

Uncertainty Results for the Probability of Raid Annihilation Measure

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ABSTRACT: *Probability of Raid Annihilation, P_{RA} , is the Navy's Measure of a single ship with its combat systems to detect, control, engage and defeat a specified raid of threats within a specified level of probability in an operational environment. Threat performance and combat system performance both can vary significantly with natural environment conditions so the P_{RA} federation incorporates these effects.*

In an earlier paper (01F-SIW-077), uncertainty in the P_{RA} Measure or federation outcome was linked mathematically to uncertainty in the implemented natural environment representation. This procedure was based on the detailed documentation and analysis provided by the P_{RA} Environment Concept Model (ECM). In this paper uncertainty results are presented using this procedure. The uncertainty assessment includes a reasonable range of uncertainties in the individual environment parameters but maintains consistency within the environment representation by means of the ECM. Maintaining consistency is required because the uncertainty analysis shows that the uncertainty in P_{RA} Measure depends not only on the magnitude of uncertainty in an environment parameter but also on the overall environment representation supplied. The uncertainty analysis not only provides an assessment of how good the P_{RA} Measure is but also provides information on the best-cost benefit to improvement in the P_{RA} measure.

1. Introduction

This paper continues the development of a general methodology designed to assess uncertainties in simulations with the Probability of Raid Annihilation (P_{RA}) Federation system as a specific example. The uncertainty results shown here are for the specific link between the P_{RA} Measure and the atmospheric temperature as used in one of the ship's radars. The effect that the natural environment, specifically the atmospheric temperature, can have on the P_{RA} Measure is clearly demonstrated. Section 2, Background, provides a summary of the development to date, including a brief description of how the P_{RA} Environment Concept Model (ECM) can be used to assess the uncertainties in the P_{RA} Measure of Effectiveness. Section 3, the P_{RA} Federation, explains the simplified form of the P_{RA} ECM used here, the types of uncertainties as well as the calculations and results. Conclusions and ongoing research are summarized in Section 4.

2. Background

The Probability of Raid Annihilation (P_{RA}) is defined as the ability of a stand-alone ship to defeat a raid of anti-ship missile threats [9]. The ability to evaluate this capability is made difficult because it is not practical to Measure this probability directly. Therefore, the evaluation process is a combination of live tests and simulations. This amalgamation of various contributing elements contains a number of uncertainties. These uncertainties are the result of assumptions and constraints necessary to create a relevant model. The interaction among these components in the real world is quite complex; therefore, a method to document and combine them in the simulation is needed. The constructs available in the Unified Modeling Language (UML) are appropriate for this task.

An earlier paper [4] described a method of representing these uncertainties in a system using UML as imple-

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mented in Rational Rose. That paper reviewed a process of system documentation based on the Environment Concept Model (ECM) [2], [3] and [6]. It then proceeded to describe the use of the ECM to document and evaluate the uncertainties contained in a simulation. The initial work that led to this paper was focused on documenting information about the natural environment. This investigation quickly yielded the not surprising result that the environment cannot be considered independently from the remainder of the systems of interest. Therefore a system model of the P_{RA} evaluation process was developed. This model included the natural environment as an integral component of the system. Because of this approach the relative importance of all components of P_{RA} can be evaluated in various situations. These situations include various natural environment conditions, such as different types of radar ducting.

The current paper demonstrates, through specific examples, how changes in the temperature profile lead to changes in the P_{RA} . This is but one trace though the uncertainty description. Due to the capability to document the interaction of components using the Rational Rose

implementation of UML, the example presented here is extensible and is the subject of work in progress.

3. Probability of Raid Annihilation Federation

3.1 P_{RA} Federation and the Environment

A simplified view of the P_{RA} Federation is shown in Figure 1. The single ship is defending itself against a single threat or multiple threats. The ship defense behavior is considered in three components: detect, control and engage. Threat behavior and ship behavior, including detect, control, and engage, are all affected by the natural environment. The natural environment includes the fundamental parameters such as temperature and pressure as well as effects such as sea clutter and RF propagation. The environment database for a particular runtime application includes the numerical values actually used. The P_{RA} Measure will depend on the implemented environment parameters and effects as well as the numerical values used.

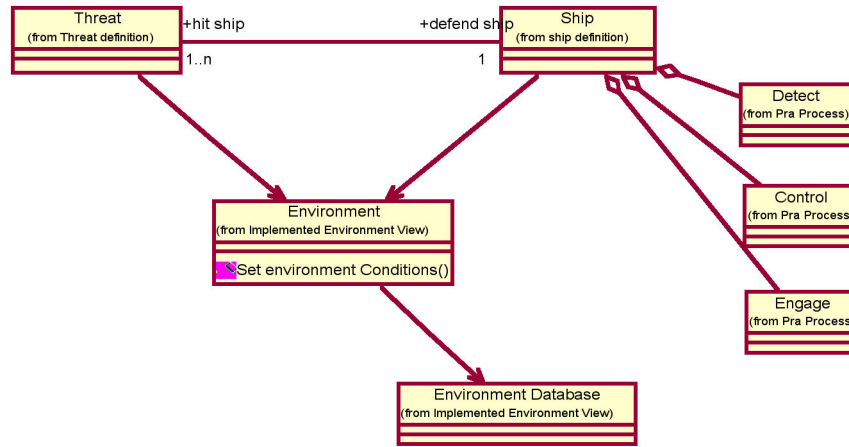


Figure 1

The P_{RA} Measure is a complex function of the Probabilities for Detection, Control and Engagement. A typical relation is given as:

$$P_{RA} = \prod_{i=1}^T [1 - \prod_{j=1}^W (1 - PD_{ij} \bullet PC_{ij} \bullet PE_{ij})] \quad (\text{Eq. 1})$$

where i is the threat number in the raid, T is the total number of threats in the raid, j is the weapon number and W is the total number of weapons in the combat suite and

where

- PD_{ij} = Probability that sensor j detects threat i at sufficient range,
- PC_{ij} = Probability that combat system j functions properly to conduct engagement on the threat i ,
- PE_{ij} = Probability that weapon j killed or neutralized threat i .

The actions during a runtime application for the P_{RA} evaluation are illustrated in Figure 2. For a given threat, there is no overlap in time for the sequence of actions.

Each step must be completed before the next can be started. The threat must be launched before it can be detected. The threat must be detected before it can be controlled and it must be controlled before it can be engaged. For a successful engagement, the ship defenses must either destroy the threat, a hard kill, or jam its sensors so

it misses the ship, a soft kill. For a given runtime application then, either the threat is eliminated, hard or soft kill, or it hits the ship. The probabilities of detection, control and engagement, however, are much more complex functions that this activity diagram suggests.

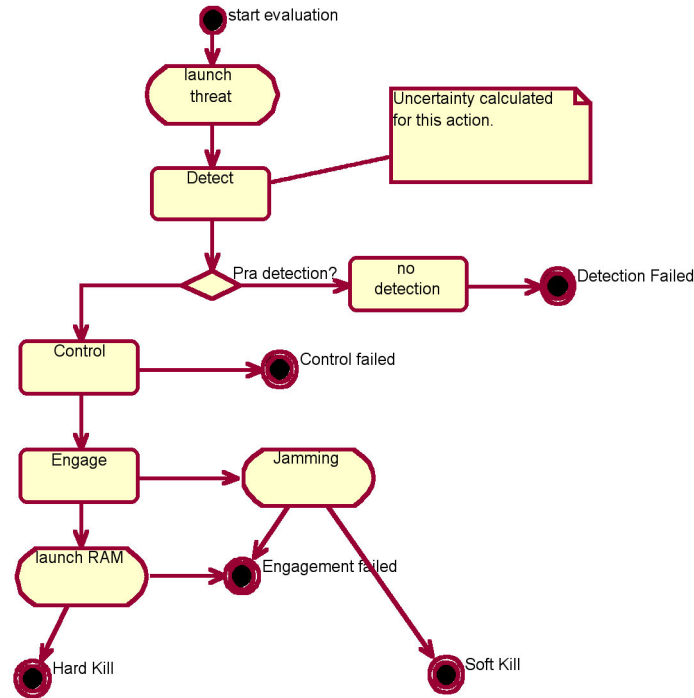


Figure 2

Detection of the threat depends on the sensors available on the ship. One of these sensors is the SPS-48E radar. As shown in Figure 3, the radar performance, in turn, depends on radar refractivity, atmospheric loss and sea clutter. For the uncertainty results shown here, atmospheric loss and sea clutter will be held fixed. Only uncertainty in the refractivity is considered and only that due to uncertainty in the temperature profile as it affects ducting or bending of the electromagnetic waves will be examined. In general, the uncertainty in the magnitude of the temperature is less important than the uncertainty in the location and size of the *gradients in the temperature profile* for it is the *gradients in refractivity* that determine the ducting.

In summary, the P_{RA} Measure depends on the atmospheric temperature so any uncertainty in the atmospheric temperature causes a corresponding uncertainty in the P_{RA} Measure. In particular, the uncertainty in the temperature affects the uncertainty in the modified refractivity and therefore in the ducting of the radar signal. The uncertainty in the ducting affects the uncertainty in the signal to noise ratio and therefore in the Probability of Detection. And, finally, the uncertainty in the Probability of Detection affects the uncertainty in the P_{RA} Measure. Many paths link the atmospheric temperature to the P_{RA} Measure but the selection of one path is done both for presentation clarity and to permit the careful examination of each separate link in the entire P_{RA} Federation.

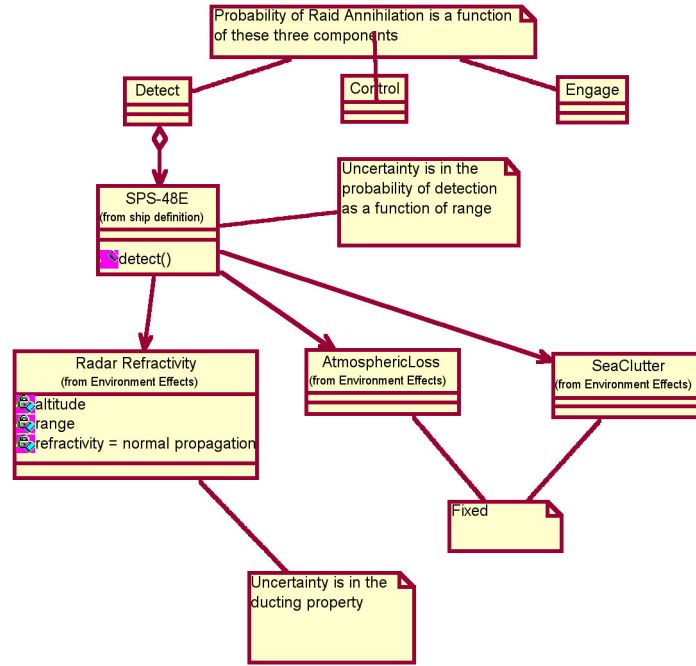


Figure 3

3.2 Uncertainty

The Fall SIW paper [4] discussed two causes of uncertainty in the environment: observation or instrument error and representation or model limitations. These two types of uncertainty exist in each environment scenario and therefore in each simulation run to calculate the P_{RA} measure. However, there is another source of uncertainty in the P_{RA} measure. The P_{RA} Measure derived from a P_{RA} Federation run will vary with the environment scenario that is supplied for that run. The environment scenario varies considerably with location and time. Each P_{RA} Federation run will produce a single number for the P_{RA} Measure but the P_{RA} Measure can be different for each environment scenario used. The **range** of environment scenarios used in the P_{RA} Federation runs will determine the **range** in values for the P_{RA} measure.

This use of the term uncertainty to cover variability as well as inherent errors is not universal. The important issue to assess is how well the P_{RA} Measure can be determined. And that assessment should include an evaluation of the range of conditions under which a ship will be operating when it defends itself against a threat.

To illustrate the link between the range in the environment scenarios and the range in the value of the P_{RA} Federation, consider the atmospheric temperature. The atmospheric temperature varies with location and time. The **range** of temperature **variation** depends on the location, time period and local weather conditions. For exam-

ple, the day-night variation is near-surface temperature for many geographical areas rarely exceeds 30 °F unless a strong front moves through the region. Note that the **range** of day-night variation decreases with increasing altitude, affecting the gradients in temperature. What is the corresponding range in the P_{RA} measure? The procedure to answer that question is presented in Subsection 3.3.

The determination of the uncertainties in the P_{RA} Measure requires a full analysis of the possible sources of uncertainty in the P_{RA} Federation. The first step will be to analyze the uncertainties due to the environment representation. To illustrate how this will be done, this paper provides some preliminary results showing how the temperature profile alters the P_{RA} Measure through the Probability of Detection for a generic radar.

3.3 Calculations and Results

For these calculations a single threat is assumed. Also, the Probability of Detection PD is assumed to be due to only one radar while the Probability of Contact and the Probability of Engagement PE or kill will both be set to one. With these assumptions, Eq. 1 becomes

$$P_{RA} = PD_{11} \quad (\text{Eq. 2})$$

The Probability of Detection for radar is a function of the radar signal to noise ratio, P_r / N_s , defined as follows:

$$\frac{P_r}{N_s} = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 K T B N_f L_s L_p} \quad (\text{Eq. 3})$$

where

- | | | |
|-----------|---|--------------------------------------|
| P_t | = | transmitted power |
| G_t | = | transmitting antenna gain |
| G_r | = | receiving antenna gain |
| λ | = | wavelength |
| σ | = | radar-target cross-section of target |
| R | = | radar range |
| K | = | Boltzmann's constant |
| T | = | temperature |
| B | = | receiver noise bandwidth |
| N_f | = | receiver noise figure |
| L_s | = | system loss |
| L_p | = | two-way propagation loss |

This version of the radar signal to noise ratio is a general one. The specific form changes with the type of radar, the type of target, and the intervening medium as well as the assumptions and approximations made. The intervening medium for the P_{RA} Federation is the atmosphere, including the gases, water vapor, ice crystals and any other particulates. The atmosphere may absorb, scatter, reflect and bend electromagnetic waves sent by the transmitter.

In order to assess the uncertainty or range in the P_{RA} Measure due to the range of possible environmental scenarios, it will be necessary to perform the complex calculations for refractivity, Probability of Detection and Signal-to-Noise ratio for a wide range of environment scenarios. Fortunately as series of tactical decisions aids involving radar and radar performance have been developed by the Navy.

Due to the importance of radars and radar performance to the Navy's fleet operations, Space and Naval Warfare Center, San Diego, has developed the Advanced Refractive Effects Prediction System (AREPS) [7]. For a given radar, the AREPS software calculates and displays refractivity, modified refractivity and gradient ducts as a function of height. The software handles range-dependent and range-independent refractivity with various ocean and terrain path options. AREPS also calculates and displays the Probability of Detection, signal-to-noise ratio and propagation loss versus height, range and bearing from the radar transmitter. The AREPS software offers considerable flexibility to the user in setting the parameters for the radar transmitter as well as the natural environment input. (Complete details about the AREPS software, including the composite propagation model used, are found in [8]. The AREPS software is available as a CD-ROM or can be downloaded from <http://sunspot.spawar.navy.mil>.)

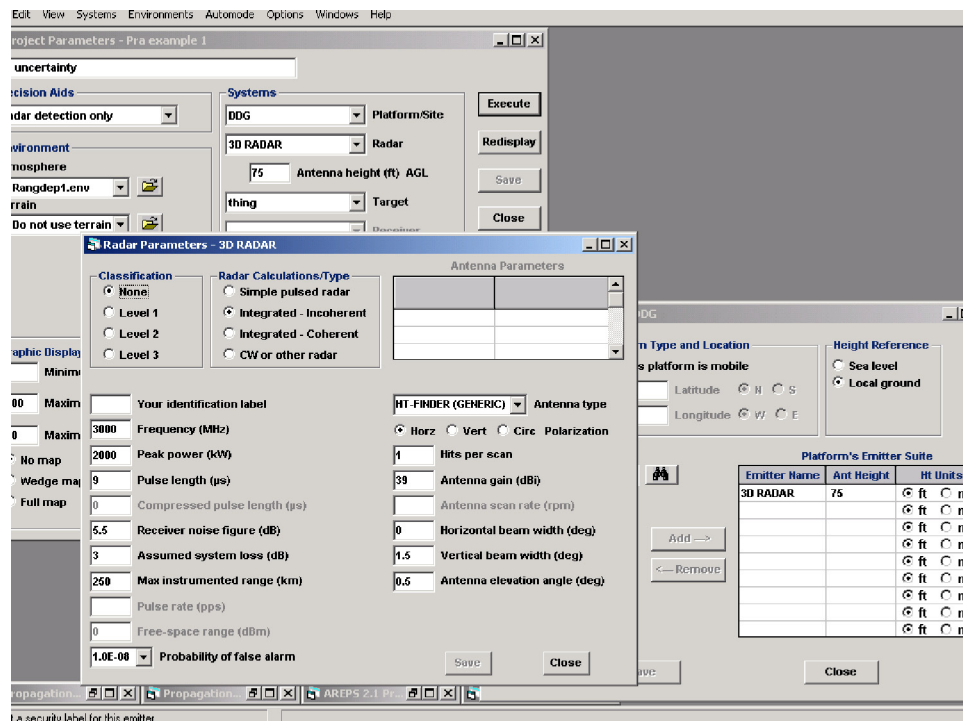


Figure 4

For this paper, a generic radar is used, not one with the exact characteristics of the SPS-48E. The AREPS screen

for the radar parameters is shown in Figure 4. Note the number of parameters that have to be specified for the

calculations. Some parameters such as antenna height are fixed for a given radar and ship. Others such as frequency may be set to different values for a given radar but are fixed in this paper. Full evaluation of the P_{RA} Measure must consider the range possible for each parameter as well as the uncertainty in the value set for the parameter but the uncertainty in some parameters is much greater than in others. For example, the uncertainty in the set frequency is much less than that in the assumed system loss. Further discussion on the radar parameters and options can be found in [8], pp. 57 – 67.

The AREPS software supports a wide range of options for the environment, including climatology, World Meteorological Organization data, and other sources using Fleet Numerical Meteorological and Oceanography Center data. In addition, the user may provide custom environment data to study specific conditions or phenomena such as ducting. Options and detailed guidance on handling the environment in a consistent manner are provided in [8], pp 89 – 150.

For microwave frequencies and below, the refractivity N is given ([1] and [8]) as

$$N = \frac{77.6p}{T} + \frac{3.73 \cdot 10^{-5}e}{T^2} \quad (\text{Eq. 4})$$

where

- e = the partial pressure of water vapor in millibars,
- p = barometric pressure in millibars, and
- T = absolute temperature in degrees Kelvin.

The barometric pressure p and the partial pressure of water vapor e decrease with height while the temperature T for a *standard atmosphere* [10] decreases more slowly with height in the troposphere. However, the, the linear decrease of temperature with height given in standard atmosphere is a long term average. Individual observations show that the lapse rate can vary considerably from that of a *standard atmosphere* and frequently reverses sign when the atmospheric temperature actually increases with height. Such an increase is referred to as a temperature inversion. Temperature inversions are common when there is strong nighttime surface cooling.

To study ducting, the modified refractivity M is used. For the altitude h in feet,

$$M = N + 0.048h \quad (\text{Eq. 5})$$

Both the refractivity and modified refractivity for a standard atmosphere are shown in Figure 5. The gradient conditions and location of ducts are shown as a function of height on the right hand side of the Figure 5.

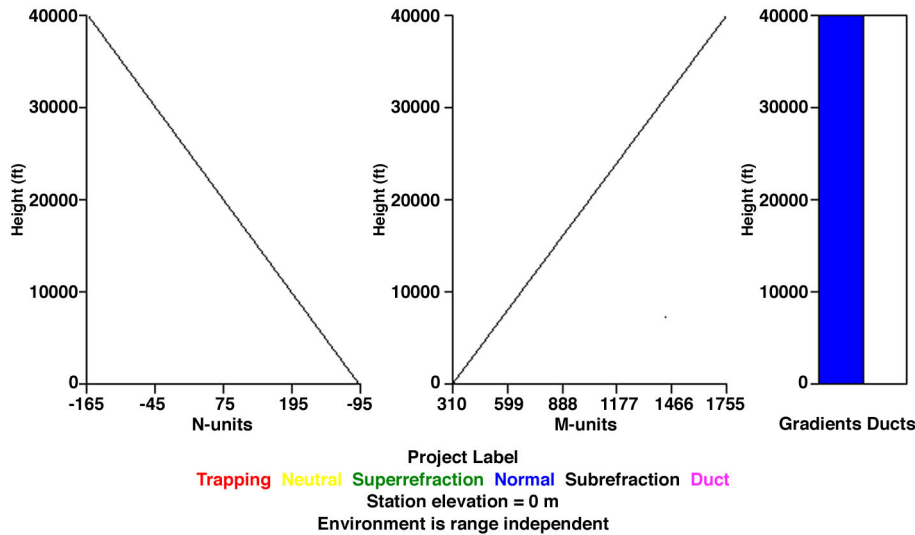


Figure 5

The refractivity N or modified refractivity M is a measure of the bending that occurs as the electromagnetic waves travel through the atmosphere. The gradient of modified refractivity determines whether ducting is present. If the

M gradient is negative then trapping occurs. For a *standard atmosphere* the M gradient is 118 M/km, which is considered to be within normal refractive conditions with no ducts present.

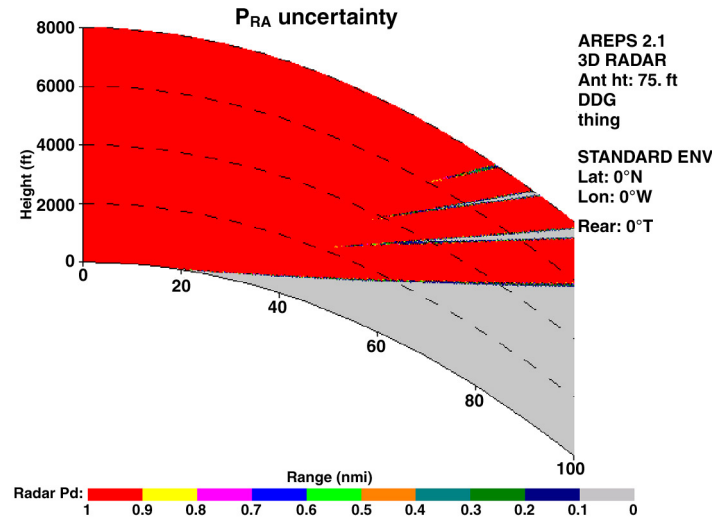


Figure 6

The corresponding Probability of Detection PD for the modified refractivity plotted in Figure 5 is shown in Figure 6. The modified refractivity is assumed to be range independent. That is, the pressure p , temperature T and partial pressure of water vapor e do not vary horizontally, only vertically. Not surprisingly, for a given height, the PD decreases with increasing range. For ranges of less than 20 nmi the PD is greater than 90% at all altitudes. Beyond 20 nmi there is a sharp demarcation between a PD of $> 90\%$ and a PD of $< 10\%$ with the height of demarcation increasing with increasing range. A target such as an aircraft that wished to avoid detection should fly in at as low an altitude as possible to delay or avoid

detection. This tactic is well known as “flying under the radar” and may be feasible if there is no surface ducting.

Surface cooling, among other conditions, can lead to conditions where the atmospheric temperature increases with height immediately above the earth’s surface. It is also common for the temperature lapse rate to change magnitude fairly sharply, another condition that can lead to ducting. The modified refractivity M and the associated ducting are shown for a typical atmospheric profile in Figure 7. The surface ducting is due to the increase of temperature at the surface and the upper level duct is due to the change in the lapse rate and hence the M value.

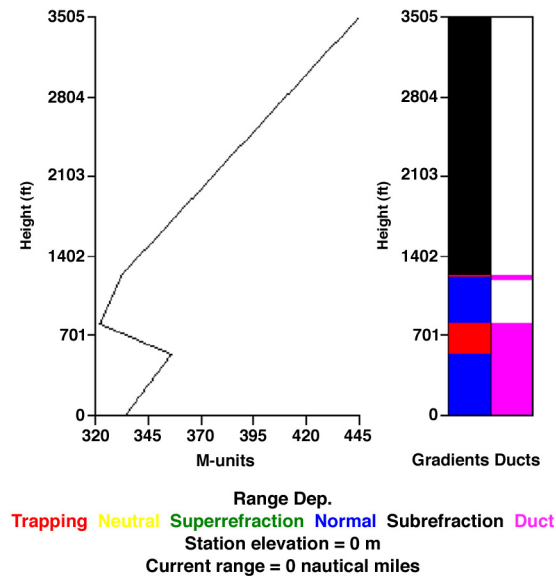


Figure 7

How does the presence of a surface duct alter the PD ? The PD values shown in Figure 8 correspond to the M values shown in Figure 7. The effect of the surface duct is quite striking. At and near the surface, the PD is now greater than 90% out to 100 nmi. A threat can no longer slip in “below the radar.”

The two examples provided here are indicative how different but common values in the temperature profile can significantly change the PD and hence the P_{RA} measure. However, the situation is even more complex than illus-

trated. For these two cases, the assumption has been made that the environment is range-independent, that is, the temperature profile does not change with horizontal location. How good is this assumption? Consider the results of a field experiment to measure directly the change in the modified refractivity M from near shore to 30 nm offshore. (Figure 9.) The position and size of the surface duct, as indicated by the regions where M decreases with height, change considerably over 30 nmi. This range-dependence is being evaluated in ongoing work.

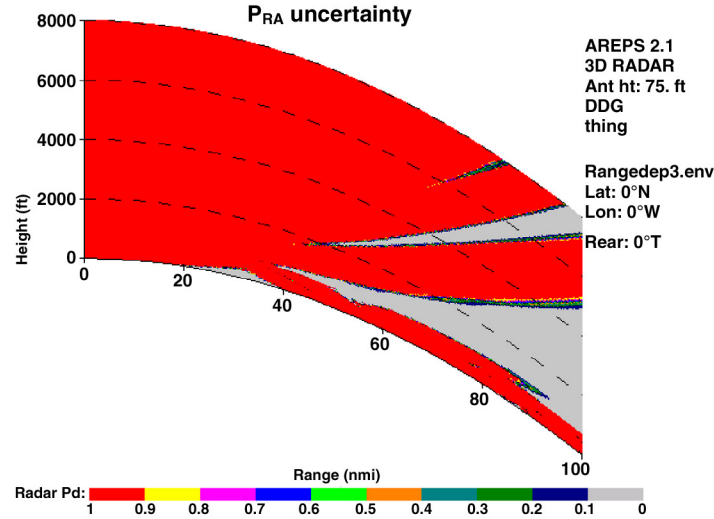


Figure 8

Incorporating realistic environment representations is sometimes viewed as degrading the overall performance of the platforms and weapons in simulations. This example shows the contrary. The Standard Atmosphere is an average of common conditions; individual profiles are generally more complex and frequently include ducts. The Standard Atmosphere, however, actually degrades the radar performance as compared to that of a profile with ducting. Tactics are developed with consideration of how the environment can be used as an asset. Simulations should reflect this operational consideration.

The complexity of simulations frequently result in the environment effects being ignored or handled with climatology or a standard atmosphere. This approach is justified by the argument that detailed environment information is not available for operations anyway. In fact, considerable research is underway for methods to obtain detailed, high quality meteorological measurements onboard ship. For details on one promising approach, involving the SPY-1 radar and the Tactical Environmental Processor (TEP), see [5].

Rather than ignoring the critically important environmental effects, simulation developers can incorporate the available techniques to handle the environment efficiently and effectively. By assessing how the environment affects uncertainty in the P_{RA} Measure, the P_{RA} Federation can be used to determine the best cost benefit to improve the P_{RA} Measure.

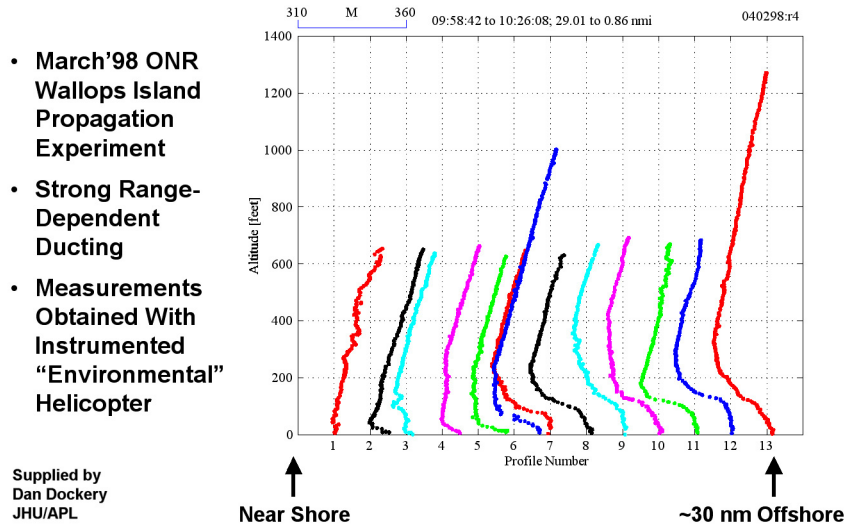
5. Conclusions

The Environment Concept Model (ECM) for the Probability of Raid Annihilation (P_{RA}), developed to capture all components of the P_{RA} Federation, is being used to develop requirements, assist in the VV&A process, and to assess uncertainties in the P_{RA} Measure. In this paper a simplified version of the P_{RA} ECM is used to show how the information captured is used to assess how the P_{RA} Measure is affected by surface temperature and the associated ducting conditions.

The simple examples provided here show how strongly common variability in the atmospheric temperature profile affects radar performance. This dependency is well

known. What is new here is the methodology to assess how this variability and dependency affects the results from simulations. The ability to assess the uncertainty in simulation results will greatly improve their immediate usefulness and aid in improving their value in the future.

Work is underway to assess the effect of a broad range of meteorological conditions on the P_{RA} Measure. These detailed results will be presented at Fall SIW 2002.



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