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Error Probability for Direct-Sequence Spread-Spectrum Multiple-Access Communications Over Nonselective and Frequency-Selective Rician Fading Channels

by

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ERROR PROBABILITY FOR DIRECT-SEQUENCE SPREAD-SPECTRUM MULTIPLE-ACCESS COMMUNICATIONS OVER NONSELECTIVE AND FREQUENCY-SELECTIVE RICIAN FADING CHANNELS

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ABSTRACT

An accurate approximation is obtained for the average probability of error in an asynchronous binary direct-sequence spread-spectrum multiple- access communications system operating over nonselective and frequency- selective Rician fading channels. The approximation is based on the integration of the characteristic function of the multiple-access interference which now consists of specular and scatter components. For non-selective fading the amount of computation required to evaluate this approximation grows linearly with the product KN, where K is the number of simultaneous transmitters and N is the number of chips per bit. For frequency-selective fading the computational effort grows linearly with the product KN^2. The resulting probability of error is also compared with an approximation based on the signal-to-noise ratio. Numerical results are presented for specific chip waveforms and signature sequences.

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I. INTRODUCTION

This paper is concerned with the average probability of error for asynchronous binary direct-sequence spread-spectrum multiple-access (DS/SSMA) communications over nonselective and frequency-selective Rician fading channels. The results presented represent a generalization of the analysis of [7] which considered only additive white Gaussian noise (AWGN) channels.

The system model is described in detail in [6]. Throughout this paper we restrict attention to binary DS/SSMA systems with phase-shift-keyed (PSK) modulation and chip waveforms which are pulses of arbitrary shape with duration equal to the inverse of the chip rate. However our results can be extended (using the results of [7]) to include other forms of direct-sequence modulation like quadriphase-shift-keying (QPSK), offset QPSK, and minimum-shift-keying (MSK).

The channel model considered in this paper is the Rician or specular-plus-Rayleigh fading channel model described in [1] for nonselective fading, and in [5] for selective fading. In the second case the fading is assumed to be frequency-selective [3]. Although our results can be extended to time-selective, as well as doubly-selective wide-sense-stationary uncorrelated-scattering (WSSUS) fading channels ([2], [5]), we restrict attention to frequency-selective channels, since these channels often arise in practice and, as shown in [5], single user DS/spread-spectrum systems outperform biphase PSK systems over such channels.

In [5] the average signal-to-noise ratio at the output of the receiver of a binary DS/SSMA system operating over time and frequency-selective Rician fading channels was evaluated. In [4] the moment space bounding

technique of [12] was applied to bound the probability of error of such systems. However the results were limited because of unsurpassable difficulties in evaluating high order moments of the interference. For the same reason, the approximation based on the Gauss Quadrature Rule, which was used in [10] - [11] for DS/SSMA systems over AWGN channels, is not computationally attractive in this case, since it requires calculation of moments which involve high order channel autocovariance functions that may not be measureble or otherwise available. Similarly, the bounds of [13] were based on convexity properties of the complementary error function which are valid only in the AWGN case. Finally, the results for two users of [14] for nonselective fading channels and [15] for time-selective fading channels appear difficult to extend so that they provide an efficient and accurate computational method for the multi-user case.

In this paper we obtain an approximation with a computational requirement which for nonselective fading is linear in the product KN and for frequency-selective fading is linear in the product KN^2, where K is the number of simultaneous transmitters and N is the number of chips per bit. This approximation is based on the integration of the characteristic function of the multiple-access interference component of the output of the correlation receiver. This component consists now of a fixed (or specular) part and a random (or scatter) part. This method, which we refer to as the characteristic-function method, was applied to DS/SSMA systems in [7] and DS/SS systems with specular multipath fading in [9] where it was shown to provide very accurate approximations to the average probability of error. Moreover, as shown in [8], any prespecified degree of accuracy can be achieved by using this approximation to obtain an expansion point for a

Taylor series representation of the actual probability of error. In the selective fading case, however, it is computationally untractable to evaluate the moments of the interference required for the series expansion method, therefore, in the sequel we consider only the characteristic-function method.

We should also point out at this point that the DS/SSMA system under consideration employs conventional matched filter receivers to combat rather than utilize the interference (multiple-access faded and nonfaded components) by discriminating against it. This is in contrast to the work of [17] - [18] where diversity reception is used to combine the contributions of the faded (reflected in those cases) paths and use them in the binary decision.

This paper is organized as follows. Nonselective Rician channels are considered in Section II, and frequency-selective Rician channels are treated in Section III. In each of these sections the fading channel model is first presented. Next the characteristic-function method is applied to each particular model and the computational requirements are considered. Finally a simpler approximation which is based on the average signal-to-noise ratio is cited for the sake of comparison. In Section IV numerical results are presented for specific chip waveforms and signature sequences.

II. PERFORMANCE OF DS/SSMA COMMUNICATIONS OVER NONSELECTIVE RICIAN FADING CHANNELS

A. A System and Channel Model

The system model is described in detail in [6] and [9]. Here we repeat the basic elements of this model so that the concepts and notation which are necessary for the rest of the paper are introduced.

The k-th transmitted signal for a binary DS/SSMA sy;stem with PSK modulation and arbitrary chip waveform can be expressed as

$$s_k(t) = \mathrm{Re}\{x_k(t)\ \exp(j2\pi f_c t)\} \qquad (1)$$

where Re stands for real part, $j = \sqrt{-1}$, f_c is the common carrier frequency, and the DS-spreaded signal $x_k(t)$ is given by

$$x_k(t) = \sqrt{2P}\ b_k(t)\Psi(t)a_k(t)\ \exp(j\theta_k). \qquad (2)$$

In (2) P is the power in each of the K transmitted signals; the equal power assumption is made for convenience in presenting numerical results, it is not necessary for the methods considered in this paper. The phase angle θ_k introduced by the modulator is uniformly distributed in $[0,2\pi]$. The shaping waveform $\Psi(t)$ that appears in (2) is periodic with period T_c (the duration of a chip), it is defined by $\Psi(t) = \psi(s)$ for $s = t(\mathrm{mod}\ T_c)$ where $\psi(s)$ is a chip waveform of arbitrary shape (see [6] and [9]) which satisfies a time-limiting constraint and an average energy constraint. The data signal $b_k(t)$ consists of rectangular pulses of duration T which take values +1 and -1 with equal probability. The ℓ-th pulse has amplitude $b_\ell^{(k)}$. The information sequence $(b_\ell^{(k)})$ is modeled as an i.i.d. sequence. The code waveform $a_k(t)$ is a periodic sequence of rectangular pulses of amplitude +1 or -1 and duration T_c. The j-th code pulse has amplitude $a_j^{(k)}$. We assume that there are N code pulses in each data pulse ($T = NT_c$) and that the period of the signature sequence $(a_j^{(k)})$ is N.

For a system with K asynchronous simultaneous transmitted signals, the received signal at a receiver which is interested in the i-th signal is given by

$$r(t) = \sum_{k=1}^{K} y_k(t-\tau_k) + n(t) \tag{3}$$

In (3) τ_k is the time delay for the communication link between the k-th transmitter and the i-th receiver. The process n(t) is a white Gaussian noise process with two-sided spectral density $N_0/2$. Finally, $y_k(t)$ is the fading channel output to input $s_k(t)$.

A Rician nonselective fading channel is described by the following input-output relationship

$$y_k(t) = s_k(t) + \mathrm{Re}\{\gamma_k A_\ell^{(k)} \exp(j\theta_\ell^{(k)}) x_k(t) \exp(j2\pi f_c t)\} \tag{4}$$

for $\ell T \leq t < (\ell + 1)T$. In (4) γ_k is a nonnegative real number and the nonnegative random variable $A_\ell^{(k)}$ satisfies the normalization constraint

$$E\{[A_\ell^{(k)}]^2\} = 1. \tag{5}$$

The attenuation of the signal strength due to the fading during the time interval $[\ell T, (\ell +1)T)$ is thus represented by $\gamma_k A_\ell^{(k)}$, and the phase shift due to the fading is denoted by $\theta_\ell^{(k)}$. Therefore, for $\gamma_k > 0$ the output signal is the sum of a nonfaded version of the input signal (specular component) and a nondelayed faded version of the input signal (scatter component). For $A_\ell^{(k)}$ Rayleigh distributed and $\theta_\ell^{(k)}$ uniformly distributed in $[0,2\pi]$ this model is discussed in [1]. Notice that from (4) and (3) the channel model reduces to the AWGN model if $\gamma_k = 0$. Similarly for $\gamma_k \to \infty$ (very weak specular component in comparison to the faded component) the channel reduces to the Rayleigh nonselective channel.

Regarding the statistical dependence of the attenuation in different data bit intervals, we will consider the cases: (i) $A_\ell^{(k)} = A_m^{(k)}$ for several values

of adjacent ℓ and m (the length of the burst is immaterial for our analysis as long as it is longer than three adjacent bits) and (ii) $A_\ell^{(k)}$ and $A_m^{(k)}$ are independent if $\ell \neq m$. Case (i) corresponds to a system with no interleaving (or partial interleaving) and a channel with fading statistics which remain invariant over the duration of several data bits. An example of case (ii) arises in a system which is fully-interleaved or a fast fading channel. Similar restrictions are imposed on the phase sequence $(\theta_\ell^{(k)})$.

The i-th receiver is matched to the i-th transmitter signal; it is assumed capable of acquiring time and phase synchronization (coherent demodulation) with the nonfaded component of the i-th signal. Therefore, we may measure time delays and phase angles relative to τ_i and θ_i, respectively, and thus set $\tau_i = \theta_i = 0$. If there is not a nonfaded component we assume that the phase of the faded component of the Rician nonselective fading channel [see (3)] changes very slowly so that the receiver can again acquire time and phase synchronization. In this case we should let $\theta_i + \theta_0^{(i)} = 0$.

Once time and phase acquisition is completed the received signal is passed through a correlation receiver which outputs

$$Z_i = \int_0^T r(t)\Psi(t)a_i(t)\cos(2\pi f_c t)dt. \qquad (6)$$

If Z_i is positive the receiver will decide that a 1 was sent, otherwise it will output a -1. In practical spread-spectrum systems, the common carrier frequency f_c and bit duration T are such that $f_c >> T^{-1}$; consequently, the double-frequency terms in the integrand of (6) may be ignored.

Finally, concerning the interaction among the different communication links between the users, we may measure the delays τ_k with respect to τ_i, set $\tau_i = 0$, and assume that for $k \neq i$ $\tau_k \pmod T$ is uniformly distributed in $[0,T]$. Then the

delays τ_k, the phase angles θ_k, and the data streams $(b_\ell^{(k)})$ are assumed to be mutually independent for a given k, as well as for any two different transmitters. Finally, we assume that all communication links fade independently; therefore, all the random variables and/or random processes which characterize the fading are independent for any two different links.

B. Evaluation of Error Probability via the Characteristic-Function Method

By ignoring the double-frequency terms in (6) we may write Z_i as

$$Z_i = (1/2\ E_b T)^{1/2}\ (b_0^{(i)} + \eta_i + \mathcal{I}_i), \tag{7}$$

In (7) $E_b \triangleq PT$ is the received energy per bit for the nonfaded component, the first term inside the parenthesis is the desired signal component (i.e., the i-th information signal for $0 \leq t < T$), η_i represents the normalized contribution of the AWGN and of the faded component of the i-th signal, and $\mathcal{I}_i$ stands for the normalized multiple-access interference (which now has nonfaded and faded components) due to all the other users besides the i-th user. We actually have that

$$\eta_i = \eta + \gamma_i F_i \tag{8}$$

and

$$\mathcal{I}_i = \sum_{k \neq i} (I_{k,i} + \gamma_k F_{k,i}), \tag{9}$$

where η is a zero-mean Gaussian random variable with variance $N_0/2E_b$, F_i is the normalized interference due to the faded component of the i-th signal, $I_{k,i}$ and $F_{k,i}$ represent normalized interference due to the nonfaded and faded components of the k-th ($k \neq i$) signal.

Due to the additive form of the multiple-access interference [see (9)], the characteristic-function method is recommended for the evaluation of the probability of error for the DS/SSMA systems under consideration. As in [7] we can write $\bar{P}_{e,i}$, the average probability of error for the i-th

receiver (the averge to be considered with respect to all te random variables introduced by the transmitters and the channel) as

$$\bar{P}_{e,i} = 1/2\ P\{Z_i \leq 0 | b_0^{(i)} = +1\} + 1/2P\{Z_i > 0 | b_0^{(i)} = -1\}$$
$$= 1/2 - 1/2P\{-1 < \eta_i + \mathcal{I}_i \leq 1\}. \tag{10}$$

where η_i and $\mathcal{I}_i$ were defined above. Let ϕ_{η_i} and ϕ_i denote the characteristic functions of the random variables of η_i and $\mathcal{I}_i$, respectively. Since the distributions of η_i and $\mathcal{I}_i$ considered in this paper are symmetric, the characteristic functions ϕ_{η_i} and ϕ_i are real-valued even functions. Therefore as in [7] - [9] we can write

$$\bar{P}_{e,i} = \bar{P}_{e,i}^{0} + \pi^{-1}\int_0^{\infty} u^{-1}(\sin u)\ \phi_{\eta_i}(u)\ [1-\phi_i(u)]du \tag{11}$$

where $\bar{P}_{e,i}^{0}$ is the average probability of error in the absence of multiple-access interference (i.e., when $\mathcal{I}_i = 0$ or $K = 1$). Because of (9) and the independence assumptions of Section II.A above we can write for each real u

$$\phi_i(u) = \prod_{k\neq i} E\{\exp[ju(I_{k,i} + \gamma_k F_{k,i})]\}, \tag{12}$$

which facilitates the computation of the characteristic function of the multiple-access interference.

For the Rician nonselective fading channel of Section II.A we can easily derive that

$$F_i = b_0^{(i)} A_0^{(i)} \cos(\theta_0^{(i)}), \tag{13a}$$

$$I_{k,i} = T^{-1}[b_1^{(k)} R_{k,i}(\tau_k) + b_2^{(k)} \hat{R}_{k,i}(\tau_k)]\cos\theta_k', \tag{13b}$$

and

$$F_{k,i} = T^{-1}[b_1^{(k)} A_1^{(k)}\ R_{k,i}(\tau_k)\cos(\theta_k' + \theta_1^{(k)})$$
$$+ b_2^{(k)} A_2^{(k)}\ \hat{R}_{k,i}(\tau_k)\cos(\theta_k' + \theta_2^{(k)})], \tag{13c}$$

where

$$\theta_k' = \theta_k - 2\pi f_c \tau_k.$$

Recall that $\tau_i = \theta_i = 0$ so that $\theta_i' = 0$. The arguments of the partial continuous crosscorrelation functions $R_{k,i}$ and $\hat{R}_{k,i}$ (defined in [6]) should be considered modulo T. Let $\underline{b}_k = (b_1^{(k)}, b_2^{(k)})$ denote the pair of the $(\lambda_k - 1)$-th and λ_k-th consecutive data bits (suppose that for $k \neq i$ $\lambda_k T \leq \tau_k < (\lambda_k + 1)T$). If, for simplicity, we assume that the $A_\ell^{(k)}$'s are Rayleigh distributed in $[0, 2\pi]$, then since $\theta_k' + \theta_\ell^{(k)} \pmod{2\pi}$ is also uniformly distributed in $[0, 2\pi]$ the random variables F_i and $F_{k,i}$ are conditionally zero-mean Gaussian when conditioned on $b_0^{(i)}$ and $(\underline{b}_k, \tau_k)$, respectively.

Next we proceed to the evaluation of the characteristic functions of η_i and I_i which are necessary for the computation in (11). Let σ_i^2 denote the variance of η_i defined by (8), then

$$\sigma_i^2 = (2E_b/N_0)^{-1} + 1/2\gamma_i^2, \tag{14}$$

$$P_{e,i}^0 = Q(\sigma_i^{-1}), \tag{15}$$

where Q is the complementary error function, and

$$\phi_{\eta_i}(u) = \exp(-1/2u^2\sigma_i^2). \tag{16}$$

To proceed further we need to evaluate the variance of the conditionally Gaussian random variable $F_{k,i}$ given by (13c). It turns out that for (i) constant fading

$$\mathrm{Var}\{F_{k,i} | \underline{b}_k, \tau_k\} = 1/2\ T^{-2}[b_1^{(k)} R_{k,i}(\tau_k) + b_2^{(k)} \hat{R}_{k,i}(\tau_k)]^2, \tag{17}$$

and for (ii) independent fading

$$\mathrm{Var}\{F_{k,i} | \tau_k\} = 1/2T^{-2}\{R_{k,i}^2(\tau_k) + \hat{R}_{k,i}^2(\tau_k)\}. \tag{18}$$

For $\ell T_c \leq \tau < (\ell+1)T_c$ and any arbitrary function h define

$$\sigma^2(\ell, h, \tau) = 1/2\ T^{-2}[h(\ell+1)R_\psi(\tau) + h(\ell)\hat{R}_\psi(\tau)]^2, \tag{19}$$

$$\sigma_{k,i}^2(\ell;\tau) = 1/2\ T^{-2}[R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau)], \tag{20}$$

and

$$\tilde{f}(u;\ell,h;\tau) = \frac{2}{\pi}\int_0^{\pi/2} \cos\{\frac{u}{T}[h(\ell+1)R_\psi(\tau) + h(\ell)\hat{R}_\psi(\tau)]\cos\theta\}d\theta. \qquad (21)$$

In (19) and (21) R_ψ and $\hat{R}_\psi$ are the continuous partial autocorrelation functions of the chip waveform. The functions $R_{k,i}$ and $\hat{R}_{k,i}$ depend on them and on the discrete crosscorrelation function of the k-th and i-th signature sequences $C_{k,i}$. Then after some straightforward but cumbersome manipulations, (12) becomes

$$\phi_i(u) = \prod_{k\neq i}\{(2N)^{-1}\sum_{\ell=0}^{N-1} T_c^{-1}\int_0^{T_c}\{\tilde{f}(u;\ell,\theta_{k,i};\tau)\exp[-1/2u^2\gamma_k^2\sigma^2(\ell,\theta_{k,i};\tau)] + \tilde{f}(u;\ell,\hat{\theta}_{k,i};\tau)\exp[-1/2u^2\gamma_k^2\sigma^2(\ell,\hat{\theta}_{k,i};\tau)]\}d\tau\} \qquad (22a)$$

for (i) constant fading, and

$$\phi_i(u) = \prod_{k\neq i}\{(2N)^{-1}\sum_{\ell=0}^{N-1} T_c^{-1}\int_0^{T_c}[\tilde{f}(u;\ell,\theta_{k,i};\tau) + \tilde{f}(u;\ell,\hat{\theta}_{k,i};\tau)] \cdot\exp[-1/2u^2\gamma_k^2\sigma_{k,i}^2(\ell;\tau)]d\tau\} \qquad (22b)$$

for (ii) independent fading.

If the channel encountered by the i-th transmitted signal is a Rayleigh nonselective fading channel (i.e, there is no non-faded component of the i-th signal) we need to modify the above results in the following way. We start with finite γ_i and define $\bar{E}_b \triangleq (1 + \gamma_i^2)PT$ to be the total received energy at the i-th receiver in the absence of AWGN and multiple-access interference (i.e., K = 1), and $\bar{\gamma}_k^2 = (1 + \gamma_k^2)/(1 + \gamma_i^2)$ to be the ratio of the total received energy from the k-th signal to the total received energy from the i-th signal. For large γ_i, we assume (see Section II.A) that the receiver acquires time and phase synchronization with the faded component of the i-th signal, so that we may set $\theta_i + \theta_0^{(i)} = 0$ and together with the definitions above write (7) and (8) - (9) as

$$Z_i = (1/2\bar{E}_b T)^{1/2} \left[\eta + \frac{1}{\sqrt{1+\gamma_i^2}} (b_0^{(i)}\cos\theta_i + \gamma_i b_0^{(i)} A_0^{(i)}) + \sum_{k\neq i} \frac{\bar{\gamma}_k}{\sqrt{1+\gamma_k^2}} (I_{k,i} + \gamma_k F_{k,i})\right]. \tag{23}$$

If we fix $\bar{E}_b$ and let $\gamma_i \to \infty$ (23) reduces to

$$Z_i = (1/2\bar{E}_b T)^{1/2}[b_0^{(i)} A_0^{(i)} + \eta_i + \mathcal{I}_i] \tag{24}$$

where $A_0^{(i)}$ is Rayleigh distributed (with second order moment 1), $\eta_i = \eta$ is zero-mean Gaussian with variance $N_0/2\bar{E}_b$, and $\mathcal{I}_i$ is given by

$$\mathcal{I}_i = \sum_{k\neq i} \frac{\bar{\gamma}_k}{\sqrt{1+\gamma_k^2}} (I_{k,i} + \gamma_k F_{k,i}) \tag{25}$$

where $I_{k,i}$ and $F_{k,i}$ are as defined in (13b) - (13c).

A discussion of the computational requirements for formulas of the type of (11) can be found in [7] - [8] and since the situation here is very similar with that treated there, it will not be repeated. However, it is worth observing that since there is a summation of N terms and a product of K terms in (22a) - (22b) the computational effort for this method grows linearly with the product KN.

Starting from (24) and proceeding as for (10), we find that $P_{e,i}(A_0^{(i)})$ the conditional probability of error given $A_0^{(i)}$ can be expressed initially as

$$P_{e,i}(A_0^{(i)}) = 1/2 - 1/2\ P\{-A_0^{(i)} < \eta_i + \mathcal{I}_i \leq A_0^{(i)}\}, \tag{26}$$

and finally [cf. (11)] as

$$P_{e,i}(A_0^{(i)}) = P^0_{e,i}(A_0^{(i)}) + \pi^{-1} \int_0^\infty u^{-1}\sin(A_0^{(i)}u)\phi_{\eta_i}(u)[1 - \phi_i(u)]du. \tag{27}$$

Since η_i is now zero-mean Gaussian with variance

$$\sigma_i^2 = (2\bar{E}_b/N_0)^{-1} \tag{28}$$

$\phi_{\eta_i}(u)$ is given by (16) with σ_i^2 defined by (28) and

$$P_{e,i}^{0}(A_0^{(i)}) = Q(A_0^{(i)}/\sigma_i). \tag{29}$$

To obtain the average probability of error $\bar{P}_{e,i}$ we should average the second member of (27) with respect to the Rayleigh distributed random variable $A_0^{(i)}$. The result (see Appendix A of [9] for the proof) is

$$\bar{P}_{e,i} = \bar{P}_{e,i}^{0} + 1/2\pi^{-1/2} \int_0^{\infty} \exp(-1/4u^2)\phi_{\eta_i}(u)[1 - \phi_i(u)]du \tag{30}$$

where

$$\bar{P}_{e,i}^{0} = 1/2\{1-[1 + (\bar{E}_b/N_0)^{-1}]^{-1/2}\} \tag{31}$$

and $\phi_i(u)$ is given by (22a) and (23b) for case (i) constant fading and (ii) independent fading, respectively, evaluated [because of (25)] at $\bar{\gamma}_k u/\sqrt{1+\gamma_k^2}$ instead of u.

C. Approximation Based on Average Signal-to-Noise Ratio

For the sake of comparison we next cite an approximation based on SNR_i the average signal-to-noise ratio at the output of the i-th receiver. As suggested in [6]

$$SNR_i \triangleq E\{Z_i\}/[Var\{Z_i\}]^{1/2} \tag{32}$$

which in our case becomes

$$SNR_i = [Var\{\eta_i\} + \sum_{k\neq i} Var(I_{k,i} + \gamma_{k,i}F_{k,i})]^{-1/2}. \tag{33}$$

For Rician nonselective fading (33) reduces to

$$SNR_i = \{(2E_b/N_0)^{-1} + 1/2\gamma_i^2 + \sum_{k\neq i} (1+\gamma_k^2)T^{-3}[\mu_{k,i}(0)m_\psi + \mu_{k,i}(1)m'_\psi]\}^{-1/2} \tag{34}$$

and the resulting approximation to the average error probability is

$$\bar{P}_{e,i}^{G} = Q(SNR_i). \tag{35}$$

In (34) the function $\mu_{k,i}(n)$ (see [6]) depends on the discrete crosscorrelation function $C_{k,i}$ while m_ψ and m'_ψ (again see [6]) depend on R_ψ and $\hat{R}_\psi$. For Rayleigh nonselective fading (33) becomes

$$SNR_i = \{(\bar{E}_b/N_0)^{-1} + \sum_{k\neq i} \bar{\gamma}_k^2 T^{-3}[\mu_{k,i}(0)m_\psi + \mu_{k,i}(1)m'_\psi]\}^{-1/2} \qquad (36)$$

and the resulting approximation is

$$\bar{P}_{e,i}^{G} = 1/2\{1-[1+(SNR_i)^{-2}]^{-1/2}\}. \qquad (37)$$

Notice that, in contrast to the approximation based on the characterisitic-function method, the approximations obtained via the signal-to-noise ratio do not ditinguish between the constant and independent fading cases considered in Section II.A.

III. PERFORMANCE OF DS/SS COMMUNICATIONS OVER FREQUENCY-SELECTIVE RICIAN FADING CHANNELS

A. Channel Model

The input-output relationship for a wide-sense-stationary uncorrelated-scattering (WSSUS) frequency-selective fading channel can be expressed as

$$y_k(t) = s_k(t) + \mathrm{Re}\{u_k(t-\nu_k T)\exp[j2\pi f_c(t-\nu_k T)]\} \tag{38}$$

where

$$u_k(t) = \gamma_k \int_{-\infty}^{\infty} h_k(t,\zeta)x_k(t-\zeta)d\zeta. \tag{39}$$

The fading process $h_k(t,\zeta)$ (which can be thought of as the time-varying impulse response of a low pass filter) is a zero-mean complex Gaussian random process wich has autocovariance

$$E\{h_k(t,\zeta)h_k^*(s,\xi)\} = g_k(\zeta)\delta(\zeta-\xi), \tag{40a}$$

and satisfies the normalization constraint

$$\int_{-\infty}^{\infty} g_k(\zeta)d\zeta = 1, \tag{40b}$$

and the necessary condition for stationary bandpass processes (see [2], and the discussion in Section 2.2 of [4])

$$E\{h_k(t,\zeta)h_k(s,\xi)\} = 0. \tag{40c}$$

The first term of (38) is again termed the specular component while the second term in (38) is now called the diffuse faded component. If γ_k is a positive real number, the channel is a Rician fading channel. If $\gamma_k \to \infty$ the faded component becomes dominant and the channel is a Rayleigh fading channel. This model is basically described in detail in [2] - [3], in particular the Rician fading channel is described in [4] - [5]. Here we consider a slightly more general model since for the Rician case we allow [see (38)] an average delay $\nu_k T$ (ν_k is a nonnegative integer) between the nonfaded specular component and the diffused faded component.

The WSSUS fading model can be thought of arising when the transmitted signal encounters a slowly-moving scattering random medium which can be modeled as a layered scatterer consisting of a large number of layers of infinitesimal thickness. It, is therefore, a frequency-selective channel, i.e., it is dispersive only in frequency. The high-frequency (HF) ionospheric and the microwave tropospheric scatter channels fit the above description; Section 2.1 of [4] provides more examples of such channels.

We impose a limitation on the selectivity of the channel in order to facilitate subsequent analysis. In particular, as in [5] and [3] it is assumed that

$$g_k(\zeta) = 0 \qquad \text{for } |\zeta| > T \tag{41}$$

which is a constraint on the frequency selectivity of the channel that allows us to restrict attention to the intersymbol interference from the two adjacent data bits.

With regard to the average delay $\nu_k T$ between the nonfaded specular component and the faded scatter component of $y_k(t)$ in (38), we will consider the two extreme cases (i) $\nu_k = 0$ (no delay) and (ii) $\nu_k \geq 3$ (large delay). The choice of the integer 3 as the lower bound for ν_k in (ii) is justified by the fact that, in view of (41), it uncorrelates the contributions of the nonfaded and faded components at the output of the receiver.

B. Evaluation of Error Probability via the Characteristic-Function Method

The output of the matched filter Z_i is again given by (7) where η_i and I_i are given by (8) and (9), respectively. The random variable η in (8) is again zero-mean Gaussian with variance $N_0/2E_b$; the nonfaded component of the interference (due to the k-th signal) $I_{k,i}$ is given by (13b), but the faded

components F_i and $F_{k,i}$ of (8) - (9) are now found [combine (38) - (39) with, (3) and (6)] to be

$$F_i = \mathrm{Re}\{\tilde{F}_i\}, \tag{42a}$$

$$\tilde{F}_i = T^{-1} \int_{-\infty}^{\infty} h_i(t-\nu_i T,\zeta)\Gamma_i(t,\zeta+\nu_i T)\exp(j\Phi_i)d\zeta dt, \tag{42b}$$

$$F_{k,i} = \mathrm{Re}\{\tilde{F}_{k,i}\}, \tag{42c}$$

$$\tilde{F}_{k,i} = T^{-1} \int_0^T \int_{-\infty}^{\infty} h_k(t-\tau_k-\nu_k T,\zeta)\Gamma_{k,i}(t,\zeta+\tau_k+\nu_k T)\exp(j\Phi_k)d\zeta dt, \tag{42d}$$

where

$$\Gamma_{k,i}(t,\zeta) = a_k(t-\zeta)a_i(t)b_k(t-\zeta)b_i(t)\Psi(t-\zeta)\Psi(t)$$

and

$$\Phi_k = \theta_k - 2\pi f_c(\tau_k+\nu_k T).$$

The function $\Gamma_{i,i}$ is denoted by Γ_i. Since h_k is a complex Gaussian random field, the random variables F_i and $F_{k,i}$ are conditionally Gaussian random variables (conditioned on time delays, phase angles, and data streams). Recall that $\nu_k T$ ($1 \leq k \leq K$) accounts for the average delay between the nonfaded and the diffused components of the channel output (see Section III.A)

Next, we proceed to the evaluation of the average error probability $\bar{P}_{e,i}$ via the characteristic-function method. As in the nonselective fading case, we need to evaluate $\bar{P}_{e,i}$, ϕ_{n_i}, and ϕ_i for use in (11). We follow the same steps as before. The random variable F_i is zero-mean conditionally Gaussian with conditional variance

$$\mathrm{Var}\{F_i|\underline{b}_i'\} = 1/2\ E\{\tilde{F}_i\tilde{F}_i^*\}$$
$$= T^{-2} \int_0^T g_i(\zeta)[\ R_i^2(\zeta) + \hat{R}_i^2(\zeta)+(b_{1'}^{(i)}+b_{2'}^{(i)})R_i(\zeta)\hat{R}_i(\zeta)]d\zeta. \tag{43}$$

where $\underline{b}_i' = (b_{1'}^{(i)}, b_{2'}^{(i)})$ denotes the pair of the (ν_i-1)-th and ν_i-th consecutive data bits of the i-th data stream. To derive (43) we used

(42a), (42b), (40a) - (40c), and (41). The Gaussian random variable η_i has then conditional variance

$$\mathrm{Var}\{\eta_i|\underline{b}_i'\} = (2E_b/N_0)^{-1} + \gamma_i^2\mathrm{Var}\{F_i|\underline{b}_i'\}. \tag{44}$$

Then, if we define

$$[\sigma_i^{(1)}]^2 = (2E_b/N_0)^{-1} + \gamma_i^2\int_0^T g_i(\zeta)[R_i^2(\zeta)+\hat{R}_i^2(\zeta)]d\zeta \tag{45a}$$

$$[\sigma_i^{(2)}]^2 = (2E_b/N_0)^{-1} + \gamma_i^2T^{-2}\int_0^T g_i(\zeta)[R_i(\zeta)+\hat{R}_i(\zeta)]^2d\zeta \tag{45b}$$

$$[\sigma_i^{(3)}]^2 = (2E_b/N_0)^{-1} + \gamma_i^2T^{-2}\int_0^T g_i(\zeta)[R_i(\zeta)-\hat{R}_i(\zeta)]^2d\zeta, \tag{45c}$$

we can write

$$P_{e,i}^{\eta} = \sum_{j=1}^{3} c_jQ([\sigma_i^{(j)}]^{-1}) \tag{46}$$

and

$$\phi_{\eta_i} = \sum_{j=1}^{3} c_j \exp\{-1/2[u\sigma_i^{(j)}]^2\} \tag{47}$$

where $c_1 = 1/2$ and $c_2 = c_3 = 1/4$. Note that these two results are independent of the events $\nu_i = 0$ (no delay: case (i) of Section III.A or $\nu_i \geq 3$ (large delay: case (ii) of Section III.A.

The next step is to evaluate ϕ_i through (12) with $I_{k,i}$ and $F_{k,i}$ given by (13b) and (42c) - (42d), respectively. We first compute the conditional variance of $F_{k,i}$, this turns out to be

$$\begin{aligned}\mathrm{Var}\{F_{k,i}|\underline{b}_k',\tau\} = 1/2T^{-2}\{&\int_0^{\tau} g_k(\zeta-\tau+T)[b_{1'}^{(k)}R_{k,i}(\zeta)+b_{2'}^{(k)}\hat{R}_{k,i}(\zeta)]^2d\zeta\\ &+\int_0^T g_k(\zeta-\tau)[b_{2'}^{(k)}R_{k,i}(\zeta) + b_{3'}^{(k)}\hat{R}_{k,i}(\zeta)]^2d\zeta\\ &+\int_{\tau}^T g_k(\zeta-\tau-T)[b_{3'}^{(k)}R_{k,i}(\zeta) + b_{4'}^{(k)}\hat{R}_{k,i}(\zeta)]^2d\zeta\}.\end{aligned} \tag{48}$$

In (48) $\underline{b}_k' = (b_{1'}^{(k)}, b_{2'}^{(k)}, b_{3'}^{(k)}, b_{4'}^{(k)})$ $(k \neq i)$ is the quadruple which consists of the (μ_k-2)-th, (μ_k-1)-th, μ_k-th, and (μ_k+1)-th consecutive bits of the k-th data stream, $\mu_k \triangleq \lambda_k + \nu_k$, and $\tau = \tau_k(\mathrm{mod}\ T)$ where

$\ell T_c \leq \tau < (\ell+1)T_c$. The conditional variance about takes on 8 different values which we denote by $[\sigma_{k,i}^{(j)}(\ell;\tau)]^2$ and $[\hat{\sigma}_{k,i}^{(j)}(\ell;\tau)]^2$ for $j = 1,2,3,4$. Then the characteristic function ϕ_i is derived as

$$\phi_i(u) = \prod_{k\neq i}\{(8N)^{-1}\sum_{\ell=0}^{N-1} T_c^{-1}\int_0^{T_c}[\tilde{f}(u;\ell,\theta_{k,i};\tau)\sum_{j=1}^{4}\exp\{-1/2[u\gamma_k\sigma_{k,i}^{(j)}(\ell;\tau)]^2\}$$

$$+ \tilde{f}(u;\ell,\hat{\theta}_{k,i};\tau)\sum_{j=1}^{4}\exp\{-1/2[u\gamma_k\hat{\sigma}_{k,i}^{(j)}(\ell;\tau)]^2\}]d\tau\} \tag{49a}$$

for (i) no delay ($\nu_k = 0$, all k), and

$$\phi_i(u) = \prod_{k\neq i}\{(8N)^{-1}\sum_{\ell=0}^{N-1} T_c^{-1}\int_0^{T_c}[\tilde{f}(u;\ell,\theta_{k,i};\tau) + \tilde{f}(u;\ell,\hat{\theta}_{k,i};\tau)]$$

$$\cdot \sum_{j=1}^{4}[\exp\{-1/2[u\gamma_k\sigma_{k,i}^{(j)}(\ell;\tau)]^2\}$$

$$+ \exp\{-1/2[u\gamma_k\hat{\sigma}_{k,i}^{(j)}(\ell;\tau)]^2\}]d\tau\} \tag{49b}$$

for (ii) large delay ($\nu_k \geq 3$, all k). In case (i) $I_{k,i}$ [given by (13b)] and $F_{k,i}$ above are correlated [$\underline{b}_k' = (b_1^{(k)}, \underline{b}_k^{(k)}, b_{4'}^{(k)})$]. In case (ii) $I_{k,i}$ and $F_{k,i}$ are uncorrelated ($\underline{b}_k$ and $\underline{b}_k'$ have no components in common). These facts are reflected in the form of ϕ_i in (49a) and (49b) (the function $\tilde{f}$ and the sum with the four terms are the characteristic functions of $I_{k,i}$ and $F_{k,i}$ given τ respectively). The function $\tilde{f}$ is as defined in (21).

To see on the computational effort grows linearly with KN^2, notice that in (49a) - (49b), besides the product of K-1 terms and the summation of N terms, the integrals split into summations with a total of N terms as well [e.g., $\int_0^{\tau} = \sum_{n=0}^{\ell=1}\int_{nT_c}^{(n+1)T_c} + \int_{\ell T_c}^{\tau}$, for $\ell T_c \leq \tau < (\ell+1)T_c$].

C. Approximation Based on the Average Signal-to-Noise Ratio

The average signal-to-noise ratio in this case is given by [cf. (34)]

$$SNR_i = \{[\sigma_i^{(1)}]^2 + \sum_{k\neq i} (1+\gamma_k^2)[\mu_{k,i}(0)m_\psi + \mu_{k,i}(1)m'_\psi]\}^{-1/2} \quad (50)$$

where $\sigma_i^{(1)}$ is given by (45a) above. This is a slight generalization of the results of [5] for arbitrary chip waveforms. Notice that SNR_i and the resulting approximation (35) cannot distinguish between cases (i) and (ii) (no delay versus large delay) of the fading model of Section III.A.

IV. NUMERICAL RESULTS AND CONCLUSIONS

All the signatures sequences employed in this section are AO/LSE m-sequences of lengths N=31 and 127. In particular, for a DS/SSMA system with K users the first K sequences of Figures A.1.(a) (N=31) and A.1.(b) (N=127) of [15] are used.

For $\bar{P}_{e,i}$, the approximation to the error probability which is based on the characteristic-function method, we use Simson's integration technique with the same parameters as in Section III of [7] (L=20, $\varepsilon<10^{-14}$, $n_\theta \approx 10$, $n_\tau=10$, and n=20). We set i=1 and consider $\bar{P}_{e,1}$ for the error probability of the receiver matched to the first signal.

First, we present numerical results for DS/SSMA systems with nonselective Rician and Rayleigh fading channels. In Table 1 the Gaussian approximation to the error probability $\bar{P}^{G}_{e,1}$ and the approximations $\bar{P}^{(i)}_{e,1}$ and $\bar{P}^{(ii)}_{e,1}$ obtained by the characteristic-function method are compared for a DS/SSMA system with sequences of length 31 and K = 2, 3, and 4 simultaneous users. The communications link between the k-th transmitter and the receiver matched to the first signal is a Rician nonselective fading channel with relative power of the faded component $\gamma_k^2=\gamma^2=.1$, $1\leq k\leq K$. The error probabilities $\bar{P}^{(i)}_{e,1}$ and $\bar{P}^{(ii)}_{e,1}$ correspond to the cases of (i) slow fading and (ii) fast fading discussed in Section II.A. We observe that the Gaussian approximation is somewhat optimistic when compared with the more accurate approximations based on the characteristic function but still in good agreement with them. Notice that slow fading causes a slightly worse performance of the DS/SSMA system than fast fading. Recall that the Gaussian approximation results in the same value of the error probability for both slow and fast fading.

In Figure 1 we plotted both $\bar{P}_{e,1}^{(i)}$ (for slow fading) and $\bar{P}_{e,1}^{G}$ versus E_b/N_0 for a DS/SSMA system with K=3, N=31, and Rician nonselective fading. The relative power of all faded components are γ^2=.01, .05, .1, .2, and .4. Notice that as γ^2 increases the system degrades gracefully. For a nonselective fading channels DS/SS modulation enables the matched filter receiver to discriminate effectively against the nonfaded and faded components of the interfering signals but not against the faded component of the desired signal. This explains why the error probability is already larger than 10^{-3} for γ^2 = .1, .2, and .4. Finally, notice that the accuracy of the Gaussian approximation improves as γ^2 increases.

In Table 2 we compare the performance of DS/SSMA systems with rectangular and sine chip waveforms. The sine chip waveform used is $\psi(t) = \sqrt{2} \sin(\pi t/T_c)$, $0 \leq t \leq T_c$. The system parameters are K=3, and N=31. The Rician nonselective slow fading channel parameter takes on the values γ^2 = .01 and .1. For moderately heavy fading (γ^2=.1) the rectangular chip waveform results in a slightly better performance than the sine chip waveform as both the Gaussian approximation $\bar{P}_{e,1}^{G}$ and the more accurate approximation $\bar{P}_{e,1}^{(i)}$ indicate. For light fading (γ^2=.01) the Gaussian approximation still shows better performance for the sine chip waveform for all values of E_b/N_0; however, the more accurate approximation indicates that the sine chip waveform outperforms the rectangular chip waveform only for low signal-to-noise ratios; the situation is reversed for higher signal-to-noise ratios.

In Table 3 we present results for a DS/SSMA system with K=3, N=31 and Rayleigh nonselective fading channels with relative power of faded components taking values $\bar{\gamma}^2$=.01, .1, and .2. The system performance has now

degraded considerably and higher signal-to-noise ratios were considered. Notice that the Gaussian approximation is conservative and for large values of $\bar{\gamma}^2$ is off the actual value by one order of magnitude. On the other hand the values of the accurate approximation for (i) slow fading and (ii) fast fading do not differ in the first four significant digits.

Next, we consider DS/SSMA systems operating through Rician frequency-selective channels. In Table 4 the Gaussian approximation is compared to the approximation obtained via the characteristic-function method for the cases (i) zero average delay between nonfaded and faded components and (ii) large delay (see Section III.A) and DS/SSMA systems with K=3 and N=31 and 127. The relative power of the diffuse component is $\gamma^2=.4$ for all signals. Notice that the system performance for case (ii)gives slightly lower error probability than case (i) due to the more extensive randomization involved. The Gaussian approximation results in an optimistic estimate of the error probability for all cases but it is satisfactorily close to the more accurate results. Recall that the Gaussian approximation takes the same value for the aformentioned cases (i) and (ii).

In Figure 2 $\bar{P}^{G}_{e,1}$ and $\bar{P}^{(i)}_{e,1}$ are plotted versus E_b/N_0 for DS/SSMA systems with N=31 and K=2, 3, and 4. The autocovariance function of the frequency-selective WSSUS channel is triangular: $g_k(\tau) = \frac{1}{T}(1 - \frac{|t|}{T})$ for $|t| \leq T$ and 0 otherwise. The relative power of all the faded components is $\gamma^2=.4$. Notice that for an error probability of 10^{-5} the Gaussian approximation is off by almost 1 dB for K=3. For fixed E_b/N_0 the Gaussian approximation is satisfactory for all cases as it is also indicated by the results of Table 4.

Finally, in Figure 3 we have plotted $\bar{P}_{e,1}^{(i)}$ versus E_b/N_0 for a DS/SSMA system with K=3 and N=31 and relative power in the diffuse component taking values γ^2=0., .1, .2, .3, and .4. Notice the graceful degradation of the system performance as γ^2 increases. The value γ^2=0. corresponds to DS/SSMA with an AWGN channel. A comparison of Figures 1 and 3 verifies that DS/SS modulation is much more efficient against frequency-selective fading than against nonselective fading.

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Table Captions

Table 1. Probability of error for DS/SSMA systems with Rician nonselective fading (N=31, γ^2=.1).

Table 2. Probability of error for DS/SSMA systems with different chip waveforms and Rician nonselective fading (K=3, N=31).

Table 3. Probability of error for a DS/SSMA system with Rayleigh nonselective fading (K=3, N=31).

Table 4. Probability of error for a DS/SSMA system with frequency-selective fading (K=3, γ^2=.4).

Table 1. Probability of error for DS/SSMA systems with Rician nonselective fading (N=31, γ^2=.1).

	K=2			K=3			K=4		
E_b/N_0	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$
6	1.04	1.04	1.04 ($\times 10^{-2}$)	1.25	1.26	1.26 ($\times 10^{-2}$)	1.53	1.54	1.54 ($\times 10^{-2}$)
8	3.83	3.86	3.86 ($\times 10^{-3}$)	5.27	5.37	5.35 ($\times 10^{-3}$)	0.73	0.75	0.74 ($\times 10^{-2}$)
10	1.36	1.40	1.39 ($\times 10^{-3}$)	2.23	2.34	2.31 ($\times 10^{-3}$)	3.58	3.80	3.76 ($\times 10^{-3}$)
12	5.18	5.43	5.37 ($\times 10^{-4}$)	1.02	1.12	1.10 ($\times 10^{-3}$)	1.93	2.16	2.11 ($\times 10^{-3}$)
14	2.26	2.45	2.40 ($\times 10^{-4}$)	5.37	6.25	6.05 ($\times 10^{-4}$)	1.18	1.39	1.35 ($\times 10^{-3}$)
16	1.17	1.32	1.28 ($\times 10^{-4}$)	3.29	4.04	3.86 ($\times 10^{-4}$)	0.82	1.02	0.97 ($\times 10^{-3}$)

Table 2. Probability of error for DS/SSMA systems with different chip waveforms and Rician nonselective fading (K=3, N=31).

	$\bar{\gamma}^2$=.01					$\bar{\gamma}^2$=.1				
	rect		sine			rect		sine		
E_b/N_0(dB)	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$		$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	
6	5.19	5.25	4.92	4.99	($\times 10^{-3}$)	1.25	1.26	1.21	1.22	($\times 10^{-2}$)
8	1.06	1.11	0.95	1.02	($\times 10^{-3}$)	5.27	5.37	4.97	5.09	($\times 10^{-3}$)
10	1.52	1.78	1.24	1.61	($\times 10^{-4}$)	2.23	2.34	2.04	2.17	($\times 10^{-3}$)
12	1.70	2.56	1.20	2.41	($\times 10^{-5}$)	1.02	1.12	0.91	1.03	($\times 10^{-3}$)
14	1.80	4.07	1.05	4.28	($\times 10^{-6}$)	5.37	6.25	4.63	5.67	($\times 10^{-4}$)
16	1.23	0.83	1.03	1.00	($\times 10^{-6}$)	3.29	4.04	2.76	3.66	($\times 10^{-4}$)

Table 3. Probability of error for a DS/SSMA system with Rayleigh nonselective fading (K=3, N=31).

	$\bar{\gamma}^2=.01$			$\bar{\gamma}^2=.1$			$\bar{\gamma}^2=.2$		
$\bar{E}_b/N_0$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$
10	6.42	6.43	6.43 ($\times 10^{-2}$)	6.47	6.51	6.51 ($\times 10^{-2}$)	6.53	6.60	6.60 ($\times 10^{-2}$)
20	2.35	2.34	2.34 ($\times 10^{-2}$)	2.55	2.43	2.43 ($\times 10^{-2}$)	2.74	2.53	2.53 ($\times 10^{-2}$)
30	0.85	0.78	0.78 ($\times 10^{-2}$)	1.34	0.87	0.87 ($\times 10^{-2}$)	1.72	0.98	0.98 ($\times 10^{-2}$)
40	4.37	2.59	2.59 ($\times 10^{-3}$)	1.14	0.35	0.35 ($\times 10^{-2}$)	1.56	0.46	0.46 ($\times 10^{-2}$)
50	3.70	0.89	0.89 ($\times 10^{-3}$)	11.2	1.83	1.83 ($\times 10^{-3}$)	15.6	2.87	2.87 ($\times 10^{-3}$)
∞	36.2	1.07	1.07 ($\times 10^{-4}$)	11.2	1.05	1.05 ($\times 10^{-3}$)	15.6	2.08	2.09 ($\times 10^{-3}$)

Table 4. Probability of error for a DS/SSMA system with frequency-selective fading (K=3, $\gamma^2=.4$).

	N=31				N=127			
E_b/N_0(dB)	$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$		$\bar{P}^G_{e,1}$	$\bar{P}^{(i)}_{e,1}$	$\bar{P}^{(ii)}_{e,1}$	
6	6.34	6.40	6.40	($\times 10^{-3}$)	3.20	3.20	3.20	($\times 10^{-3}$)
8	1.57	1.62	1.62	($\times 10^{-3}$)	3.78	3.82	3.82	($\times 10^{-4}$)
10	3.10	3.42	3.41	($\times 10^{-4}$)	1.83	1.97	1.97	($\times 10^{-5}$)
12	5.54	6.97	6.95	($\times 10^{-5}$)	3.31	3.89	3.89	($\times 10^{-7}$)
14	1.05	1.61	1.59	($\times 10^{-5}$)	1.91	3.22	3.20	($\times 10^{-9}$)
16	2.47	4.75	4.73	($\times 10^{-6}$)	0.44	1.02	1.01	($\times 10^{-10}$)

Figure Captions

Figure 1. Probability of error for a DS/SSMA system with Rician nonselective fading (K=3, N=31, and γ^2=.01, .05, .1, .2, and .4)

Figure 2. Probability of error for a DS/SSMA system with frequency-selective fading (N=31, γ^2=.4, and K=2,3, and 4).

Figure 3. Probability of error for a DS/SSMA system with frequency-selective fading (K=3, N=31, and γ^2=0., .1, .2, .4, and .6)

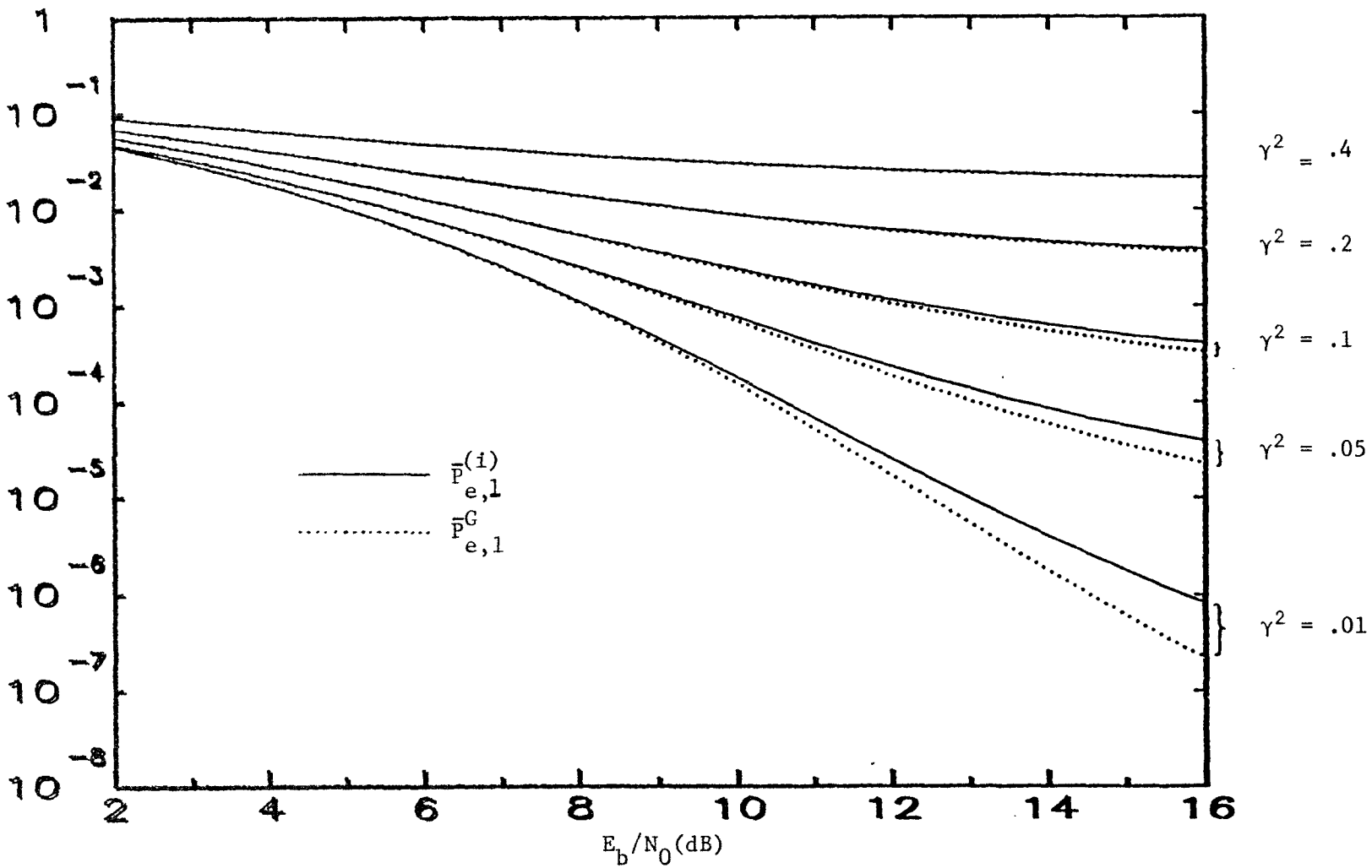

Figure 1. Probability of error for a DS/SSMA system with Rician nonselective fading (K = 3, N = 31, and γ^2 = .01, .05, .1, .2, and .4)

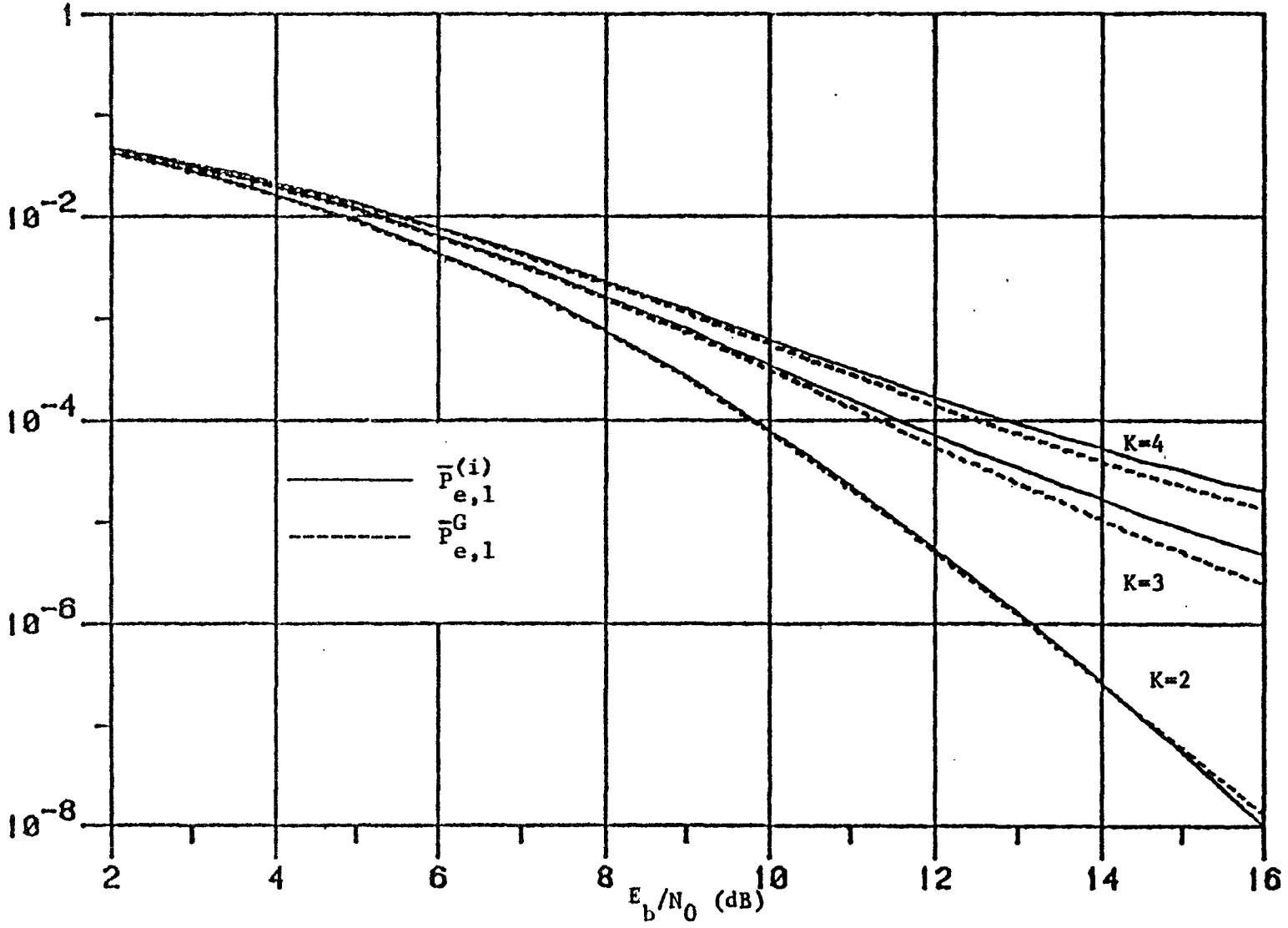

Figure 2. Probability of error for a DS/SSMA system with frequency-selective fading ($N = 31$, $\gamma^2 = .4$, and $K = 2, 3,$ and 4).

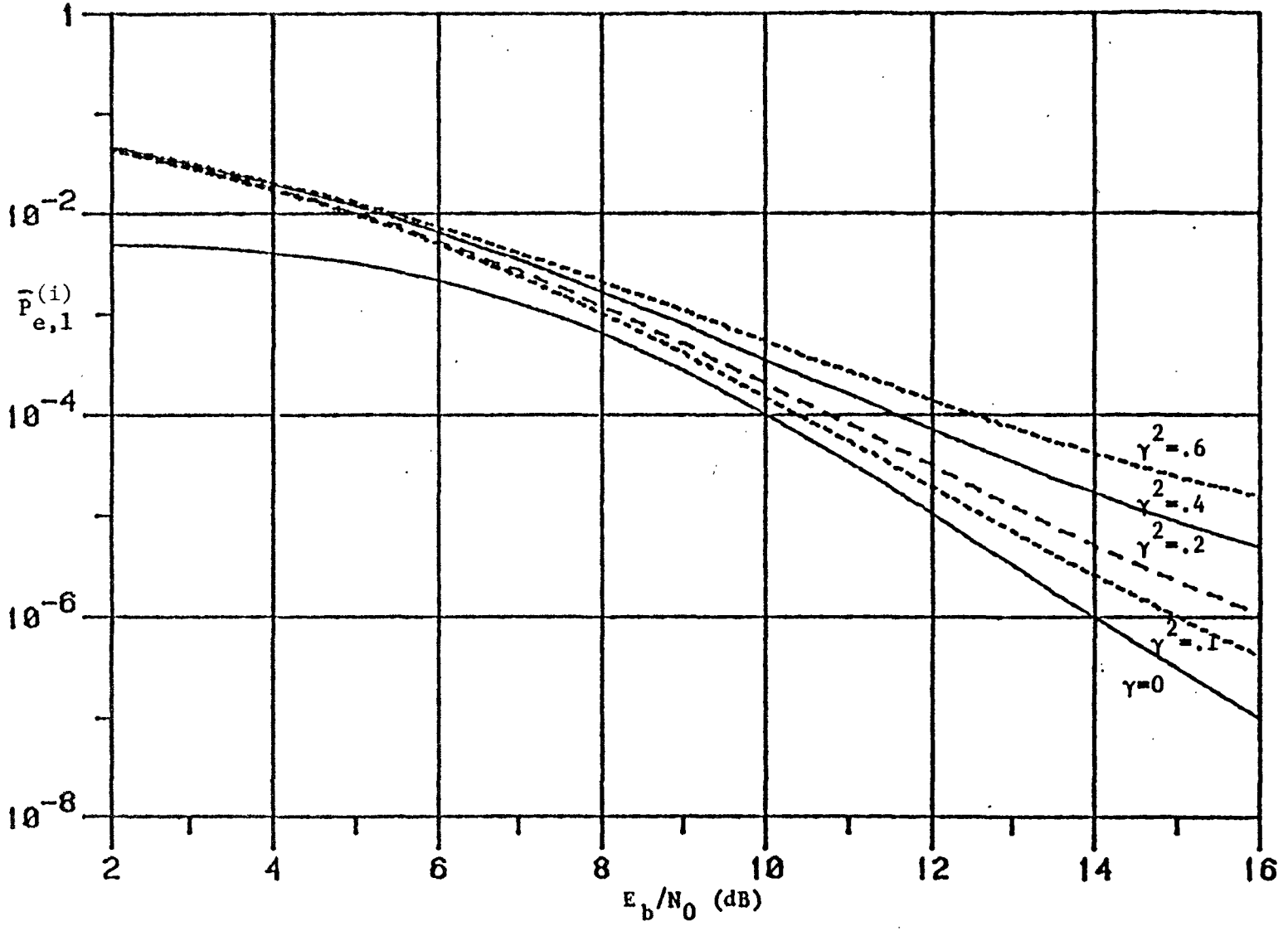

Figure 3. Probability of error for a DS/SSMA system with frequency-selective fading (K = 3, N = 31, and γ^2 = 0., .1, .2, .4, and .6)