

Performance Analysis of Adaptive Polarization Filters

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Abstract—In the dynamic electromagnetic environments, a polarization estimator is usually utilized by an adaptive polarization filter to acquire the knowledge of the polarization of the interference field, then optimal polarization is calculated for polarization filtering. In this sense, an adaptive polarization filter can be modeled as an open loop cascading system of a polarization estimator, an optimum polarization calculator and a polarization filter. Based on the dual channel orthogonal polarization measurements, theoretical performance analysis of the polarization estimator is derived. It is proved that the polarization estimation error obeys the Rayleigh distribution approximately. Finally, by studying the transmission process of the estimation error in a polarization filter, a series of analytic formulae on the performances of adaptive polarization filters are derived, which statistically reveal the relationship between the filtering performance and the signal to noise ratio clearly.

Index Terms—radar, polarization, estimation, polarization filter, adaptive

I. INTRODUCTION

Under the condition of modern electronic warfare, environments become more and more adverse for the radar. The problem of anti-jamming is of great importance. The adaptive polarization filtering technique has been considered as a promising technology for radar anti-jamming [1~3,8,9]. In the past thirty years, lots of adaptive polarization filters have been designed for various purposes. Many of them worked well. However, their intrinsic limitations were also exposed. Hence evaluating the performance of an adaptive polarization filter becomes urgent for the polarization signal processing technique. Due to the complexity of clutter, the diversity of interference and the specialty of polarization filtering, the designation of polarization filters usually has very strong pertinence. There is still no frame theory on the performance of polarization filtering.

Essentially, polarization filters improve the qualities of the desired signals through selecting the antenna's polarization. There are mainly two classes of polarization filters. One is called Interference Suppressed Polarization Filters(ISPF)

which makes the received jamming power minimum through selecting antenna's polarization orthogonal to that of jamming. No minding the signals' polarization is one characteristic of these filters. Minimizing the jamming power is the only goal of ISPF^[1,2,9]. The other class filters such as SINR (Signal to Interference plus Noise Ratio) and PDSI (Power Difference of Signal and Interference) filters consider the polarization of both signals and jamming in the antenna's beam^[4,7].

In order to improve the quality of the received signals, the prior information about jamming must be known. In fact, the polarization of jamming is usually obtained by estimation^[4,6]. As for jamming of high polarization degree, jamming suppression ratio can be very large (infinitude in theory). However if the receiving polarization isn't orthogonal to that of interference strictly, the filtering performance will decrease rapidly^[2]. Hence, the estimation to interference polarization is a key in the filtering process.

The physical premise of obtaining the polarization of incident electromagnetic field is the dual channel orthogonal polarization measurements. Theoretically, the amplitude and phase of the measurements can represent the polarization of incidence field uniquely^[8]. In practice, the outputs of the dual channel orthogonal polarization measurements are taken as the incident waves' estimations, that is to say, the dual channel orthogonal polarization measurements can be regarded as a polarization estimator. From the viewpoint of signal estimation, it is an optimum polarization estimator, and it represents a superior limit of filtering performance^[13].

This paper is organized as follows. In Section II, the model of the polarization estimator is established. Its error based on the Stokes vector is derived. In Section III, it is proved that the polarization estimation error approximately obeys the Rayleigh distribution. In Section IV, a series of analytic formulae on the performances of adaptive polarization filters are derived, which stochastically reveal the relationship between the filtering performance and the signal to noise ratio clearly.

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II. THE POLARIZATION VECTOR ESTIMATION BASED ON THE DUAL CHANNEL ORTHOGONAL POLARIZATION MEASUREMENTS

Select horizontal and vertical linear polarization $(\hat{\mathbf{h}}, \hat{\mathbf{v}})$ as polarization basis, and assume any a planar incident wave can be represented as $\mathbf{s} = [s_h, s_v]^T$ based on $(\hat{\mathbf{h}}, \hat{\mathbf{v}})$. Here the symbol “ T ” represents transpose. Suppose this wave is received by dual channel orthogonal polarization measurements. Then the output is a complex vector and is written as $\mathbf{x}$. Neglecting the dual channel inconsistency, coupling and nonlinearity, the signal model can be represented:

$$\mathbf{x} = \mathbf{s} + \mathbf{n} \quad (1)$$

Where $\mathbf{n}$ is the thermal noise of the measurement system. In fact, the output is directly looked as the estimation of the incident signal vector, that is:

$$\hat{\mathbf{s}} = \frac{\mathbf{x}}{\|\mathbf{x}\|} = \frac{\mathbf{s} + \mathbf{n}}{\|\mathbf{s} + \mathbf{n}\|} \quad (2)$$

Where $\hat{\mathbf{s}}$ denotes the estimation of the incident wave, $\|\cdot\|$ is the 2 norm of complex vector. Obviously $\|\hat{\mathbf{s}}\| = 1$. To simple, suppose the power of the signal is unit, that is $\|\mathbf{s}\| = 1$. Hence $\mathbf{s}$ can be written

$$\mathbf{s} = [\cos\gamma, \sin\gamma e^{j\phi}]^T, \gamma \in [0, \frac{\pi}{2}], \phi \in [0, 2\pi] \quad (3)$$

Where (γ, ϕ) is called polarization phase descriptor^[8].

In order to analyze the performance of the estimator, the difference between the estimation polarization $\hat{\mathbf{s}}$ and the real polarization $\mathbf{s}$ must be measured. From the relationship of the Stokes vector and electric field vector, it is known:

$$\mathbf{J}_s = R(\mathbf{s} \otimes \mathbf{s}^*)$$

$$\hat{\mathbf{J}}_s = R(\hat{\mathbf{s}} \otimes \hat{\mathbf{s}}^*)$$

Where $\mathbf{J}_s$ and $\hat{\mathbf{J}}_s$ are Stokes vector of the true polarization and estimation polarization, respectively. The symbol “ $\otimes$ ” and “ $*$ ” denote Kronecker product and conjugating respectively. R is written as:^[8]

$$R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}$$

Since $\|\mathbf{s}\| = \|\hat{\mathbf{s}}\| = 1$, $\mathbf{J}_s = [1, \mathbf{g}_s^T]^T$, $\hat{\mathbf{J}}_s = [1, \hat{\mathbf{g}}_s^T]^T$, in which $\mathbf{g}_s$ and $\hat{\mathbf{g}}_s$ are called the Stokes Subvector of true polarization and estimated polarization. Obviously, $\mathbf{g}_s$ and $\hat{\mathbf{g}}_s$ are both real three dimension vector. In addition, they

satisfy $\|\mathbf{g}_s\| = \|\hat{\mathbf{g}}_s\| = 1$ and are mapped to two points on the Poincaré polarization sphere^[8], as illustrated in Fig.1.

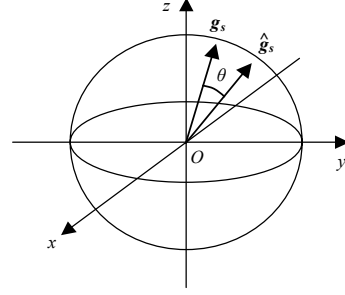


Fig.1 the vector presentation of true polarization and estimated polarization on Poincaré sphere

Suppose θ is the angle between true polarization $\mathbf{g}_s$ and estimated polarization $\hat{\mathbf{g}}_s$. It is the natural scale for the performance of polarization estimator. It can be easily proved that:

$$\frac{1}{2}(1 + \cos\theta) = |\mathbf{s}^H \hat{\mathbf{s}}|^2 \quad (4)$$

The symbol “ H ” is complex conjugate transposition. It shows that the angle between true polarization and estimated polarization can be calculated by the inner product of electric field vector. Substitute Equ. (2) into above formula, then we get:

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{\|\mathbf{n}\|^2 - |\mathbf{n}^H \mathbf{s}|^2}{1 + \mathbf{n}^H \mathbf{s} + \mathbf{s}^H \mathbf{n} + \|\mathbf{n}\|^2} \quad (5)$$

Suppose the SNR of measure system is high, that is $\|\mathbf{n}\| \ll \|\mathbf{s}\| = 1$. Then Equ. (5) can be rewritten as:

$$\sin^2\left(\frac{\theta}{2}\right) \approx \mathbf{n}^H \mathbf{W} \mathbf{n} \quad (6)$$

Where $\mathbf{W} = \mathbf{I}_2 - \mathbf{s} \mathbf{s}^H$, $\mathbf{I}_2 = \text{diag}\{1, 1\}$. Since $\|\mathbf{s}\| = 1$, the matrix $\mathbf{W}$ must be nonnegative definite Hermitian matrix. In fact, substitute Equ. (3) into the expression of $\mathbf{W}$, then Unitary diagonalize $\mathbf{W}$, we get:

$$\mathbf{W} = \mathbf{U}^H \mathbf{\Lambda} \mathbf{U} \quad (7)$$

in which $\mathbf{\Lambda} = \text{diag}\{1, 0\}$, $\mathbf{U}$ is a Unitary matrix, its expression is

$$\mathbf{U} = \begin{bmatrix} \sin \gamma e^{j\phi} & -\cos \gamma \\ \cos \gamma & \sin \gamma e^{-j\phi} \end{bmatrix} \quad (8)$$

Substitute Equ.(2) into Equ.(6), then $\sin^2 \frac{\theta}{2} \approx \mathbf{m}^H \mathbf{\Lambda} \mathbf{m}$.

Where $\mathbf{m} = \mathbf{U} \mathbf{n}$ is the Unitary transform of the noise output vector. Since $\|\mathbf{n}\| = \|\mathbf{m}\| \ll 1$, then $\theta \ll 1$, then

$$\theta \approx 2\sqrt{\mathbf{m}^H \mathbf{\Lambda} \mathbf{m}} \quad (9)$$

From Equ.(6) and Equ.(9), the θ is dependent on two factors: one is the system output noise, another one is the signal polarization parameter whose effect is embodied in

matrix W . According to Equ.(9), we will analyze the statistical property of polarization estimation error.

III. PERFORMANCE ANALYSIS OF THE POLARIZATION ESTIMATOR

For radars operating above VHF/UHF frequencies, regardless of clutter and interference, the output noise vector $\mathbf{n}$ of dual channel is mainly resulted from reception thermal noise which can often be viewed as band limited white noise and obeys Gauss distribution. Suppose $\mathbf{n}$ is a complex Gauss vector with zero mean and covariance matrix $\Sigma_n = \langle \mathbf{n}\mathbf{n}^H \rangle$, Here the symbol of $\langle \cdot \rangle$ denotes statistical expectation. Denote $\mathbf{m} = U\mathbf{n}$, so $\mathbf{m}$ is a complex Gauss vector with zero mean too and its covariance matrix is $\Sigma_m = U \Sigma_n U^H$.

Denote $\mathbf{m} = [m_1, m_2]^T = [x_1 + jy_1, x_2 + jy_2]^T$, m_1 is a gaussian complex variable, according to stochastic process theory, m_1 is a narrowband gaussian noise actually, x_1 and y_1 are two real low frequency orthogonal process^[11]. If the covariance of m_1 is σ_{m11} , then the covariance of x_1 and y_1 are $\sigma_{m11}/2$. The envelope of m_1 $A_1 = \sqrt{x_1^2 + y_1^2}$ satisfies Rayleigh distribution, and its PDF is

$$f_{A_1}(x) = \frac{2x}{\sigma_{m11}} \exp\left(-\frac{x^2}{\sigma_{m11}}\right), \quad x \geq 0.$$

Substitute $\mathbf{m}$ into Equ.(9), then $\theta \approx 2A_1$. So θ obeys the Rayleigh distribution approximately and its PDF

$$f(\theta) = \frac{\theta}{2\sigma_{m11}} \exp\left(-\frac{\theta^2}{4\sigma_{m11}}\right), \quad \theta \in [0, \pi] \quad (10)$$

If

$$\Sigma_n = \begin{bmatrix} \sigma_{n11} & \sigma_{n12} \\ \sigma_{n21} & \sigma_{n22} \end{bmatrix} \quad (11)$$

According to the relationship between Σ_m and Σ_n ,

$$\sigma_{m11} = \sigma_{n11} \sin^2 \gamma + \sigma_{n22} \cos^2 \gamma - 2\text{Re}[\sigma_{n12} e^{j\phi}] \sin \gamma \cos \gamma \quad (12)$$

Especially, when dual channel are independent to each other, that is to say, the two polarization components of $\mathbf{n}$ are independent to each other, then $\sigma_{n12} = \sigma_{n21}^* = 0$. Based on Equ.(12), then σ_{m11} can be written as:

$$\sigma_{m11} = \sigma_{n11} \sin^2 \gamma + \sigma_{n22} \cos^2 \gamma$$

Furthermore, if the power of the two polarization components of $\mathbf{n}$ are equal, namely, $\sigma_{n11} = \sigma_{n22}$, then σ_{m11} can be rewritten as: $\sigma_{m11} = \sigma_{n11}$. Fig.2 offered the PDF curves of θ based on two different SNR. In the figure, SNR is the Signal to Noise Ratio in the output of dual channel. The stick curve is statistical graph of PDF of from 20000 Monte

Carlo experiments. Solid line is the theory curve based on Equ.(10). From Monte Carlo experiments, in higher SNR ($\geq 10\text{dB}$), the Rayleigh distribution can be good in representation of the PDF of θ .

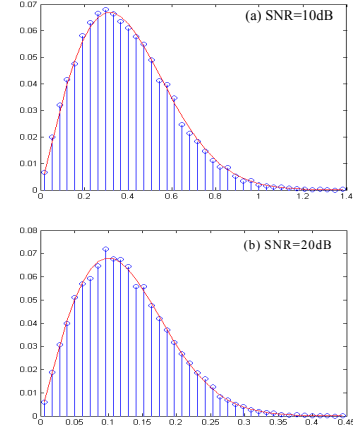


Fig 2 the PDF curve of polarization estimation error angle θ
(The signal polarization is Left Circularly and the two outputs of dual channel are independent and their powers are equal)

IV. THE PERFORMANCE ANALYSIS OF ADAPTIVE POLARIZATION FILTER BASED ON POLARIZATION ESTIMATION

The technique of polarization filtering must know the polarization of interference filed beforehand. Actually, many polarization filters in applications have an adaptive estimator, such as Nathanson's APC canceller^[14], Gherardelli's MLP APC and MLP SAPC canceller^[15,16], Pottier's optimum polarization detector^[5] and so on. These filtering often have three steps: at first, adaptive estimate the interference polarization, then calculate optimum receiving polarization, finally, filter with interference suppressed. Obviously, the estimation precision of interference will directly affect the performance of the whole filtering process.

The characteristic of polarization estimator error is analyzed in above two Sections, its influence on the performance will be studied in this Section.

Denote the estimation polarization is $\hat{\mathbf{s}}$. $\mathbf{g}_s$ and $\hat{\mathbf{g}}_s$ are respectively Stokes Subvectors of true and estimation polarization, and the angle between the two vectors is θ which satisfies $\cos \theta = \mathbf{g}_s^T \hat{\mathbf{g}}_s$. If there is only one interference source, then the process of polarization filtering can be represented as in Fig 3.

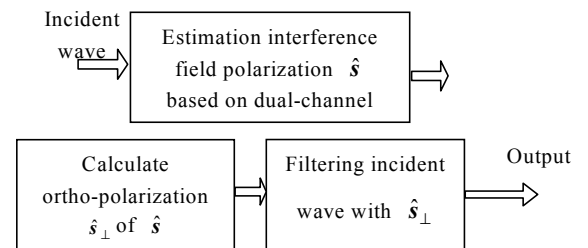


Figure 3 the model of open loop cascading system of a adaptive polarization filter

Define $\hat{\mathbf{s}} = [\cos \hat{\gamma}, \sin \hat{\gamma} e^{j\hat{\phi}}]^T$, then its orthogonal polarization is $\hat{\mathbf{s}}_{\perp} = [-\sin \hat{\gamma}, \cos \hat{\gamma} e^{-j\hat{\phi}}]^T$. Obviously, its Stokes Subvector satisfies $\hat{\mathbf{g}}_{\perp} = \Lambda_{12} \hat{\mathbf{g}}_s$. Where $\Lambda_{12} = \text{diag}\{-1, -1, 1\}$. For the limit of polarization estimation precision, $\hat{\mathbf{s}}_{\perp}$ isn't always orthogonal with $\mathbf{s}$ rigidly. That is to say, there must be a little interference signal leak into the radar receiver, it is called interference remainder:

$$P_r = |\hat{\mathbf{s}}_{\perp}^T \mathbf{s}|^2 = \frac{1}{2} \hat{\mathbf{J}}_{\perp}^T U_4 \mathbf{J}_s = \frac{1}{2} (1 + \hat{\mathbf{g}}_{\perp}^T \Lambda_3 \mathbf{g}_s)$$

Where $\hat{\mathbf{J}}_{\perp} = R(\hat{\mathbf{s}}_{\perp} \otimes \hat{\mathbf{s}}_{\perp}^*) = [\mathbf{1}, \hat{\mathbf{g}}_{\perp}^T]^T$ is Stokes vector of $\hat{\mathbf{s}}_{\perp}$ correspondingly, $U_4 = \text{diag}\{1, 1, -1\}$, $\Lambda_3 = \text{diag}\{1, 1, -1\}$.

Because of $\hat{\mathbf{g}}_{\perp} = \Lambda_{12} \hat{\mathbf{g}}_s$ and $\cos \theta = \mathbf{g}_s^T \hat{\mathbf{g}}_s$ then formula above can be written as:

$$P_r = \frac{1}{2} [1 + (\Lambda_{12} \hat{\mathbf{g}}_s)^T \Lambda_3 \mathbf{g}_s] = \frac{1}{2} (1 - \hat{\mathbf{g}}_s^T \mathbf{g}_s) = \frac{1}{2} (1 - \cos \theta) \quad (13)$$

It is a very significant conclusion. It indicates that interference remainder is only decided by the estimation error of polarization for all adaptive ISPF. It also proves that the polarization estimation is the key of polarization filter design. In addition, it presents a simple mathematical model for adaptive polarization filter performance analysis. For example, we can easily get the probability property of P_r only if the statistical property of θ is known. In the following, some analytic formulae will be deduced. They are universal for all adaptive polarization filters and offered a super limit for their performance.

When the interference is strong and thermal noise is weak, polarization estimation error angle θ obeys Rayleigh distribution approximately, and its PDF is offered by formula (10). Because the relationship between interference remainder P_r and θ is monotonously descending function, then the PDF of P_r can be derived from formula below.

$$f_{\eta}(\eta) = f_{\theta}(\theta) \left| \frac{d\theta}{d\eta} \right| = f_{\theta}(\theta) \frac{2}{\sin \theta}$$

where $\eta = P_r$. Substitute $\theta = \arccos(1 - 2\eta)$ and $\sin \theta = 2\sqrt{\eta - \eta^2}$ into above formula, then

$$f_{\eta}(\eta) = \frac{\arccos(1 - 2\eta)}{2\sigma_{m11}\sqrt{\eta - \eta^2}} \exp \left\{ -\frac{[\arccos(1 - 2\eta)]^2}{4\sigma_{m11}} \right\}, \eta \in [0, 1] \quad (14)$$

Since $\|\mathbf{s}\| = 1$, the remainder power of interference P_r (or η) must be between 0 and 1, hence η can be regarded as the attenuation of interference signal through the polarization filter, or be called "polarization interference

suppression ratio". For example, a narrowband Left Handed Circularly polarization interference signal, the dual channel estimator is adopted in radar processing and polarization estimation error angle is $\theta = 5^\circ$. Based of the estimator, use orthogonal estimation polarization to suppress the interference, according to Equ.(13), we obtains $\eta = 0.002$. Namely, the polarization interference suppression ratio is -27.2dB. In some sense, Equ. (14) is the PDF function of polarization interference suppression ratio η of polarization filter. Fig. 4 presents some PDF curve of Polarization Interference Suppression Ratio. It indicates that the higher the SNR, the more η approaches zero, in other words, the better the performance of the filter is.

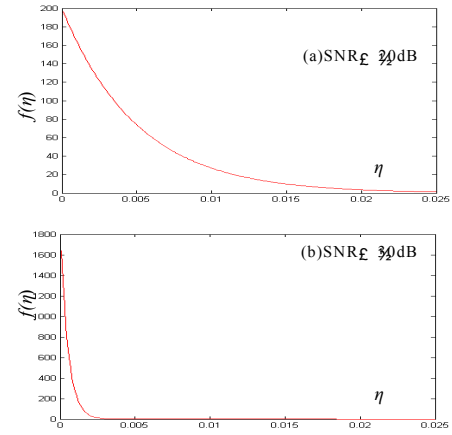


Fig. 4 the PDF curve of Interference Suppression Ratio

From the view of statistics, the PDF of polarization interference suppression ratio is a whole representation for the performance of interference suppression polarization filter. But in practice, using one or more statistical scale representing the performance is more popular. Similarity to the concepts of "Trust Level" and "Trust Interval" in statistics^[17], we choose "probability level" representing the performance of interference suppression polarization filter: given a probability level α , it corresponds to a interference suppression ratio critical value η_0 , interference suppression ratio of polarization filter η should be smaller than η_0 in probability of α , that is

$$\int_0^{\eta_0} f_{\eta}(\eta) d\eta \leq \alpha \quad (15)$$

Substitute Equ.(14) into above, then we get:

$$\eta_0 = \frac{1}{2} \left[1 - \cos \sqrt{-4\sigma_{m11} \ln(1 - \alpha)} \right] \quad (16)$$

Fig 5 presented the curve of critical polarization interference suppression ratio v.s. SNR and v.s. probability level. Fig 5(a) indicates that the relationship between η_0 and SNR is mono-descending with condition of a given a

probability level α . And when SNR is higher than 5dB, the relationship between η_0 and SNR(dB) is linear approximately. Fig 5(b) indicates that the relationship between η_0 and probability level α is mono-increasing on condition of a given SNR.

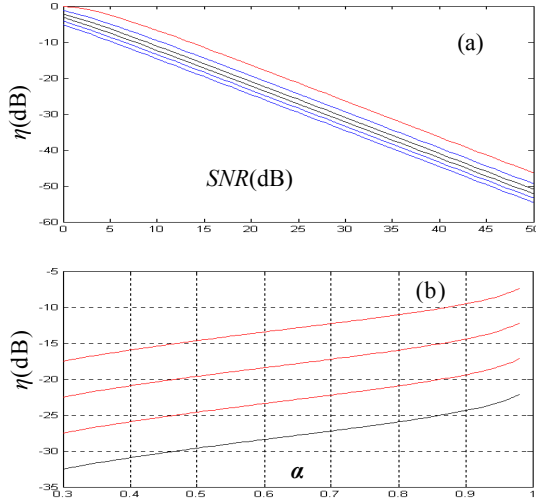


Figure 5 the curve of Critical Polarization Interference Suppression Ratio v.s. SNR (a) or probability level (b) : (a) From up to down, probability levels respectively are $\alpha = 0.99, 0.9, 0.8, 0.7, 0.6, 0.5$ (b) From up to down, SNRs respectively are SNR=10, 15, 20, 25dB

Define $SNR = \frac{\|s\|_g^2}{\|n\|_g^2}$ is the signal to noise ratio of actual output of dual channel, where the bottom label “g” denotes the actual output of the measurement system. From the suppose that the interference field has unit power, then $SNR = \frac{\|s\|_g^2}{tr \Sigma_{n,g}} = 1/tr(\Sigma_{n,g}/\|s\|_g^2)$, where the symbol of “tr” represents the trace of matrix^[10]. It can be concluded that the covariance matrix Σ_n of n is actually the standardization of the covariance matrix $\Sigma_{n,g}$ to interference power, that is $\Sigma_n = \Sigma_{n,g}/\|s\|_g^2$. Similarly, $\Sigma_m = \Sigma_{m,g}/\|s\|_g^2$, $\sigma_{m11} = \sigma_{m11,g}/\|s\|_g^2$. Especially, when the noise of dual-channel have the same characteristic and in independent, then $\Sigma_{n,g} = \sigma_{n11,g} I_2$. Since it is known

before $\sigma_{m11,g} = \sigma_{n11,g} = \frac{1}{2} tr \Sigma_{n,g}$, then

$$\sigma_{m11} = \frac{1}{2} tr \Sigma_{n,g} / \|s\|_g^2 = \frac{1}{2SNR} \quad (17)$$

Substitute Equ.(17) into Equ.(10), (14) and (16), then some relationships, which include the PDF of Polarization estimation error angle θ and polarization suppression ratio η , and the function between Polarization Interfere

Suppression Ratio critical value η_0 and Signal to Noise Ratio (SNR), are all derived.

$$f_\theta(\theta) = SNR \cdot \theta \exp\left(-\frac{SNR}{2}\theta^2\right), \quad \theta \in [0, \pi] \quad (18)$$

$$f_\eta(\eta) = \frac{SNR \cdot \arccos(1-2\eta)}{\sqrt{\eta-\eta^2}} \exp\left\{-\frac{SNR}{2}[\arccos(1-2\eta)]^2\right\}, \quad \eta \in [0,1] \quad (19)$$

$$\eta_0 = \frac{1}{2} \left[1 - \cos \sqrt{-\frac{2}{SNR} \ln(1-\alpha)} \right] \quad (20)$$

From these formulae, the relationship between the performance of estimation and Signal to Noise Ratio can be clearly known.

V. CONCLUSION

An adaptive polarization filter can be modeled as an open loop cascading system of a polarization estimator, an optimum polarization calculator and a polarization filter. Based on this model, a super limit of theoretical performance of the adaptive polarization filters was obtained and its analytical formula is presented too. The study is of great importance to the performance elevation of adaptive polarization system analysis and optimum design in adaptive polarization filter. For example, it is easily known that the estimation precision will directly limit the whole filter performance from the model of open loop cascading system and the filter performance formula. So optimizing the estimator should be important in designing the filter.

So far, adaptive polarization filters are mainly used to reject jamming with high polarization degree. Routine adaptive polarization filters aren't satisfying in suppress jamming partially polarized. This is a desiderate problem in polarization technology, and it pose new demand in evaluating the performance of adaptive polarization filters.

REFERENCES

- [1] A.J.Poelman. Virtual polarization adaptation, a method of increasing the detection capabilities of a radar system through polarization vector processing. IEE Proc. Communications, Radar and signal Processing, 128(5): 261~270,1981
- [2] Dino Giuli. Polarization diversity in radars. IEEE Proc.74 (2):245~267,1986
- [3] Wang Xuesong, Zhuang Zhaowen, Xiao Shunping, Guo Guirong. The Optimal Receive of Polarized Signals: Partially Polarized Case, Journal of Electronic Science, 20(4):468~473, 1998. (in Chinese)
- [4] R.S.Raghavan, N.Pulsone, D.J.Mclaughlin. Adaptive estimation of the polarization of a signal. IEEE Trans. ES.31(2):845~852, April 1995
- [5] E.Pottier, J.Saillard. Optimal Polarimetric Detection of radar target in slowly fluctuating environment of clutter. IEEE AES Magazine, 4~9, Nov.1990
- [6] Xu Zhenhai, Wang Xuesong, Xiao Shunping, Zhuang Zhaowen. Polarization Adaptive Recursive Filtering Algorithm. ACTA ELECTRONICA SINICA, 30(4):608~610, 2002. (in Chinese)

- [7] D.P.Stapor. Optimal receive antenna polarization in the presence of interference and noise. IEEE Trans. on AP. 43(5):473~477,1995
- [8] Zhuang Zhaowen , Xiao Shunping, Wang Xuesong. Radar polarization Information Processing and Application. Beijing: National Defence Industry Press,1999. (in Chinese)
- [9] Zhang Guoyi. Study on Polarization Anti jamming in HF Gound Wave Radar[D]. Harbin: Harbin Institute of Technology, 2002. (in Chinese)
- [10] Wang Chaoyue, Shi Rongchang. Matrix Analysis. Beijing: Beijing Institute of Technology Press,1989. (in Chinese)
- [11] Lu Dajin. Random Process and its Application. Beijing: Tsinghua University Press, 1986. (in Chinese)
- [12] M.I.Skolnik. Introduction to radar systems (3rd edition). McGraw Hill Higher Education, 2001
- [13] S. Haykin. Adaptive filter theory (3rd edition). Prentice Hall, 1996
- [14] F. E. Nathanson. Adaptive circular polarization. IEEE Int. Radar Conf., 221~225, 1975
- [15] D.Giuli, M.Fossi, M.Gherardelli. A technique for adaptive polarization filtering in radars. Proc. of IEEE Int. Radar Conf., 213~219, 1985
- [16] M.Gherardelli. Adaptive polarization suppression of intentional radar disturbance. IEE Proc. Pt. F. 137(6):407~416, 1990
- [17] Mao Shisong, Wang Jinglong. Higher Statistics. Beijing: Higher Education Press; Germen: Springer Press, 1998.7.(in Chinese)

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