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13 ABSTRACT (Maximum 200 words)

This report summarizes the work completed by the Principle Investigator in several subfields of Reliability Theory proposed for study under the support of the aforenamed ARO grant. Research advances in four specific areas are reported, and twelve research papers are cited for details. In structural reliability, new and useful properties of system signatures are developed. Examples include necessary and sufficient conditions on the signatures of two competing systems for the respective system lifetimes to be ordered. Extensions of these results allow one determine the precise number and location of the time points at which two survival functions or failure rates cross, thus making it possible to determine specific intervals of time in which one system performs better than another. In two referenced papers, new results are obtained on estimating survival and related functions in non-standard circumstances: (1) the problem of estimating the cumulative incidence functions subject to a form of stochastic ordering and (2) problems in which only autopsy data (for example, data on the survival or failure of the welded steel bars in a collapsed structure) are available for the development of inference about the underlying distributions of material strength Y and the stress X to which the material is subjected. Two research papers present solutions to problems in Reliability Economics where the aim is to resolve the tension between the performance of systems of interest and their cost. We focus on coherent systems in n independent, identically distributed components ($\sim$ F) and mixtures thereof. For a given family of criterion functions, exact, closed-form optimality results are obtained for systems of fixed but arbitrary order n. Statistically based approximations are developed and justified when the component lifetime distribution F is unknown. In two papers on Bayesian inference, we study (1) the class of prior distributions for Bayes esimators of a normal mean that provide better performance than the celebrated shrinkage estimator of James and Stein and (2) the notion of self consistency in the context of Bayes estimation. In the latter study, a prior distribution (or Bayes estimator) is said to be self consistent if the equation $E(\theta \mid T = E(\theta)) = E(\theta)$ is satisfied, where T is assumed to be a sufficient and unbiased estimator of θ. Characterization results for families of self-consistent priors are obtained. The concept of conjugacy is broadened substantially and the "threshold problem" considered by Samaniego and Reneau (JASA, 1994)) is reexamined in a wider context by identifying conditions under which the Bayes estimators relative to priors belonging to a broad conjugate and self-consistent family outperform classical procedures.

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FINAL REPORT

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With the support of ARO Contract # DAAD 19-02-1-0377, the Principal Investigator pursued research problems in four distinct areas of reliability modeling and inference. The progress made on these problems is presented in the twelve papers references below. We briefly describe here the main results obtained.

I. Structural Reliability.

Samaniego (IEEE-TR, 1985) introduced the notion of the "signature" of a coherent system in n components with i.i.d. lifetimes. In brief, the signature of the system is a probability vector whose ith element is equal to the probability that the system fails upon the failure of the ith component failure. In papers [2], [3], [4], [6] and [11], new and useful properties of signatures are developed. For example, in [6], necessary and sufficient conditions are obtained on the signatures of two competing systems for the respective system lifetimes to be ordered in a specific sense (stochastic, hazard-rate or likelihood-ratio ordering). These results extend to the more general problem of determining the precise number and location of the points in time at which two survival functions or failure rates cross, thus making it possible to determine specific intervals of time in which one system performs better than another. In [2], the exact relationship between domination theory and signatures is identified, making it possible to exploit simultaneously the computational advantages of signed dominations and the utility of signatures in the comparisons of systems and networks. The relevance and utility and interpretability of signatures in the comparison of consecutive k-out-of-n systems is demonstrated in [4], and new results on the comparative limiting behavior of survival curves and failure rate functions are obtained in [11]. A broad overview of the applicability in signatures in reliability is given in [3], a paper which also contains new developments on the relationship of system signatures to the ordering of the expected lifetimes of two systems.

II. Statistical Inference in Reliability.

In the papers [1] and [8] referenced below, new results are obtained on estimating survival and related functions in non-standard circumstances. In [1], the problem of estimating the cumulative incidence functions corresponding to two competing failure modes is considered under the assumption that the CIFs are subject to a form of stochastic ordering. Consistency and weak convergence are established for the proposed estimators. The method is illustrated on real data. The problem studied in [8] is motivated from engineering applications in which only autopsy data (for example, data on the survival or failure of the welded steel bars in a collapsed structure) are available for the development of inference about the underlying distributions of material strength Y and the stress X to which the material is subjected. In a specific modeling scenario, both classical and Bayes estimates of model parameters are developed and compared. The feasibility of drawing inferences from imperfect (yet still informative) data available at the autopsy stage is demonstrated through a detailed treatment of the exponential case.

III. Reliability Economics.

Reliability Economics is a field that can be defined as the collection of problems in which there is tension between the performance of systems of interest and their cost. Given such a problem, the aim is to resolve the tension through an optimization process that identifies the system that maximizes some appropriate criterion function (e.g., expected lifetime per unit cost). In [7], a paper that was presented at the 10th Army Conference on Applied Statistics, we describe the problem of finding the optimal system design relative to a class of criterion functions that may be viewed as generalizations of the notion of "performance per unit cost", and we outline its solution. We focus on coherent systems in n independent, identically distributed components and mixtures thereof. A system's performance and cost are represented as functions of the system's signature vector, the expected order statistics of a sample of size n from a common underlying distribution F and the costs associated with k-out-of-n systems. For a given family of criterion functions, exact, closed-form optimality results are obtained for systems of fixed but arbitrary order n. Statistically-based approximations are developed and justified when the component lifetime distribution F is unknown. Assuming the availability of auxiliary life tests on a sample of N components, the asymptotic theory of L-estimators is adapted for the purpose of proving the consistency and asymptotic normality (as N tends to infinity) of our estimators of the expected ordered failure times of the n components of the systems under study. These results lead to the identification of epsilon-optimal systems relative to the criterion function used. In [8], the mathematical derivations of the optimization results and of the asymptotic theory for the proposed estimators are presented in detail.

IV. Bayesian Inference.

While our work on the Bayesian approach to estimation has direct applicability to reliability (as evidenced, for example, in [8]), the work presented in papers [5] and [10] cited below have more general importance and applicability. In [5], under specific modeling assumptions, we characterize the class of prior distributions for Bayes esimators of a normal mean that provide better performance than the celebrated shrinkage estimator of James and Stein. This study sheds light on precisely when one would be well-advised to use Bayesian shrinkage over frequentist shrinkage. In [10], a paper presented at the 11th Army Conference on Applied Statistics, the notion of self consistency is defined in the context of Bayes estimation, relative to squared error loss, of a parameter θ of an exponential family of distributions. In this setting, a prior distribution (or Bayes estimator) is said to be self consistent if the equation $E(\theta \mid T = E(\theta)) = E(\theta)$ is satisfied, where T is assumed to be a sufficient and unbiased estimator of θ (usually, the MVUE). This condition simply states that if your experimental outcome agrees with your prior opinion about θ, then the experiment should not change your opinion about it. Surprisingly, many prior distributions, including both “objective” and proper priors, do not enjoy this property. Characterization results for families of self-consistent priors are obtained. The concept of conjugacy is broadened quite substantially by relaxing the exponentiality condition imposed on the prior model by Diaconis and Ylvisaker (Annals of Statistics,1979). The paper concludes by reexamining the “threshold problem” considered by Samaniego and Reneau (JASA, 1994)) in a wider context by identifying conditions under which the Bayes estimators relative to priors belonging to a broad conjugate and self-consistent family outperform classical procedures.

V. Papers written under the support of ARO Contract DAAD 19-02-1-0377.

[1] “Estimation of Cumulative Incidence Functions in Competing Risks Studies Under an Order Restriction”, Journal of Statistical Planning and Inference, Vol 118 (2003), 145-165 (with H. El Barmi, S. Kochar, S. and H. Mukerjee)

[2] “Linking Dominations and Signatures in Network Reliability Theory”, in Mathematical and Statistical Methods in Reliability, B. Lindquist and K. Doksum, Editors, (2003) pp. 89 – 103, Singapore: World Scientific (with P. Boland and E. Vestrup)

[3] "The Signature of a Coherent System and its Applications in Reliability" in Mathematical Reliability Theory: An Expository Perspective, Soyer R., Mazzuchi, T. and Singpurwalla, N. (Editors), 1 – 30 (2004) Boston: Klewer Academic Publishers, (with P. Boland)

[4] "Stochastic Ordering Results for Consecutive k-out-of-n:F Systems", IEEE Transactions in Reliability, 53 (2004), 7 - 10 (with P. Boland)

[5] "Bayes versus Frequentist Shrinkage in Multivariate Normal Problems," Sankhya, 66, 109 – 139, (2004) (with E. Vestrup).

[6] "Characterizations of the Relative Behavior of Two Coherent Systems via Properties of Their Signature Vectors", Proceedings of the International Conference on Distribution Theory, Order Statistics and Inference, 1 – 13, (2004) and Chapter 18, Advances in Distribution Theory, Order Statistics and Inference, N. Balakrishnan, Editor-In-Chief, Boston: Birkhauser, to appear (with H. Block and M. Dugas)

[7] On Optimal System Design under Reliability and Economic Constraints, Proceedings: the 10th Army Conference on Applied Statistics (2005) (with M. Dugas)

[8] "Estimation based on Autopsy Data from Stress-Strength Experiments", Journal of Quality Technology and Quality Management, Special Issue on Reliability, to appear

[9] "Life Testing in a Weibull Environment", in The Weibull Distribution: Theory, Methods and Applications, Balakrishnan and Basu, A. (Editors), to appear (with Y. S. Chong)

[10] "On Conjugacy and Self Consistency in Bayesian Inference", Proceedings: the 11th Army Conference on Applied Statistics, to appear (2006)

[11] "Signature-related Results on System Failure Rates and Lifetimes", submitted for publication (with H. Block and M. Dugas)

[12] "On Optimal System Design in Reliability-Economics Frameworks", submitted for publication (with M. Dugas)