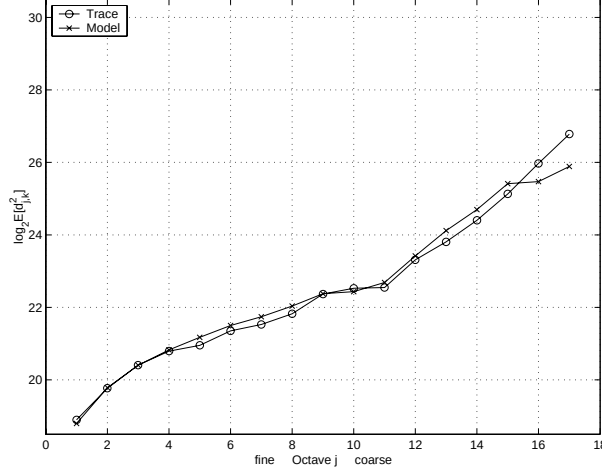


Fig. 4. Logscale Diagram of Real Trace and Synthesized Traffic

B. Higher Order Statistics

The higher order statistics are obtained by the structure function $S(q, j)$ and the partition function $T(q)$ defined in [6],

$$S(q, j) := \sum_{k=1}^{2^{(L-j)}} \|2^{-(L-j)/2} d_{j,k}\|^q$$

where $L := \log_2(\text{Data Length})$. $T(q)$ is approximated by the slope of $\log_2 S(q, j)$ when j is small. The multifractal spectrum $f(\alpha)$ is the Legendre Transform of $T(q)$:

$$f(\alpha) := \inf_q (q\alpha - T(q)).$$

The multifractal spectrum $f(\alpha)$ provides a measure of the “frequency” of the singularity exponent $\alpha(t)$ at time t . It indicates the probability of a certain value of the singularity exponent:

$$Pr[\alpha(t) = \alpha] \approx 2^{-L(1-f(\alpha))}$$

For a monofractal process, like the fractional Gaussian noise (FGN, the increment of fractional Brownian motion), its singularity exponent $\alpha(t)$ is a constant H for every t ; this might be considered as a degenerate case of multifractality. The corresponding partition function $T(q) = qH - 1$ is a linear function of q . Since the $\alpha(t)$ is equal to H for every t in FGN, its multifractal spectrum should be a single point at $(H, 1)$. We will use the FGN as the pilot process and compare the multifractal spectrum with the real and synthesized traffic. For a multifractal process, the partition function is a concave function of q and the singularity exponent $\alpha(t)$ has a wide range of values. In other words, there is a non-negligible probability that $\alpha(t)$ is equal to some specific value. Figure 5 shows the partition functions of the real trace, synthesized trace and the FGN. The concave curves of partition functions show that the real traffic and synthesized traffic are multifractal processes and the partition function of FGN is very close to a linear function due to its monofractal behavior. It is more clear to see the difference in their multifractal spectra in Fig. 6. The spectrum of FGN shows that the probability $Pr[\alpha(t) = H] \approx 1$. For real and synthesized traffic, their spectra show a rich variety of singularity exponents with a non-negligible probability. Moreover, the spectrum of our model shows not only the multifractal property but also the same shape with the spectrum of the real traffic.

V. QUEUING BEHAVIOR

After comparing the statistical properties of the real and synthesized traffic, we are also interested in their queuing behaviors. We consider a simple first come first serve queuing system (FIFO) with fixed service rate and infinite buffer

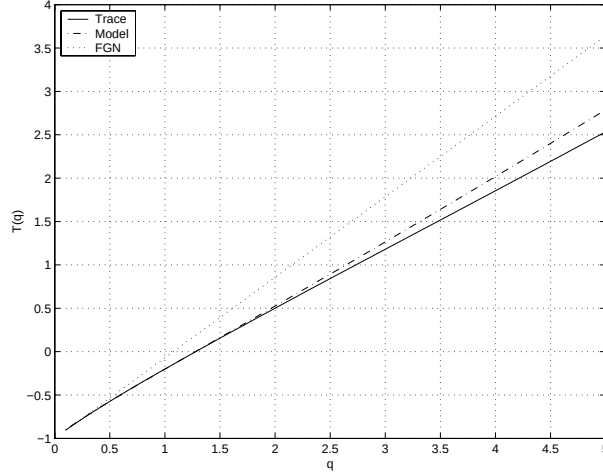


Fig. 5. Partition Function $T(q)$ of Real Trace, Synthesized Traffic and FGN

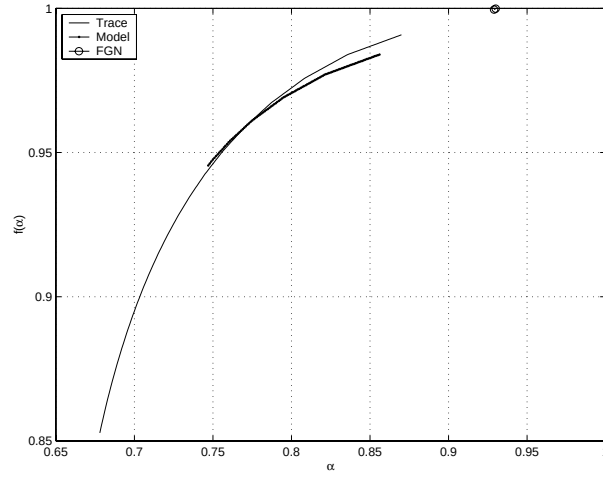


Fig. 6. Multifractal Spectra of Real Trace, Synthesized Traffic and FGN

size. The queue length distribution with different utilization is obtained by properly adjusting the service rate. Figure 7 shows the steady state queue length tail distribution $Pr[Q > b]$ with various levels of utilization $\rho = 0.6, 0.7, 0.8$ and 0.9 . When the traffic load is heavy, the real and synthesized traffic have almost the same distribution. Under light traffic load, the synthesized traffic also provides a good prediction for the queue length distribution when the queue length is less than 50K bytes. In the region of large queue lengths, the tail queue length distribution is overestimated. However, this event happens with a small probability due to the light traffic load.

VI. CONCLUSIONS

The wavelet analysis can provide a multi-resolution "lens" for traffic analysis. When we observe the traffic at large time scales, we are blind to the behavior of protocol, congestion, and network dynamics. The traditional single ON/OFF model is good enough to explain its monofractal phenomenon at large time scales. However, when we go deep into the smaller time scales, which are smaller than the average round-trip time, the constant rate assumption in the single ON/OFF model does not hold anymore. The traffic behavior in this region is strongly dependent on the round-trip time and the active time of bursts. In order to investigate the multifractal behavior at these small time scales, we suggested an open-loop traffic model with a multilevel ON/OFF structure. Based on the model parameters we estimated from a real trace, a synthesized trace was generated with fixed packet size. By using the wavelet analysis,

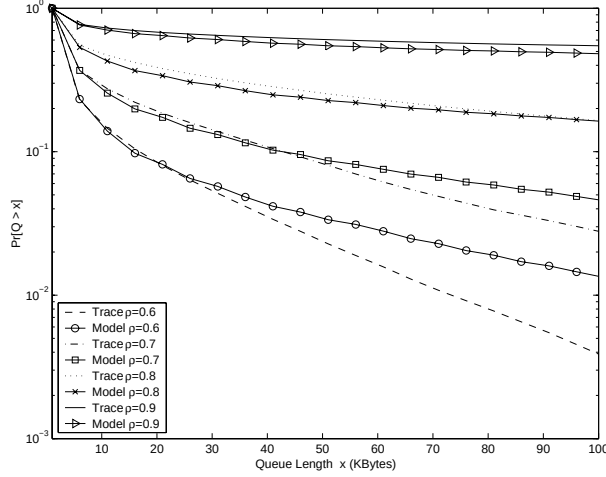


Fig. 7. Queue Length Tail distribution of Real Trace and Synthesized Traffic with Utilization $\rho = 0.6, 0.7, 0.8$ and 0.9

we demonstrated their similar behaviors in second order and higher order statistics, then compared their queue length distributions in queues with infinite buffer size. Our results suggest that the HTTP traffic might be simulated well by an open-loop traffic generator with a multilevel ON/OFF structure. Since we employ network parameters such as active time of bursts, round-trip times as the model parameters, it would be very helpful to understand the impact of the various network parameters on the statistical behavior of HTTP traffic and on the corresponding network performance.

APPENDIX

Here we verify our selection of linear region at small scales by a single level Exponential ON/OFF process $W(t)$. Let the Exponential random variables T_{21} and T_{20} be the duration of ON and OFF with mean $1/\lambda_{21}$ and $1/\lambda_{20}$ respectively. According to [8], the autocorrelation and the power spectrum density are

$$r(t) = \frac{B^2}{\lambda(1/\lambda_{21} + 1/\lambda_{20})} e^{-\lambda t} \quad (1)$$

$$S(\omega) = \int_{-\infty}^{\infty} r(t) e^{-j\omega t} dt = \frac{2B^2}{(1/\lambda_{21} + 1/\lambda_{20})(\omega^2 + \lambda^2)}. \quad (2)$$

where $\lambda = \lambda_{21} + \lambda_{20}$. Since the mother wavelet is a bandpass function, with [1] we have the approximation of $E[d_{j,k}^2]$ by assuming that $\Psi(\omega)$ is an ideal bandpass function:

$$E[d_{j,k}^2] \approx 2 \int_{\pi/2^j}^{\pi/2^{j-1}} S(\omega) 2^j \|\Psi_0(2^j \omega)\|^2 d\omega \quad (3)$$

$$\approx \frac{2^{j+2} B^2}{\lambda(1/\lambda_{21} + 1/\lambda_{20})} \left(\arctan \frac{\pi/\lambda}{2^{j-1}} - \arctan \frac{\pi/\lambda}{2^j} \right) \quad (4)$$

Since we have N connections and the ON/OFF ratio of the first level R_1 , the Logscale diagram of this short range dependent process is

$$\log_2 E[d_{j,k}^2] \approx \log_2 N R_1 + \log_2 \left\{ \frac{2^{j+2} B^2}{\lambda(1/\lambda_{21} + 1/\lambda_{20})} \left(\arctan \frac{\pi/\lambda}{2^{j-1}} - \arctan \frac{\pi/\lambda}{2^j} \right) \right\} \quad (5)$$

The Logscale diagram of this short range dependent ON/OFF process is also shown in Fig.(3) with the same mean ON time and OFF time estimated in section 4. It is clear that the short range dependence dominates the nonlinear

region on the time scales which are less than 16 msec (octave $j = 4$). The zero slope of the Logscale diagram indicates the absence of correlation on large time scales. In order to estimate the parameter α_{21} in the model without the bias from the effect of the short range dependence, Fig.3 shows that we can choose the region between the end of short range dependence and the beginning of another linear region of the upper level.

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