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Using Apparent Contrast as a Surf Zone Index

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Using Apparent Contrast as a Surf Zone Index

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Introduction and Overview

This document gives a logical outline of how the concept of Apparent Contrast can be adapted to serve as a Surf Zone Index (SZI). The resulting SZI may then be used to assess the potential mine detection performance of an arbitrary Electro-Optical Sensor in an arbitrary Surf Zone. This paper is intended to be the first in a series of papers that will get increasingly more sophisticated moving from irradiance (the simplest construct) in this paper to radiance and time-dependent constructs in later reports. This paper sketches the physical processes affecting light propagation in the Surf Zone and then shows how they can be mathematically described and combined into an apparent contrast-based Surf Zone Index.

The topics are covered in the following order:

1. A review of Contrast and Contrast Transmittance as used by the old Scripps Institution of Oceanography (SIO) Visibility Laboratory per Preisendorfer (1976)

2. Application of this approach to develop a SZI for a passive optical image (as opposed to an active or laser device) detector for the special case of bottom mines in the Surf Zone and how it affects Probability of Detection and False Alarm Rate

3. Some discussion of how this first simple introduction of SZI can be improved

Historical Contrast Studies (Preisendorfer, 1976)

S. Q. Duntley, the founder of the SIO Visibility Laboratory, extensively used the concept of Contrast to quantify the perception capability of the human eye. He used this concept in the famous Tiffany Experiments of World War II to determine ways of detecting and camouflaging aircraft in day and night operations and later during the Cold War for submarine and antisubmarine (ASW) operations and diver visibility. The concept is based on Fechner's law from psychophysics, which states that a human will detect a change in a signal in a background when that signal exceeds some threshold difference above or below the background level sensed. This law apparently works for human vision, hearing, and touch. For the human eye, this threshold is nominally 2% for photoptic vision with the cones.

(There is a concept from television engineering called the Contrast Modulation Index, which is defined as the difference between the maximum and minimum signal divided by the sum of the maximum and minimum signal. This concept is good for television and related displays but should not be mistaken for visibility contrast as will be defined.)

Classical Contrast using Radiances or Power/Steradian/Area

The classic setting is an observer viewing a target against a background through a medium of arbitrary turbidity. The scattering of light from the turbid medium produces "space light" or $L^* = L^*(z)$, forcing the observer to see through a veiling light field like a fog. This "fog" increases with range, z, to the target and/or turbidity. This case describes underwater or through the atmosphere viewing. To simply introduce the relevant concepts, only solar lighting and horizontal lines of sight will be considered. (The lighting could be artificial, in which case, the definition gets more complicated.)

Let’s introduce the concept of Inherent Contrast. The *Inherent* radiance from the target at a distance of z = 0 is L_{t0} and the *Inherent* radiance from the adjacent background at the same distance is L_{b0}. Then, the *Inherent Contrast* of the target against the background is defined as

$$C_0 \equiv \frac{L_{t0} - L_{b0}}{L_{b0}}. \quad (1)$$

As z = 0 between the observer and the target, there is no veiling space light or L*(0) = 0. Now when the observer is a distance, z, from the target, the *Apparent* target radiance seen by the observer is

$$L_{tz} = L_{t0} e^{-cz} + L*, \quad (2)$$

where L_{t0} exp (-cz) is the Inherent radiance from the target attenuated by the beam or volume attenuation coefficient, c, and L*(z) is the space light in the space between the observer and the target. Similarly, the *Apparent* background radiance is

$$L_{bz} = L_{b0} e^{-cz} + L*. \quad (3)$$

(Note: The optical property c is used because the problem generally treated dealt with the visual image of a target. If a large object was to be seen with no regard for imaging detail, then k, the diffuse imaging coefficient, could replace c.) Now the *Apparent Contrast* of the target as seen by the observer at a distance, z, from the target, has the same form as Inherent Contrast and is

$$C_z \equiv \frac{L_{tz} - L_{bz}}{L_{bz}} = \frac{(L_{t0} e^{-cz} + L*) - (L_{b0} e^{-cz} + L*)}{L_{b0} e^{-cz} + L*}. \quad (4)$$

Doing a little algebraic rearranging, the Apparent Contrast becomes

$$C_z \equiv \frac{L_{t0} - L_{b0}}{L_{b0}(1 + L* e^{cz} / L_{b0})} = C_0 \frac{1}{(1 + L* e^{cz} / L_{b0})} \equiv C_0 Y, \quad (5)$$

which has become the original Inherent Contrast multiplied by a factor, Y. In turn, Y is defined as the Contrast Transmittance Function. Explicitly,

$$Y \equiv (1 + L* e^{cz} / L_{b0})^{-1}. \quad (6)$$

Note physically how this function works. First, either because the distance, z, increases or because scattering in the medium increases, space light increases. This increase, in turn, causes the Contrast Transmittance, Y, to decrease, and less contrast is transmitted. Similarly, if c, the volume attenuation increases, Y also decreases. The utility of Y is obvious. Given the space light, the distance, the beam or volume attenuation coefficient, c, and the background radiance, the contrast transmittance for a given setting may be calculated and then used to find the Apparent Contrast, C_z, at other ranges of an object of Inherent Contrast, C_0.

Toward a Surf Zone Index

Overview of the Important Surf Zone Processes

The ability to detect bottom objects in the Surf Zone using an airborne passive optical sensor largely depends upon three processes active in the region. (In the following atmospheric transmission, effects will be assumed negligible, which may often be the case for a low-flying aircraft.) The first process is the reflectivity and transmissivity of the surface. Surface reflectivity will vary from 0 to 1, depending on the amount of whitecaps, foam, and bubbles present. When the reflectivity is high, i.e., the water surface is covered with white caps, there will be little chance that an optical system can see the bottom. The second process is the clarity of the water. The following will use two measures of water clarity. Light coming through the surface and irradiating a target on the bottom will be attenuated by the diffuse attenuation coefficient, k. Light reflected from the target and from the background, which returns to the imaging receiver's detector plane, will be attenuated by c, the volume or beam attenuation coefficient. The depth of the bottom, z_B, will multiply the c and k coefficients in the exponential attenuation law. The third process is the inherent reflectivity contrast of the target against the bottom background. A target will be undetectable if its reflectivity is very close to that of the reflectivity of the bottom. Biological growth may cause this to occur over time. (Time-dependent detection will be considered in a later paper on mines lying on the bottom.)

General Apparent Contrast for the Surf Zone

The case of a passive optical imaging sensor will be the only one considered in this first treatment. Such a sensor would be the Littoral Airborne Sensor–Hyperspectral (LASH) mine countermeasures (MCM) device or PAR Government Systems' 4-wavelength multispectral imager. This first treatment will consider irradiance, E or Power/Area, as opposed to the radiance, L, of the previous section. (Note that E = L x a solid angle.) An irradiance treatment is sufficient to impart the physics of the method. Later treatments will consider radiance, time dependence, and other more complicated variables. Consider the geometry of figure 1. Here, ρ_T is the reflectivity of the target, ρ_B is the reflectivity of the bottom, z_B is the depth of the bottom or alternately the thickness of the water over the bottom, and E_o is the incident solar and sky irradiance on the surface of the water.

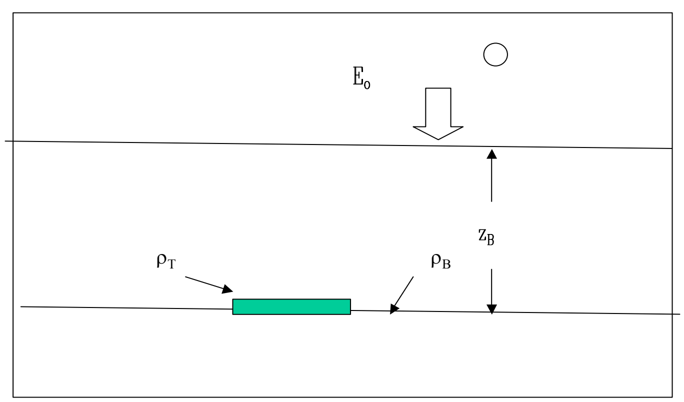

Figure 1. Surf Zone Geometry.

For this geometry, the Inherent Contrast of the target is then given as

$$C_0 \equiv \frac{\rho_T - \rho_B}{\rho_B}.$$

With this Inherent Reflectivity Contrast defined, the Apparent Reflectivity Contrast will now be developed. Now the irradiance reaching the target and the bottom at depth, z_B, passes through the surface and the water and is given by $E_o T_s e^{-kz_B}$, where T_s is the transmittance of the sea surface. Assuming that the sea surface has no emissivity at optical wavelengths, sea surface transmittance is related to the sea surface reflectance, R_s, as $T_s = (1 - R_s)$. When the sea surface is nearly covered with foam and has a high reflectivity, the sea surface transmittance is correspondingly low, with little light getting through. (A more thorough treatment of R_s, the irradiance surface reflectance, including the effects of air/water, whitecap, and foam reflectance, is given in Walker [1994].) This irradiance is incident upon the scatterers in the water volume above the target and produces a veiling space irradiance, simply given as E* = E*(z). (One explicit form of E*(z) will be presented later.) The light strikes the target and the adjacent bottom with irradiance reflectivities of ρ_T and ρ_B respectively. The component of image forming irradiance seen by an imager above the surface is given for the target as

$$E_T' = E_o T_S^2 e^{-(k+c)z_B} \rho_T , \tag{7}$$

and the image forming irradiance from the adjacent bottom seen by the imager is correspondingly given by

$$E_B' = E_o T_S^2 e^{-(k+c)z_B} \rho_B . \tag{8}$$

Now the total irradiance E_T coming from the target and falling on the imager detector is the sum of the image component, E_T', plus that from the veiling water irradiance, $E^*(z_B)$, and that from the sea surface, E_S, i.e.,

$$E_T = E_T' + E^*(z_B) + E_S . \tag{9}$$

Similarly, the corresponding total irradiance, E_B, from the bottom is given as

$$E_B = E_B' + E^*(z_B) + E_S . \tag{10}$$

The Apparent Contrast seen at the imager is

$$C_A = \frac{E_T - E_B}{E_B} . \tag{11}$$

Substituting explicitly for E_T and E_B and introducing the original reflectivity Inherent Contrast, C_0, we get Apparent Contrast in a form implicitly introducing Y, the Contrast Transmittance function, i.e.,

$$C_A = \frac{E_T' - E_B'}{E_B' + E^* + E_S} = \frac{E_o T_S^2 \exp(-[k+c]z_B) \times (\rho_T - \rho_B)}{E_o T_S^2 \exp(-[k+c]z_B) \times \rho_B + E^* + E_S} \equiv \left(\frac{\rho_T - \rho_B}{\rho_B} \right) \times Y \equiv C_o Y . \tag{12}$$

Explicitly,

$$Y = \left(1 + \frac{E^* + E_S}{\rho_B E_o T_S^2 \exp(-[k+c]z_B)}\right)^{-1} . \tag{13}$$

Let's examine what this treatment has accomplished. First, the Apparent Contrast has been factored into two terms. The first term, Inherent Contrast, contains the target effect only. The second term, the Contrast

Transmittance, contains all other factors describing the environment, including lighting, water turbidity, depth, and surface white cap and foam conditions. Both terms include the bottom reflectivity. Thus, the use of apparent contrast neatly separates the environmental conditions that will be unique to each Surf Zone environment from the target reflectivity, which will initially be defined by outside forces. This initial target reflectivity will likely change with time, depending upon the extant local biological and physical conditions present.

Again looking at Y, the Contrast Transmittance, it qualitatively mirrors one's physical intuition for the optical detection of mines in the Surf Zone. Y (and correspondingly C_A) decreases as light reflected from the surface (e.g., whitecaps and foam) increases, as light scattered from the water increases, and as the target and bottom depth increases. The effect of the bottom reflectivity, ρ_B, is best understood by examining its role in the Inherent Contrast. Now the limits of ρ_B are 0 for a perfectly black bottom to 1.0 for a perfectly white 100% reflecting bottom. If a target of reflectivity in the range of 8 to 15% is chosen, then at the extremes of 0 and 100% bottom reflectivity, the inherent target to bottom contrast is respectively $C_0 = (\rho_T - 0)/0 = \infty$ for $\rho_B = 0$ and $C_0 = (\rho_T - 1)/1 \approx -1$ for $\rho_B = 1$. Note the sign change that occurs in inherent contrast when the target and background have similar reflectance values. Detection based on contrast alone would be difficult, if not impossible, in these circumstances.

A Specific Example of Surf Zone Apparent Contrast

Having presented a definition of Surf Zone Apparent Contrast, it is instructive to now employ a simple light transport model to examine this approach in more detail. In the following, a simple irradiance model will be used. Explicit expressions will be given for the space light, E*, for the scattered light from the water volume and for E_S, the light reflected from the sea surface using the irradiance model. The model is a simple variant of models used by Preisendorfer, et al., to gain physical insight into hydrooptical processes before proceeding into more rigorous two-flow irradiance models and radiative transport models (RTE) involving radiance, coherence, polarization, etc. (Note that in the future these more sophisticated models will be used to apply the Surf Zone Apparent Contrast approach to the Rapid Overt Airborne Reconnaissance (ROAR) system itself or others of interest.)

Again, the geometry is as shown in figure 1. As before, the target and bottom components of image forming irradiance seen by an imager above the surface are respectively,

$$E_T^{'} = E_o T_S^2 e^{-(k+c)z_B} \rho_T \text{ and } E_B^{'} = E_o T_S^2 e^{-(k+c)z_B} \rho_B . \tag{14}$$

Similarly, the total irradiance E_T from the target and the bottom falling on the imager detector is the sum of the image component, E_T', plus that from the veiling water irradiance, $E^*(z_B)$, and the sea surface irradiance, E_S, i.e.,

$$E_T = E_T^{'} + E^*(z_B) + E_S \text{, and } E_B = E_B^{'} + E^*(z_B) + E_S , \tag{15}$$

respectively. Now the sea surface irradiance is simply the product of the input sun/sky irradiance times the sea surface irradiance reflectance, or explicitly,

$$E_S = E_o R_S . \tag{16}$$

An explicit form for the water-scattered light can be obtained by assigning a backscatter coefficient of b_b (1/m) to the assumed vertically homogeneous water. This coefficient forms an element of differential backscattering, which is integrated from the surface to the bottom. Finally, the scattered light is transmitted through the air/sea interface on its way to the imager. Thus, the scattered water space light is explicitly,

$$E^* = \frac{T_S^2 E_o b_b}{2k}\left(1 - \exp(-2kz_B)\right). \quad (17)$$

By definition, the Apparent Contrast seen at the imager is

$$C_A = \frac{E_T - E_B}{E_B} = \frac{E_T^' - E_B^'}{E_B^' + E^* + E_S} \equiv \left(\frac{\rho_T - \rho_B}{\rho_B}\right) \times Y \equiv C_o Y\,. \quad (18)$$

Upon substituting the explicit forms for E* and E_S into this expression and simplifying, the Surf Zone Contrast Transmittance, Y, now has the explicit form,

$$Y = \left[1 + \frac{(b_b/2k)}{\rho_B}\left(\exp[(c+k)z_B] - \exp[(c-k)z_B]\right) + \frac{R_S/T_S^2}{\rho_B}\exp[(c+k)z_B]\right]^{-1}. \quad (19)$$

Employing the hyperbolic sine function,

$$Y = \left[1 + \frac{(b_b/k)}{\rho_B}\left(\exp[cz_B]\sinh[kz_B]\right) + \frac{R_S/T_S^2}{\rho_B}\exp[(c+k)z_B]\right]^{-1}. \quad (20)$$

Again, Y, the Contrast Transmittance, qualitatively mirrors physical intuition for the optical detection of mines in the Surf Zone. Consider the surface reflectance term. Note that $R_S + T_S = 1$, i.e., when the surface is clear of white caps, foam, etc., $R_S \sim 0$, most incident light is transmitted through the surface, and $R_S/T_S^2 = 0$. The surface does not affect the imaging light. When the surface is covered with white caps, etc., little if any light gets through, $R_S \sim 1$, most incident light does not penetrate the surface, and $R_S/T_S^2 = \infty$. Correspondingly, Y and C_A go to 0. The water term shows that Y and C_A decrease with increasing water turbidity—c, k, and b_b—and with the depth of the bottom. The effect of the bottom reflectivity, ρ_B, is simply that Y and C_A increase over a highly reflecting bottom, and decrease over one of low reflectance.

Thus, for this simple model, contrast transmittance and apparent contrast again give a physically intuitive description of the Surf Zone optics. The use of more sophisticated light models may be used to further refine this approach.

Using Surf Zone Apparent Contrast to Determine False Alarm Rates and Probability of Detection

The Detection Signal-to-Noise Ratio (SNR) and the resulting Probability of Detection, P_D, and False Alarm, P_{FA}, are the measures of primary importance in evaluating the performance potential of an Airborne Optical System to detect mines in the Surf Zone. The following shows, *in principle*, how the foregoing SZI based on Apparent Contrast will be used to relate the Surf Zone parameters of wave heights, breaking waves and white water, water clarity, and bottom reflectivity and the target reflectivity to SNR, P_D, and P_{FA}.

In the following, the most fundamental definition of SNR will be used to demonstrate the utility of the method. Thus detection system parameters such as quantum efficiency, field of view, integration time, collection area, etc. will be omitted for this treatment. (Note that these system parameters can easily be included, when a given system is to be evaluated in an arbitrary Surf Zone environment.) SNR will be defined *in the most fundamental manner* as the ratio of signal energy to noise energy coming from the target and background and impinging on the detector surface. To make a smooth transition to detection theory, the square root of SNR will be employed. The signal, i.e., the increase and decrease of target irradiance above or below the background, E_B, is defined as $\Delta S_E = E_T^{'} - E_B^{'}$, where E_T' and E_B' have been previously defined. (The signal power is the square of this quantity, but is not used here.) Note that the statistics of a photon counting process are Poisson in nature. One characteristic of a Poisson process is that the variance equals the mean value of a distribution. Thus, in the present case, the noise power is related to the mean of the total background irradiance, E_B, as $\sigma_E^2 = \overline{E}_B$, or $\sigma_B = \overline{E}_B^{1/2}$ is the standard deviation of the noise. Now, by definition, $SNR^{1/2} = \frac{\Delta S}{\sigma}$. Using the relations developed in the previous section, this $SNR^{1/2}$ is expressed in terms of Apparent and Inherent Contrast and Contrast transmittance as

$$SNR^{1/2} = \frac{\Delta S}{\sigma} = \frac{E_T^{'} - E_B^{'}}{\overline{E}_B^{1/2}} = C_A \overline{E}_B^{1/2} = C_0 Y \overline{E}_B^{1/2}. \tag{21}$$

(Note that this treatment is valid for either negative or positive contrasts. The easiest way of handling the signs of contrasts is to use absolute values. This will be demonstrated later when calculations are made.)

It is a short step from here to develop the detection and false alarm probabilities. Let the irradiances with no target present be distributed with a probability density function, $p_{0r}(E)$. Then, using the Neyman–Pearson criterion to find a threshold level, E_{th}, the relation,

$$P_{FA} = \int_{E_{th}}^{\infty} p_0(E)dE \equiv \Phi, \tag{22}$$

is set equal to a desired Probability of False Alarm, Φ, corresponding to a given False Alarm Rate (FAR). E_{TH} is then dithered and the integral evaluated until the integral gives the desired Φ and the appropriate E_{TH}. Similarly, when a target is present, the irradiances are distributed with a probability density function, $p_1(E)$. With the FAR-defined E_{TH}, the Probability of Detection is obtained from this distribution as

$$P_D = \int_{E_{th}}^{\infty} p_1(E)dE. \tag{23}$$

Now, let's look at a specific example of Gaussian-distributed background and background + target irradiances, i.e.,

$$p(E_i)dE_i = \frac{1}{\sqrt{2\pi}\sigma} \exp[-(E_i - \overline{E}_i)^2 / 2\sigma^2]dE, \tag{24}$$

where i = 0 or 1, standing for either background only or target +background cases. These distributions are shown in figure 2.

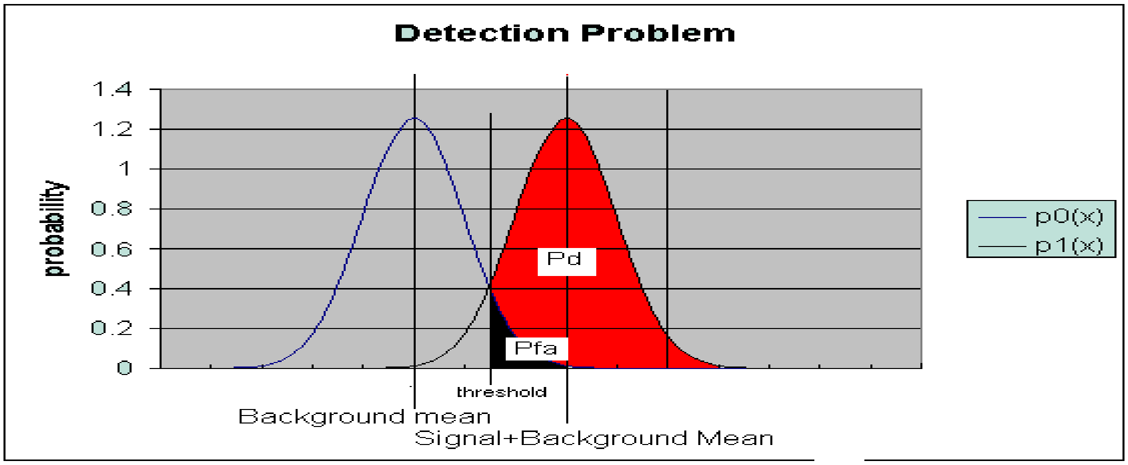

Figure 2. Gaussian distributions for Detection.

This figure shows the Probability of False Alarm as the area of the integral in the p_0 distribution above E_{TH}. Similarly, the Probability of Detection is the area of the integral in the p_1 distribution, also above E_{TH}. The assumptions here are that the addition of the small amount of target irradiance to the background does not significantly affect the variance of the background, thus allowing the two distributions to have the same variance with different means. A trick shown in Whalen in evaluating these integrals for P_{FA} and P_D and explicitly showing their dependence on $SNR^{1/2}$ is to transform them into normal distributions with new limits of integration, depending on whether p_0 or p_1 is evaluated. The normal distribution has zero mean and unity variance and its density function is defined as

$$p_N(u)du = \frac{1}{\sqrt{2\pi}} \exp[-u^2/2]du\,. \tag{25}$$

Consider P_{FA}. The transformation of E into the normal variable u is $u = \frac{E - \overline{E}_B}{\sigma_B}$, where $\overline{E}_B$ is the mean of the previously defined total background irradiance given earlier. Then, the expression for P_{FA} becomes

$$P_{FA} = \frac{1}{\sqrt{2\pi}} \int_{\frac{E_{TH}-\overline{E}_B}{\sigma_B}}^{\infty} \exp[-u^2/2]du = \Phi. \tag{26}$$

Note the normalized threshold limit, $u_{TH} = \frac{E_{TH} - \overline{E}_B}{\sigma_B}$, will be set to achieve the desired probability of false alarm, Φ, a constant, as before.

Similarly, consider the normalized P_D. The difference between the p_0 and the p_1 distributions is only in their mean values, and this difference is important. The mean of the p_1 distribution is the sum of the mean of the p_0 distribution plus the signal, ΔS, i.e.,

$$\overline{E}_{B+T} = \overline{E}_B + \Delta S = \overline{E}_B + (E_T^{'} - E_B^{'}). \tag{27}$$

Here the transformation becomes

$$u = \frac{E - \overline{E}_{B+T}}{\sigma_B} = \frac{E - \overline{E}_B - (E_T^{'} - E_B^{'})}{\sigma_B} = \frac{E - \overline{E}_B}{\sigma_B} - \frac{(E_T^{'} - E_B^{'})}{\sigma_B} \equiv \frac{E - \overline{E}_B}{\sigma_B} - \frac{\Delta S}{\sigma_B}. \tag{28}$$

Now applying this transformation to obtain the normalized integral and to obtain the P_D threshold value, $u_{P_D TH} = \frac{E_{TH} - \overline{E}_{B+T}}{\sigma_B}$, the probability of detection becomes

$$P_D = \frac{1}{\sqrt{2\pi}} \int_{\frac{E_{TH} - \overline{E}_{B+T}}{\sigma_B}}^{\infty} \exp[-u^2/2] du = \frac{1}{\sqrt{2\pi}} \int_{\frac{E_{TH} - \overline{E}_B}{\sigma_B} - \frac{\Delta S}{\sigma_B}}^{\infty} \exp[-u^2/2] du \,. \tag{29}$$

Noting that the term, $\frac{E_{TH} - \overline{E}_B}{\sigma_B} = u_{TH}$, comes from the P_{FA} treatment and is simply the threshold value previously obtained using the Neyman–Pearson criterion and the $P_{FA} = \Phi$ constraint and that the $SNR^{1/2} = \Delta S/\sigma_B$, then the lower limit for the P_D integral is $u_{TH} - SNR^{1/2}$. $SNR^{1/2}$ is related to the Surf Zone Contrast transmittance function, Y, the background irradiance, and the inherent contrast of the target to the bottom previously shown as

$$SNR^{1/2} = \frac{\Delta S}{\sigma} = \frac{E_T^{'} - E_B^{'}}{\overline{E}_B^{1/2}} = C_A \overline{E}_B^{1/2} = C_0 Y \overline{E}_B^{1/2} .. \tag{30}$$

Thus, the P_D expressed in terms of the Surf Zone Index becomes

$$P_D = \frac{1}{\sqrt{2\pi}} \int_{u_{TH} - C_0 Y E_B^{1/2}}^{\infty} \exp[-u^2/2] du \,, \tag{31}$$

with the integral's lower limit being the Neyman–Pearson false alarm threshold minus the square root of the SNR represented as a function of the SZI, Y, i.e., u_{TH} - $C_o Y E_B^{1/2}$, as shown above. Probability of Detection is thus explicitly a function of the SZI.

A Sample Calculation

A rough calculation was performed using the method previously defined to verify its utility in evaluating the performance of a very generic passive (sunlight/skylight illumination only) imaging detector in detecting an object on the bottom in the Surf Zone. An Excel spread sheet was used in this calculation. A target of reflectivity, ρ_T , sat on the bottom of the Surf Zone at a depth of 10 feet or 3.048 m or so. The bottom was given a reflectivity of ρ_W . The nominal value for target reflectivity was 0.12 or 12% and the nominal bottom reflectivity used was 0.30 or 30%. The Inherent Contrast of the target to the bottom is as previously defined $C_0 = (\rho_T - \rho_W)/\rho_W$, which, for this case, was 0.6 or –60%. (One calculation was done for the case of an extremely black target of 0 reflectivity against a lighter bottom. This calculation gave the limiting inherent contrast of –1.0.) The solar/sky irradiation incident on the sea surface taken from Walker (1994) was 1957 watts/m^2/micron at a center wavelength of 530.5 nm. Calculations were done for five values of sea surface reflectivity, i.e., 0, 25, 50, 75, and 100% reflectance corresponding to conditions ranging from no white water, surface foam, and bubbles to complete white water coverage.

Calculations were done for Jerlov (1976) water types ranging from Oceanic II and III through Coastal 1 to 9. Additionally, two special cases were considered based on past and planned Office of Naval Research (ONR) measurement sites. These were Hawaii (HI) and Duck, North Carolina, and were also examined for the target depth dependence, which will not be reported here. Relevant optical properties for the calculations include k, c, and b_b, respectively, the diffuse and beam or volume attenuation coefficients and the back scatter coefficient. Variable k was obtained directly from the Jerlov tables. Coefficient c was approximated by the old Duntley rule of thumb of c = k/3. The backscatter coefficient was obtained from k and c using the Honey–Wells approximation and the Jerlov observation that $b_b = 0.02$ b, i.e., $b_b = 0.02 \times 6/5(c - k)$. The resulting optical properties for $\lambda = 532$ nm are shown in Table 1. Hawaiian waters are considered to be Jerlov Ocean II (perhaps a bit too clear for reality), while Duck is considered to be worse than Jerlov Coastal 9 (also perhaps a little too clear, i.e., actual Duck waters were generally more turbid than shown.)

Table 1. Jerlov Water Type Optical Properties.

Jerlov Type	II/HI	III	1	3	5	7	9	Duck-like
k (1/m)	0.0796	0.1171	0.127	0.1972	0.3072	0.4816	0.738	1
c (1/m)	0.2388	0.3513	0.381	0.5916	0.9216	1.4448	2.214	3
bb (1/m)	0.0038208	0.005621	0.006096	0.009466	0.014746	0.023117	0.035424	0.048

The calculations were initiated by first calculating the specific background irradiance, E_B, for the given background reflectivity value and the in-water scattered irradiance, E*, for each of the water types. Next the surface reflected irradiance, E_S, was calculated for each of the surface reflectivities from 0 to 100% in 25% increments. These irradiances were combined to obtain the contrast transmittance, Y, for the 10-foot depth target and each water type. This Contrast Transmittance was multiplied by the inherent contrast of the target ($C_0 = -0.6$) to obtain the apparent contrast of the target at 10-foot depth. The resulting Contrast Transmittances (or Surf Zone Indices) and the Apparent Contrasts are shown in table 2. Note that Y, the Contrast Transmittance or Surf Zone Index, ranges from 0 to 1 and that the resulting absolute value of the Apparent Contrasts are always less than the absolute value of the Inherent Contrast.

Table 2. Surf Zone Index and Apparent Contrast for Jerlov Water Types.

Water Type	II	III	1	3	5	7	9	Duck-like
Contrast Transmittance aka. Surf Zone Index								
Surface Reflectance								
0%	92.5%	85.5%	83.1%	61.7%	25.9%	3.6%	0.2%	0.0%
25%	20.0%	13.6%	12.2%	5.5%	1.5%	0.2%	0.0%	0.0%
50%	5.4%	3.5%	3.1%	1.3%	0.3%	0.0%	0.0%	0.0%
75%	0.9%	0.6%	0.5%	0.2%	0.1%	0.0%	0.0%	0.0%
100%	0.000	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Apparent Contrast of target of Inherent Contrast = -0.6								
Surface Reflectance								
0%	-0.555	-0.513	-0.499	-0.370	-0.155	-0.022	-0.001	0.000
25%	-0.120	-0.082	-0.073	-0.033	-0.009	-0.001	0.000	0.000
50%	-0.032	-0.021	-0.018	-0.008	-0.002	0.000	0.000	0.000
75%	-0.006	-0.004	-0.003	-0.001	0.000	0.000	0.000	0.000
100%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Figure 3 plots the values for Surf Zone Index, Y, as a function of sea surface reflectivity for each Water Type.

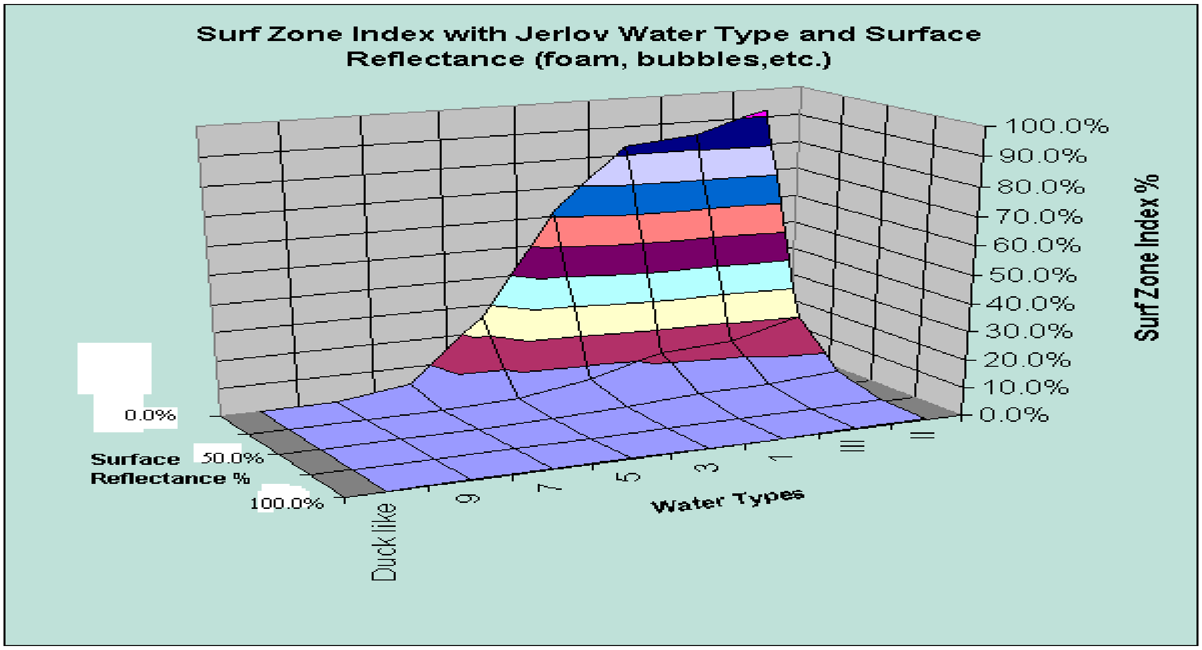

Figure 3. Surf Zone Index, Y, as a function of sea surface reflectivity for each Water Type.

The next step in the study was the calculation of SNRs using the Surf Zone Indices (table 3.) Per the preceding formulas, $SNR^{1/2}$ was calculated by multiplying the absolute value of the apparent contrast by the square root of the background irradiance, i.e.,

$$SNR^{1/2} = |C_A| \overline{E}_B^{1/2} = |C_0| Y \overline{E}_B^{1/2} . \tag{32}$$

Table 3. Sample Square Root of SNR for Jerlov Water Types.

Water Type	II	III	1	3	5	7	9	Duck-like
Surface Reflectance	Square Root of Signal-to-Noise Ratio							
0%	8.606	6.582	6.112	3.434	1.137	0.146	0.006	0.000
25%	3.004	1.970	1.759	0.772	0.205	0.025	0.001	0.000
50%	1.035	0.662	0.587	0.252	0.066	0.008	0.000	0.000
75%	0.217	0.137	0.122	0.052	0.014	0.002	0.000	0.000
100%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

With $SNR^{1/2}$ calculations done, the corresponding probabilities of detection are then evaluated following McDonough and Whalen (1995) by first using the Neyman–Pearson criterion to ascertain the appropriated threshold value to achieve a given False Alarm Probability. Threshold values were found for $P_{FA} = 0.1$, 0.01, 0.001, and 0.0001. With these thresholds and the calculated $SNR^{1/2}$, i.e., $u_{TH} - C_o YE_B{}^{1/2}$, the corresponding Probabilities of Detection are found using the following integral,

$$P_D = \frac{1}{\sqrt{2\pi}} \int_{u_{TH} - C_0 Y E_B^{1/2}}^{\infty} \exp[-u^2/2] du , \tag{33}$$

where SNR has been written in terms of Surf Zone Index, etc. These P_D's are given in table 4 for P_{FA}'s of 0.1 and 0.01.

Table 4. Probability of Detection for two values of P_{FA} for Jerlov Water Types.

Water Type							
	II	III	1	3	5	7	9	Duck-like
Surface Reflectance	Probability of Detection for P_{FA} = 0.1							
0%	100.0%	100.0%	100.0%	99.9%	44.3%	12.8%	10.1%	10.0%
25%	99.3%	83.5%	75.1%	30.6%	14.1%	10.5%	10.0%	10.0%
50%	40.3%	26.8%	24.4%	15.2%	11.2%	10.2%	10.0%	10.0%
75%	14.4%	12.7%	12.3%	11.0%	10.3%	10.1%	10.0%	10.0%
100%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Surface Reflectance	Probability of Detection for P_{FA} = 0.01							
0%	100.0%	100.0%	100.0%	94.2%	11.7%	1.5%	1.0%	1.0%
25%	83.2%	36.1%	28.6%	6.0%	1.7%	1.1%	1.0%	1.0%
50%	9.9%	4.8%	4.1%	1.9%	1.2%	1.0%	1.0%	1.0%
75%	1.7%	1.4%	1.4%	1.2%	1.0%	1.0%	1.0%	1.0%
100%	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%

Figure 4 is a plot of the Detection Probabilities for P_{FA}'s of 0.1 and 0.01 and the corresponding Surf Zone Index, Y, for the water types. The cases plotted are for surface reflectivity = 0, i.e., there are no white caps or foam.

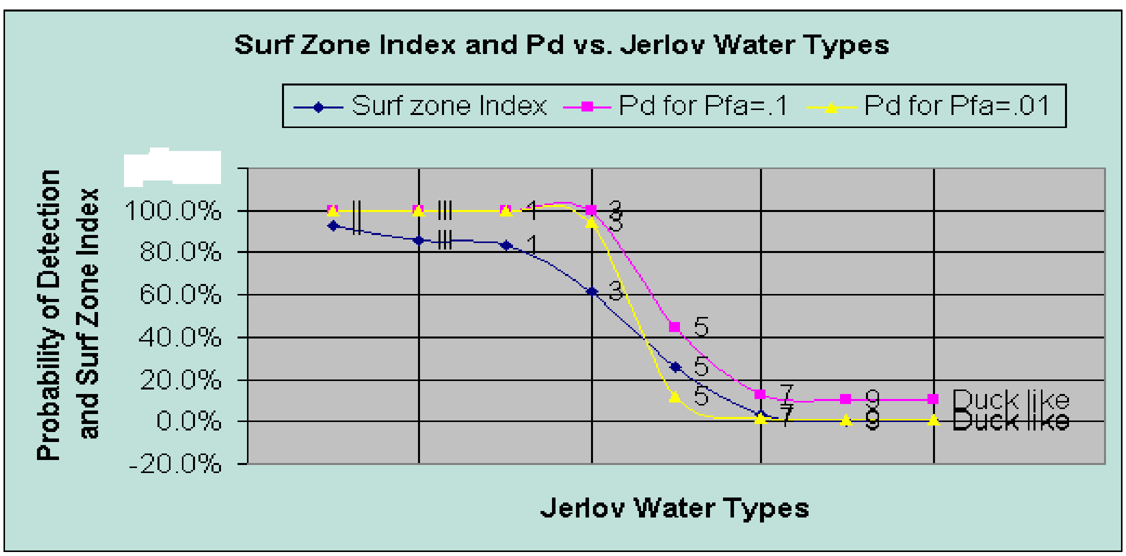

Figure 4. P_D's and Surf Zone Index for Water Types (no surface reflectivity, i.e., no white caps, etc.)

Concluding Remarks and Future Work Suggestions

A Surf Zone Index (Y = SZI) has been developed using an approach based on classical contrast transmittance theory. A simple irradiance treatment of the SZI has been given in this report to demonstrate the principle of the method. This treatment has considered a very generic irradiance-imaging detector.

The SZI treatment developed has several favorable features. First, it separates the effects of the environment from the target. These environmental factors include water turbidity, water depth and wave height, sea surface white caps and foam, bottom reflectivity, and incident light levels. Secondly, because its range is from zero to one, it gives a very intuitively satisfying quantification as a measure of performance capability in a given environment. If SZI is close to zero, system performance will be severely limited. This limitation can be caused by dirty water or extreme surf, etc., but will appear quantitatively in the SZI. The larger the SZI, the better the expected performance. When SZI = 1, the Surf Zone conditions are like looking though clear air. Thirdly, SZI is shown to be directly relatable to SNR and thence to Detection and False Alarm Probabilities. It is thus immediately relevant to a system operator and mission planner. Finally, SZI may be generalized through a treatment as a function of random variables to arrive at confidence bounds and other statistical measures of performance and environmental fluctuations.

Future work will address more complicated passive radiance imagers and Light Detection and Ranging (LIDAR) imaging systems with range gating. More complicated radiative transport models together with imaging system models should be used for this future work. However, the principle that has been illustrated in this document should generally apply to all future treatments.

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