New Binary Complementary Codes Compressing a Pulse to a Width of Several Sub-pulses

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***Abstract*— In this paper, we propose new binary complementary codes, which compress a pulse to a width of several sub-pulses, and survey all combinations of the binary complementary codes with several code lengths. We are able to find a large number of binary complementary codes and complete complementary codes. Furthermore, we show that longer such complementary codes can be acquired using the expansion method shown by Golay. In addition, we show that the auto-correlation sidelobe of an individual code for the proposed binary complementary code pair is smaller than that of the conventional.**

***Index Terms*— radar, pulse compression, complementary code, complete complementary code, cross correlation**

I. Introduction

To improve the range resolution of radar while maintaining a good signal-to-noise ratio (S/N), it is necessary to decrease the pulse width and to increase the peak power. There are cases, however, in which the peak power cannot be increased, such as in phased array radars using semiconductor devices, and millimeter-wave radar. The pulse compression technique has been investigated as a remedy in such cases. However, unnecessary sidelobes are present before and after the compressed pulse, and there have been efforts to reduce those sidelobes. Applying the complementary code is one of the most effective methods to eliminate sidelobes when the Doppler effect is negligible [1].

The complementary code consists of a pair of codes, which have the property that the sum of 2 auto-correlation functions has a sidelobe level of zero. In an actual radar system, each of the coded pulses are transmitted separately in time or frequency, and they are added after each is auto-correlated at the receiver.

The first application of the complementary code was to infrared multislit spectrometry. Following that, there were attempts to apply it to the radar; in fact, the complementary code has been utilized for the mesosphere-stratosphere-troposphere (MST) radar [10],[11]. Nowadays, there are many attempts to apply it to communication technology [3]. In this study, new binary complementary codes are proposed, and their applications are considered, mainly focusing on radar.

Golay firstly proposed the binary complementary code [2] in addition to introducing the expansion methods, by which another complementary code of length $N\times2^m$ (m is integer) is constructed by a complementary code of length N. The binary complementary codes are available in lengths of 1, 2, 4, 8, 10, 16, 20, 26, 32, 40, 52, 64, 80, 100 (up to 100). However, available code lengths are limited, and the number of available codes in each length is also limited [2],[5]. Sivaswamy and Frank proposed the multi-phase complementary code [4],[5], which have more code pairs than the binary complementary code. However, the multi-phase code has the problem of the complexity of the hardware for the digital decoder.

It should be noted, however, that all of those studies consider the coded pulse that compresses the pulse to a single sub-pulse width. On the other hand, it has been shown by authors that the code compressing a pulse to a width of several sub-pulses could have some merits as a standard pulse compression code [8]. Furthermore, it has been shown that the 4-phase complementary code compressed to a width of several sub-pulses has far more code pairs than a pulse compressed to the width of a single sub-pulse. Moreover, by using such 4-phase complementary codes, it is possible to reduce the sidelobe produced by the Doppler frequency; however, the binary complementary code that compresses a pulse to a width of several sub-pulses has not yet been investigated.

This study introduces new binary complementary codes that compress a pulse to the width of several sub-pulses. We then show that more code pairs exist in such binary complementary codes than in conventional binary complementary codes, and also indicate that these code pairs can compose longer complementary codes of the proposed type using the expansion method proposed by Golay. As a result, we find that there are more binary complementary codes of the proposed type than of the conventional type. Another important advantage of the proposed binary complementary codes introduced in this paper is that there are more complete complementary codes [6], which consist of a pair of complementary codes and the sum of two cross-correlation functions between the mate codes that are always zero for any time shift, than in the conventional type. It is also shown that, in the proposed complementary codes, there are networks connected by the relationship between two complementary codes that jointly construct a complete complementary code. This fact means that it is possible to eliminate or suppress the mutual interference even on the same frequency by using such proposed complementary codes on several adjoining radars and synchronizing the pulse transmission. Namely, the effective use of the frequency resource is possible.

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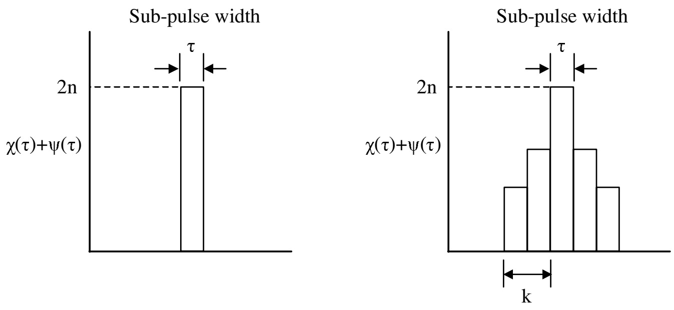

(a) Conventional code (b) Proposed code

Fig. 1 Compressed pulse waveform.

Furthermore, we show that the auto-correlation sidelobe of an individual code for the proposed complementary code pair is smaller than that for the conventional type. Using such proposed complementary codes, it is possible to reduce the sidelobe produced by the Doppler frequency.

II. PRINCIPLE

The auto-correlation functions of two codes, $A_n=(a_1,a_2, \dots ,a_n)$ and $B_n=(b_1,b_2, \dots ,b_n)$, are defined as Eq.(1) and Eq.(2) respectively.

$$\chi(\tau)=\sum_{i=1}^{n} A_i \cdot A^*_{i+\tau} \qquad \tau \geq 0 \tag{1}$$

$$\psi(\tau)=\sum_{i=1}^{n} B_i \cdot B^*_{i+\tau} \qquad \tau \geq 0 \tag{2}$$

The conventional binary complementary code is defined by Eq.(3). Namely, the sum of two auto-correlation functions is zero for any time shift τ except $\tau=0$:

$$\chi(\tau)+\psi(\tau)=\begin{cases} 0, & \text{for all } \tau>0 \\ 2n, & \text{for } \tau=0 \end{cases} \tag{3}$$

where τ is the time shift and * is a complex conjugation.

This paper proposes new binary complementary codes that compress a pulse to a width of several sub-pulses. The new binary complementary codes is defined by Eq.(4).

$$\chi(\tau)+\psi(\tau)=\begin{cases} zero, & \text{for all } \tau>k \\ nonzero, & \text{for all } k \geq \tau>0 \\ 2n, & \text{for } \tau=0 \end{cases} \tag{4}$$

The spectrum bandwidth of the proposed binary complementary code is narrower than that of the conventional binary complementary code, because the compressed pulse width is wider. Figure 1(b) shows the pulse shape compressed by the proposed code (that by the conventional code is shown in Fig.1(a)), where k is the number of sub-pulses as indicated in Fig.1(b), and if m is the number of sub-pulse, then k=(m-1)/2.

III. PROPOSED BINARY COMPLEMENTARY CODE

This paper considers binary complementary codes. The binary sets are $(0, \pi)$, and the total number of two-code combinations is $2^n(2^{n-1})/2$. The binary complementary codes that compress a pulse to a width of several sub-pulses, namely satisfying Eq.(4), are investigated. First of all, such binary complementary codes of short code length are surveyed for all code combinations.

A pair of binary codes whose length is 3 can be written by Eqs.(5) and (6),

$$A=\{e^{j\phi_0}, e^{j(\phi_0+\phi_1)}, e^{j(\phi_0+\phi_1+\phi_2)}\} \tag{5}$$

$$B=\{e^{j\theta_0}, e^{j(\theta_0+\theta_1)}, e^{j(\theta_0+\theta_1+\theta_2)}\} \tag{6}$$

where ϕ_0, ϕ_1, ϕ_2 are the phase differences between neighbor sub-pulses of codes A, while θ_0, θ_1, and θ_2 are those of code B. Their auto-correlation functions, $\chi(\tau)$, $\psi(\tau)$, can be written by Eqs.(7) and (8), respectively.

$$\begin{bmatrix}\chi(1)\\ \chi(2)\end{bmatrix}=\begin{bmatrix} e^{j\phi_1} & e^{j\phi_2} \\ 0 & e^{j(\phi_1+\phi_2)}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}\psi(1)\\ \psi(2)\end{bmatrix}=\begin{bmatrix} e^{j\theta_1} & e^{j\theta_2} \\ 0 & e^{j(\theta_1+\theta_2)}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} \tag{8}$$

Condition (4) can then be written as Eqs.(9) and (10).

$$\chi(1)+\psi(1)=e^{j\phi_1}+e^{j\phi_2}+e^{j\theta_1}+e^{j\theta_2} \neq 0 \tag{9}$$

$$\chi(2)+\psi(2)=e^{j(\phi_1+\phi_2)}+e^{j(\theta_1+\theta_2)}=0 \tag{10}$$

Equation (9) means that the compressed pulse is composed of three sub-pulses. In that case, the value of k is 1.

A pair of binary codes whose length equals 4 is similarly expressed as Eqs.(11) and (12).

$$A=\{e^{j\phi_0}, e^{j(\phi_0+\phi_1)}, e^{j(\phi_0+\phi_1+\phi_2)}, e^{j(\phi_0+\phi_1+\phi_2+\phi_3)}\} \tag{11}$$

$$B=\{e^{j\theta_0}, e^{j(\theta_0+\theta_1)}, e^{j(\theta_0+\theta_1+\theta_2)}, e^{j(\theta_0+\theta_1+\theta_2+\theta_3)}\} \tag{12}$$

There are two cases satisfying Eq.(4).
One is given by Eqs.(13), (14), and (15).

$$\chi(1)+\psi(1)=e^{j\phi_1}+e^{j\phi_2}+e^{j\phi_3}+e^{j\theta_1}+e^{j\theta_2}+e^{j\theta_3} \neq 0 \tag{13}$$

$$\chi(2)+\psi(2)=e^{j(\phi_1+\phi_2)}+e^{j(\phi_2+\phi_3)}+e^{j(\theta_1+\theta_2)}+e^{j(\theta_2+\theta_3)}=0 \tag{14}$$

$$\chi(3)+\psi(3)=e^{j(\phi_1+\phi_2+\phi_3)}+e^{j(\theta_1+\theta_2+\theta_3)}=0 \tag{15}$$

In this case, the number of the one-side neighbors is 1, namely k=1.

The case of k=2 is given by Eqs.(16), (17), and (18).

$$\chi(1)+\psi(1)=e^{j\phi_1}+e^{j\phi_2}+e^{j\phi_3}+e^{j\theta_1}+e^{j\theta_2}+e^{j\theta_3} \neq 0 \tag{16}$$

$$\chi(2)+\psi(2)=e^{j(\phi_1+\phi_2)}+e^{j(\phi_2+\phi_3)}+e^{j(\theta_1+\theta_2)}+e^{j(\theta_2+\theta_3)} \neq 0 \tag{17}$$

$$\chi(3)+\psi(3)=e^{j(\phi_1+\phi_2+\phi_3)}+e^{j(\theta_1+\theta_2+\theta_3)}=0 \tag{18}$$

Similarly, it is possible to obtain the conditions for a longer complementary code of the proposed type.

Table 1 Number of binary complementary codes and complete complementary codes.

Code length	Conventional codes (k =0)	Proposed codes				
		k=1	k=2	k=3	k=4	k=5
3	0	4 (2)	-	-	-	-
4	4 (2)	2 (1)	4 (2)	-	-	-
5	0	0	24 (12)	4 (2)	-	-
6	0	16 (8)	2 (1)	20 (12) *	24 (12)	-
7	0	0	0	108 (54)	16 (8)	16 (8)
8	24 (12)	8 (8) *	32 (16)	50 (25)	50 (24)	80 (42)
9	0	0	0	0	404 (202)	28 (14)
10	16 (8)	8 (4)	120 (60)	32 (16)	58 (29)	146 (84) *
11	0	0	0	0	0	1172 (586)
12	0	128 (64)	8 (8) *	144 (88) *	224 (112)	362 (181)

m (n) : *m*-value and *n*-value are the number of complementary codes and complete complementary codes, respectively.
* : includes connection of complete complementary codes.
k : includes the number of one-sided sub-pulses.

Table 1 shows the number of code pairs satisfying those conditions, which are obtained using a computer. The value in parentheses indicates the number of complete complementary codes, which are described further in Section 5-A. The conventional binary complementary code is then indicated at the case of k=0, which is compressed to a single sub-pulse. If k=1, then the number of sub-pulses for a mainlobe is 3, and similarly, if k=2 and k=3, it is 5 and 7, respectively. This paper investigates binary complementary codes whose code lengths range from 3 to 12 and have a k of less than 5. It should be noted that there exist more binary complementary codes of the proposed type than do those of the conventional type. Consequently, the increase in code necessary for selecting appropriate jamming and mutual interference between neighboring radars is actually reduced [5].

An example of the proposed binary complementary code is shown as follows.

[Code length=12 (k=1)]
A=(0, 0, 0, 0, 0, 0, 0, π, π, π, 0, 0)
B=(0, 0, 0, 0, π, 0, π, π, 0, 0, π, π)

The compressed pulse waveform is shown in Fig. 2.

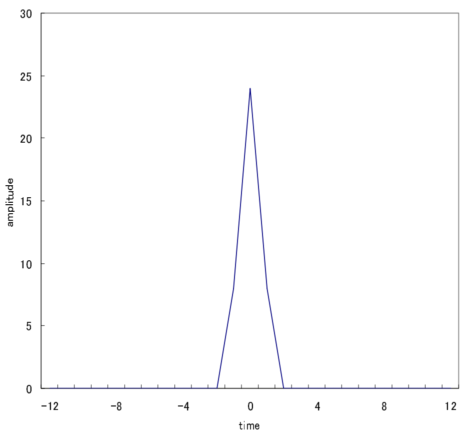

Fig. 2 Output waveform for proposed complementary code with code length 12 (k=1).

IV. Expansion Of Code Length

In section 3, we investigated the binary complementary codes of the proposed type of which length is from 3 to 12, and found that the ones of longer code length are necessary for the actual application. It may be possible to acquire even longer binary complementary codes by an exhaustive computer search as used in section 3. However, the calculation time will be excessive when the code length is longer than 10. Therefore, in this section, we discuss whether it is possible to obtain long binary complementary codes of the proposed type using the expansion method introduced by Golay.

When a complementary code of length n, $A_n=(a_1,a_2,\ldots,a_n)$ and $B_n=(b_1,b_2,\ldots,b_n)$, is given, a complementary code of length 2n, $\{C_{2n}, D_{2n}\}$, which is shown in Eqs.(19) and (20), or Eqs.(21) and (22), will be obtained by the Golay's method [2].

$$C_{2n} = (a_1, a_2, \ldots , a_n, b_1, b_2, \ldots , b_n) \quad (19)$$
$$D_{2n} = (a_1, a_2, \ldots , a_n, (-b_1), (-b_2), \ldots , (-b_n)) \quad (20)$$
or
$$C_{2n} = (a_1, a_2, \ldots , a_n, b_1, b_2, \ldots , b_n) \quad (21)$$
$$D_{2n} = (a_1, (-b_1), a_2, (-b_2), \ldots , a_n, (-b_n)) \quad (22)$$

However, this is the method for conventional binary complementary codes. It should therefore be confirmed whether the Golay's method could be also applicable for the propoesd binary complementary code by which a pulse is compressed to a width of several sub-pulses. However, it is easily found that the expansion method given by Eqs.(19) and (20) can be applied to the proposed binary complementary code, whereas that given by Eqs.(21) and (22) cannot, because in the case of the method given in Eqs.(21) and (22), the mainlobe of the compressed pulse is divided with some separated peaks.

The compressed pulse width is scarcely changed by using the expansion method of Eqs.(19) and (20), because there is barely any change to the spectrum bandwidth of the code. This method can be applied repeatedly. The code length will be about $n \times 2^m$ if the expansion method is applied m times. Since the number of binary complementary codes obtained by the expansion depends on the number of the code pairs, it is then found that more long binary complementary codes exist for the proposed type than for that of the conventional type,

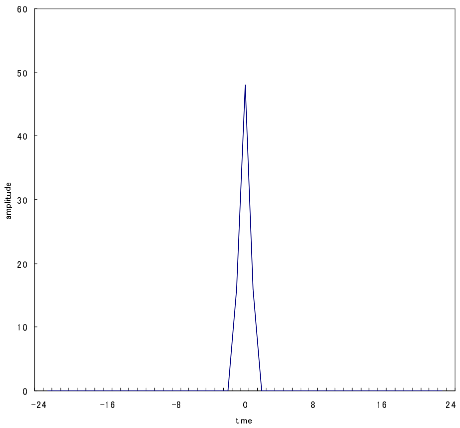

Fig. 3 Output waveform for proposed complementary code with code length 24 (k=1).

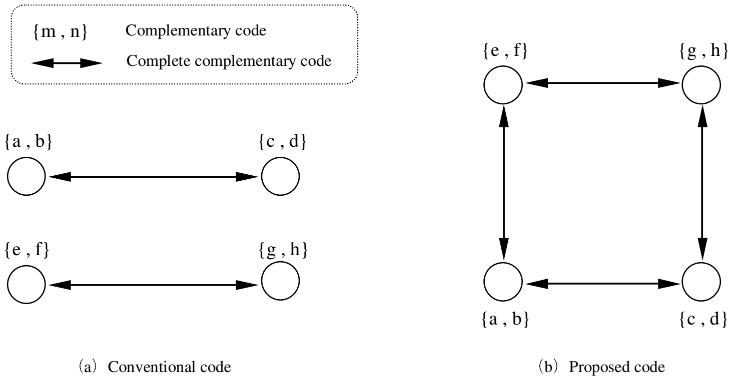

Fig. 4 Connection structure of complete complementary codes.

which is a very important advantage of the proposed binary complementary code. An example code for which the expansion method is applied is as shown below, where {A, B} is the base code pairs and {C, D} is the expanded code. The code length of A and B is 12 (k=1), and the code length of C and D is 24 (k=1).

A = $(0, 0, 0, 0, 0, 0, 0, \pi, \pi, \pi, 0, 0)$
B = $(0, 0, 0, 0, \pi, 0, \pi, \pi, 0, 0, \pi, \pi)$
C = $(0, 0, 0, 0, 0, 0, 0, \pi, \pi, \pi, 0, 0, 0, 0, 0, 0, \pi, 0, \pi, \pi, 0, 0, \pi, \pi)$
D = $(0, 0, 0, 0, 0, 0, 0, \pi, \pi, \pi, 0, 0, \pi, \pi, \pi, \pi, 0, \pi, 0, 0, \pi, \pi, 0, 0)$

Figure 3 shows the pulse waveform compressed by the expanded example code.

V. Characteristics Of Proposed Binary Complementary Codes

A. Complete Complementary Codes

A complete complementary code consists of a pair of complementary codes and includes the feature that the sum of two cross-correlation functions between the mate codes is always zero for any time shift [6]. A pair of complementary codes, $\{A_n, B_n\}$ and $\{A'_n, B'_n\}$, is then called complete complementary code if it satisfies Eqs.(23) and (24).

$A_n = (a_1, a_2, \ldots, a_n)$ $\quad$ $B_n = (b_1, b_2, \ldots, b_n)$
$A'_n = (a'_1, a'_2, \ldots, a'_n)$ $\quad$ $B'_n = (b'_1, b'_2, \ldots, b'_n)$

$$C(\tau) = \sum_{i=1}^{n} A_i \cdot A'^*_{i+\tau} + \sum_{i=1}^{n} B_i \cdot B'^*_{i+\tau} \tag{23}$$

$$C(\tau) = 0 \quad \text{for } \tau = 0 \sim n-1 \tag{24}$$

It is possible to reduce mutual interference by using complete complementary codes [5].

The number of complete complementary codes among all code pairs counted in Section 3 is shown in parentheses in Table 1. The conventional binary complementary codes have lengths of 4, 8 and 10, when surveyed up to a length of 12. The lengths of complete complementary codes are also 4, 8 and 10, respectively. On the other hand, the proposed binary complete complementary codes are found in all code lengths ranging from 3 to 12, and there exist many complete complementary code pairs. As shown in Table 1, it should be noted that there are actually more complete complementary codes of the proposed type than of the conventional type, which is another important advantage of the proposed binary complementary code. An example of a complete complementary code of the proposed type is shown as follows:

[Code length=6 (k=1)]
A = $(0, 0, 0, 0, 0, \pi)$ $\quad$ B = $(0, \pi, \pi, 0, 0, 0)$
A' = $(0, 0, 0, \pi, \pi, 0)$ $\quad$ B' = $(0, \pi, \pi, \pi, \pi, \pi)$

It is clear that the code expanded by Eqs.(19) and (20) has the characteristics of a complete complementary code, that is to say, the complete complementary code expanded by Eqs.(19) and (20) also becomes a complete complementary code with a longer code length.

Furthermore, the proposed binary complementary codes have a unique feature in that they construct a network connected by the relationship between complete complementary codes. Figure 4(a) shows the structure connected by the complete complementary codes in the conventional type, while Fig.4(b) shows the structure connected by the proposed binary complete complementary codes. The symbol {m , n} represents a complementary code, and the symbols m and n represent each component code. The symbol ↔ represents the relationship between two complementary codes that jointly construct a complete complementary code. It is noted that the network is constructed by the proposed binary complete complementary code, only on the code lengths of 6 (k=3), 8 (k=1), 10 (k=5), 12 (k=2), and 12 (k=3). (The symbol * denotes the relationship shown in Table 1.)

The symbol ↔ shown in Fig.4(b) indicates the complete complementary codes of which the cross-correlation is zero for all time shifts. It is then possible to eliminate or suppress the mutual interference even on radars with the same transmitted frequency by using the proposed complete complementary codes. This results in the effective use of the frequency resources.

Table 2 Sp/Mp ratio of individual code of binary complementary code pair for compression ratio and code length.

Complementary codes		Compression ratio	Sp/Mp ratio of auto-correlation of individual code
Proposed code	Length 12, *k*=1	8.0	-15.6 dB (=2.0/12)
Conventional code	Length 8	8.0	-12.0 dB (=2.0/8)
Proposed code	Length 24, *k*=1	16.0	-15.6 dB (=4.0/24)
Conventional code	Length 16	16.0	-14.5 dB (=3.0/16)

B. Characteristics of Doppler frequency

1) Peak-sidelobe to peak-mainlobe ratio

Using a complementary code, it is possible to eliminate sidelobes when the Doppler frequency is very small. However, when the Doppler frequency is fairly large, there is a problem that sidelobes appear due to the Doppler frequency. This effect can be reduced, however, by using a complementary code with a smaller auto-correlation sidelobe of individual code [5]. Figure 5 shows the auto-correlation of individual code for the proposed binary complementary code given in Section 3. Table 2 shows the compression ratios and the minimum S_p/M_p ratio (peak-sidelobe to peak-mainlobe ratio) surveyed for the compressed waveform of the individual code. The compression ratio is defined as the ratio of the transmitted pulse width to the compressed pulse width, and the compressed pulse width is defined as the width between two points where the value of the auto-correlation function is half the center peak value [7],[8],[9]. The S_p/M_p ratio of the individual code in Table 2 for the proposed code is the smallest among all the proposed binary complementary codes with code length 12 and code length 24 expanded from the complementary codes of code length 12 in Table 1 by applying Golay's method once. Similarly, the S_p/M_p ratios for the conventional codes are also the smallest among all the conventional binary complementary codes of code length 8 and code length 16 expanded from the complementary codes of code length 8 by Golay's method.

The compression ratio of the proposed complementary code is 8.0, whose code length is 12 at k=1. Table 2 indicates that the minimum S_p/M_p ratio for the proposed complementary code is –15.6 dB, and the minimum S_p/M_p ratio for the conventional complementary code is –12.0 dB, suggesting that the characteristic for the Doppler frequency shift is improved by using the proposed complementary codes.

The proposed binary complementary codes in Table 2 are shown in the following.

[Proposed complementary code, code length=12 (k=1)]
A = (0, 0, 0, 0, 0, 0, 0, π, π, π, 0, 0)
B = (0, 0, 0, 0, π, 0, π, π, 0, 0, π, π)
[Conventional complementary code, code length=8]
A = (0, 0, 0, π, 0, π, 0, 0)
B = (0, 0, 0, π, π, 0, π, π)
[Proposed complementary code, code length=24 (k=1)]
A = (0, 0, 0, 0, π, 0, π, π, 0, 0, π, 0, π, π, π, 0, 0, 0, 0, 0, 0, 0)
B = (0, 0, 0, 0, π, 0, π, π, 0, 0, π, π, π, π, 0, 0, 0, π, π, π, π, π, π, π)
[Conventional complementary code, code length=16]
A = (0, 0, π, 0, 0, 0, 0, π, 0, 0, π, 0, π, π, π, 0)
B = (0, 0, π, 0, 0, 0, 0, π, π, π, 0, π, 0, 0, 0, π)

2) Total sidelobe power to mainlobe power ratio

Here, we conduct a quantitative evaluation to examine the effect of the actual use for the Doppler frequency. The mainlobe power P_m and the total sidelobe power P_S of the proposed binary complementary codes are defined by Eqs.(25) and (26), respectively [9].

$$\begin{aligned} P_m = 2 \int_0^{(k+1)T_s} \int_{-\infty}^{\infty} & |\chi_A(\tau,0) + \chi_B(\tau,0)|^2 \\ & \times S(\omega - \omega_d) \cos^2(\omega T/2) \\ & + |\chi_B(\tau,0) + \chi_A(\tau,0)|^2 \\ & \times S(\omega - \omega_d) \sin^2(\omega T/2) d\omega d\tau \end{aligned} \quad (25)$$

$$\begin{aligned} P_S = 2 \int_{(k+1)T_s}^{NT_s} \int_{-\infty}^{\infty} & 4|\chi_A(\tau,0)|^2 S(\omega - \omega_d) \\ & \times \sin^2(\omega T/2) d\omega d\tau \end{aligned} \quad (26)$$

Here, χ_A , χ_B are the ambiguity functions of the complementary code, which consists of each code A and B (See Appendix-1), while $S(\omega)$ and ω_d are the power spectrum of the echo from the target and the Doppler frequency, respectively. The interpulse period is represented by T, T_S is the sub-pulse duration, N is the number of sub-pulses, and k is the number of one-side neighbor sub-pulses. The conventional binary complementary code is present in the case of k=0 [10]. The relation between the P_S/P_m ratio and the Doppler frequency is shown in Fig.6, where $S(\omega) = 1/\sqrt{2\pi\sigma^2} \cdot \exp(-\omega^2/2\sigma^2)$, σ=0, and time and frequency are normalized by the sub-pulse duration Ts and the reciprocal of the sub-pulse duration 1/Ts, respectively. It should be noted that the P_S/P_m ratio of the proposed binary complementary codes improves by –6.7 dB when the compression ratio is 8.

VI. Discussion

The proposed binary complementary codes whose code lengths range from 3 to 12 are investigated in this paper. We found that the proposed binary complementary codes have more code pairs than the conventional binary complementary codes, and is thought that this occurs by relaxing the conditions for the compressed pulses. The condition is to compress a pulse to several sub-pulses. Furthermore, the condition carries the advantage that more complete complementary codes are found and that complementary codes with smaller auto-correlation sidelobes of an individual code are acquired. We also note that the proposed binary complementary codes improves the S_P/M_P ratio and the characteristics for the Doppler frequency shift.

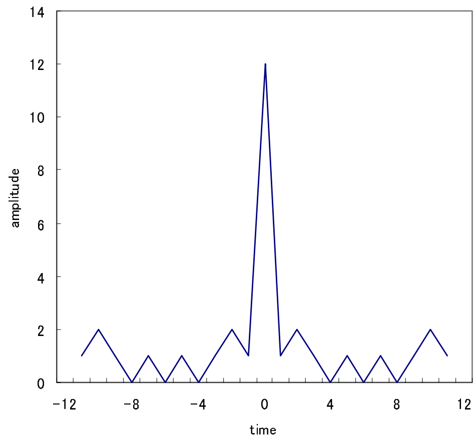

Fig.5 Auto-correlation for one code of complementary pair with code length 12 (k=1).

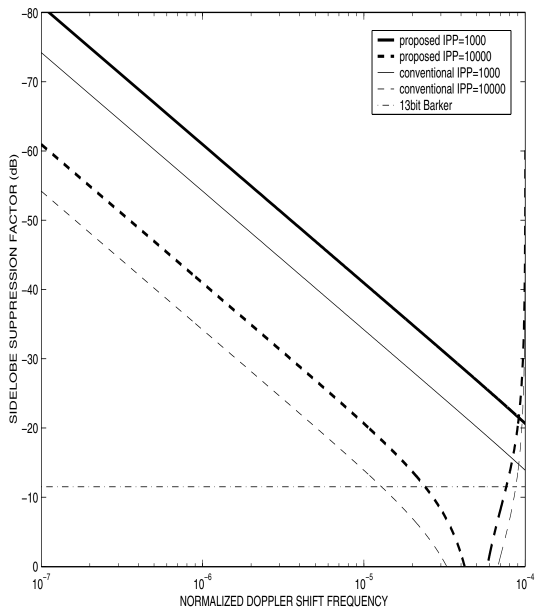

Fig.6 Ps/Pm ratio versus the Doppler shift frequency for the interpulse period IPP of 1,000 and 10,000. (Compression ratio=8)

VII. CONCLUSIONS

We introduced new binary complementary codes that can compress a pulse to the width of several sub-pulses. Such binary complementary codes of code lengths ranging from 3 to 12 were surveyed, and we found that more of the proposed binary complementary codes exist than do conventional binary complementary codes. Another important result introduced in this paper was that there are also a large number of complete complementary codes in the proposed binary complementary codes. Consequently, the available code choices for interference and jamming increase. Furthermore, we also confirmed that longer complementary codes can be acquired by using the method shown by Golay.

It was also shown that, with the proposed binary complementary codes, a network connected by the relationship of the complete complementary code between two complementary codes can be constructed. This fact means that, by using such proposed binary complementary codes in several adjoining radars with pluse transmission synchronized with each other, it is possible to eliminate or suppress the mutual interference, even in the radars operating on the same frequency. We also shown that the auto-correlation sidelobe of individual code for the proposed binary complementary code is smaller than that for the conventional. As a result, we confirmed that the effect of sidelobe supression for the Doppler frequncy shift is improved by using the proposed complementary code.

APPENDIX

1. Ambiguity function

The ambiguity function $\chi(\omega)$ is defined by Eq.(A.1) [13].

$$\chi(\tau,\omega) = \int_{-\infty}^{\infty} A(t)\cdot A^*(t+\tau)e^{j\omega t}dt \qquad \text{(A.1)}$$

where A is code sequence, ω is the Doppler frequency, and τ is the time shift.

The ambiguity function consisting of two codes, A and B, can be expressed as follows:

$$\chi(\tau,\omega) = \chi_A(\tau,\omega)e^{-j\omega T/2} + \chi_B(\tau,\omega)e^{-j\omega T/2} \qquad \text{(A.2)}$$

where T is the sub-pulse duration [10].

REFERENCES

[1] M.I.Skolnik, Radar handbook, second edition, McGraw-Hill, New York, 1990.
[2] M.J.E.Golay, “Complementary series,” IRE Trans. Information Theory, vol.IT-7, pp.82-87, April 1961.
[3] Richard van Nee, “OFDM codes for peak-to-average power reduction and error correction,” Proc. IEEE Globecom, pp.740-744, Nov. 1996.
[4] R.Sivaswamy, “Multiphase complementary codes,” IEEE Trans. Information Theory, vol.IT-24, no.5, pp.546-552, 1978.
[5] R.L.Frank, “Polyphase complementary codes,” IEEE Trans. Information Theory, vol.IT-26, no.6, pp.641-647, 1980.
[6] N.Suehiro, “Complete complementary code composed of N-multiple-shift orthogonal sequences,” Trans. IECE of Japan (in Japanese) , vol.J65-A, no.12, pp.1247-1253, 1982.
[7] W.S.Burdic, Radar signal analysis, Prentice-Hall, Englewood Cliffs, 1968.
[8] R.Sato, M.Shinriki, “Time sidelobe reduction technique for binary phase coded pulse compression,” Trans. IEICE of Japan (in Japanese), vol.J83-B, no.3, pp.352-360, March 2003.
[9] H.Takase, M.Shinriki, R.Sato, “4-phase complementary codes compressed to several sub-pulses,” Trans. IEICE of Japan (in Japanese), vol.J85-B, no.8, pp.1434-1444, Aug. 2002.
[10] K.Wakasugi and S.Fukao, “Sidelobe properties of a complementary code used in MST radar observations,” IEEE Trans.Geosci.Remote Sensing, vol.GRS-23, no.1, pp.57-59, Jan. 1985.
[11] T.Takano, T.Sato, M.Kashimoto, M.Murada, Radio instrumentation and navigation in space (in Japanese), pp.26-27, Corona publishing, Tokyo, 2000.
[12] M.I.Skolnik, Introduction to radar systems, third edition, McGraw-Hill, New York, 2001.