MAGNETIC ACTUATOR IN SPACE AND APPLICATION FOR HIGH PRECISION FORMATION FLYING

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Electromagnetic (EM) actuators in space applications are not a new idea, but they are most of the time associated to low Earth orbit missions, where the on-board magnetic moment interacts with the Earth magnetic field. More recently EM actuators have been studied in the context of *formation flying* as a way to generate inter-spacecraft force and torque in order to control the formation geometry (relative position and attitude of the spacecrafts). With respect to other possible actuators (like FEEP and cold-gas thrusters), EM actuators do not require propellant (hence ensuring longer lifetime) and do not cause contamination problems, but are effective only in a limited range of distances (due to the magnetic field decreasing as $1/r^3$).

In this paper we consider a setup where a spacecraft (called the *hub*) generates the magnetic field, while the other spacecrafts (called the *flyers*) modify their on-board magnetic moment in order to generate the desired force and torque. We show that if the magnetic field generated by the hub is constant, it is not possible to generate *any* combination of force and torque (there are forbidden *directions*, independently of the magnitude of the desired force and torque) and we present a possible solution based on a rotating field. The main idea is to have a *time-varying* magnetic field generated by the hub (chosen as a rotating magnetic dipole) and time-varying magnetic moments on the flyers. The variation law on each flyer's magnetic moment is computed in order to obtain the desired force and torque *on average* (over one rotation period). With such approach, the *instantaneous* force and torque may differ from the desired ones, these discrepancies being "filtered out" by the spacecraft inertia.

Finally we present a preliminary sizing of the actuators based on classical and superconducting coils, for typical mission requirements : this underlines the technological challenges to be faced in order to build the proposed kind of actuator.

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1 Magnetic actuators basics

1.1 Notations

A given vector quantity will be denoted in bold (**x**) in the text and with an overhead arrow in the equations. Its module will be plain (x) in the text and in the equations and the corresponding unitary vector will be noted $\mathbf{u_x}$ (with an overhead arrow in the equations). Dot and cross products are noted respectively · and ×. The vacuum permeability is noted μ_0 and is $4p10^{-7}$ H/m.

1.2 Dipolar field and interactions

Considering a dipolar magnetic moment **M** located in the origin of a given reference frame, the magnetic field at the point **r** is given by :

$$\vec{B}(\vec{M},\vec{r}) = \frac{m_0}{4p}\frac{1}{r^5}\left[3(\vec{M}\cdot\vec{r})\vec{r} - r^2\vec{M}\right] = \frac{m_0}{4p}\frac{M}{r^3}\left[3(\vec{u}_M\cdot\vec{u}_r)\vec{u}_r - \vec{u}_M\right]$$

If a second dipolar magnetic moment is located at **r**, the interaction force and torque are :

$$\begin{aligned}
&\vec{T}(\vec{m},\vec{M},\vec{r}) = \vec{m}\times\vec{B}(\vec{M},\vec{r})\\
&\vec{F}(\vec{m},\vec{M},\vec{r}) = grad(\vec{m}\cdot\vec{B}(\vec{M},\vec{r})) = (\vec{m}\cdot grad)\vec{B}(\vec{M},\vec{r})
\end{aligned}$$

Developing the expressions gives :

$$\begin{aligned}
\vec{T} &= \frac{m_0}{4p}\frac{3(\vec{M}\cdot\vec{r})(\vec{m}\times\vec{r}) - r^2(\vec{m}\times\vec{M})}{r^5}\\
&= \frac{m_0}{4p}\frac{Mm}{r^3}\left[\vec{u}_m\times\left(3(\vec{u}_M\cdot\vec{u}_r)\vec{u}_r - \vec{u}_M\right)\right]\\
\vec{F} &= \frac{m_0}{4p}\frac{r^2(\vec{M}\cdot\vec{r})\vec{m} + r^2(\vec{m}\cdot\vec{r})\vec{M} + [r^2(\vec{M}\cdot\vec{m}) - 5(\vec{M}\cdot\vec{r})(\vec{m}\cdot\vec{r})]\vec{r}}{r^7}\\
&= \frac{3m_0}{4p}\frac{Mm}{r^4}\left[(\vec{u}_M\cdot\vec{u}_r)\vec{u}_m + (\vec{u}_m\cdot\vec{u}_r)\vec{u}_M + \left((\vec{u}_M\cdot\vec{u}_m) - 5(\vec{u}_M\cdot\vec{u}_r)(\vec{u}_m\cdot\vec{u}_r)\right)\vec{u}_r\right]
\end{aligned}$$

It is obvious that given a value of **B** it is not always possible to chose a value of **m** that generates any desired value of **T** and/or **F**. When **M** and **m** are oriented as **r**, **B** is oriented as **r** too, the torque is null and the module of the force (repulsive if the directions of **M** and **m** are opposite each other, attractive if they are equal) is :

$$F = \frac{3m_0}{2p}\frac{Mm}{r^4}$$

1.3 Two space applications

Magneto torque bars (MTBs) are frequently used to control the attitude of low Earth orbit satellites : the on-board magnetic moment is modified by a control law and interacts with the Earth's magnetic field. The interaction torque is used to control the attitude of the satellite (or to dump the rotation speed of the satellite after separation from the launcher). The magnetic field seen by the satellite is time-varying (due to the relative motion between the Earth and the satellite) and different control laws take this into account in different ways.

In the NASA *Terrestrial Planet Finder* (TPF) mission, n satellites are aligned and the entire formation is rotating around the central point : radial forces between satellites are necessary to compensate the centrifugal force and to ensure relative positioning. Magnetic actuators have been proposed for such task and dedicated control laws have been studied. It has been shown that the magnetic actuators could also be used to *spin* the formation : in this case however, the magnetic moments are not co-linear with respect to the relative position vectors, hence each spacecraft sees a disturbance torque that must be compensated to avoid attitude modifications.

2 Proposed solution

2.1 Concept

The proposed solution consists in a hub satellite generating a time-varying magnetic field and N flyer satellites. The magnetic moment of each flyer is computed in order to produce the desired torque and force on average, knowing the variation law of the magnetic field generated by the hub and the relative position of the flyer with respect to the hub.

In this paper we focus on one of the simplest implementation : the hub is located in the origin of the reference frame and carries a magnetic momentum **M(t)**, constant in module but rotating around the origin at constant rotation rate ? , while a single flyer located at **r** carries the magnetic moment **m(t)**, varying in a *synchronous* way. In Cartesian coordinates :

$$\vec{M}(t) = M\begin{bmatrix}\cos(wt)\\ \sin(wt)\\ 0\end{bmatrix} = \vec{M}(M,w,t)\ ,\ \ \vec{m}(t) = \begin{bmatrix}mc_x\cos(wt)+ms_x\sin(wt)\\ mc_y\cos(wt)+ms_y\sin(wt)\\ mc_z\cos(wt)+ms_z\sin(wt)\end{bmatrix} = \vec{m}(\vec{p},w,t)$$

The parameters of the variation law of the flyer's moment are $mc_{x/y/z}$ ans $ms_{x/y/z}$ and are noted **p** in the rest of the document. Plugging the expressions of **M(t)** and **m(t)** in the expressions of **B**, **T** and **F** we get :

$$\begin{aligned}\vec{B}(\vec{r},\vec{M}(t)) &= \frac{m_0}{4p}\frac{1}{r^5}\left[3(\vec{M}(t)\cdot\vec{r})\vec{r} - r^2\vec{M}(t)\right] = \vec{B}(\vec{r},M,w,t)\\ \vec{T}(\vec{m}(t),\vec{B}(t)) &= \vec{m}(t)\times\vec{B}(t) = \vec{T}(\vec{p},\vec{r},M,w,t)\\ \vec{F}(\vec{m}(t),\vec{B}(t)) &= grad(\vec{m}(t)\cdot\vec{B}(t)) = \vec{F}(\vec{p},\vec{r},M,w,t)\end{aligned}$$

The above *instantaneous* torque and force are then analytically integrated over one period to obtain the expression of their average value :

$$\begin{aligned}\vec{T}_A &= \frac{1}{T}\int_0^T \vec{T}(\vec{p},\vec{r},M,w,t)dt = \vec{T}_A(\vec{p},\vec{r},M)\\ \vec{F}_A &= \frac{1}{T}\int_0^T \vec{F}(\vec{p},\vec{r},M,w,t)dt = \vec{F}_A(\vec{p},\vec{r},M)\end{aligned}$$

It turns out that the above expressions of the *average* torque and force can be rewritten as :

$$\begin{bmatrix} \vec{T}_A \\ \vec{F}_A \end{bmatrix} = C(\vec{r}, M) \cdot \vec{p}$$

where C(**r**,M) is a 6 by 6 matrix depending only on **r** and M. It is at this point clear that the above relation can be used to compute **p** given an *average required* torque and force :

$$\vec{p}(t) = C^{-1}(\vec{r}(t), M) \cdot \begin{bmatrix} \vec{T}_{AR}(t) \\ \vec{F}_{AR}(t) \end{bmatrix}$$

The above relationship is only valid where C is non-singular and one can compute (the Cartesian components of **r** being x, y and z) :

$$\det(C(\vec{r}, M)) = \frac{27M^6(x^2+y^2)z}{64(x^2+y^2+z^2)^{11}}$$

The above expressions allows to deduce the singular configurations, namely all the points in the XY plane (the plane in which lies the **M(t)** vector) and all the points of the Z axis (the axis of rotation of **M(t)**). Close to these singular configurations there are combinations of required average force and torque that needs an extremely large magnetic momentum to be obtained.

2.2 Remarks

In presence of multiple flyers the situation does not change too much, but it is necessary to avoid inter-flyer magnetic interaction. Since the maximum interaction force/couple depends on the product M·m, it is possible to put small magnetic moments on the flyers and a big one on the hub (this assumes the flyer-flyer distance to be comparable to the hub-flyer one).

Notice also that the hub will see the sum over all the flyers of the interaction torque and force and it will move and rotate accordingly : inertial actuators (like thrusters) are hence required to compensate these movements, but they can be much more coarse since the only objective is to stay away from singular configurations. This because the mission requirement is to maintain relative attitude and positioning of the flyers, the hub being not part of the scientific instrument of the mission.

All the computations rely on the exact knowledge of the magnetic field : the dipolar approximation used is accurate only in *far field*, i.e. when r is much bigger than the characteristic length of the magnetic dipole that generates the field. Otherwise the average torque/force may be affected by an error (that could be compensated by the outer control loop). Notice also that the system requires the knowledge of hub-flyer relative position and attitude and a synchronisation (the flyer's magnetic moment law must be synchronised to the hub magnetic moment rotation).

3 Preliminary sizing

In the following paragraphs we will show some very preliminary computations in order to highlight the technological challenges involved in the realisation of the proposed solution.

3.1 Overall sizing

Supposing that the magnetic moment of the flyer is generated by 3 orthogonal moments, we focus on one of them (let's take the x axis) and we obtain the following relations between the 2-norm of the averaged desired torque/force and the x-axis magnetic momentum of the flyer :

$$\left\| \begin{bmatrix} mc_x \\ ms_x \end{bmatrix} \right\|_2 = \left\| \left(C(\vec{r}, M)^{-1} \right)_X \cdot \begin{bmatrix} \vec{T}_{AD} \\ \vec{F}_{AD} \end{bmatrix} \right\|_2 \leq \left\| \left(C(\vec{r}, M)^{-1} \right)_X \right\|_2 \cdot \left\| \begin{bmatrix} \vec{T}_{AD} \\ \vec{F}_{AD} \end{bmatrix} \right\|_2$$

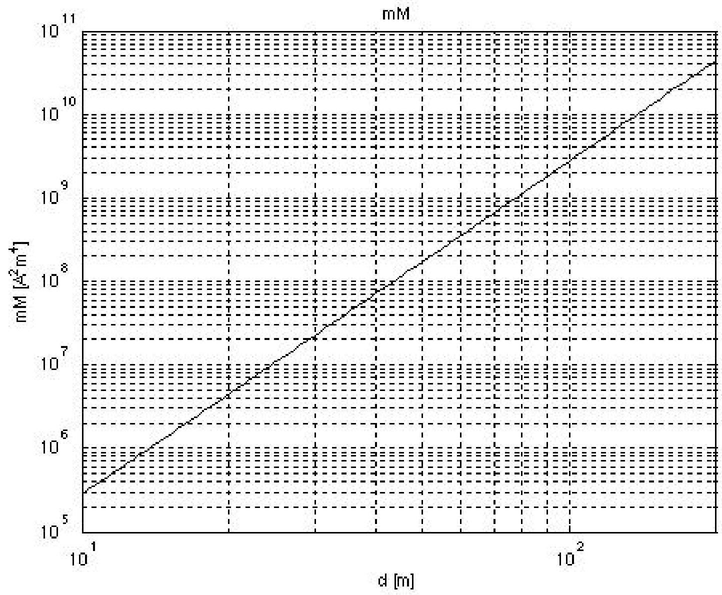
The C(**r**,M) matrix is affine in M (C(**r**,M) = M·C(**r**,1)) : the first quantity we want to size is the product m·M with respect to the hub-flyer distance and the required force/torque. As stated before; when the two magnetic moments are aligned, the interaction force module is :

$$F = \frac{3m_0}{2p} \frac{Mm}{r^4}$$

This value is an upper bound with respect to what we can obtain with our solution because we are dealing with non-aligned moments (at least most of the time) and because we aim at producing force and torque. In fact, we can still use the above formula introducing a correction factor C_F :

$$F / T = \frac{3m_0}{2p} \frac{Mm}{r^4} \frac{1}{C_F^2}$$

The correction factor depends on the distance and on the flyer elevation angle, but for a given angle is pretty constant with respect to the distance (it becomes large in the near field, but there our computations are no longer valid) : we can consider it to be in the [4.0-5.5] range for an elevation angle in [30°-75°], the minimum (and hence the optimum) being roughly located at 60°.



As an example, we choose a flyer located at 50m from the hub and at an elevation angle of 60°. We would like to obtain an average torque and force vector with a 2-norm equal to 1^{E}-6. Our computations give a value of the m·M product of roughly $1.^{E}8$ $(A\cdot m^2)^2$ and supposing the M/m ratio equal to 100, we obtain M = $1.^{E}5$ $A\cdot m^2$ and m = $1.^{E}3$ $A\cdot m^2$.

3.2 Flyer magnetic moment sizing

The magnetic moment of the flyer is smaller and is time varying : because of that we choose "classical" coils (and not superconducting ones), with or without a magnetic core.

3.2.1 Flat coil without ferromagnetic core

For a single coil of radius R and with a current I obtained as the product of the coil wire section by the current density J ($I = J\cdot S_C$) the magnetic moment is :

$$M = (pR^2)(JS_C)$$

In fact, we suppose that the coil wire with section S_C is built with N thinner wires with section S_W and a filling ratio of 1 (optimistic), so that $N = S_C/S_W$. In order to compute the inductance of the coil we use the formula of the inductance of a single current loop multiplied by N^2. Expressing all this in term of M, R, J and S_W we get :

$$S_C = \frac{M}{pR^2 J} \qquad N = \frac{S_C}{S_W} = \frac{M}{pR^2 J S_W}$$

$$R_w = 2r_r \frac{M}{RJS_W^2} \qquad M_w = 2r_m \frac{M}{RJ} \qquad P_w = 2r_r \frac{MJ}{R}$$

$$L_1 = m_0 R\left(\log(8\frac{R}{r}) - \frac{7}{4}\right) = m_0 R\left(\log\left(8pR^2\sqrt{\frac{J}{M}}\right) - \frac{7}{4}\right)$$

$$L = N^2 L_1 = \frac{M^2}{(JS_W)^2 p^2 R^3}\left(\log\left(8pR^2\sqrt{\frac{J}{M}}\right) - \frac{7}{4}\right)$$

where RW, MW and PW are respectively the wire resistance, mass and dissipated power, $?_m$ is the wire density and $?_r$ is the wire resistivity. A sizing could be the following :

M	[Am^2] :	1,00E+03	Sc	[m^2] :	3,18E-04
J	[A/m^2] :	1,00E+06	r	[m] :	1,01E-02
R	[m] :	1,00E+00	N	[-] :	1,06E+02
Wire res.	[Ohmm] :	1,70E-08	Lw	[m] :	6,67E+02
Wire den.	[Kg/m^3] :	8,90E+03	Mw	[Kg] :	1,78E+01
Wire section	[m^2] :	3,00E-06	Iw	[A] :	3,00E+00
			Rw	[Ohm] :	3,78E+00
			Pw	[W] :	3,40E+01
			L	[H] :	3,77E-02

In the presented sizing (M = $1^{E}6Am^2$), the coil radius being 1m, the coil will probably run around the spacecraft : it will be made with 100 turns of $6mm^2$ copper wire for a total length of 600m and a weight of 18Kg, for a dissipated power of 34W.

3.2.2 Tall coil with ferromagnetic core

If we introduce ferromagnetic material in the coil core the sizing is slightly different : in order to be sure to have uniform magnetisation of the core we prefer a “tall” coil. This increases the mass but also creates a stronger field in the coil due to the high magnetic permeability of the ferromagnetic material (μ_R). Now the interesting parameters to look at are the mass and volume of the magnetic core (M_C, V_C) and the number of coils per meter (n). The design parameters are M, $?_C$ (core density), B_{SAT} (saturation field in the core), μ_R, I (the current in the coils) and ß (the ratio between the coil height and the coil diameter) :

$$V_C = m_0 \frac{M}{B_{SAT}} \quad , \quad n = \frac{B_{SAT}}{m_0 m_R I}$$

$$L = \frac{B_{SAT} M}{m_R I^2} \quad , \quad R = \sqrt[3]{m_0 \frac{M}{B_{SAT}\, 2pb}}$$

M	[Am^2] :	1,00E+03
Core den.	[Kg/m^3] :	8,00E+03
Core Bsat	[T] :	2,00E+00
Mur	[H/m] :	4,00E+03
I	[A] :	1,00E+01
Beta (>2)	[-] :	5,00E+00
Wire res.	[Ohm.m] :	1,70E-08
Wire den.	[Kg/m^3] :	8,90E+03
Wire section	[m^2] :	3,00E-06

Core vol.	[m^3] :	6,28E-04
Core mass	[kg] :	5,03E+00
n	[1/m] :	3,98E+01
L	[H] :	5,00E-03
R	[m] :	2,71E-02
H	[m] :	2,71E-01

N	[-] :	1,08E+01
Lw	[m] :	1,84E+00
Mw	[Kg] :	4,92E-02
Rw	[Ohm] :	1,04E-02
Pw	[W] :	1,04E+00

In this case (M = $1^{E}3Am^2$), the magnetic core has a 5Kg mass (27cm height and 2.7cm radius) and the wire dissipated power will be 1W : with respect to the previous case, size and power consumption are much smaller (hence we can hope to produce larger M within reasonable mass/power limitations).

3.3 Hub magnetic moment sizing

For the hub magnetic moment we present here two solutions based on superconducting coils. In order to obtain the rotating field, one possibility is to have a constant magnetic moment on the hub and spin the hub itself. Another possibility is to have a rotating magnetic moment, but this seems complicate to obtain with superconducting coils.

3.3.1 Long superconducting coil

Superconducting materials allow large currents to flow without any resistance. They lose their properties above critical values of temperature, current density and magnetic field (the three quantities define a critical surface). We consider a “tall” empty cylinder (outer radius R_O, inner radius R_I). The magnetic field in the middle of the cylinder (minimum) is given by :

$$B_0 = JR_I F(a,b) \quad , \quad a = \frac{R_O}{R_I} \quad , \quad b = \frac{h}{2R_I}$$

$$F(a,b) = m_0 b \ln\left(\frac{a + \sqrt{a^2 + b^2}}{1 + \sqrt{1 + b^2}} \right)$$

Expressing everything as function of M, B_0, ß and J we get :

$$V_C = \frac{m_0 M}{B_0} \quad , \quad R_I = \sqrt[3]{\frac{m_0 M}{B_0 2pb}}$$

$$h = 2R_I b \quad , \quad a = F^{-1}\left(\frac{B_0}{JR_I}, b\right)$$

$$R_O = aR_I \quad , \quad E = \frac{1}{2} M B_0$$

M [Am^2] :	1,00E+06	Core vol. [m^3] :	3,14E-01
B0 [T] :	4,00E+00	Ri [m] :	2,92E-01
Beta (>2) [-] :	2,00E+00	H [m] :	1,17E+00
J [A/m^2] :	2,00E+09	Mag energy [J] :	2,00E+06
SC den. [Kg/m^3] :	8,00E+03	Alpha [-] :	1,01E+00
		Ro [m] :	2,94E-01
		RT [m] :	1,78E-03
		SC vol. [m^3] :	3,84E-03
		SC mass [Kg] :	3,07E+01

In this sizing (M = $1^E 6Am^2$), the cylinder has an outer radius of 294mm, an inner radius of 292mm (radial thickness is almost 2mm) and is 1.17m long, with a mass of 30Kg.

3.3.2 Flat superconducting coil

In the case of a "flat" superconducting coil, the sizing is done just as a classical coil. The only difference (on top of the fact that the resistance is 0) is that the field in the core cannot be considered uniform (as in the case of a long coil) : since it is maximum close to the "wire", we need to check that it value does not exceed the critical value of the material.

M [Am^2] :	1,00E+06	Sc [m^2] :	3,18E-03
J [A/m^2] :	1,00E+08	r [m] :	3,18E-02
R [m] :	1,00E+00	N [-] :	1,06E+03
Wire res. [Ohmm] :	0,00E+00	Lw [m] :	6,67E+03
Wire den. [Kg/m^3] :	8,90E+03	Mw [Kg] :	1,78E+02
Wire section [m^2] :	3,00E-06	Iw [A] :	3,00E+02
		Rw [Ohm] :	0,00E+00
		Pw [W] :	0,00E+00
		L1 [H] :	4,79E-06
		L [H] :	5,39E+00
		Mag energy [J] :	2,43E+05
		Alpha [-] :	1,07E+00
		Beta [-] :	6,58E-02
		B0 [T] :	5,25E-01
		Bw [T] :	2,00E+00

In this case ($M = 1^E6Am^2$) the 1m radius coil is built with 1000 turns of $3mm^2$ superconducting wire: the wire is 6Km long and weights 178Kg. The magnetic field in proximity of the wire is 2T.

4 Conclusions

The proposed actuation system is challenging : it requires relative position and attitude determination, it provides “only” *average* force and couple, it delivers instantaneous components to be filtered out by the inertia. It is also very challenging on the technological side : having large magnetic moment on-board may force the introduction of constraint in materials and on the integration phase. The use of superconducting materials also requires cryogenic technology and has many unexplored difficulties (how to start/stop the large currents circulating in the coils, safety issues, cost and total mass trade-offs). Anyway, magnetic actuation could be an alternative to other micro actuators for precision formation flying mission: their main disadvantage is the $1/r^3$ dependency of the magnetic field, limiting the range of operation in terms of force/torque and distance. But compare to other micro actuator base on thrusters they offer fully scaleable actuators by design, they don't create accommodation constraint due plume effect, contamination, charging effect, etc…