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THESIS

SALARY AUCTIONS AND MATCHING AS INCENTIVES FOR RECRUITING TO POSITIONS THAT ARE HARD TO FILL IN THE NORWEGIAN ARMED FORCES

by

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March 2006

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A significant number of positions in the Norwegian Armed Forces that are open for assignment are not filled because they do not receive any qualified applicants. Over the last five years, over 30 percent of the announced job vacancies have been unfilled. This thesis explores two different areas of research to help remedy this; auction theory and assignment market mechanisms. Auction and assignment market theory and practice are examined to reveal how these mechanisms might provide incentives and improve the quality of military assignments.

This research finds that both of these mechanisms fall short when used independently. Auction theory is problematic when both sides of the market have preferences over the outcome; assignment models are problematic when there are system level concerns about which jobs remain unfilled. This thesis introduces a hybrid solution, containing elements of both auction theory and assignment markets, which has the potential to improve the current matching process. This research improves our knowledge and understanding about both of these research areas, and their interactions.

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SALARY AUCTIONS AND MATCHING AS INCENTIVES FOR RECRUITING TO POSITIONS THAT ARE HARD TO FILL IN THE NORWEGIAN ARMED FORCES

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ABSTRACT

A significant number of positions in the Norwegian Armed Forces that are open for assignment are not filled because they do not receive any qualified applicants. Over the last five years, more than 30 percent of the announced job vacancies have been unfilled. This thesis explores two different areas of research to help remedy this; auction theory and assignment market mechanisms. Auction and assignment market theory and practice are examined to reveal how these mechanisms might provide incentives and improve the quality of military assignments.

This research finds that both of these mechanisms fall short when used independently. Auction theory is problematic when both sides of the market have preferences over the outcome; assignment models are problematic when there are system level concerns about which jobs remain unfilled. This thesis introduces a hybrid solution, containing elements of both auction theory and assignment markets, which has the potential to improve the current matching process. This research improves our knowledge and understanding about both of these research areas, and their interactions.

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I. INTRODUCTION

A. BACKGROUND

”Never have I seen home and the living life of home so wide, so clear and so close, as just from the distant and under the absence.”[1]

Henrik Ibsen, 1828 – 1906

This quote is meant to illustrate one of the purposes that justify the idea of traveling to a foreign country far from home to undergo education. Also, this represents a part of the background for this research. By being distanced from the military organization back home one starts reflecting differently than when occupied in everyday life at home. Also, being abroad has made it relatively uncomplicated to obtain an unbiased view with regard to the chosen topic for this thesis. Being in a different environment, learning to know how other organizations perceive theories and practice applications, and comparing this to the situation back home, has also enriched the learning process of this research.

“Every post is honorable in which a man can serve his country.”[2]

George Washington, 1732 – 1799

“There must be some other stimulus, besides love for their country, to make men fond of service…”

George Washington, 1732 – 1799

These two quotations have their origin about 230 years ago; however they are still valid today. The content is related to incentives for serving in the military, and the two sentences might represent quite opposite ideas of motivation and incentives. In the recent

[1] In Norwegian: “Aldrig har jeg set hjemmet og hjemmets levende liv så fyldigt, så klart og så nær ind på, som netop fra det fjerne og under fraværelsen.”

[2] In a letter to Benedict Arnold, September 14, 1775

years the Armed Forces in several countries, including both the USA as well as Norway, have gone through significant changes. These changes are seen in different areas, including a downsizing in the number of personnel, and an increased focus on cost reduction. Since 2003, the US Navy has experimented with Assignment Incentive Pay (AIP). The objective has been twofold; to assign personnel to hard-to-fill positions, and also to get an improved match between each position and each individual employee. The experiment has been done by using least cost calculations, so apart from filling positions and matching individuals with jobs, the objective has also included the desire to save costs.

Over the years the Norwegian Armed Forces have experienced similar problems as the American forces related to filling all positions that are announced for employment at each assignment cycle. In the new organizational context there is an increased focus on the well-being of each individual employee. At the same time the organization shrinks, so does the personnel redundancy. Consequently, it is of great importance that the employer is able to allocate the right employee to each position in the organization. In this context, the military ought to look at new mechanisms in the assignment process to be able to obtain this goal. In this respect, it would be natural to consider mechanisms like auctions and matching. Use of these mechanisms might generate applicants for positions that traditionally have been hard to fill, and also give information to the employer with regards to the strengths of preferences among the employees.

B. PURPOSE

The main theme of this study is market design. A market is a venue for buyers and sellers; as Lieberman and Hall define it, "it is a group of buyers and sellers with the potential to trade"[3]. All markets, including the labor market, are characterized by suppliers and consumers, and the process of finding equilibrium. The context of this research is the labor market in the Norwegian Armed Forces. The focus is on the permanent assignment process, where military officers are allocated to vacant positions in the military organization. This market has the same main ingredients as all other

[3] Marc Lieberman and Robert Hall, *Introduction to Economics* (Mason, OH, USA: South-Western College Publishing, 2000), 42.

markets. Furthermore, it is characterized by one consumer who is the employer, and many suppliers who are the officers who offer their labor.

In the current assignment process, the employer has the final word on which officer wins each position. The process in general involves allocating the most qualified applicant to each position. During the latest years, a significant number of positions open for assignment have not been filled because they have not received any qualified applicants. As such, there should be room for improvement in this process. The main question that underlies this study is the following: Is there any appropriate mechanism that might be introduced to improve the current assignment process?

A recent study analyzing the introduction of AIP into the US Navy concluded that this service could transform its current system through the use of incentives. In general, AIP should be more cost-effective than the current system; although, for some individual positions, it may not be. Further, the conclusion stated, "Given the adequate incentives, sailors will volunteer for billets that are traditionally hard to fill".[4] Based on this, it is important to analyze if the theories of auctions and matching may fit the context of assignments in the Norwegian Armed Forces, and further, which mechanism that might be the most appropriate solution to introduce.

C. RESEARCH QUESTIONS

1. Primary Research Question

- To what extent can auctions and matching be utilized as incentives in the recruiting process for permanent assignments in the Norwegian Armed Forces, and what would be the most appropriate auction or matching mechanism for use as an incentive for this organization?

2. Secondary Research Questions

- How does the assignment process work in the Norwegian Armed Forces, and what are the main problems related to the current process?

[4] Golding, Heidi L. W., Eric W. Christensen and Diana S. Lien, *Transforming the Assignment System: Will Incentives Reduce Critical Shortages?* (Alexandria, VA, USA: Center for Naval Analyses, 2002).

- Which different auction designs exist, and how can auctions be used in an assignment setting?
- Which different matching designs exist, and how can matching be used in an assignment setting?

D. SCOPE AND LIMITATIONS

1. Scope

The scope of this study includes: (1) an overview of the current assignment process in the Norwegian Armed Forces, including a presentation of problem areas related to the current process, (2) an overview of auction theory, and application of auctions, focusing on an assignment setting, (3) an overview of the mechanisms of matching, and application of this kind of mechanisms, focusing on an assignment setting, and (4) a recommendation for a hybrid solution to use in the assignment process in the Norwegian Armed Forces.

2. Limitations

This research does not consider circumstances of this topic that are related to roles and interests of stakeholders like unions and politicians. Also, it is outside the scope of this research to analyze to what degree any specific position is hard-to-fill in the Norwegian military (i.e. which specific positions are consistently announced for employment without getting any qualified applicants). The recommended mechanism is proven theoretically. However, it is outside the scope of this thesis to examine the model through tests and/or experiments, to see if it works in a practical setup.

The auction theory that is part of this research is limited to one-sided auctions, meaning auctions in which there is one single seller and many buyers, or there are many sellers and only one buyer. Two-sided auctions, with many sellers and many buyers, are not discussed.

The discussion of matching mechanisms is limited to a basic introduction to the topic. For a more in-depth study refer to the NPS Master's thesis by Paul Robards on two-sided matching[5].

E. EXPECTED BENEFITS OF THE STUDY

This study will provide general insights into both theories as well as applying auction theory and matching mechanisms. Further, this research will help improve our knowledge and understanding about both of these research areas, and their interactions. It will also provide a suggestion for an appropriate mechanism to consider for the assignment process in the Norwegian Armed Forces.

F. ORGANIZATION OF THE THESIS

In Chapter II, the current assignment process in the Norwegian military organization is presented. Chapter III provides an overview of auction theory, while Chapter IV concentrates on practical application of auctions. Chapter V reviews main elements of the mechanisms of matching, followed by a discussion of practical uses of matching in Chapter VI. Chapter VII describes the recommended hybrid mechanism, while Chapter VIII contains a summary, conclusions and recommendations for further research.

[5] Paul A. Robards, *Applying Two-Sided Matching Processes to the United States Navy Enlisted Assignment Process* (Naval Postgraduate School, Monterey, CA, USA, 2001).

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II. THE ASSIGNMENT PROCESS IN THE NORWEGIAN ARMED FORCES

A. BACKGROUND

After the Viking age, in which Norway was a superpower, the royal line died out in 1387. For Norway, this started a long period as the weaker part of a union with Denmark. Denmark-Norway sided with Napoleon in the Napoleonic Wars, and in 1814 Norway was ceded to Sweden. However, Norway claimed her independence, and got her own *Constitution (Grunion*) on 17 May 1814. Norway stayed in the union with Sweden until independence was finally declared in 1905; during these years the *Constitution*, along with independent institutions, were in effect. Derived from the *Constitution*, a *Conscription Law (Vernepliktsloven*) was established in 1854. From that time forward, conscription has been the cornerstone of Norway's military system.

The Armed Forces in Norway consist of three major groups; officers, other ranks including enlisted personnel, and civilian employees. The overall peacetime strength is approximately 14,600, divided into Army (7,000), Navy (5,000), Air Force (2,000), and Home Guard (600)[6]. The military is currently in a process of downsizing the services, and the peacetime strength numbers will continue to decrease towards the end of 2008.

Military employees are mainly officers. There is no separate system of non-commissioned officers. Still, there are non-commissioned officers in the Armed Forces, as all officer candidates have to serve one year as a non-commissioned officer before they get promoted to the lowest officer rank. However, these non-commissioned officers are regulated through the same set of rules as the officer corps. The enlisted military personnel are recruited through the conscription system and they serve for a maximum period of 12 months. This means there is no voluntary enlisted force as in the American military services. The majority of the civilian employees are hired in one specific position at one particular unit or station. Consequently, the career of a civilian employee in general does not include a methodical change of position and geographical location. The pattern of the officers' careers seem to be somewhat similar to the current situation

6 *Norwegian Defence 2005* (Oslo, Norway: The Royal Norwegian Ministry of Defence, 2005).

for American military officers; it is a lifestyle with frequent changes of position, often including geographic relocation, and for many individuals reflecting a desired vertical career path.

B. THE CURRENT RULES REGULATING THE ASSIGNMENT PROCESS

The relationship between the main rules regulating the employment of Norwegian Armed Forces' personnel is outlined in Figure 1, below.

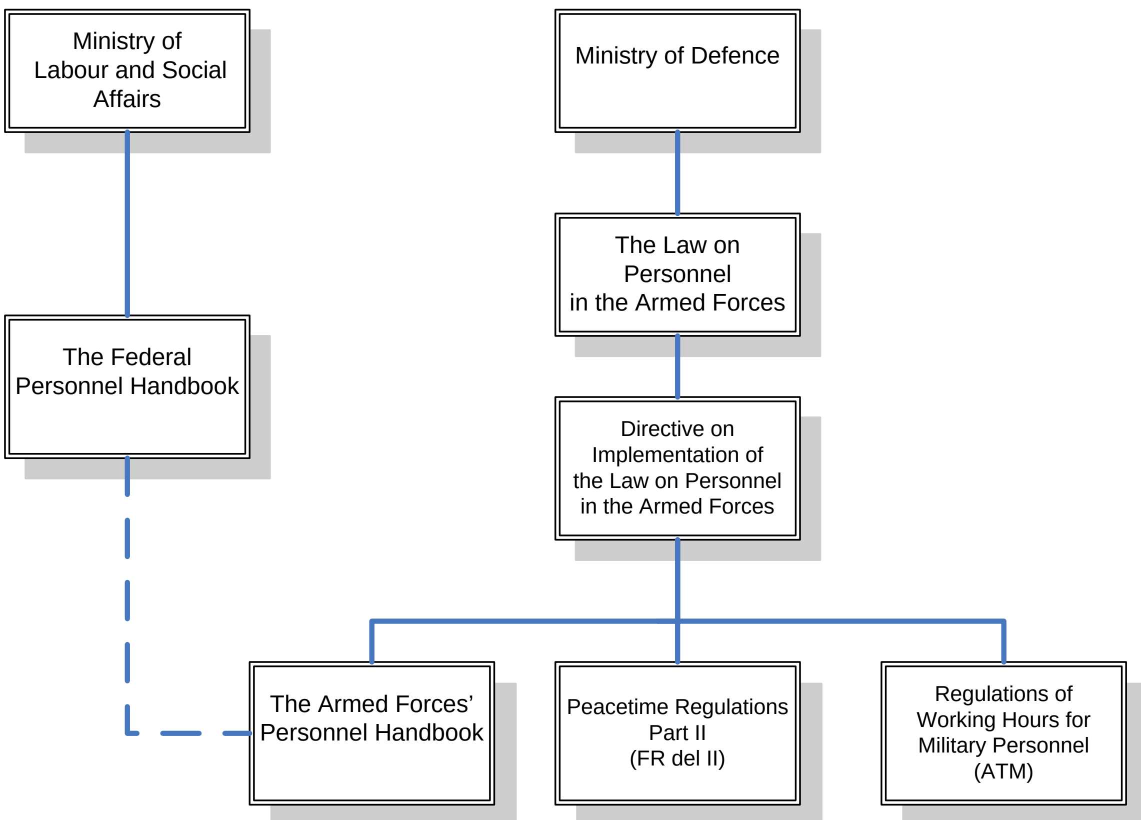

Figure 1. Relationship between the main set of rules regulating the employment of personnel of the Norwegian Armed Forces

The Law on Personnel of the Armed Forces (Lov om personell i Forsvaret)[7] is the overarching regulation concerning all Armed Forces' personnel. *The Federal Personnel Handbook* (*Statens personellhåndbok)* consists of regulations valid for all federal

[7] *Lov Om Personell i Forsvaret*, (2004).

employees, including the military. *The Armed Forces' Personnel Handbook (Forsvarets Personalhåndbok,* or *FPH)*[8] is derived from these two regulations, and also draws on other federal laws[9]. *FPH* applies to all employees of the Armed Forces. Part B of this handbook; *FPH Part B* regulates all military officers and non-commissioned officers. This part of the handbook describes the employment for these groups in detail, including regulations for hiring, education, promotion, dismissal, and the assignment process.

C. OUTLINE OF THE CURRENT ASSIGNMENT PROCESS

1. General

The goal of the assignment process is to secure qualified Armed Forces personnel in all duty positions. Officers' assignments are divided into three categories:

- Permanent
- Temporary
- Special assignments.

The latter involves mainly short term assignments, like assignments to participate in a particular exercise or an international operation. Temporary assignments can be used with or without the consent of the individual. However, the time frame of such assignments is not supposed to exceed one year. The process of such assignments is continuous, and these individual assignments are mostly decided at the local level, usually without involving central authorities in the decision-making. These assignments are not limited by specific deadlines or a set number of assignment cycles per year.

The permanent assignment category is the main form of assignment. This category is to be used to as large a degree as possible. The process of the permanent assignments is the focus for the discussion to come.

[8] Forsvarsdepartementet, *Forsvarets Personellhåndbok, Del B, Forvaltning Av Befal* (Oslo, Norway: Forsvarsdepartementet, 2005).

[9] These include The Administration Law [*Forvaltningsloven*], The Civil Servant Law [*Tjenestemannsloven*], and The Work Environment Law [*Arbeidsmiljøloven*]

2. Permanent Assignments

Defence Staff Norway (DEFSTNOR) is the staff of the Chief of Defence. DEFSTNOR is responsible for prioritizing and balancing the overall resources to be assigned to operational activities, the production of forces, and the provision of support services. DEFSTNOR is organized as a joint staff and individual service staffs. The joint staff consists of a central staff and a number of specialist and professional staffs. Permanent assignments, as the main form of assignment for military officers, is centrally organized and coordinated for all military services by the Personnel- Financial- and Control Staff (Personell- Økonomi- og Styringsstaben), abbreviated FST/PØS, in DEFSTNOR[10].

The detailed process of permanent assignment is described in *FPH Part B*, 4.2.5. The outlined process encompasses the ranks from O1 – O5 (Second Lieutenant through Lieutenant Colonel / Commander Senior Grade)[11]. Each cycle starts with all duty stations notifying FST/PØS of which positions are vacant or to be vacant, and as such requested to be announced for permanent employment. FST/PØS collects all incoming requests. In most cases too many positions are reported for announcement, compared to the inventory of officers available in the current cycle (the actual number varies). Then FST/PØS prioritizes the requests and decides which positions will be announced, and which will not. The next step is the announcement itself; usually there is a three week deadline for the receipt of applications. Applications are electronically registered at each duty station, and the local level prioritizes internal candidates who have applied for the same position.

After the deadline, the majority of the work is executed by officers at FST/PØS. They collect all applications with all attachments that are submitted. They gather relevant background information on all applicants, prioritize all candidates for each position, and prepare a written recommendation. This recommendation is presented to the assignment board ("Forsvarssjefens råd i tilsettings- og disponeringssaker"). The

[10] "The Present Organisation of the Armed Forces´ Political and Military Leadership," http://www.mil.no/languages/english/start/facts/article.jhtml?articleID=32061 (accessed October 26, 2005).

[11] There exists a separate process for ranks O6 (Colonel / Commander) and above, which will not be covered in this text.

assignment board is the body that makes all decisions on permanent assignments, as well as decisions on suspension and dismissal of military personnel.

During a calendar year there are usually two to four of these cycles[12]. In 2003, three cycles were held. In 2004 and 2005, four cycles were held each year, and three cycles are planned for 2006. The workforce involved in the permanent assignment cycles includes approximately 14-16 officers in FST/PØS; four each from the Navy and Air Force, and six to eight from the Army. These officers work full time with the permanent cycles. Additionally, the Air Force involves its five KARIUS[13] staff officers as necessary; the other services also bring in assistance as required. The assignment board itself includes seven people[14].

D. PROBLEM AREAS WITH THE CURRENT ASSIGNMENT PROCESS

There are some problem areas in the current process of permanent assignments. One of these is the fact that, as of today, not all vacant positions are announced for permanent employment at the time they become vacant. FST/PØS has a capacity constraint on the number of positions they can announce in each cycle. Their capacity is mainly limited by their allocated workforce, and also based on additional regulations attached to each cycle. In the announcement process, NATO positions and other international commitments always have first priority, along with some other positions that are assessed as essential to fill. This again omits other positions with a lower priority, and often more regular positions at domestic units are left out of the announcements. It might actually take several cycles before a position is announced. During this time, most of these positions are filled temporarily. The process of recruiting individual officers to such temporary assignments often involves a significant amount of extra work for those involved.

If a position is included in the announcement cycle, there is still no guarantee that it will be filled. If there are no applicants for a position, no individual can be

[12] Forsvaret [Norwegian Defense], *Overordnede Retningslinjer for Disponeringsomgangen 2006* (Oslo, Norway: Forsvarsstaben/PØS, 2005).

[13] Career Planning officer [*Karriereutviklingssystem – KARIUS*]

[14] Forsvarsdepartementet, *Forsvarets Personellhåndbok, Del B, Forvaltning Av Befal*, 4.2.6.1

permanently hired into it until it is announced once more. Consequently, it has to be announced at the next cycle, unless an individual has approved to fill the position temporarily for a period of time. Actually, such positions represent an accumulating problem for the employer. In general, positions without applicants have to be announced again, at least one more time, in order to be filled permanently. However, with a constraint on the number of positions that might be announced per cycle, FST/PØS may have to prioritize between announcing a position for a second or third time, or announcing another vacant position; a vacant position with no applicants in one cycle might take up valuable space in a later cycle.

Another element is the "lag" connected to the process. There are a limited number of cycles each year. If an officer applies for and is appointed to a new position, he leaves a "hole" at his old position in the organization. In some cases, in particular for positions with a time limit, replacements can be planned in due time. However, in most cases there is a lag from the time a position becomes vacant until a permanent replacement has been hired and reports to this position. As such, there is an on-going lag, as each vacant position that gets filled leaves another hole somewhere else in the organization.

The imperfections with this process might affect the workplace in different ways. In a worst case scenario, readiness levels, as well as the quality of the tasks that are being performed, might suffer from shortcomings in personnel strength. On the personal level, officers and civilian employees might experience decreases in motivation and job satisfaction if one's unit experiences problems with recruiting for different positions. In particular, this could be a concern for employees in close proximity to the holes in the organization, like co-workers and employees that have to assume responsibility for the tasks belonging to a position that is vacant. The motivation of employees sitting in positions related to planning and distribution of personnel could also be influenced. The responsibilities related to recruiting individuals to hard-to-fill positions is often time consuming, and at times it might be a very challenging task to find any qualified candidates at all. At least one service, the Air Force, often involves its centrally located KARIUS staff officers in this work. The KARIUS officer is a key person in the personnel planning and distribution process. Being responsible for one special field only (e.g. pilots, force support, etc) this officer usually has an updated overview of the workforce in his special field. The KARIUS officer cooperates closely with the personnel departments at each duty station. Frustration is observed at those units that frequently experience significant holes in their organization. The KARIUS officer himself is in most cases involved in finding candidates to temporarily fill vacant positions. As such, he also experiences the extra work load related to this process himself[15].

The military organization has been, and still is, experiencing a major change, involving downsizing, merging of some units, and closure or realignment of others. A large portion of the duty positions in the military organization have been renamed in this process; some have even been renamed more than once over the last few years. Consequently, it is beyond the scope of this thesis to investigate the degree to which any particular position is continually without applicants for permanent employment.

Based on all factors above, it would seem essential for the Armed Forces to permanently fill as many positions as possible the first time they are announced. However, the facts from the latest years illustrate a situation that does not agree with this assertion. Table 1 below illustrates the total number of positions that were announced in each cycle over the last five years. Further, it shows the number of positions without any applicants, as well as the number of positions that are "not assigned", which means there have been applicants, but none of them have been qualified. Together, the "no applicants" and "no assignment" numbers indicate the proportion of positions that are not filled in each cycle.

[15] *The author has first hand experience in the KARIUS duty position, from 2002-03. From 2000-02, the author also worked as Staff Officer Personnel at a unit experiencing this problem.*

Cycle #	Number of announced positions*	"No applicants"		"No assignment"		Sum % "No applicants" + "No assignment"
Runde #	**Antall kunngjorte stillinger***	**"Ingen søkere"**		**"Ingen tilsetting"**		**Sum % "Ingen søkere" + "Ingen tilsetting"**
		#	**%**	**#**	**%**	
2001 / 1	52	0	0,000	16	0,308	0,308
2001 / 2+3	856	101	0,118	165	0,193	0,311
2002 / 1	249	32	0,129	42	0,169	0,297
2002 / 2A+2B	2.895	335	0,116	738	0,255	0,371
2002 / 3	8	0	0,000	1	0,125	0,125
2003 / 1	539	95	0,176	137	0,254	0,430
2003 / 2	923	138	0,150	243	0,263	0,413
2003 / 3	591	118	0,200	115	0,194	0,394
2004 / 1	618	125	0,202	139	0,225	0,427
2004 / 2	453	52	0,115	117	0,258	0,373
2004 / 3	1.679	232	0,138	150	0,089	0,228
2004 / 4	302	53	0,175	73	0,243	0,417
2005 / 1	320	30	0,094	60	0,188	0,281
2005 / 2	179	13	0,073	18	0,101	0,173
2005 / 3	598	37	0,062	117	0,196	0,258
2005 / 4	56	0	0,000	9	0,161	0,161
2006 / 1	802	18	0,022	98	0,122	0,145
Sum	6 546					
Average	385		**0,104**		**0,197**	**0,301**
Weighted average			**0,124**		**0,206**	**0,330**

Table 1. Trends in permanent assignments[16]

* : Number does not include announced positions that are withdrawn during the cycle (Inneholder ikke stillinger som er utlyst og deretter trukket i løpet av den aktuelle runden)

The numbers in Table 1 form a relatively clear picture. Over the last five years, on average 30.1 percent of the positions announced for permanent employment have not been filled due to a lack of qualified applicants. The weighted average gives a more

[16] The data in Table 1 has been provided by Maj L. Hjerpsted, FST/PØS

precise number than the basic average. This number measures the number of positions in each cycle, and as such reflects the size of each cycle. The weighted average is even bigger than the basic average, showing 33.0 percent over the last five years. This means that for cycles with a large number of positions, the proportion of positions that are not filled is larger than in the small cycles. From the employer's standpoint, it ought to be an obvious challenge to find solutions that could improve this situation. Among those incentives that could be considered relevant in this respect are salary auctions and matching.

E. CHAPTER SUMMARY

This chapter presented some relevant data on the Norwegian Armed Forces, along with an overview of the existing set of regulations governing the assignment process. Further, the categories of assignment, permanent, temporary, and special assignments were presented with a focus on the current process for permanently assigning military officers.

The main problem areas related to the permanent assignment process were also outlined. First is the capacity constraint on number of positions that might be announced in each cycle, which results in vacant positions queuing up as opposed to being announced at the time they become vacant. Second is the "lag" which is created in the organization as an officer applies for and is appointed to a new position. In addition, there is no guarantee that there will be any applicant for an announced position, so one hard-to-fill position might occupy valuable space in more than one cycle. Finally, the problems related to the assignment process might have negative consequences for motivation and job satisfaction among those who experience and work with these tasks first hand.

In conclusion, some key statistics from the recent years' permanent assignment cycles were presented. This data identifies the proportion of positions that were announced for permanent employment but were not filled due to lack of qualified applicants. The number is significant; the weighted average is 33.0 percent over the last five years, while the simple average is 30.1 percent over the same time period.

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III. AUCTION THEORY

A. BACKGROUND

The encyclopedia states that, *"The word auction is derived from the Latin "augere", meaning increase, or "augment" (the participle form being "auctus"). It is a form of sales contracting where several individuals present are encouraged to bid and overbid for an object or a right"*[17]. A definition using different wording states that, "*An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants*"[18].

Basically, an auction is a unique set of rules allocating scarce resources based on price making by competition of buyers or sellers for the right to purchase or sell. This is an alternative to other selling mechanisms, like bargaining or a fixed, posted price. In a transaction method like an auction, one side of the market generally plays a very active role, while the other side ordinarily plays a passive role[19]. It is important to note that the root of the word auction – *auctio* – is misleading, since far from all auctions have ascending price schemes[20].

Auctions as a market institution actually existed by 500 B.C. In Babylon at that time, women were sold on an annual auction, "on condition that they be wed"[21]. In 193 A.D., the whole Roman Empire was sold by such a transaction[22]. In newer eras, world famous auction enterprises like Sotheby's[23] and Christies[24] started their businesses in 1744 and 1766, respectively. These companies started the development of the so-called

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17 *Aschehougs Konversasjonsleksikon* (Oslo, Norway: Aschehougs forlag, 1939).

18 R. Preston McAfee and John McMillan, "Auctions and Bidding," *Journal of Economic Literature* XXV (Summer 1987), 701.

19 Ralph Cassady Jr, *Auctions and Auctioneering* (Berkeley and Los Angeles, CA, USA: University of California Press, 1967), 8.

20 Ibid., 7

21 Ibid., 26

22 Ibid., 29

23 "Sothebys.Com," http://search.sothebys.com/about/corporate/as_corphistory.html (accessed October 3, 2005).

24 "Christies.Com," http://www.christies.com/history/overview.asp (accessed October 3, 2005).

English auction. In the recent years, in particular following the introduction and expansion of different services on the Internet in the 1990s, auctions have become very common for a wide spectrum of the population in most countries. Companies like *eBay* auction a wide variety of products; these companies have experienced a massive expansion in their sales volumes and profits. A wide variety of products and services are traded by auctions. This includes art and antiques, real estate, bonds and stocks, drilling rights, flowers and fish, treasury bills, houses, and even assignments of workers.

B. KEY TERMINOLOGY

Before introducing the different types of auctions, it is important to explain some key features of auction mechanisms.

1. Forward vs Reverse Auctions

a. Forward Auctions

A forward auction involves one single seller and more than one potential buyer. In forward auctions, the seller is the bid-taker and the potential buyers are the bidders. In this format, the bids increase and the winner is the individual who presents the highest bid. An example might be an open antique auction in which several bidders compete for an antique statue. The bids increase and in the end the winner is the one with the highest bid. A forward auction might also be of the sealed-bid type, where each bidder submits one written bid. As above, the winner is the bidder submitting the highest bid.

b. Reverse Auctions

A reverse auction is the opposite of a forward auction; there are many sellers and only one buyer. Contrary to the forward auction, the single buyer is the bid-taker and the sellers are the bidders. This mechanism is characterized by the fact that the lowest bid is the winning bid. An example of a reverse auction could involve a procurement situation. If a company was to outsource cleaning its office building, it would ask cleaning agencies to bid for doing this job. This would allow the cleaning agencies to state the amount of money they would need to receive to accept the job. The winner would be not the bidder who submitted the highest bid, but rather the agency bidding the lowest amount to accept the task.

The main distinctions between forward and reverse auctions are viewed in Table 2, below.

	Forward auction	Reverse auction
Number of Buyers	More than 1	1
Number of Sellers	1	More than 1
Bidders	Buyers	Sellers
Bid-taker	Seller	Buyer
Winner	Highest bid	Lowest bid

Table 2. Distinctions between forward and reverse auctions

2. Basic Auction Models

A key feature of auctions is the presence of asymmetric information. In most cases the participants in an auction have different knowledge about the item. In addition each individual's taste and personal preferences are important. This results in different measures of value for the item. Details about taste and preferences are often not known to other than the bidder himself. This situation creates uncertainty, which again might influence the bidder's behavior.

The basic theoretical auction model consists of two polar cases. On the one extreme, all bidders' true valuation of an object is identical. This is called common values. On the other extreme is the case where each bidder's value is completely independent from the value of his competitors, which is called private values. The two theoretical models are to be interpreted as polar cases. In reality, however, most auction situations are likely to contain aspects of both.

a. Common Value Model

In the Common value (CV) model, the actual value of the object is the same for all bidders. However, the bidders have different private information about the actual real value. This means that the bidders do not know the actual value of the object. An example could be an oil deposit in the North Sea. The value of this oil field depends on the amount of oil that exists at the bottom of the sea, and the bidders may have received different signals about the geological conditions, but the true value of the oil field is approximately the same for all bidders. If a bidder learned about another bidder's valuation of this oil field, he would consider this as useful information, and he would probably change his estimate of the value in light of this. This is contrary to the Private value model, in which a bidder's value of an object would be unaffected by learning information about the preferences of other bidders.

b. Independent Private-Values Model

In the Independent private-values (IPV) model, the situation is opposite of the CV model. Each bidder knows his own value for the item at the time he is bidding. This value is not shared with the other bidders. Valuations are drawn independently from each other, and there is uncertainty about the other bidders' values. However, one's value is not affected by any information about other bidders' values. Still, as pointed out by McAfee and McMillan, a bidder might change his bid based on this new information; this would be for strategic reasons and not because his value of the object has changed[25]. Antiques bought by collectors (not for resale) and sports tickets represent common examples of items having independent private-values.

This can be illustrated by an auction for a ticket to the Ice-hockey Final in the Winter Olympics. There are three individuals participating in the auction. The first individual is not particularly interested in sports, while the second person prefers Skiing or Biathlon. The third person is a real ice-hockey fan and his favorite team has just qualified for the Olympics Finals. It should be obvious that the third person values the ticket more than the two other individuals. This person's value does not change whether

[25] McAfee and McMillan, *Auctions and Bidding*, 705

or not he learns the values of the two other bidders. However, this knowledge might change his bidding strategy even though his value of the ticket remains the same.

C. THE DIFFERENT TYPES OF AUCTIONS

A vast number of books and research studies are available on the topic of auctions, most of them published since the 1960s. Based on the literature, a wide variety of selling formats fit under the term auction. There are “deadline” auctions, which are commonly used by Internet auction sites. Here, the individual with the highest bid at a certain declared deadline wins the auction. A “candle” auction has a random stopping time, and the winner is the person with the highest bid at the time the candle burns out. The winner of an auction could pay a price averaging all the bids, or a price equal to the third-highest bid. The range of possibilities is wide. However, in his book *Auction Theory,* Vijay Krishna has outlined the four major auction types used when a unique item is to be bought or sold, as shown below in Figure 2[26]. These are the main formats addressed in this study.

	Open Format	**Sealed-bid Format**
First-price	**Dutch Auction**	**First-Price Auction**
Second-price	**English Auction**	**Second-Price (Vickrey) Auction**

Table 3. Auction formats

1. Dutch Auction

The Dutch auction takes its name from the flower markets in the Netherlands, where the auctions are characterized by a clock announcing a descending price of the offered products. Since the first flower auctions were held in this market in 1911, inconceivable amounts of flowers have been traded using this transaction method. At the Aalsmeer Flower Auction, the largest flower auction in the world, a total of 19 million

[26] Vijay Krishna, *Auction Theory* (San Diego, CA, USA: Academic Press, 2002), 5-6.

flowers and two million potted plants are traded each day. These premises include a total of 13 auction clocks, divided between five auction halls, all operating simultaneously[27].

The price scheme in the forward Dutch auction format is announced by an auctioneer, or some kind of electronic device, like the previously mentioned clock. Since the format of this auction type is one with descending prices, the initial price is very high. The price decreases, and the prices enunciated are invitations for the potential buyers to bid. Following the decreasing prices, the first bidder is the highest one, and thus the successful one. The winner has to pay the current price, meaning the price shown by the clock at the time the bid was given. In order to maximize his expected profit, which in a forward auction is buyer value minus price paid, each buyer has to consider his own value of the product, but also the value of other buyers and the expectation of the other buyers' bidding strategies[28]. Consequently, potential buyers compete against their perception of the competition they face in each individual auction. To win the auction, the buyer should claim the product before any other competitor, yet below his maximum value for the product that is offered. The key aspect with this auction system is that the initial call is sufficiently high to permit the bidder with the highest value to register his maximum bid. At the same time, if speed of the auctions is important, the seller's initial price should not be set too high, as this would delay the auction process[29]. The Dutch auction could also work in reverse format. In this case, the bid-taker is now the buyer and the price starts at a low level and rises gradually. The bidders/sellers compete for the right to sell their product and the winner is the first seller to place a bid.

2. First-price Sealed-bid Auction

The first-price sealed-bid (FPSB) auction is a commonly used and pretty straightforward auction type; potential buyers submit sealed bids and the bidder who submits the highest bid (lowest bid in reverse mode) receives the item and pays the

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[27] "Alsmeer Flower Auction," http://www.aalsmeer.com/contentpagina.asp?cnt_id=193%20&MainMenu=Aalsmeer+Flower+Auction&SubMenu=History (accessed October 10, 2005).

[28] William Vickrey, "Counterspeculation, Auctions, and Competitive Sealed Tenders," *The Journal of Finance* 16, no. 1 (March 1961), 21.

[29] Cassady Jr, *Auctions and Auctioneering*, 60-63.

amount bid. Conceptually, this auction format has similarities to the Dutch auction as there is no observed information on competitors and their bidding strategies. Before turning in the sealed-bid, bidders have to consider their own value of the item offered, and estimate both the competitors' value of the product and their bidding strategies. This is required for the bidders to maximize their expected profit[30].

In this type of auction, with a finite number of bidders competing for an asset, the expected winning bid is lower than the expected value of the asset for the winner, leaving some expected profit for the winner. Furthermore, as the number of bidders increases, the expected value of the winning bid also increases. This improves the result for the seller, but reduces the expected profit for the auction winner[31].

3. English Auction (Open outcry ascending bid auction)

Historically, the English auction is the auction type that is most widely used in a majority of countries, and also the type that most people seem to connect with the term "auction"[32]. This auction type uses an ascending-bid format, being the converse of the Dutch auction. An auctioneer is in charge of the bidding process. He seeks an initial bid from the assembled buyers and then calls successively higher prices until only one bidder remains. In this process, a bidder can revise his bid and increase it relative to other bids. This is a difference from the FPSB auction, where each bidder only submits one bid. When nobody raises the current bid, the remaining bidder wins the auction, paying the price he bid. The winning price is equal to the second-highest bidder's value, plus one bid increment to guarantee victory. However, the size of the extra bid increment varies from case to case; depending on the situation it could be anything from one cent to a large amount of money. Irrespective of the size of the extra bid increment, this auction type has some equivalent characteristics to the second-price sealed-bid (SPSB) auction, which is discussed below.

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[30] Vickrey, *Counterspeculation, Auctions, and Competitive Sealed Tenders*, 21.

[31] Kenneth R. French and Robert E. McCormick, "Sealed Bids, Sunk Costs, and the Process of Competition," *The Journal of Business* 57, no. 4 (October 1984), 418.

[32] Cassady Jr, *Auctions and Auctioneering*, 56.

In reverse mode, the auctioneer, now representing the buyer, starts by calling out a high price. The sellers bid successively lower prices until only one of them, the winning bidder, remains. Consequently, the winning bid is the lowest bid.

In the forward English auction, each bidder's dominant strategy is to keep bidding until the price reaches his maximum willingness-to-pay (WTP) for the product, and then drop out of the auction. If a buyer drops out before his value is reached, he will forego potential gains. If he continues bidding above his maximum WTP, this will clearly create a loss. The winner wins before the auction reaches his dropout price. Apart from the winner, all bidders effectively reveal their full value for the good through their respective dropout points[33]. In the ascending-bid auction, buyer competition is at its maximum intensity[34]. Still, one's optimal bidding strategy does not depend on the bids submitted by one's rivals.

Even though the principles are very simple, the English auction system is very complex, and the system is very structured with regards to price intervals, approved bidding signals and the withdrawal of auction items. The auctioneer has a significant amount of responsibility, both in executing the auction according to set procedures and in recognizing only one potential buyer at each level, even though there might be several bids at the different price levels.

4. Second-price Sealed-bid (Vickrey) Auction

SPSB auctions are commonly called Vickrey auctions, after William Vickrey[35]. Vickrey was an early proponent of auctions. In his seminal 1961 article, Vickrey described the three different types of auctions that have been outlined so far in this chapter. Additionally, he argued that, "*since it has been shown that the Dutch auction has certain characteristics in some circumstances that may be considered disadvantageous as compared with the more certainly Pareto-optimal results of the*

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33 Vickrey, *Counterspeculation, Auctions, and Competitive Sealed Tenders*, 14.

34 Cassady Jr, *Auctions and Auctioneering*, 56-60.

35 "William Vickrey Bio," http://www.nap.edu/html/biomems/wvickrey.html (accessed October 5, 2005).

progressive auction, it is of interest to inquire whether there is not some sealed-bid procedure that would be logically isomorphic to the progressive auction"[36].

Based on this, he outlined a fourth auction format. This is a sealed-bid auction where the winner does not pay the price of the highest bid, but an award price equal to the second highest bid price. In reverse mode, the lowest bidder wins, paying the second lowest bid price. Vickrey argued that this sort of auction might increase the total welfare, which equals the sum of consumer and producer surplus, as compared to a sealed-bid first-price auction. His main argument was that the "top-price" auction with sealed bids is not optimal with regards to asymmetry among the bidders, errors in evaluation, or mistakes in strategy[37]. The SBSP auction does not involve any evaluation of the market and other bidders; each bidder can solely concentrate on his own value of the offered product.

This auction format is an example of a mechanism which elicits an individual's true WTP, or value. By bidding above his own WTP, an individual runs the risk that someone else will bid above his WTP, and he will buy the object only at a loss; if an individual bids below his own value for the object, he runs the risk of someone else buying the item at a lower price than the amount he is actually willing to pay. In the Vickrey auction, it is in every individual's best interest to state a truthful bid, equal to one's value of the object that is offered. This auction type is also socially efficient, maximizing efficiency in resource allocation. The object goes to the person with the highest WTP, and this person pays the social opportunity cost, which is the second highest bid.

D. SIGNIFICANT FACTORS WHEN DECIDING AUCTION FORMAT

A seller might have different objectives for holding an auction. The main objective may be to maximize his personal profit, or it might be to maximize welfare or social efficiency. Dependent on his purposes, the seller faces different choices regarding auction format. Which type of auction should the seller use to generate the highest possible price on his item? Following the Revenue Equivalence Theorem, which will be

[36] Vickrey, *Counterspeculation, Auctions, and Competitive Sealed Tenders*, 20.

[37] Ibid., 21.

explained below, it actually does not matter which auction design is chosen. In theory, each of the four auction forms yields on average the same revenue for the seller. However, many auctions fail to satisfy the criteria of this theorem. Additionally, it is a fact that bidders act differently with different auction designs. Klemperer points out two main features to bear in mind. These two are the auction's robustness against collusion and its attractiveness to potential bidders. It is also important to note that there is not "one size fits all". Local circumstances matter in the practical design of auctions, and "the devil is in the details"[38]. So, regardless of the main objectives, an auction holder should choose the auction format carefully. The main factors to consider in auction design are outlined below.

1. Revenue Equivalence

The Revenue Equivalence Theorem is discussed by several auction economists. Among these are McAfee and McMillan[39]. They refer to the *Benchmark model,* which is an auction based on the following assumptions:

1. The bidders are risk neutral.
2. The Independent private-values assumption applies.
3. The bidders are symmetric (their values are drawn from similar probability distributions).
4. Payment is a function of bids alone.

According to the Benchmark model, if these four criteria are satisfied, it actually does not matter which auction design is chosen. In theory, each of the four main auction forms yields <u>on average</u> the same revenue for the seller. It is important to note that this revenue equivalence is based on expected outcomes. The basis for this theorem follows below.

The different auction types are already described. The SPSB and the English auctions both ensure that the winner pays an amount equivalent to the second highest

[38] Paul Klemperer, "What really Matters in Auction Design," *Journal of Economic Perspectives* 16, no. 1 (Winter 2002), 187.

[39] McAfee and McMillan, *Auctions and Bidding*, 706-707.

bidder's value. In the English auction, this winning price might be increased by one extra bid increment, just to secure the victory. In general, these two auction types yield the same revenue.

The FPSB and the Dutch auctions also yield identical outcomes. The reason for this is the fact that the bidders face an identical situation in these two auction settings; the bidder has no information concerning his competitors' values and bidding strategies. Each bidder decides his bid without knowing the other bidders' decisions and the winner pays a price equal to his own bid[40]. Klemperer points out the fact that the winner in these two auction types will try to bid in such a way that he just slightly outbids the second highest bid. By doing so, he will maximize his profit[41]. If all bidders follow this strategy, these auctions will also produce revenues close to the second highest bidder's valuation.

Based on the above, all four auction types yield similar results. Again, these are average outcomes and require satisfying all four criteria of the Revenue Equivalence Theorem. However, most real-world auctions fail to satisfy one or more of these assumptions. Consequently, auction design does matter in reality.

2. Risk Tolerance among the Bidders

As pointed out above, the predominant difference between the first- and second-price auction types is information that the bidders have about competing bidders' values for the product and their bidding strategies. In the Dutch and FPSB auctions, bids must be placed in absence of any information on the competing bidders. The English auction is open in all respects, as every bid is observed by all participants. In the SPSB auction, as a result of its features, this information is not relevant for how a bidder decides to bid.

Risk tolerance, which describes an individual's attitude toward risk, has three possible outcomes; risk aversion, risk neutrality and risk seeking. These preferences are best described through an example. You are offered a gift of $10,000 in cash. However, at the same time you are asked if you would rather make a gamble. The gamble includes

[40] McAfee and McMillan, *Auctions and Bidding*, 707.

[41] Paul Klemperer, "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys* 13, no. 3 (July 1999), 232-233.

flipping a fair coin. If you win, you will receive not $10,000, but rather $20,000. If you lose, you do not receive anything. Would you accept this offer? In fact, most people would say no. The reason is that the majority of people are risk averse. This means that the idea of ending up with no gain is more painful than the benefit from an extra $10,000 gain, compared to the safe $10,000 alternative. For a risk-seeking person, the situation is opposite; the gain of a specified dollar amount increases welfare more than a loss of the same dollar amount decreases welfare. If you are indifferent to the coin toss, you are risk neutral. This means that a gain or a loss of a specified dollar amount produces an equal increase or decrease in welfare[42].

If the potential bidders in an auction are risk neutral, then under reasonably general conditions they will behave the same no matter which auction design is chosen. However, the situation is different if the buyers are risk averse. Risk averse individuals behave more cautiously than risk neutral individuals when they face uncertainty. In first-price auctions, there is more uncertainty because there is less information available for the bidders compared to an English or SPSB auction. A risk averse buyer seeks to reduce the level of uncertainty. The level of uncertainty will decrease as the probability of winning the auction increases. Consequently, the risk averse buyer in a first-price forward auction will increase his bid; this decreases his possible profit but at the same time increases the probability of winning the auction. Bidding more aggressively is the key to increasing the chance to win. As a result, a first-price forward auction with risk averse buyers will have a higher expected winning price than an English auction[43].

Vickrey pointed out that asymmetries among bidders may lead to inefficient allocations in a FPSB or Dutch auction, and the revenue could be either better or worse than that from an English or SPSB auction[44]. In general value environments, the effects of risk aversion are ambiguous and no clear conclusion seems to exist. However, in most auctions efficient allocation is not the focus; most auctions are not driven by the welfare of consumers or society as a whole, but rather by the sellers' and buyers' profits.

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[42] Jeffrey H. Moore and Larry R. Weatherford, *Decision Modeling with Microsoft Excel, Sixth Edition* (Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 2001), 412-414.

[43] R. Preston McAfee and John McMillan, *Incentives in Government Contracting* (Toronto, Canada: University of Toronto Press, 1988), 19-20.

[44] Vickrey, *Counterspeculation, Auctions, and Competitive Sealed Tenders*, 8-37.

3. Collusion

In general it is in the seller's interest to intensify competition; buyers would seek the opposite, reducing competition to keep prices under control. A major fear in auction design is the risk that participants collude either explicitly or tacitly to avoid bidding up prices[45]. The risk of collusion is different across the different auction types. In a sealed-bid auction, collusion among the participants is much harder than in an open auction. Since sealed-bid auctions do not use open bidding to signal, there is no communication between the participants in the bidding process. In open auctions, in particular the English auction, signals can be sent between the bidders. This can take the form of signaling through your bid, if the item at hand has a large value for you, or retaliation against bidders who do not cooperate, making them pay more than they otherwise would have paid for products that they in fact value more than you[46].

According to Cassady, collusive activity in auctions is not very common. However, an auctioneer has several options if he becomes aware of such activity. He could set a reserve price. This term will be explained further in the next section. Another option is to refuse to put an item up for sale if the price is artificially depressed, or withdraw the product and put it up for sale on a later occasion. Yet another option is to refuse bids from certain participants, if these are known to be part of a collusion ring. The auctioneer might also counter collusion by running up the price using phantom bids, or bids from nonexistent buyers, although these measures could be looked upon as unethical[47].

Even though collusive behavior might be considered morally wrong, in most cases it is hard to challenge legally. As Klemperer points out, rather than trying to outlaw such behavior by regulating auctions more closely, and thus reducing the flexibility for the bidders, it is much better to solve this problem with better auction design[48].

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[45] Klemperer, *What really Matters in Auction Design*, 170.

[46] Ibid., 179.

[47] Cassady Jr, *Auctions and Auctioneering*, 189-192.

[48] Klemperer, *What really Matters in Auction Design*, 172.

4. Reserve Price

If there is no reserve price, or price floor, in an auction, the seller takes whatever price the auction generates for his product. Such a case might result in the seller "giving his product away", if the highest bid is far less than the perceived value of the product. To guarantee a minimum profit, the seller might set a reserve price, which means that he discards all bids if they are too low. However, setting a reserve price on an auction item does not guarantee that the product will be sold at this price, or at any price. On the contrary, if the reserve price is set at a level which is perceived as too high, the reserve price might make the item harder to sell. If the reserve price is set higher than the seller's valuation of the item, but this is also higher than all bidders' valuations, then the seller loses his sale, even though some bidders might have been willing to pay more than the item was worth to the seller. Consequently, a reserve price may be useful, but it should be applied with care[49].

As pointed out above, reserve prices might be a useful tool where one suspects collusion. If an item for sale has no minimum price, potential buyers might make an arrangement where only one of them submits a bid, presumably a very low one. In this case, the winning bidder might win the auction at an extremely low price, gaining revenue at the expense of the seller. To avoid this situation, the seller can introduce a reserve price, which is the lowest acceptable sales price. In this case, the potential revenue for the colluding buyer is significantly less than without the reserve price, and it is less tempting for the bidders to cooperate. Cassady uses antiques sales as an example to emphasize this point. In certain geographic areas, like London, reserve prices are commonplace at antiques auctions. In other areas, like Beverly Hills, they are unusual. The difference stems mainly from the fact that most bidders are dealers in the British antiques auctions, who could easily collude; a majority of the bidders are private collectors in Beverly Hills, who could not as easily collude[50]. Thus, depending on the circumstances, setting a reserve price can be an effective approach to preventing collusive activity.

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[49] Cassady Jr, *Auctions and Auctioneering*, 226-230.

[50] Cassady Jr, *Auctions and Auctioneering*, 229.

5. Private Information

There are several aspects and attributes related to information, which can affect an auction in many ways. Some authors even hold bidders' private information as the most important factor affecting their strategic behavior[51]. In this thesis, the discussion of private information is limited to the properties that are assessed as essential for the current context.

In an auction setting, the seller might hold relevant information about the item that is for sale, which is only known by him and not by the potential buyers. This private information might be affiliated with the buyers' estimated value of the object. Such information might include knowledge about the product itself or information about the quantity that is available for sale. Additionally, this could involve sales data from previous similar auctions or relevant data about the potential buyers, including the number of expected buyers and characteristics of this group.

If the seller has any such information, he has to decide whether or not to reveal it to the potential buyers. Such information could have at least two different effects. Bidders might tend to underestimate the value of an object. If information changes the bidders' perception of value, particularly increasing the value, the seller should share the information to increase his revenue. However, if the bidders tend to overestimate their value of the object, revealing information would have the opposite effect, and it is not in the seller's interest to reveal.

Klemperer observes the fact that if information increases uncertainty, then bidders in a FPSB and Dutch auction will tend to bid more aggressively if they are risk-averse. In a forward auction, a slight increase in a player's bid slightly increases his probability of winning at the cost of slightly reducing the value of winning. In this case it is good for the seller to reveal his private information, as this will cause the bids to be higher than they would be otherwise.

[51] Robert Wilson, "Strategic Analysis of Auctions" in *Handbook of Game Theory*, vol. 1 ed. (Amsterdam, Holland: Elsevier Science Publishers, 1992), 227-279.

Milgrom refers to a *Linkage Principle*[52]. This principle holds that if the price paid by the buyer can be more effectively linked to exogenous variables that are affiliated with the bidder's private information, then generally the seller is better off and the bidders worse off. An example is using royalties in the selling of a publication right, which links the price paid to the actual value. This will increase the seller's surplus. In general, this implies that a seller would be better off by adopting a policy of always revealing his private information, rather than never revealing information. However, as already stated the seller's objective in many cases is not primarily to maximize profits. Dependent on the context, the seller should consider carefully which information to be revealed.

6. Number of Bidders

Bidders constitute a crucial element in auctions. Without competition between bidders, the auction will not be successful. In general, the seller's revenue increases in a first-price auction as the number of bidders increases. In fact, it might be more helpful for the seller to expand the audience of bidders, rather than using time and energy to evaluate which auction format to use, or calculating a reserve price[53]. However, it is important to differentiate between first-price and second-price auctions. Research shows that in FPSB and Dutch auctions, bidders tend to bid more aggressively as the number of rivals increase. English and SPSB auctions are different; bidders generally do not change their bids as number of rivals increases[54]. Bidding more aggressively is not just a general reflex to increased competition, but rather a feature that differs between the different auction types.

Research also shows interesting results considering uncertainty regarding the number of bidders in an auction. With risk averse bidders, the seller's revenue in private-value first-price auctions is greater if the actual number of bidders is concealed rather

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[52] Paul Milgrom, "Auctions and Bidding: A Primer," *Journal of Economic Perspectives* 3, no. 3 (Summer 1989), 16.

[53] Klemperer, *Auction Theory: A Guide to the Literature*, 239.

[54] John H. Kagel and Alvin E. Roth, *The Handbook of Experimental Economics* (Princeton, NJ, USA: Princeton University Press, 1995), 514-517.

than revealed. With risk neutral bidders, the revenue does not depend on whether or not the actual number of bidders is revealed[55]. In second-price auctions, based on the dominant strategy of bidding one's true value for an object, uncertainty with regards to number of bidders should not have any effect.

7. Other Instruments

The seller has some additional instruments when designing auctions. He might charge a fixed and non-refundable entry fee to all bidders for the right to participate. An entry fee works similarly to setting a reserve price, as it excludes all buyers with a low value for the product. An entry fee could be used to exclude unwanted bidders from entering an auction. An example could be a procurement auction, where the item purchased is very complex, and processing bids is very time-consuming and costly for the bid-taker. In this case, the bid-taker would try to avoid "unnecessary" bids from bidders that are not serious players in the respective market. An entry fee could deter such players, and at the same time increase the serious players' commitment. However, there is a balance, as an entry fee should not be set so that it discourages serious bidders and restricts the competition. The situation is otherwise if the setting is to fill vacant positions in an organization, and the goal is to fill as many positions as possible. Then an entry fee would probably discourage individuals from participating rather than increasing their interest, since this fee involves a non-refundable expense and no guarantee of any personal return. Clearly, the size of the entry fee also affects how the fee is perceived by the possible bidders. As for the reserve price, the entry fee should be applied with care.

Another instrument is for the seller to limit the time for submitting bids. Limiting the timeframe of the auction limits the opportunity for the buyers to get acquainted and evaluate each others' potential strategies. In an assignment setting, a time limit would reduce the individuals' possibilities of doing research on the body of bidders and investigating other bidders' valuation. As such, a time limit would increase the uncertainty among the bidders and lead to more aggressive bidding, favoring the employer. There is an obvious disadvantage with the time limit, which is particularly

[55] John H. Kagel and Alvin E. Roth, *The Handbook of Experimental Economics* (Princeton, NJ, USA: Princeton University Press, 1995), 516-517.

visible in a military job assignment auction, where the workforce is dispersed over a large geographic region. If there is a time limit in an assignment cycle, there might be individuals who are not informed about the auction, or not physically able to submit their bids. In this case, the employer should inform potential bidders thoroughly in advance to ensure that as many participants as possible are informed about the auction to come.

Both a seller and a buyer also have the opportunity to use middlemen in an auction transaction. Middlemen are much more common in open auctions (i.e. Dutch and English auctions), as compared to sealed-bid auctions, where the involved parties in most cases represent themselves. As an example, middlemen are frequently used in auctions where the bidders do not want to reveal their identity, like in an antique auction. A middleman could also be used if the bidder himself is not able to be present during the auction, such as daily auctions for products like fish or flowers. Here, middlemen operate on behalf of different companies in these businesses. Middlemen usually have detailed knowledge about the item for sale. They also need to know the item's valuation by the bidder or bidders they represent, so they know the maximum limit for their bids. In most sealed-bid auctions, where the bidders are only allowed to submit one bid, the bidders represent themselves in the process. In the next chapter this thesis will discuss possible auction formats for an assignment setting. However, it would seem instinctive that most individuals would prefer to represent themselves in an auction where future job options were at stake. Still, if an individual is unable to submit a bid in an assignment auction, for instance due to his present geographic location, a middleman could be used to enable this individual's participation in the auction.

E. CHAPTER SUMMARY

This chapter first outlined auction theory in general. Then some key terminology was introduced, including differentiating forward from reverse auctions and Private-values from Common value models. Further, the four main types of auctions were presented. These are the Dutch auction, the first-price sealed-bid auction, the English auction, and the second-price sealed-bid auction. Finally, some design factors relevant in choosing an auction format were described. These include the Revenue Equivalence Theorem, risk tolerance among the bidders, collusion, reserve price, private information, number of bidders, and other instruments.

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IV. APPLICATION OF AUCTIONS

Theory is based on perfect rationality, and thus the basic concepts of a theory are often easy to understand. On the other hand, human behavior is limited to rational thought, and often does not follow what is supposed to carry along the optimal result for an individual. Therefore, theoretical analysis has to be complemented by experimental research and analysis of how individuals actually tend to act in different settings. This chapter will first present examples of frequent uses of the different types of auctions. Following this, the emphasis will be on relevant considerations in auction design for the context of military assignments. This includes an analysis of how individuals tend to behave in different settings of this context. The discussion in this chapter will end with a recommendation for why an auction alone is not recommended as the most appropriate mechanism for use as an incentive in the assignment process in the Norwegian Armed Forces.

A. COMMON USES OF THE DIFFERENT AUCTION TYPES

1. Dutch Auction

As outlined in Chapter III, the Dutch auction is the design used for trading flowers at the markets in the Netherlands. This auction design is also commonly used at fish markets around the world. In a forward mode, which is its most common format, this auction type usually involves one seller and many potential buyers. In the reverse format, it mainly involves one buyer and many potential sellers.

2. First-price Sealed-bid Auction

The FPSB auction is generally used in cases where the object is large or very valuable. Given an environment characterized by a forward auction with one single seller and at most a few potential buyers, or a reverse auction with a few potential sellers and one single buyer, this auction type is also frequently used[56].

Examples using FPSB auctions include when the government is selling a valuable right, like mineral rights on government-owned land, and when government contracts are offered for procurement, including purchasing or developing new equipment and technology for the military, or for construction, repairs, and maintenance of public property. The FPSB auction is also common for trading securities, and it was traditionally used in the famous weekly Treasury bill auction in New York[57].

3. English Auction

The English auction is commonly used for trading a wide variety of goods. This includes products like antiques, artwork, tobacco, and livestock. The English auction is also a widespread alternative to the Dutch auction at fish markets. Being the converse of the Dutch auction, this auction type is characterized by one seller and many potential buyers in forward mode. In reverse mode the situation is opposite; one buyer and many potential sellers.

4. Second-price Sealed-bid Auction

As mentioned at an earlier stage, this auction format is also commonly called Vickrey auctions, after William Vickrey. Even though he discussed this auction type thoroughly, Vickrey himself did not give any practical examples of this auction type, and the format was considered rare. However, as Lucking-Reiley reveals in his 2000 article[58], such auctions were actually introduced in the stamp collector market as early as in the early 1900s, being the main format used by stamp auctioneers since the 1930s. Even though perceived as rare, this auction format is actually commonly used today. This includes the mechanism used at different Internet auction sites, including *eBay*.

[56] Richard Engelbrecht-Wiggans, Martin Shubik and Robert M. Stark, *Auctions, Bidding, and Contracting: Uses and Theory* (New York, NY, USA: New York University Press, 1983), 50.

[57] McAfee and McMillan, *Incentives in Government Contracting*, 168.

[58] David Lucking-Reiley, "Vickrey Auctions in Practice: From Nineteenth-Century Philately to Twenty-First-Century E-Commerce," *Journal of Economic Perspectives* 14, no. 3 (Summer 2000), 183-192.

Here, products are traded by what appears to be English auctions. However, bidders can use a computer program, popularly called "proxy bidding", to bid on their behalf. This involves having a computer program raise one's bid by the minimum increment, within the bidder's limits. In effect, this is a second-price auction in which the amount bid is equivalent to the limit set by a bidder[59]. Consequently, the concept of Vickrey auctions is frequently used, although the format is not always easily recognized or obvious.

B. FORMAT CONSIDERATIONS FOR AUCTIONS IN THE SETTING OF PERMANENT ASSIGNMENTS IN THE NORWEGIAN ARMED FORCES

By looking at the key terminology of auctions, which was presented in Chapter III, some conclusions with regards to auction design in the context of this thesis can already be drawn. First, an assignment auction will be a reverse auction. The situation would be similar to the procurement example discussed in the previous chapter. Being the employer, the Armed Forces would be the single buyer and bid-taker, while the body of eligible officers represents the sellers. The officers will place their bids stating how much money they would need to receive in salary for accepting a particular position. In general, the officers are bidding against one another for the opportunity to sell their right to be employed, and the employer, at least in theory, would accept the lowest offer as the winning bid. However, as will be discussed further, there might exist conditions that cause the employer to choose differently from its theoretically optimal choice. Second, an assignment auction will be an IPV auction. The situation would be similar to the ice-hockey ticket example in Chapter III. To apply or place a bid for an unpopular position does, to a large degree, rest on independent private values. Personal preferences and taste, including family situation and geographic ties, desire for a vertical career, skills, education level, civilian opportunities and other characteristics do influence how each individual employee would evaluate the opportunity of bidding for a vacant position. The position's value to one bidder is unaffected by its value to other bidders, though information about the value to others might strategically affect one's bid.

In theory, an employer could use any of the four main auction types for assignment purposes. All of the designs are simple and robust, and given a situation of

59 Krishna, *Auction Theory*, 9.

revenue equivalence, where all the conditions of the *Benchmark Model* are satisfied, they all even yield the same results, on average. However, as presented above, each different auction type seems to work well for different environments. Additionally, the different designs would involve different expected behavior from the bidders. It is also a question whether using any of the auction formats would improve results compared to the current assignment system. Prior to introducing a salary auction, the employer would have to analyze relevant factors, such as transaction costs, probabilities of collusion, and what information is and should be available for the different parties involved. Even more important is which goal that should be chosen as decision criterion for the auction design. This goal could be efficiency, cost effectiveness, equity, practicality, and manipulability. Equity might have different definitions. In this context, two different meanings of the expression will be discussed; equity in relation to salary and equity with regards to surplus for the bidders. In the following, these decision criteria will be discussed relative to the different auction types and to the current context of salary auctions.

To make it easier for the reader to follow the discussion, a matrix with the main findings follows at the next page, prior to the discussion. Each factor is measured for each auction type. All of the results are represented by a symbol in the matrix. The symbols are meant to indicate how well each factor performs relative to each auction type.

	Efficiency	Cost Effective-ness	Equity (Pay)	Equity (Surplus)	Practicality	Manipula-bility
Dutch						
English						
First-price Sealed-bid						
Second-price Sealed-bid						

Table 4. Salary auction format considerations

1. Efficiency

Efficiency in this context means economic, or social, efficiency. This involves maximizing the joint surpluses for the bid-taker and the bidders, and the notion of getting the "right" individual to win the auction. Social efficiency is optimized at the point that maximizes the sum of the bid-taker's surplus and the winning bidder's surplus. The "right" individual is the one with the highest personal value for the current position. In this context, efficiency has nothing to do with pay in the form of salary levels.

The different auction formats involve different ways of selecting the winner. Also, the optimal strategies for the bidders vary in the different formats. In the English auction, the employee ought to drop out as soon as the bid passes his true valuation. Equally, an individual's valuation, or WTP, corresponds to the bid he ought to submit in a SPSB auction. In both the Dutch auction and the FPSB auction, the bidders have to consider the values and strategies of their competitors, and their bids are a trade-off between the expected surplus and their probability of winning. Even so, in all these cases the winner would be the individual submitting the lowest bid. Moreover, the theory suggests that the expected result of a salary auction would be the same for all formats. The winning bid is expected to be at the level of the second lowest value.

However, there is one thing that separates the SPSB auction from the other formats. As discussed earlier, the dominant strategy in a SPSB auction is to bid exactly one's value of the object. Even so, bidders frequently deviate from this strategy. Many economists have expressed surprise to this point. One of the suggested reasons for this is that the dominant strategy is not very transparent, and for the uninformed bidder it might seem more sensible to overbid one's value to win the auction[60]. The consequence of this is that those individuals, who actually win such an auction by overbidding their true WTP, do this only with a loss. Because we do not know how people will behave, revenue equivalence fails and so does efficiency. Based on this, the SPSB auction is not assessed as equally good as the other formats with regards to efficiency.

2. Cost Effectiveness

The cost effectiveness objective is seen from the bid-taker, or the employer's perspective. This party will try to reduce the costs of a salary auction as much as possible. In practice, this means keeping the salaries which are based on auctions as low as possible. Achieving this involves choosing the auction format that transfers the most surplus from the bidder to the bid-taker. According to the Revenue Equivalence Theorem, all auction designs yield the same average result. This implicitly means that the surplus for the bid-taker will be the same under all designs. Therefore, as a starting point, all of the designs have the same status in Table 4. However, most real auctions, including a salary auction, fail to satisfy all four of the necessary Revenue Equivalence assumptions. Three of these assumptions will likely hold in this kind of context. These include the assumption that IPV applies, which was described earlier in this chapter. Further, that payment is a function of bids alone, which means that the value of the winning bid is the only factor relevant when the winner is determined. Additionally, the assumption saying that the bidders are symmetric. Even though the bidders' values may differ, there is no reason to think that they are not drawn from similar probability distributions. The values and distribution of the draw may vary from auction to auction; still the probability distributions will not vary.

60 Kagel and Roth, *The Handbook of Experimental Economics*, 513.

The status for the last assumption, saying that the bidders are risk neutral, is more questionable. The majority of people are risk averse. There exists no research that justifies a claim that military officers of the Norwegian Armed Forces are characterized differently in this respect compared to the rest of the population. Research related to use of auctions as an incentive for sailors to separate from the US Navy indicates that sailors follow the same trend. There is no reason to expect this group to be different from other people; meaning they might also be risk averse, as opposed to risk neutral.

Risk tolerance is discussed in Chapter III. According to this theory, in a situation where the risk neutral bid-taker faces risk averse bidders, the risk averse bidders will bid more aggressively to increase their probability of winning the auction. This behavior would shift surplus value to the employer, since more aggressive bids would cause the winning bid to decrease. Consequently, for the bid-taker to minimize his cost and maximize his surplus, he ought to choose the FPSB or Dutch auction over SPSB or English auctions[61].

Another fact that might support the employer's choice of a FPSB auction in this context is that with an increased number of bidders, the expected value of the winning bid decreases. This means that, as the number of bidders rises, a higher portion of the surplus is expected to come to the employer. However, if the number of bidders is expected to be very low, the advantage for the employer would be smaller, as the winning bid is expected to be higher.

Risk aversion would favor the FPSB auction as the preferable format for the employer relative to cost effectiveness. In this perspective, the other formats are considered as equal.

3. Equity (Salary)

First, equity might mean pay. In this respect, every individual with similar background (i.e. education level, experience, years of service) and doing similar jobs should get paid the same salary. Within one auction the second-price formats are equitable in the meaning of salary, since all winners in one specific auction are paid the

[61] Klemperer, *Auction Theory: A Guide to the Literature*, 234.

same wage. Across different auctions, the second-price formats would not be equitable. One of the hallmarks of an auction is that the price is set by competition, in this context competition between employees. This means that there is no fixed, posted price, and each individual auction will yield a result independent of other current or former auctions. In practice, five different persons with identical backgrounds could work in an office, doing the same job. Based on a second-price auction format, the individuals who got their jobs in the same auction will have the same salary. However, if they got their positions based on winning different auctions, their salaries could be different. The first-price auction formats are inequitable both within an auction, as well as across auctions. As such, the SPSB and English auctions are preferable if the main objective of an assignment process is to achieve a state of equity under this definition. Still, being inequitable across auctions, none of the auction formats are perfectly fit for this context.

4. Equity (Bidders' surplus)

The second definition of equity involves equalizing the officers' surplus values. Since officers have different valuations of different positions, this does not mean that they should have identical salaries, however, each individual should have a salary that corresponds to his value. As such, this definition of equity means to make the bidders equally happy. In an English and SPSB auction all winners receive the wage corresponding to the first excluded bidder. This means the pay would be equal, but the surpluses would be different. In the Dutch and FPSB auctions the winner gets a salary corresponding to his bid. This should be the reservation wage plus one increment. Assuming that the bidders have similar risk aversion, the results from the Dutch and FPSB auctions would involve more equal surplus values for the winners.

5. Practicality

In reality, both of the open auction formats, Dutch and English auctions, do not seem very practical for an employer to use for assignments purposes. To choose this format, all bidders would need to be gathered, if not in person then at least by a representative for each individual bidder. For most enterprises it would not be feasible to arrange such an auction. For an employer like the military, which has its employees scattered at numerous geographical locations in different countries all around the world, this solution would be both very difficult and expensive to carry out.

Both the sealed-bid formats would be easy to conduct in practice. The bidders would just need a format for how to submit their bids, and a known timeframe for the auction process, including a due time for when to submit their bids. However, there is one thing that separates the two sealed-bid auction types in this respect. A majority of people are familiar with FPSB auctions and know the basic mechanisms involved in this design. In a FPSB salary auction, the winner is the one with the lowest bid, and the dollar payment will equal the dollar value of his bid. For a first-time user these mechanisms are relatively easy to comprehend. Compared to this, the mechanisms related to a SPSB auction are harder to understand. Additionally, this design is not so extensively known to most people. Due to this, it would require more education of the employees by their employer before implementing the SPSB design. Among other things, this would include education in strategic behavior, such as the notion of bidding one's value for an object. Even though this fact would be relatively easy to explain in theory, experiments show that individuals still do not tend to comply with their optimal behavior. Therefore, from a practical standpoint, a FPSB auction would be a better design than the SPSB auction.

6. Manipulability

Manipulative activity could be explained as the opposite of truth revelation, meaning activity that is performed by an individual or group of individuals to improve the personal result by not acting truthfully. Manipulating activity among participants of an auction could happen in different ways. It could involve cooperation among bidders, or individual bidders behaving in ways other than what they are expected to. Another form of manipulating behavior could be that the bid-taker acts to provoke some kind of reaction from the bidders.

As described in Chapter III, the fear of collusion is higher in the Dutch and English auctions than in the sealed-bid formats. In the open auctions, the participants can observe the behavior of their competitors and transmit signals in this process. The probability of such activity occurring would depend both on the number of bidders and how familiar they are with each other. If they are given time to prepare strategies prior to an auction, the bidders might cooperate to improve their personal results. This could be done by revealing their values for different positions to one another, and by doing this in a way that distributes the offered positions between them. If the pool of bidders is large, then it would be more difficult to involve all bidders in this activity, and thereby be harder to obtain effective collusive activity. In the longer term, the pool of employees would learn the mechanism more and more, and also become more familiar with how to take advantage of this. The most significant difference between the Dutch and English auction in this respect is that in the Dutch auction it is too late to adjust one's bid for the current auction as soon as another participant makes his move.

In the sealed-bid formats, it is harder to collude. There is no communication between the bidders during the process. However, if the pool of bidders is very small and the bidders are familiar with each other, collusion could also be feasible in the sealed-bid formats. In the context of this thesis, it is not very likely that all potential bidders would be familiar with all the other participants. This means that an attempt to collude would have a very small probability of success; in most cases some bidders are unknown to the others. Most likely these bidders would jeopardize the collusive activity, either deliberately or not. Overall, the possibility of collusion is an argument against both of the open auction formats.

Manipulation is not restricted to the bidders' side of the "equation". Some research suggests that manipulation by the bid-taker is one of the reasons that SPSB auctions are not used even more. Rothkopf, Teisberg and Kahn[62] proposed two main concerns: first, bidders may fear that truthful information is revealed to third parties with whom the bidder will interact on later occasions; second, bidders may fear that the auctioneer might cheat. Since the auctioneer knows each bidder's maximum WTP, which in the salary auction is equivalent to the lowest acceptable salary, the auctioneer would have an incentive to pretend there is a second-lowest bid just above the lowest bid, to keep the salary as low as possible. With the employer being a public enterprise like

[62] Michael H. Rothkopf, Thomas J. Teisberg and Edward P. Kahn, "Why are Vickrey Auctions Rare?" *The Journal of Political Economy* 98, no. 1 (February 1990), 101-103.

the military, one would expect the probability of manipulation by the bid-taker to be very small. Still, the perception of the bid-taker among the employees is important relative to how they bid. Consequently, even though the probability of manipulation by the employer is small, some situational context could lead the bidders to believe manipulation existed, which would influence on their behavior. This is an argument disfavoring the SPSB auction.

As discussed in Chapter III, an employer who fears collusion or other manipulating activity, which might lead to over-paying for a specific position, can use different mechanisms. Rather than considering altering the auction design, a simple measure would be to set a reserve price at the maximum WTP for this position. Other mechanisms that would not involve large expenses are restrictions in the time available for submitting bids and considering what private information to reveal to the bidders. If the employer needed to fill a large number of positions in one particular assignment cycle, one would expect the bidders to seek more surplus value the more bids are to be accepted (and positions filled).

Overall, the probabilities of manipulating activities would favor the use of the sealed-bid auction formats, particularly the FPSB auction, as opposed to the open formats, where the English auction has the most disadvantages.

C. COMPLICATIONS WITH USE OF AUCTIONS IN THE ASSIGNMENT CONTEXT

In theory, an auction could be introduced as an incentive to improve the assignment process of the Norwegian Armed Forces. Such a mechanism would bring along some positive effects. This would include filling vacant positions in the organization, under the assumption that the employer was willing to pay a certain price. Additionally, an auction would tell us the price of each position at that time. As the discussion above outlines, the process of considering auction formats includes trade-offs between the employer’s different goals, and also between the goals of the employees. It is important to bear in mind that the bidders will act differently depending on the auction format and other factors, like their risk tolerance and their perception of the environment of the current auction. If the values of the bidders were believed to be strongly correlated, then the value of the bids would probably not differ to a large degree. This would favor the use of a FPSB auction, because risk averse bidders would tend to bid close to their reservation price. However, if the format was a FPSB auction, and the employees had a perception that there were few potential bidders, then they would have an incentive to seek a bigger surplus value, as the probability of winning would be higher than if there were a larger number of employees competing. Based on the factors discussed, the sealed-bid auction formats seem most suitable in the context of military assignments in Norway, with the FPSB preferable to the SPSB auction. However, although such a mechanism would carry along positive effects, there are complications involved. Furthermore, the main problem at hand will still not be solved.

Some major complications arise with auction mechanisms applied to the assignment context. First, auction theory assumes that in a salary auction the bidder with the lowest bid wins. In an assignment setting, one bidder might submit bids on more than one job. However, each bidder can not get more than one job, even if he submits the lowest bid for more than one position. This situation introduces the concept of multi-object auctions. This concept recognizes that more than one object is for sale. A multi-object auction behaves similarly to a single-object auction, as long as two assumptions hold; that each seller (buyer) can sell (buy) more than one object, and that the valuations for the objects are independent. Still, because each bidder in an assignment auction can only win one position, bidders may have an incentive to overstate their minimum willingness-to-accept (WTA), which is their reservation wage. The reason is that there is a high probability that the mechanism will be "forced" to allocate them to a position. Particularly, this would occur in the case when the number of open positions is near, at, or above the number of bidders. An example could be a case with 10 open positions, and 10 qualified officers bidding for these jobs. If an applicant knows that the employer has to fill all the open positions, then he would have an incentive to bid higher than he would do otherwise. In this example, if the employer had to fill all the 10 open positions, all the 10 applicants would win one position, regardless of how high their bids were. The applicant may not necessarily know whether or not the employer has to fill all the openings. Still, if he knew that he was one of only a few qualified individuals, the incentive to bid aggressively would remain present, as he would still know that the chance of winning one of the open positions was good. The outcome in this example would be more expensive for the employer than it needed to be.

The second complication involves preferences. Auctions assume one-sided preferences. Consequently, in a pure salary auction, the dollar value of the salary bids is the only criterion deciding which employee gets which job. In reality, however, both the individual bidders and the employer have preferences. An employee might have strong preferences over the position to which he is assigned, and this should be reflected in his bidding. For the bid-taker, the situation is similar. Even though employees submit bids of different values, the employer in most cases cares about the value of the bids and also about which employee ends up assigned to which position. The auction mechanism does not consider this aspect, as the individual with the lowest bid is the winner regardless of the employer's preferences.

One way of introducing the employer's preferences in this context is to use a quality or fitness variable combined with bid values to determine auction winners. For example, the employer might rate bidders not just according to their cost or wage, but rather according to some cost vs. benefit analysis (for example, value/quality minus cost). In such a scenario, however, high quality bidders have an incentive to overstate their WTA. The reason is that if an individual knows that his fitness score is high, there is an incentive for this bidder to try to maximize his personal surplus by submitting a higher bid than his true valuation of the position. This deficiency will be proven for a generic case in the Appendix.

Another problem in this context is the case where a position is offered, and an individual does not submit a bid at all. This could indicate that this individual is willing to take this job for zero payment. On the other hand, it could also mean that this person is not willing to take this job at all. In this setting, the employer would not know, and it might make it harder for the employer to get an overview of the employee's preferences.

Due to the complications that are discussed, there are obvious limitations to how much gain an auction can provide in this setting. One proposed solution to problems like this is a two-sided matching mechanism. The next chapters of this thesis will focus on this mechanism.

D. CHAPTER SUMMARY

This chapter first outlined common uses of the different auction types. Then it focused on auctions as an incentive in the Norwegian Armed Forces' assignment process. Different auction formats were considered and discussed relative to the different auction outcomes. These outcomes included efficiency, cost effectiveness, equity in relation to bidder salary and surplus, practicality, and manipulability. Following this presentation, two major complications were described: first, in an assignment auction, each employee can not win more than one auction, even though he might have the lowest bid for more than one job. Consequently, there is an incentive for the bidders to overstate their WTA, rather than submit a truthful valuation; second, auctions assume one-sided preferences, meaning that the employer, representing the vacant positions, does not have preferences over which employee gets which job. If a quality variable is introduced in combination with bid value to determine the auction winner, then high quality bidders again have an incentive to maximize their surplus by overstating their WTA. Based on this, an auction mechanism is insufficient as an incentive in this setting.

V. MATCHING THEORY

A. BACKGROUND

A main function of many markets and social processes is to match one kind of agent with another. The main difference between a matching market and other markets is the fact that each object that is bought and sold is unique. Furthermore, the seller and buyer have preferences, and care about the item that is traded. This means that the mechanism of matching, contrary to auctions, is designed to handle two-sided preferences. The term "matching" refers to the bilateral nature of the exchange in these markets; if you work for a firm, then this firm also employs you[63]. One of the characteristics regarding matching is the fact that theory and use are closely linked. Matching theory saw its first light around the same time as the auction theory in the early 1960s. The practical application of matching had actually started earlier; graduating physicians were assigned to hospitals in the USA using a matching mechanism as early as 1951. Even though theory and use are closely linked together, it is sensible to present the basic theory before analyzing practical application of these mechanisms. This chapter will present the basic theoretic foundation of matching. Then the next chapter will focus on application.

There are two main matching models; "one-sided" and "two-sided". In both models, an agent from one side of the market can only be matched with an agent or agents from the other side. These models will be presented in the following sections.

B. ONE-SIDED MATCHING

A "one-sided" matching market involves a market with two disjoint sets of agents where one of these sets has preferences over the outcomes of a matching process, while the other set of agents does not have any preferences. The situation can be described by the case of an employer distributing 30 offices between 30 employees. The offices do not have any preference over which employee gets allocated to which office. On the other

[63] Alvin E. Roth and Sotomayor, Marilda A. Oliveira, "Two-Sided Matching" in *Handbook of Game Theory with Economic Applications*, Volume 1 ed. (San Francisco, CA, USA: Elsevier Science Publishers B.V., 1992), 486.

hand, the employees do have preferences over the office to which they are allocated. One-sided matching gets difficult if more than one agent has the same set of preferences. If each of the 30 employees is allowed to rank the offices, there is a possibility that more than one might rank the same office as number one. If so, the desires of the employees are correlated. In this case, the employer cannot make an efficient decision about who gets each office based on this ranking alone. Most likely, the employer will have to introduce other factors in this decision process, like seniority, age, physical handicaps, special needs, or other factors. Alternatively, the decision could be made by drawing lots.

C. TWO-SIDED MATCHING

A market is “two-sided” if it is represented by two distinct categories of agents, and both of the two categories have preferences. This could be students and colleges, workers and firms, or marriageable men and women. Labor markets represent a natural application of two-sided matching models. Illustrated through the relationship between workers and firms, the situation is different from one-sided matching because both sides care; the workers have preferences over the firms where they wish to be employed, and firms have preferences over workers who they wish to employ[64].

The two prominent two-sided matching models are one-to-one and many-to-one matching models, which will both be discussed below. The goal of these processes is the same as in one-sided matching; to find a stable match. A one-to-one match is stable if it cannot be mutually improved upon by any pair of agents. Similarly, in many-to-one matching a match is considered stable if the assignment of workers to firms is acceptable for both parties, such that no pair of worker and firm not matched to each other would both prefer such a match to their current assignment.

To determine an acceptable match, it is necessary to know the preferences of each individual agent over agents from the other category of the market. In the worker/firm example, each individual worker must indicate his preferences over the firms for which he would like to work, and each firm has to indicate its preferences over workers they

[64] Alvin E. Roth, "Matching," http://kuznets.fas.harvard.edu/~aroth/match.html (accessed November 6, 2005).

would like to employ. These lists of preferences do not need to be complete. However, the more exhaustive the lists, the more they improve the owner's chances of being matched.

1. One-to-one Matching Models

One-to-one matching means that each agent on one side of the market can be matched to at most one agent on the other side. Two different cases exist of one-to-one matching; with and without "sidepayment". The case without sidepayment is called *the marriage problem*[65], while the case with sidepayment is called *the assignment game*[66].

a. The Marriage Problem

The most adequate analogy of *the marriage problem* is the market of marriageable men and women. In this context, there are two options for each agent; either to be matched with one agent of the opposite gender, or remain unmatched (i.e. single). All agents have a complete preference ordering over the other set of agents. For the men, this ranking includes all acceptable women in the population. Those women with whom a man would not under any circumstance be matched constitute the non-acceptable women. A man would rather stay unmatched rather than being matched with a woman of the non-acceptable category. This means that his list of acceptable women might be empty.

An outcome of this kind of matching produces a subset of the men to be matched in monogamous marriages with a subset of the women. The remainder of the agents is left unmarried. Using the men's preferences as foundation for the matching can produce a different result compared to using the women's preferences; only when there is one unique stable set of marriages do the two procedures produce the same result. Gale

[65] D. Gale and L. S. Shapley, "College Admissions and the Stability of Marriage," *The American Mathematical Monthly* 69, no. 1 (January 1962), 9-15.

[66] L. S. Shapley and M. Shubik, "The Assignment Game I: The Core," *International Journal of Game Theory* 1 (1972), 111-130.

and Shapley[67] proved that no matter which set of preferences is being used, the set of stable matches will be nonempty. This means that there will always exist at least one stable matching for every marriage market.

Another feature is that if the men's preference ordering is used for the matching, it will produce a stable set of outcomes which is optimal for the men. If, however, the women do the proposing, it may produce another stable set of outcomes, this one being optimal for the women. If the women propose, the procedure is exactly the same as if the men propose; the difference in procedure is only which set of agents take the initiative. There does not exist any other outcome compared to the men (women) optimal solution that at least one man (woman) would prefer and no man (woman) would oppose (i.e. has the lowest average rank-order preference for men (women) of any stable match). Roth[68] shows that the optimal stable match for the men is the worst stable outcome for the women, and vice versa. Gale and Sotomayor[69] prove that adding more women to a matching population can never make any man in this population worse off, it can only improve their outcome.

There does not exist any stable matching procedure for this problem which makes it a dominant strategy for all agents on both sides of the market to state their true preferences. However, the set of agents whose preference list is used as foundation for the matching procedure should always state their true preferences. The process of matching men with women does not need to be based on either of the sides' preference lists, as it is possible to obtain stable matches without solely favoring one side's preferences in the process. For any set of men and women, no matter which preference pattern they have, there is always at least one stable matching[70].

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67 Gale and Shapley, *College Admissions and the Stability of Marriage*, 9-15.

68 Alvin E. Roth, "The College Admissions Problem is Not Equivalent to the Marriage Problem," *Journal of Economic Theory* 36, no. 2 (August 1985), 277-288.

69 David Gale and Marilda Sotomayor, "Ms. Machiavelli and the Stable Matching Problem," *The American Mathematical Monthly* 92, no. 4 (April 1985), 261-268.

70 Gale and Shapley, *College Admissions and the Stability of Marriage*, 9-15.

b. The Assignment Game

Shapley and Shubik have characterized one-to-one matching with sidepayment as a two-sided market where the product comes in large, indivisible units. Examples of such products might be cars or real estate for private homes. The product is exchanged for money, and each player in this market supplies or demands exactly one unit, meaning each producer supplies exactly one item, while each consumer demands exactly one item. The products do not need to be identical, and the same unit may have different values to different participants. The key difference from *the marriage problem* is that an additional payment from either the producer or the consumer to third parties is allowed. These are called "side payments". Using the example of private homes, *the assignment game* might include the transfer of any house from an owner to a buyer, and in addition the transfer of money between any of the players, including both buyers and sellers. *The assignment game* involves profitable interaction between the different players, where the goal of each player is to maximize his welfare. In Shapley and Shubik's model, the optimal solution does not involve third-party payments. Still, there exist stable solutions both with and without the side payment[71].

2. Many-to-one Matching Model: *The College Admissions Problem*

Many-to-one matching differs from one-to-one matching in that agents on one side of the market can be matched with more than one agent of the other category. Usually, one side of the market consists of institutions, while the other side consists of individuals; colleges admit many students, firms hire many workers, and hospitals employ many interns, all at the same time[72]. The situation is described by Gale and Shapley[73] in a model called *the college admissions problem*. Here, one set of agents (colleges) could be matched with more than one agent on the other side (students). Each college has a limit, or quota, for how many students it might accept. Each student has a rank order list over the set of colleges he would consider attending. An outcome of a

71 Shapley and Shubik, *The Assignment Game I: The Core*, 111-130.

72 Alvin E. Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis* (New York, NY, USA: Cambridge University Press, 1990), 123.

73 Gale and Shapley, *College Admissions and the Stability of Marriage*, 9-15.

matching process assigns each student to at most one college, and each college to at most its quota of students. The rest of students and vacant college seats remain unmatched.

Some of the results discussed for the marriage problem are valid for *the college admissions problem* as well. This includes the theorem stating that no matter which set of preferences is being used, there will always be at least one stable matching for every market. Also, for each of the agents' preference orderings (i.e. colleges' or students' preferences) there does exist an optimal set of outcomes which is stable[74]. However, as discussed by Roth, the agents in the many-to-one matching scenario face some incentives that are different compared to *the marriage problem*. In the many-to-one matching model, the colleges not only have preferences over students, but also have preferences over outcomes, as each outcome represents one of the possible sets of students. Following this, each college has to compare different sets of students. A college has what is called responsive preferences over its assignments if it prefers the assignment containing the more preferred student for any two assignments that differ in only one student. Roth shows that if a college has responsive preferences, the college-optimal stable outcome, which is based on the college's preference list, may not after all be the most preferred outcome to the college. Consequently, there exists no stable matching procedure that makes it a dominant strategy for colleges to state their true preferences. With the college-optimal procedure, there is no dominant strategy for the students to state their true preferences, either. For the students, however, there exists no other outcome which all students would prefer to the student-optimal outcome. Following this, there is a dominant strategy for all students to state their true preferences as long as the student-optimal procedure is used[75].

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[74] Gale and Shapley, *College Admissions and the Stability of Marriage*, 9-15.

[75] Roth, *The College Admissions Problem is Not Equivalent to the Marriage Problem*, 277-288.

D. OPTIMAL MATCHING PROCEDURES

1. Optimal Matching Procedures in a One-to-one Market

Gale and Shapley describe a procedure for obtaining matches in one-to-one markets. The males do the proposing in their procedure. The procedure, which is called a deferred-acceptance-procedure, is conducted as follows:

> To start, let each boy propose to his favorite girl. Each girl who receives more than one proposal rejects all but her favorite from among those who have proposed to her. However, she does not accept him yet, but keeps him on a string to allow for the possibility that someone better may come along later.
>
> We are now ready for the second stage. Those boys who were rejected now propose to their second choices. Each girl receiving proposals chooses her favorite from the group consisting of the new proposers and the boy on her string, if any. She rejects all the rest and again keeps the favorite in suspense.
>
> We proceed in the same manner. Those who are rejected at the second stage propose to their next choices, and the girls again reject all but the best proposal they have had so far.
>
> Eventually, every girl will have received a proposal, for as long as any girl has not been proposed to there will be rejections and new proposals, but since no boy can propose to the same girl more than once, every girl is sure to get a proposal in due time. As soon as the last girl gets her proposal the "courtship" is declared over, and each girl is now required to accept the boy on her string.[76]

As mentioned above, Gale and Shapley further proved that this procedure yields a solution of stable matches. With this procedure, the only difference between a male- and a female-optimal algorithm is limited to which set of agents do the proposing. Consequently, there is no question of which procedure to use, but rather which set of agents should get an optimal outcome. As an alternative to maximizing the outcome for one set of agents at the expense of the other set of agents, there are other solutions that also obtain stable matches. This could be done by accounting for the weights of the agents' preferences.

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[76] Gale and Shapley, *College Admissions and the Stability of Marriage*, 12-13.

2. Optimal Matching Procedures in a Many-to-one Market

In a many-to-one market, the situation is more complicated than in *the marriage market*. This implies that the procedure used for obtaining a student-optimal solution is different from the one that yields a firm-optimal solution. However, both procedures start similarly, with each set of agents submitting rank order lists. Here, the agents also indicate unacceptable matching candidates. The rank order lists are submitted to the central clearinghouse. Following this, the clearinghouse edits the lists, as follows:

> … by removing from each hospital program's rank-order list any student who has marked that program as unacceptable, and by removing from each student's list any hospital which has indicated he is unacceptable…. The edited lists are thus rank orderings of acceptable alternatives.[77]

From this point, the procedure differs, relative to which model is used. Gale and Shapley describe the student-optimal procedure. This is an extension of the one-to-one deferred-acceptance-procedure described above. It works like this:

> First, all students apply to the college of their first choice. A college with a quota of *q* then places on its waiting list the *q* applicants who rank highest, or all applicants if there are fewer than *q*, and rejects the rest. Rejected applicants then apply to their second choice and again each college selects the top *q* from among the new applicants and those on its waiting list, puts these on its new waiting list, and rejects the rest. The procedure terminates when every applicant is either on a waiting list or has been rejected by every college to which he is willing and permitted to apply. At this point each college admits everyone on its waiting list and the stable assignment has been achieved.[78]

An early example of a model utilizing the firm-optimal solution is the centralized clearinghouse that was introduced in the early 1950s as a mechanism to distribute medical interns to hospitals in the USA. At the time of introduction of this new mechanism, this market was characterized by instability and disorder. Decisions were made by both hospitals and interns based on incomplete information, which resulted in a lot of inefficient outcomes and unhappiness for both parties. The new clearinghouse was in reality a many-to-one model of two-sided matching. The procedure used was called

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77 Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 135.

78 Gale and Shapley, *College Admissions and the Stability of Marriage*, 13-14.

the National Intern Matching Program (NIMP) algorithm. Following the initial editing, cited above, Roth describes this procedure as follows:

> These lists are entered into what may be thought of as a list-processing algorithm consisting of a *matching phase* and a *tentative-assignment-and-update phase*. The first step of the matching phase (the *1:1* step) checks to see if there are any students and hospital programs which are top-ranked in one another's ranking. (If a hospital H_i has a quota of q_i then the q_i highest students in its ranking are top-ranked.) If no such matches are found, the matching phase proceeds to the *2:1* step, at which the second ranked hospital program on each student's ranking is compared with the top-ranked students on that hospital's ranking. At any step when no matches are found, the algorithm proceeds to the next step, so the generic *k:1* step of the matching phase seeks to find student-hospital pairs such that the student is top-ranked on the hospital's ranking and the hospital is *k*th ranked by the student. At any step where such matches are found, the algorithm proceeds to the tentative-assignment-and-update phase.
>
> When the algorithm enters the tentative-assignment-and-update phase from the *k:1* step of the matching phase, the *k:1* matches are tentatively made, i.e., each student who is a top-ranked choice of his *k*th choice hospital is tentatively assigned to that hospital. The rankings of the students and hospitals are then updated in the following way. Any hospital which a student s_j ranks lower than his tentative assignment is deleted from his ranking (so the updated ranking of a student s_j tentatively assigned to his *k*th choice now lists only his first *k* choices) and student s_j is deleted from the ranking of any hospital which was deleted from s_j's ranking (so the updated rankings of each hospital now include only those applicants who haven't yet been tentatively assigned to a hospital they prefer). Note that, if one of a hospital's top-ranked candidates is deleted from its ranking, then a lower-ranked choice moves into the top-ranked category, since the hospital's updated ranking has fewer students, but the same quota, as its original ranking. When the rankings have been updated in this way, the algorithm returns to the start of the matching phase, which examines the updated rankings for new matches. Any new tentative matches found in the matching phase replace prior tentative matches involving the same student. (Note that new tentative matches can only improve a student's tentative assignment, since all lower ranked hospitals have been deleted from his ranking.) The algorithm terminates when no new tentative matches are found, at which point tentative matches become final. … Any student or hospital position which was not tentatively

> matched during the algorithm is left unassigned, and must make subsequent arrangements by directly negotiating with other unmatched students or hospitals[79]

The NIMP algorithm was implemented long before Gale and Shapley introduced their deferred-acceptance algorithm. Still, as shown by Roth, these two procedures are in fact equivalent. The case of the NIMP algorithm will be discussed further in Chapter VI.

E. BASIC PROPERTIES OF MATCHING

From the matching theory, there are some main properties to bear in mind. Although mentioned in the text above, some of these terms will be further outlined in the following sections.

1. Stability

The main objective of all matching mechanisms is to match agents from the different categories with one another in a way that is as optimal as possible, and moreover, in a way that is also durable, or stable. The outcome of a matching process will be different, depending on which set of preferences is used as the foundation for the matching. Only in one case will the different preferences produce an identical outcome; if there is only one unique set of stable matches. A stable match means that the result can not be mutually improved upon by any pair of agents involved in the ongoing matching process. It is important to notice that there might be more than one stable outcome from such a process. Using either set of agents' preference lists as the basis will produce a set of stable matches. In one-to-one matching models the result will be optimal for those agents whose preferences are used. However, stability might also be produced by other combinations that do not rely solely on one side's preference list. For all of the market types described above, stable matches are likely to occur. If the agents' preferences are highly correlated the set of stable matches will be small. This might be illustrated by a

[79] Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 135. A more detailed discussion of matching is found in Robards (2001).

situation in which many workers desire the same job or jobs, and many firms desire the same workers. In the opposite situation, where the preferences are not highly correlated, the set of stable matches will be larger.

2. Length of Preference List and Market Size

Computational matching experiments have been performed both on simple models and on more complex ones. The hypothesis has been that both the sizes of the market and the preference lists matter for the probability of being matched successfully. These experiments, by use of randomly generated preferences, indicate that the size of the set of stable matchings grows large as the market grows large. However, the sample size has to be n = 1,000 firms and workers in order for 90 percent of these to have different stable matching assignments. In reality, no worker or firm submits such a long rank order list. With a limit on these rank order lists the number of stable matches shrinks rapidly as the number of workers and firms grows. The results are similar for a large complex market as for smaller markets, leading to the conclusion that the set of stable matches is a function of the market size and length of preference lists[80]. The relationship between list length and probability of being matched is empirical. An applicant who restricts his rank order list to only a small number of acceptable jobs is less likely to get a match than an applicant who ranks a larger number of possible jobs. The same condition applies for the firms' side of the market. This would seem to be intuitively obvious, and has been supported by analyses of match results over many years. One example of such a study is the NPS thesis developed by Ng and Soh[81].

Length of preference lists and pool size are factors that are closely related in a matching process. Correlation of preferences is another term related to this. Given an employer seeking stable matches, an increase in the pool size of potential employees does not necessarily improve the outcome. The reason for this is correlation of preferences.

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80 Alvin E. Roth, "Game Theory as a Tool for Market Design," http://kuznets.fas.harvard.edu/~aroth/design.pdf (accessed January 17, 2006).

81 Hock Sing Ng and Cheow Guan Soh, *Agent-Based Simulation System: A Demonstration of the Advantages of an Electronic Employment Market in a Large Military Organization* (Naval Postgraduate School, CA, USA, 2001).

Preferences are correlated if the extra individuals that are added to an existing pool of employees have similar preferences as the individuals already in the pool. If so, the only change following the addition to the pool is that there are even more individuals competing for the same positions. In fact, this situation would reduce the percentage of stable matches in the current process, as the proportion of successful, stable matches relative to the pool size would decrease. One way of avoiding this is by also looking at the diversity of preferences among the participants. Allowing only individuals with certain preferences into the existing pool this could reduce correlation. However, the practical sides related to this could be complicated. Another possible option is to increase the list lengths of the participants. Based on what is discussed above, increasing a participant's list length would never reduce the probability of being matched for this individual. As mentioned earlier, the list of preferences for each participant in a matching market does not need to be complete. However, it is in the individuals' own interest to prepare as exhaustive a list as possible. The main reason for this is the fact that an extended list involves added matching opportunities if this individual's higher options are already occupied; consequently, the more exhaustive the list, the higher the probability for the owner being matched.

3. Gaming

In the context of matching, every participant has preferences over the participant or participants to which one can be matched. Therefore, an obvious question is if there is any incentive for the players to engage in strategic behavior, including misstating their preferences rather than telling the truth and submitting truthful valuations. The answer to this question is yes. With almost every mechanism that produces stable outcomes it will be in the interest of some participant or participants not to state their true preferences. To start with the opposite, Roth shows that it is a dominant strategy (i.e. best response to all possible sets of strategy choices) for *the marriage problem* to always state truthful bids for the set of agents whose preferences are the basis for the matching, as no other stable solution will make any of these agents better off. However, there does not exist any stable matching procedure which makes it a dominant strategy for all participants of all sets of agents to state truthful bids[82]. This means that in every stable matching mechanism, some of those agents whose preferences are only used to break ties will actually be able to improve their outcome by not acting honestly. In theory, a set of agents might promote any possible set of stable matchings through falsifying their preferences appropriately. Their challenge is how to achieve this practically. For *the college admissions problem*, the situation is different. Here, it is not a dominant strategy for the institutions (i.e. colleges, firms) to state their truthful valuations even when their preference list is used as foundation for the matching. This means that there will be institutions that could be better off by appropriately misrepresenting their preferences in all stable mechanisms.

Matching is not a perfect theory. Players might manipulate outcomes by gaming, in order to achieve a better individual result. However, playing a strategic game is often not very easy, and usually it is hard to predict which gaming strategy is optimal to maximize one's individual profit. In this context, it would be essential to have information about the other participants' valuations and the mechanism that is to be used in the matching process. One more thing complicates the picture for the gaming participant. Matching is a dynamic game, as the participants' behavior in this market context changes as the agents interact and get used to the market's characteristics. Consequently, there is no guarantee for an improved outcome for individuals who commit to gaming. In reality one is usually better off being truthful, by stating a truthful valuation. This is shown in research both on a simple and on a more complex market. Roth states this in relation to the experiments which were mentioned in the section on stability above. Commenting on the results of these experiments, he says that: "the small size of the set of stable matchings implies that there is virtually no opportunity for applicants to profitably manipulate their preference lists, regardless of what stable matching mechanism is employed"[83].

[82] Roth, *The College Admissions Problem is Not Equivalent to the Marriage Problem*, 280.

[83] Roth, *Game Theory as a Tool for Market Design*, 11.

One additional fact of interest supports this view. This condition, presented by Gale and Sotomayor[84], is valid for both *the marriage problem* and many-to-one matching. It is shown that the individuals who remain unmatched in a matching process will still be the same regardless of which set of preference lists is used in this process. In the example with workers and firms, this means that no matter which list is used for the matching, either the worker-optimal or the firms-optimal, the same individual workers would end up without a job. Additionally, even though this would include different sets of individual workers, each firm would end up with the same number of workers in all cases. Following this, it would seem as the wise solution for participants to submit truthful valuations. Additionally, if an individual is in doubt about his preferences, there is always the option to renege a match, meaning to remain single.

4. Polarization

Both Gale and Shapley (*the marriage problem*) and Shapley and Shubik (*the assignment game*) demonstrated an interesting property of matching. This is described as a general coincidence of interest among players on the same side of the market, and a polarization of interest between the two sides of the market. The effect of this property is that a matching mechanism that results in an optimal stable solution for one set of agents implicitly is the worst stable situation for the other set of agents. However, even though it involves a "worst case scenario" for one set of agents, it still represents a stable outcome. In situations where matching is to be used, there will always be a discussion of whose interests should be prioritized. In assigning workers to firms, should the workers' or the firms' preferences be attached the most significance? While doing this assessment, it is important to bear in mind that the solution does not need to be either one of the optimal solutions. Most matching processes hold other combinations which would also result in stable outcomes. However, these solutions will represent "in-between"solutions, and not be optimal for any of the involved sets of agents.

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84 Gale and Sotomayor, *Ms. Machiavelli and the Stable Matching Problem*, 261-268.

F. CHAPTER SUMMARY

This chapter gave a fundamental overview over matching theory. First and foremost, it presented the different basic matching models, encompassing one-sided matching and two-sided matching. The emphasis was on the different two-sided matching models. These include two different one-to-one matching models; *the marriage problem* and *the assignment game*, and additionally a many-to-one model called *the college admissions problem*. The discussion included optimal matching procedures both for the one-to-one market as well as the many-to-one market.

This chapter also discussed some of the main properties related to matching, and how these affect a matching process. The properties presented are stability, list length and size of the market, gaming, and polarization.

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VI. APPLICATION OF MATCHING

The previous chapter presented the theoretical foundations for matching mechanisms. This chapter will focus on relevant examples from different applications of such mechanisms. These examples will be used to illustrate matching in the context of the assignment process in the Norwegian Armed Forces.

For the discussion to come, it is important to distinguish the process of assignment from the process of discharge, or separation. There exists a significant amount of research related to separation incentives, including the use of matching mechanisms. However, this research can not be directly applied in the case of assignments. In a separation case, an employer wants to get rid of a number of people, for example 50 workers. If the employer allows the employees to place bids in a separation pay auction, then in general the employer does not care which employees he gets rid of. The 50 lowest bids are accepted independently of which employee placed which bid. Here, an auction mechanism would suffice and be appropriate. The assignment setting is different. In this case, both sides have preferences; the employees have preferences over the different positions, and the employer has preferences over the different employees. Additionally, there are other relevant factors to be considered in the process, like each individual employee's knowledge, skills and ability (KSA), fitness reports[85], and social circumstances. As concluded in Chapter IV, this makes a pure auction insufficient. In the following, the objective of the analysis will be to reveal if a matching mechanism could be used in the military assignment process in Norway, either as an appropriate mechanism on its own or as a supplement to other procedures.

Matching in this context would be two-sided. Dependent on the characteristics of the market, the model could be either a one-to-one or a many-to-one model. Specifically, this relates to whether or not each position is to be considered as an individual agent with individual preferences over the employees (one-to-one model), or if there are groups of positions that seek identically qualified officers, and therefore have the same set of preferences (many-to-one model). Below, the two different two-sided models will be

[85] In Norway: Tjenesteuttalelser

discussed, related to the context of permanent assignments in the Norwegian Armed Forces. Following this, other format considerations will be discussed.

A. THE NORWEGIAN MILITARY ASSIGNMENT PROCESS AS A ONE-TO-ONE MARKET

In any matching model for the market of permanent assignments in the Norwegian Armed Forces, each applying officer has to submit a rank order list of all acceptable positions. If this process is framed as a one-to-one market, then each vacant position also has to prepare a rank order list of the employees. The employer, on behalf of each position, has to determine the eligibility of each candidate, and how they are to be internally ranked. There are cases where units have several identical positions announced, and they seek more than one employee with identical qualifications. If the one-to-one model is used, each position needs to be treated on its own. This could involve a significant amount of extra work, as compared to the solution that all identical positions are treated as one group. Examples could be a unit announcing five similar staff officer positions, or when the Staff College announces 30 vacant student positions for the coming year. With the one-to-one model, each of these positions would need to be managed individually, rather than jointly. For the officers, this situation could involve frustration. Each individual applicant would need to ensure that all positions of interest were included on his rank order list. He would also need to spend time prioritizing positions that seem identical, and which in some cases actually would be identical.

If the assignment market is a one-to-one market, then the parallel could be drawn to the marriage market, where marriageable men and women meet to create stable marriages. At first sight, this market and its agents do fit into the marriage model. However, looking more closely, there is one major characteristic of the assignment market that does not seem to fit the marriage context. To illustrate this, let us return to Roth's discussion of the NIMP algorithm. As presented in Chapter V, this algorithm was used to match the market of medical interns to hospitals in the USA. In his discussion, Roth describes one major factor that separates a one-to-one from a many-to-one market in this respect; in the one-to-one market the firms have preferences only over individuals. In the many-to-one market the employers, in Roth's case hospitals, have preferences over groups of individuals rather than only preferences over the individual medical interns. It would be natural to assume that the Norwegian Armed Forces as an employer in general is no different from the hospitals announcing for interns. In both cases, the employers seek the best qualified individuals for their vacancies. Further, as Roth also showed, the hospitals did care not only about individuals, but they also had preferences over outcomes. This means they had preferences over which groups of interns were assigned. Similarly, it would seem sensible that the Armed Forces would care which possible sets of officers that were assigned to a unit, rather than only the individuals. If this assumption holds, it would not be correct to interpret the assignment process as a one-to-one model.

B. THE NORWEGIAN MILITARY ASSIGNMENT PROCESS AS A MANY-TO-ONE MARKET

Framed as a many-to-one model, the military assignment process in Norway would be similar to colleges recruiting students or hospitals hiring medical interns. As the one-to-one model, each officer has to submit a rank order list of all acceptable positions. The firms now have preferences over groups rather than just over individuals. Preferences over groups may involve a balance of skills across the group; for hospitals one good clinician and one good researcher might be better than two top clinicians or researchers. The military might want some employees that are good leaders, some good performers, and some with a high level of technical skills. This means that the assignment process in the Norwegian Armed Forces in theory fits the framework of a many-to-one model of two-sided matching. If the military as an employer was to consider applying a two-sided matching mechanism, they would face the choice between different possible designs. It has to be emphasized that the outcomes of a matching process do depend on which design is chosen. A student-optimal procedure leads to a different result compared to the outcome produced by the firm-optimal solution. Still, they both lead to stable matches. There are other options to choose from as well, in addition to these two polar procedures. In the National Football League, the worst team in the league gets the first priority for choosing new players prior to the next season. Some years ago, the US Naval Academy used a procedure where they allowed the best student to pick his branch first[86]. There is no answer as to what procedure is "right" to choose. For the purpose of this analysis, examples of the two polar cases will be analyzed, involving the firm-optimal and student-optimal solutions.

1. Firm-optimal Solution

As presented in Chapter V, the NIMP algorithm is one example of a firm-optimal many-to-one matching model. Introduced in the 1950s, this mechanism helped stabilize the market for distributing medical interns to hospitals in the USA. As mentioned, the procedure helped create order and stability in this market, which also provided increased satisfaction among both sets of agents. The NIMP procedure worked well until the 1970s. However, at that time the social evolvement included the trend where dual-career households became more and more frequent; this trend became more prevalent for the medical intern market. For the NIMP matching mechanism, the issue of married couples introduced instability. The initial solution was to update the algorithm, allowing couples to submit rank order lists of pairs of positions. In practice, one of the individuals of each couple had to be designated the "leading member", still each member continued to submit individual preference lists. First, the leading member was processed and matched to a position during the normal procedure. Then, a match was sought for the partner within the same geographical area.

One major problem with this procedure was that it did not take into consideration the couple's preferences over pairs of positions. This means that the leading member could get her top choice, while her partner got his 10th priority. Depending on their preferences and that this couple had a choice, they might have chosen another solution. The preferred solution could involve both partners accepting their third or fifth priority in another geographical location. Consequently, as Roth describes, "in the hospital-intern problem with couples, the set of stable matchings may be empty"[87]. Following this complication with the procedure, the participation rate for couples in this voluntary centralized clearinghouse was significantly lower than for single medical school

86 Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 86.

87 Ibid., 141.

graduates. According to Roth, the problem of processing married couples with this algorithm still seems to be unresolved. Actually, in large markets containing married couples it might even be hard to determine if a stable matching exists[88].

The problems related to the NIMP algorithm eventually caused its retirement. The NIMP program had been renamed twice, reflecting changes in the structure of postgraduate medical training. First, in 1968, it was named the National Intern and Resident Matching Program (NIRMP); then, in 1978, it became the National Residence Matching Program (NRMP)[89]. In parallel to the developments of the program during the 1970s, designers worked with a new algorithm. This new NRMP algorithm was also primarily, although not entirely, firm-proposing. Still, its design and implementation was performed without reference to the original NIMP mechanism. Even though the designers of the NRMP algorithm were not familiar with the work of Gale and Shapley, the NRMP algorithm was in fact an implementation of their algorithm. As both the NIMP and the NRMP algorithms were equivalent to the deferred-acceptance algorithm, they were not completely independent mechanisms[90]. Since its original design, the NRMP algorithm evolved considerably, introducing enhancements like different match variations. These include the reversion of unfilled positions from one program to another, matching applicants to two consecutive positions in one match using both primary and supplemental rank order lists, and restricting matches to even or odd numbers for a program. Furthermore, the algorithm better accommodated matching couples.

By the mid-1990s, new obstacles arose in this market. Students started to lose confidence in the matching mechanism, suspecting that the employers were unreasonably favored at the expense of the applicants. Also, there was speculation that applicants could “game the system” by strategically manipulating their rank order lists. This led to a new algorithm. Since 1998, the NRMP has used this new student-proposing algorithm in

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88 Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 143.

89 Alvin E. Roth, "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory," *Journal of Political Economy* 92 (1984), 991-1016.

90 Elliott Peranson, email communication, February 2006.

their matching processes. This student-optimal solution will be further discussed in section 2, below.

In the context of assignments, military officer market in Norway has obvious parallels with the medical intern market in the USA. Both markets are decentralized, with the workers spread out geographically across different locations. These workers do have preferences over different job opportunities. The units seeking workers are also dispersed, and they too have preferences, seeking the best qualified employees for their vacancies. In both these markets there exists a centralized clearinghouse that tries to match workers to the units as optimally as possible.

There is one more characteristic that is similar for these markets. Among the Norwegian military officers, as with the medical interns, there are a significant number of couples where both individuals belong in the same labor market. Based on the problem discussed above, it seems that introducing a firm-optimal solution like the NIMP or the NRMP firm-proposing algorithm in the Norwegian Armed Forces would not be the appropriate thing to do. Many matching mechanisms, like the NIMP mechanism, have started as firm-optimal procedures. However, as of today, this design is much less common in use than the student-optimal solution.

2. Student-optimal Solution

As mentioned above, NRMP now use a student-optimal matching mechanism. The current algorithm was designed in the late 1990s; even though different with regards to which set of agents do the proposing, the main features of the firm-proposing NRMP model were kept. The current model is described like this:

> The NRMP matching algorithm uses the preferences expressed in the rank order lists submitted by applicants and programs to place individuals into positions. The process begins with an attempt to place an applicant into the program indicated as most preferred on that applicant's list. If the applicant cannot be matched to this first choice program, an attempt is then made to place the applicant into the second choice program, and so on, until the applicant obtains a tentative match, or all the applicant's choices have been exhausted.

> An applicant can be tentatively matched to a program in this process if the program also ranks the applicant on its rank order list, and either:
>
> The program has an unfilled position. In this case, there is room in the program to make a tentative match between the applicant and program.
>
> The program does not have an unfilled position, but the applicant is more attractive to the program than another applicant who is already tentatively matched to the program. In this case, the applicant who is the least preferred current match in the program is removed from the program, to make room for a tentative match with the more preferred applicant.
>
> Matches are "tentative" because an applicant who is matched to a program at one point in the matching process may be removed from the program at some later point, to make room for an applicant more preferred by the program, as described in the second case above. When an applicant is removed from a previously made tentative match, an attempt is made to re-match that applicant, starting from the top of his/her list. This process is carried out for all applicants, until each applicant has either been tentatively matched to the most preferred choice possible, or all choices submitted by the applicant have been exhausted. When all applicants have been considered, the match is complete and all tentative matches become final[91].

Just like the NIMP and the pre-existing NRMP algorithms, this algorithm is very similar to the Gale-Shapley algorithm presented in Chapter V. In essence, the student-proposing NRMP algorithm is equivalent to the Gale-Shapley deferred-acceptance algorithm when there are no match variations present, like couples, supplemental list matching, reversions, and so forth. The deferred-acceptance algorithm does not incorporate any of these variations, while the new NRMP algorithm does. However, in the absence of such match variations, the NRMP algorithm simplifies down to being equivalent to the Gale-Shapley algorithm. Results from computational exploration during its design, and the results from the initial years of use, indicate that the new student-proposing NRMP mechanism produces a small number of stable matches. It is also shown that the opportunity to successfully manipulate the new procedure is very small.

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[91] The National Residence Matching Program, "The NRMP Matching Algorithm," http://www.nrmp.org/res_match/about_res/algorithms.html2006) (accessed February 14, 2006).

Compared to the pre-existing NRMP procedure, the new NRMP algorithm produces very similar results, even though students do the proposing in the new procedure, as compared to the firm-proposing pre-existing model. Only about 0.1 percent of the applicants were affected by the change in algorithms, and most of these preferred the match received under the new algorithm. Equally, only 0.5 percent of the hospital programs were affected by this change, however most of the programs preferred the match given by the pre-existing algorithm. The minimal effect on the outcome by changing the proposing set of agents indicates that there was only one set of stable matchings in most years.

The scale of the current NRMP matching program can be illustrated through some statistics. Each year, approximately 20,000 hospital jobs are filled by medical interns through the NRMP. Since 1987, the NRMP matches have included an annual proportion of about three to four percent of couples amongst the matched interns. For every year in this time period, the match rates for the couples have exceeded 90 percent. The number of unmatched interns generally varies between 6.0 – 7.5 percent[92].

One more example of the student-optimal solution will be presented, to illustrate how such a mechanism improved the situation in a different labour market. This example involves students entering New York City High Schools. One significant difference from the intern market is that while interns might seek more than one job, each high school applicant is seeking no more than one school seat. In this respect, the high school market is similar to the labour market for the Norwegian military, as each officer in this system can hold no more than one position at a time.

New York City has the largest public school system in the USA, with over a million students and over 90,000 new students entering each year. Prior to introducing a matching mechanism, one third of the students were not assigned to a school on their choice list. The old system also included widespread gaming among the participants to improve their personal outcome. Gaming was not confined to students; schools also thought strategically and concealed capacity from the central clearinghouse in the

[92] Alvin E. Roth and Elliott Peranson, "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design," *The American Economic Review* 89, no. 4 (September 1999), 748-780.

process, preserving places that could be filled later[93]. Since the schools were involved in gaming, and they also had preferences over groups of potential students, this indicates that this market is a two-sided, many-to-one market. For two reasons a student-proposing solution was chosen in this context; first, this emphasized welfare for the students. Second, this algorithm made it a dominant strategy for students to state their true preferences, while no alternative solution would carry a similar strategy for the schools.

On the whole, the proposed solution was similar to the current NRMP procedure. With the new procedure, students were allowed to rank 12 choices, while they could only rank five with the old system. Further, each student was given one single offer, rather than multiple offers as in the old system. In 2003-2004, the first year of operation for this mechanism, the results showed that only approximately 3,000 students were left with no offer from any school on their rank list. This was a significant improvement to the old system. The congestion related to the old mechanism was relieved, and the new mechanism increased the incentives to submit truthful preferences for both sets of agents. This example illustrates that a stable matching mechanism has the potential to improve assignments between students and schools, and similarly, between workers and positions.

Two-sided matching is an interesting option for the context of assignments in the Norwegian Armed Forces. Both sides care about outcomes. The primary problems, like tied movers, have been addressed, and successful outcomes have been observed in similar markets, like New York City High Schools. However, some characteristics with the current process in the Norwegian Armed Forces complicate this picture. These will be introduced in section D. First, some other considerations will be discussed.

C. OTHER CONSIDERATIONS

1. Negative Consequences of Early Matching

There are more factors to consider prior to adopting a matching process. One thing to be aware of is timing of the process. Early matching of employees to positions in thin markets might have negative consequences for the assignments and efficiency. For instance, it might not be a good solution to match military officers to their first

[93] Atila Abdulkadiroğlu, Parag A. Pathak and Alvin E. Roth, "The New York City High School Match," *American Economic Review* 95, no. 2 (2005), 364-367.

assignments at an early stage of their time as cadets at the military academies, because relevant information is not available at such an early stage. The following example will illustrate this point. In the USA, college football teams play post season “bowl” games. Prior to 1992, the teams playing each other in these “bowl” games were matched during regular season, with several games left to play. By the time the regular season was finished, the teams could have altered their position in the league, so that a league winning team played a mediocre team from another league. Obviously, the matching process was not optimal. In 1992 the matching process was changed. Now, the matching procedure is not performed until the regular season is over. Consequently, the matching has improved, as is the ability to produce “championship” games[94]. This shows that time has an important effect in this matching process, and indicates the inefficiency that might be caused by early decisions[95].

This example also shows that, if possible, an employer should avoid setting assignments before relevant information is available. By postponing the matching process, additional information could help improve efficiency and output in the matchings. Even though no data exists to directly measure this phenomenon in labor markets, Frèchette, Roth and Ünver suggest that timing is critical in these markets, like hiring law clerks based on their first year grades at law school or assigning cadets at an early stage during their time at the military academies.

2. Revelation of Truthful Preferences

There is one more assumption that needs to be satisfied to ensure that a matching mechanism works properly: participants in the market must truthfully reveal their preferences. In the one-to-one model, the optimal strategies for the agents are relatively straightforward. However, the situation is more complex in the many-to-one market, which is how we have defined the process of military assignments. As discussed in Chapter V, the dominant strategy for the employees in the many-to-one market is to state

[94] The measure of output in this example has been the number of viewers on television, measured by Nielsen Media Research.

[95] Guillaume R. Frèchette, Alvin E. Roth and M. Utku Ünver, "Unraveling Yields Inefficient Matchings; Evidence from Post-Season College Football Bowls" (2004).

truthful valuations, as long as the student (worker)-optimal procedure is used. At the same time, there is no procedure that makes it a dominant strategy for the employer to always state truthful preferences[96]. This raises an important question: Will a matching mechanism in the assignment process produce optimal, stable matches? In other words, will this procedure be appropriate?

In theory, matching mechanisms work if the agents submit truthful preferences. However, in the setting of this thesis there is no procedure that encourages both the employer and the employees to submit truthful bids. This means that participants might manipulate the procedure to improve their own outcome. As such, this is a risk of introducing this mechanism to the given context.

D. COMPLICATIONS WITH USE OF MATCHING IN THE CONTEXT OF ASSIGNMENTS

As discussed above, the characteristics of assignment markets in the Norwegian Armed Forces do suggest that a matching mechanism could be an appropriate incentive, at least theoretically. The current assignment process in the Norwegian military contains some of the main elements of a two-sided matching process. At each cycle, those officers who are eligible submit assignment applications. This is essentially a rank order list over acceptable positions. For practical reasons there is usually a limitation on the allowed list length, at current set at a maximum of eight positions per cycle. After the deadline of a cycle, all applications are registered. Then an editing process follows, similar to the NIMP/NRMP procedures. For most of the announced positions, there are certain minimum qualifications that need to be filled for an officer to be allowed to include this position on his rank order list. If an officer has applied for a position for which he is not eligible, he will be assessed as unacceptable and not considered. Following this, the central clearinghouse prioritizes the qualified candidates for each position, which becomes a recommendation to the assignment board. This priority list of the candidates is essentially the rank order list for each position. FST/PØS, whose responsibilities were described in Chapter II, is similar to the central clearinghouse. The

[96] Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 131-134.

work performed here is completed manually, with no single, structured matching remedy. As such, a structured matching mechanism could be introduced as a decision support system to provide quality assurance for the assignments made by each individual officer in this process.

However, two-sided matching does not solve the main problem at hand. In a two-sided matching process, the military as an employer could not match a qualified candidate with a position unless this individual included this position on his ranking list. As long as qualified candidates have not indicated their interest for a certain position, there will be no such match. Furthermore, a position without qualified applicants will always remain unmatched. As a consequence, the announced positions that do not receive any qualified applicants would not change based on the matching mechanism alone. As discussed earlier, participants might also manipulate the procedure in order to improve their own outcome. This means that there is no guarantee that the rank order lists submitted by the agents in this market are truthful (i.e. this mechanism is not necessarily truth revealing).

There is one more shortcoming with a matching process in the context of this thesis. While a rank order list involves making priorities between positions, the ranking list does not tell anything about the owner's strength of preferences. For example, two officers might rank the same two positions A and B as number one and two, respectively, on their ranking lists. One of these officers might have a clear opinion that position A is better than B. The other officer may be more indifferent. Still, the ranking for these two officers is identical with this ranking system.

This situation is similar for positions. With a two-sided matching process, all vacant positions prioritize their applicants. The process does not include any indication of the importance of filling one position as compared to another position; all vacant positions are given equal status in this process. Since the military has no method to fill high priority positions through a matching mechanism, this makes a pure matching mechanism incomplete in this setting.

The final chapters of this research will introduce a mechanism which combines elements of both auction theory and matching mechanisms; a hybrid of an auction and matching procedure, which has the potential to solve the problem at hand, and improve the current assignment process in the Norwegian Armed Forces.

E. CHAPTER SUMMARY

This chapter started with an outline of the assignment process in the Norwegian Armed Forces, first as a one-to-one market then as a many-to-one market, the conclusion being that the military fits the framework of a many-to-one model. In the context of the latter model, examples of the polar cases, firm-optimal and student-optimal solutions, were analyzed. While the firm-optimal algorithm in earlier years used to be the model for many labor market matching processes, it is not frequently used today, and not recommended for military assignments. On the other hand, the firm-optimal solution is now frequently employed in practical matching application, and also theoretically well fit for being decision support for the military assignment process.

Subsequently, some other format considerations with regards to matching design were discussed. Included in this was the negative consequences of early matching and the problem that there in this market is no guarantee that the agents will submit truthful preferences, caused by the fact that no mechanism produces the incentives for all the involved sets of agents to submit true preferences.

Following this, two major complications related to the current context were discussed. The first one was that, due to the design of a matching process, this mechanism is limited in the respect that it does not consider those positions not included on an applicant's rank order list, and vice versa. This means that a match can only happen if an officer has included a certain position on his rank order list, and this position accepts this individual officer as a candidate. If a vacant position that is announced for employment is not included on any applicant's rank order list, then this position will end unmatched in this process. Due to this, the probability of filling hard-to-fill positions will not increase by implementing a matching procedure into the current assignment process. The second complication involves that rank order lists used in matching procedures do not tell anything about the owner's strength of preferences. For the military as the employer, this means that there is not given any indication of the importance of each announced position. Consequently, the employer has no opportunity to separate an important position from other positions, and prioritize which jobs to fill. Based on this discussion, the assessment is that the introduction of a pure matching mechanism is insufficient to solve the main problem at hand.

VII. RECOMMENDED HYBRID MECHANISM

The discussion so far has been thorough on the theories of auctions as well as matching. However, the discussion has left both auctions and matching mechanisms as insufficient if introduced independently to solve the problem at hand in this research context. At this point, it would seem sensible to combine properties from the two distinct theories into a mechanism that could fit this setting. This solution brings in elements of both auction theory and matching mechanisms, and is as such a hybrid of an auction and matching procedure. One mechanism that might be used as foundation for the recommended solution is presented by Roth and Sotomayor[97]. In the following, their mechanism will first be presented in its original form. Then, this model will be modified in three steps to a final version, which will be presented as a recommendation for the context of this research.

A. ORIGINAL, FORWARD OPEN HYBRID MECHANISM

The most widespread auction types were introduced in Chapter III and IV. However, as discussed in those chapters, no pure auction mechanism had the property to solve the problem founding this research. The mechanism presented by Roth and Sotomayor is kind of a generalized **forward, open second-price auction** (i.e. an English auction). Still, it is somewhat more advanced than those formats discussed previously. In fact, it also contains elements of matching. In this model there are a set of bidders and a set of objects. This means that the mechanism is a form of multi-object auction. The procedure of the multi-object mechanism is formally described by Roth and Sotomayor as follows:

> At the first step of the auction the auctioneer announces an initial price vector, *p(1)*, equal to the vector *c* of reservation prices. Each bidder "bids" by announcing which object or objects are in his or her demand set at price *p(1)*.
>
> Step (*t*+1): After the bids are announced, if it is possible to match each bidder to an object in his or her demand set at price *p(t)* the algorithm

[97] Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 209-212.

stops. If no such matching exists, Hall's theorem implies that there is some *overdemanded* set, that is, a set of objects such that the number of bidders demanding only objects in this set is greater than the number of objects in the set. The auctioneer chooses a *minimal* overdemanded set (i.e., an overdemanded set S such that no strict subset of S is an overdemanded set) and raises the price of each object in the set by one unit. All other prices remain at the level *p(t)*. This defines *p*(*t*+1).

It is clear that the algorithm stops at some step *t*, because as soon as the price of an object becomes higher than any bidder's valuation for it, no bidder can demand it. It follows that the final price obtained by this algorithm is a quasi-competitive price-vector. Indeed it is the minimum equilibrium price vector, although this fact is not so obvious.[98]

A demand set, which is referred to in this description, can be defined as one or more preferred objects for an individual bidder at a given set of prices for all of the objects. In the following sections flow charts will be used to illustrate this process and how it step by step is adapted into the recommended solution for this context. The original process can be graphed as follows:

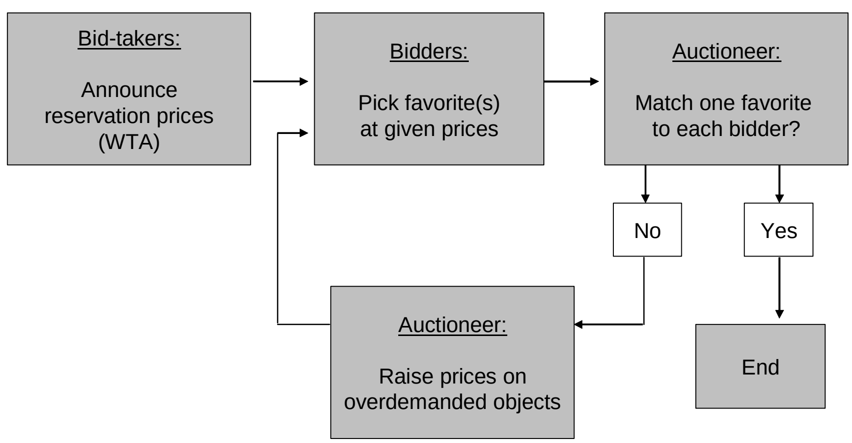

Figure 2. Original, forward open second-price hybrid mechanism

In this model each object for sale has a reservation price, which is the lowest acceptable price for selling the object. Different objects may have different prices. The first step involves the bid-takers announcing their reservation prices for their objects.

98 Roth and Sotomayor, Marilda A. Oliveira, *Two-Sided Matching - A Study in Game-Theoretic Modeling and Analysis*, 210-211.

The reservation price is the lowest acceptable selling price, which can also be called the WTA level. Then, the bidders pick their favorite or favorites at the given prices. It is important to note that each individual bidder might have multiple favorites, not restricted to one object. Following this, the auctioneer tries to match one favorite to each bidder. For those objects/bidders where this is possible, these matches are temporarily set. For those objects experiencing overdemand, the auctioneer raises the prices. Then the process returns to the point where the bidders again pick their favorites. Again, the auctioneer tries to match one favorite to each bidder. At this point, an existing match of an object/bidder might be challenged by other bidders. This means that previous matches are not final until the very end of the procedure. The process goes on until a state of equilibrium is reached. Following this model, there exists a so-called competitive equilibrium, with different prices for each object and buyers matched to objects in a one-to-one fashion. At this equilibrium, every buyer gets an object in his demand set, and additionally no unsold object has a price higher than its reservation price.

Chapters V and VI discussed matching mechanisms. As described there, a matching mechanism could have both a firm-optimal and a student-optimal solution. For Roth and Sotomayor's mechanism, it is shown that there exists a stable set of matches in the bidder-optimal solution. Following the theory from Chapter V, the dominant strategy for the bidders in both *the marriage problem* and *the college admissions problem* would be to submit truthful valuations if a bidder-optimal solution was chosen. One of the properties of a single-object second-price auction is that it in this case too is a dominant strategy to submit truthful valuations. Roth and Sotomayor show that this is also the case in the multi-object case similar to the single-object second-price auction. The bidder-optimal solution results in an optimal outcome for the bidders. Following this, the prices paid by the winning bidders represent the minimum equilibrium price. The assignment of objects to bidders involve that each buyer is assigned with exactly one object, which is always in his demand set given the set of equilibrium prices. Further, each object is assigned to at most one buyer, indicating that the number of objects has to be equal to or greater than the number of buyers.

B. FORWARD SEALED-BID MECHANISM

The first adaptation of the original model is to change it from an open auction into a sealed-bid, still second-price, auction. The flow chart for this mechanism would be as follows:

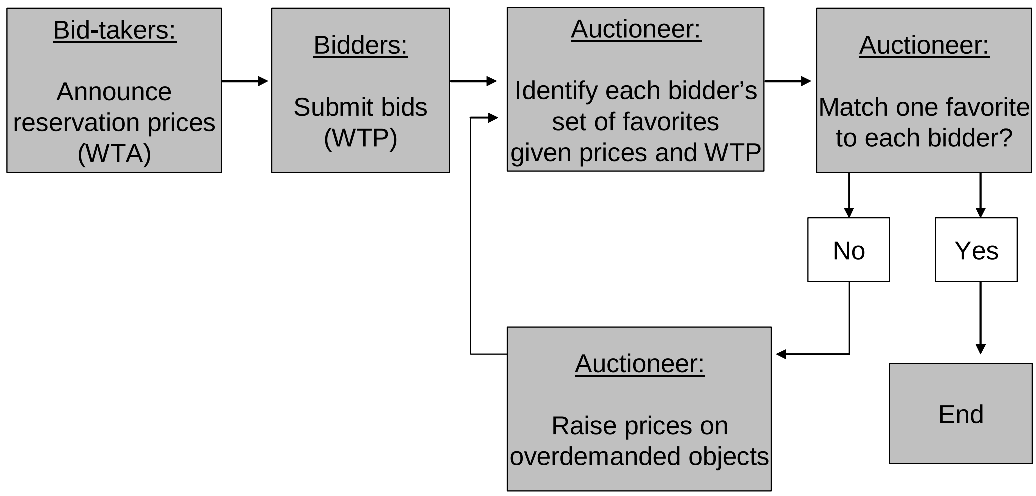

Figure 3. Forward sealed-bid second-price hybrid mechanism

This procedure starts equally as the original procedure, with the bid-takers stating their WTA. Then, the main change from the original process follows. Now the bidders submit bids for all the offered objects, not only their favorites. In this respect, their bids represent their maximum WTP for each of the available objects. Following this, the auctioneer identifies each bidder's set of favorites given the prices and given each bidder's WTP. Each bidder's favorite would be the object yielding the highest bidder surplus. The bidder surplus in this context equals the individual bidder's maximum WTP minus the current price. The matching process continues as in the original process, with the auctioneer trying to match one favorite to each bidder. If it is not possible to match each bidder with one of his or her favorites (with no two bidders being matched to the same object), then the auctioneer raises the prices of those objects with excess demand, and the process of identifying individual bidder favorites is repeated at the new price levels. When each bidder is matched to one of his favorites, the process ends.

C. REVERSE SEALED-BID MECHANISM

The second adaptation of the original model is to change it from a forward to a reverse mechanism. The reason is that in the frame of a salary auction, the idea is that the winner is represented by the lowest bid as opposed to the highest bid. The new situation could be illustrated through employees bidding for positions, rather than individuals bidding for an object. In reverse mode, the flow chart of the process would look like this:

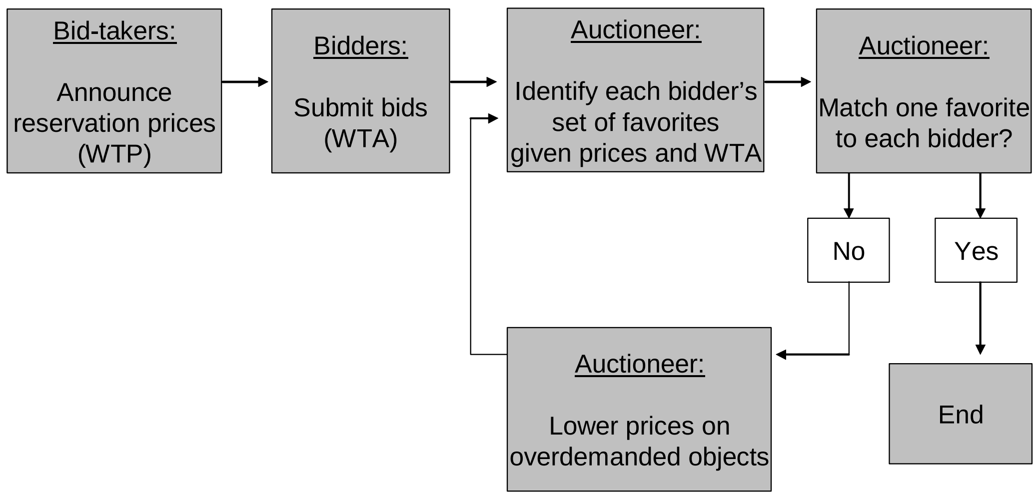

Figure 4. Reverse sealed-bid second-price hybrid mechanism

In the forward mechanisms each object for sale had a reservation price. In our new mechanism, reservation wages of each vacant position equals the reservation price of the objects. Actually both sets of agents have reservation wages. For the bid-takers, the reservation wage equals the maximum WTP for each position. For the bidders (i.e. potential employees), the reservation wage indicates the salary level that is the minimum WTA.

This procedure starts with the bid-takers stating their maximum WTP for each position. Then, the bidders submit their bids, which are their minimum WTA for each position. Following this, the procedure follows the same track as the one outlined in section B. Here, each bidder is matched to maximize the bidder surplus, which in this case equals the price minus the WTA. For those positions experiencing over-demand, meaning a surplus of applicants, the auctioneer in the reverse mode does not raise the prices (i.e. wages), rather lowers them. As for the model in the previous section, the process ends when each bidder is matched to one of his favorites.

The problems founding the models presented so far have been one-sided. In other words, the bid-taker has not cared about who wins the auction, only about the price for the object. However, in the case of a salary auction, the bid-taker, an employer, does care both about filling vacant positions and also what qualifications the employee in each position holds. This factor is not incorporated in the model in this section. Consequently, the model needs another amendment. This alteration follows in section D.

D. REVERSE SEALED-BID MECHANISM INCORPORATING MINIMUM QUALIFICATION LEVELS

The Norwegian Armed Forces' total surplus of personnel in the jobs in their organization could be illustrated through the following function:

$$S_{\text{Armed Forces}} = \sum_{\text{All jobs}} (f - w)$$

Where *S* is the surplus, *f* represents fitness, or qualifications, or value of their personnel. *w* equals wage, or the cost of having personnel in the positions. Consequently, the sum of (*f*-*w*) across all positions is the total surplus for the Armed Forces.

No known mechanism takes care of both maximizing the Armed Forces' total surplus, as well as simultaneously being truth revealing for the bidders. The reason for this is that one of the problems related to auctions, described in Chapter IV, still remains: if an individual knows that he is highly qualified for a position, he is likely to overstate his bid, rather than submit a bid equaling his reservation price. Based on this, the implication with the models so far is that fitness, or qualifications, can not factor into determining winning bids, at least not directly. Truth revelation requires that the auctioneer only compares dollar values to determine winning bids. Next, the question is how it would be possible to incorporate qualifications. A feasible method that satisfies the requirements of truth revelation would be to simply set minimum qualification levels. This means that applicants would be informed about what qualifications were necessary to hold for each position. The qualification levels would dictate whether each applicant is eligible for each position or not. After this initial screening, the qualifications would not matter for the remaining matching process. Taking all eligible applicants into consideration, the winner would be found only based on dollar values. This is not a perfect solution, as it does not necessarily maximize the total surplus for the Armed Forces. Still, it is what economists refer to as a "second best" solution. With this third, and last, adaptation from the original procedure, the recommended model would look like this:

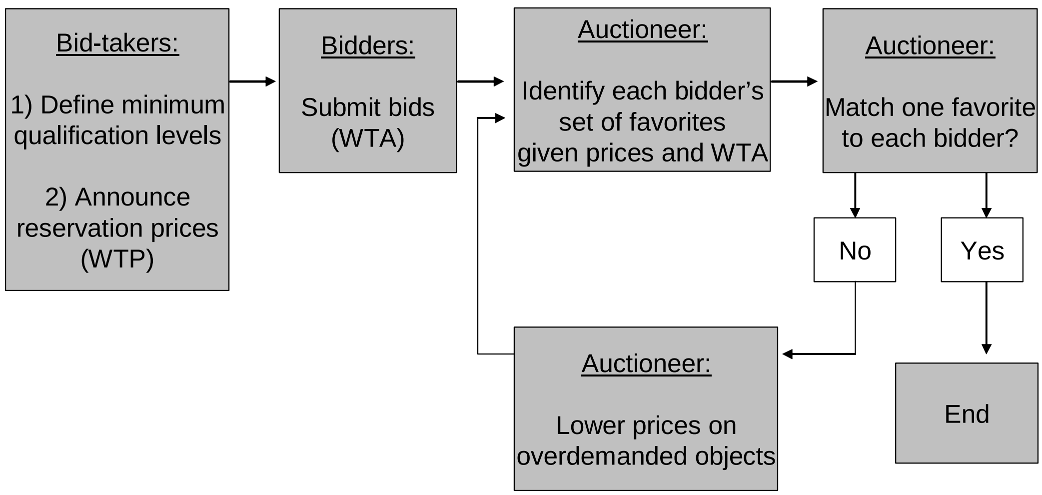

Figure 5. Reverse sealed-bid second-price hybrid mechanism incorporating minimum qualification levels

This model looks very similar to the model outlined in section C, above. As outlined, the only change is that this mechanism introduces one more set of reservation measures; the reservation fitness scores, representing the lowest acceptable level of qualifications for each position. In our setting, the modified mechanism would work as follows. The procedure would start with the employer defining reservation fitness scores, which for each position would represent the lowest level of qualifications acceptable for a candidate for this position. Then, the employer would announce the reservation prices (WTP) for all the announced positions. Following this, employees would submit their reservation prices (WTA) for those positions they were eligible for among those being announced for employment. From there, the process would follow the one outlined in the previous section. The matching process would start at the level representing the employer's maximum WTP for each announced position. At this point, each applicant would be matched to the highest surplus position or positions. For all positions experiencing over-demand, meaning more than one qualified applicant demands this position, the position would be left open in this step. Following this matching, the process would now be repeated. After a finite number of steps, all applicants will be allocated to a position on their demand set. At this point, all matches become final, and the process ends.

For positions that remain unmatched, the equilibrium price equals the reservation price (WTP), set by the bid-taker. As these positions are unmatched, it follows that at least one other option yields greater surplus for each bidder, compared to being matched with one of the unmatched positions. Following the theory of second-price auctions, the wage for each winner would be represented by the wage at which all other bidders opt out, preferring another position. Furthermore, this mechanism would be truth-revealing for the bidders.

One more thing should be emphasized with this mechanism. By starting at the employer's maximum WTP, and then adjusting the wage downwards for those positions experiencing over-demand, this is not necessarily a cost-minimizing mechanism, although it may be the minimum-cost truth-revealing mechanism. However, since the employer has increased the pay for individual positions above a standard salary, this would be an incentive for individuals to apply for positions they would otherwise not include on their rank order lists. As such, introducing this mechanism would involve an increased possibility of filling hard-to-fill positions. However, if using such a mechanism, the employer should be careful with where the minimum qualifications are set. Each position would want to maximize its "position surplus" (i.e., f-c in the function above). This could be done through two different sets of action. The reservation wage could be lowered (lowering the expected cost) or the minimum qualifications could be increased (raising the expected f). Both of these measures would raise the expected surplus for the individual position, conditional on the job being filled. However, these measures also increase the likelihood of keeping the job unfilled.

In the Norwegian Armed Forces, each position in the organization has a "job description". This description defines the main tasks for the position, and it includes “must have”-criteria and “ought to have”-criteria for the holder of the position. As such, the first step of this model is actually already taken care of. Furthermore, theoretically there are no obstacles preventing the implementation of such a mechanism in this organization. However, before considering implementing such a mechanism as a supplement to the current assignment process in the Norwegian military system, this theoretical model ought to be tested, through experiments or other manners. Additionally, there are some practical aspects that would needs attention. The main issues are mentioned as recommendations for further research, in the next chapter.

E. CHAPTER SUMMARY

This chapter first outlined an original mechanism for multi-object auctions, as presented by Roth and Sotomayor. Even though being appropriate in a situation with several bidders and several objects, this model would not fit the context of this research without any adjustments. To make the model more suited for the current context, the original model was modified in three steps before presented in the version recommended for the setting of the assignment process of the Norwegian Armed Forces. The steps involved changing the original forward, open second-price auction to being a sealed-bid, reverse mode mechanism which also incorporates minimum qualification levels for each position in the matching process. Minimum qualifications is determined on a “pass/fail-basis,” and once applicants have passed the minimum level accepted, the matching is performed only based on dollar values.

The assessment is that this model, with the outlined modifications from the original model, would hold the theoretical foundations for fitting the context of the assignment process of the Norwegian Armed Forces. If the model is found to hold in a basic setup experiment, it would have the ability of improving the current process of assigning officers to vacant positions in the Norwegian Armed Forces possible, more so than any other known mechanism.

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VIII. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

The foundation of this research is the process of permanent assignments of officers to vacant positions in the Norwegian Armed Forces, and the problem areas related to the current process. A significant number of positions that are open for assignment in this organization are not filled because they do not receive any qualified applicants. Over the last five years, over 30 percent of the announced job vacancies have been unfilled. Several problem areas have been identified, including capacity constraints for the announcement cycles, the "lag" which is created in the organization as an officer applies for and is appointed to a new position, and how the process misses an incentive for individuals to apply for hard-to-fill positions. Further, how these problem areas in a negative way might influence motivation and job satisfaction for the involved personnel.

This thesis explores two different areas of research to help remedy this context; auction theory and assignment market, or matching, mechanisms. Auction and assignment market theory and practice are examined to reveal how these mechanisms might provide incentives and improve the quality of military assignments. By implementing auctions in this context, the positive effects could involve that vacant positions in the organization got filled, under the assumption that the employer was willing to pay a high enough price for this. Additionally, an auction would tell us the price of each position at a certain time. However, there are complications related to the use of auctions. First, each employee can not win more than one auction, even though he might have the lowest bid for more than one job. Furthermore, auctions assume one-sided preferences, meaning that the employer, representing the vacant positions, does not have preferences for which employee gets which job. These deficiencies leave auctions as insufficient if implemented as an independent mechanism in the setting of this research. Matching mechanisms take into account two-sided preferences. An analysis of the market of assignments in the Norwegian Armed Forces finds that this market fits the theoretical framework of a many-to-one matching model. However, there are still complications related to introducing such a mechanism in the Norwegian military assignment process.

First, due to the design of a matching process, this type of mechanism is limited in the respect that it does not consider those positions that are not included on an applicant's rank order list, and vice versa. This means that a match can only happen if an officer has included a certain position on his rank order list, and this position accepts this individual officer as a candidate. Following this, the probability of filling hard-to-fill positions does not increase through this mechanism. Second, the rank order lists used in matching procedures do not tell anything about the owner's strength of preferences. For the military, being the employer, this means that there is not given any indication of the importance of each announced position. Consequently, the employer has no opportunity to separate an important position from other positions, and prioritize which jobs to fill. Furthermore, there is no guarantee that such a process is truth revealing, meaning that the agents' rank order lists are based on true valuations. Based on this, matching mechanisms, just like auctions, fall short when used independently to solve the main problem at hand.

The complications outlined above leads this thesis to the introduction of a hybrid solution, containing elements of both auction theory and matching. This mechanism is based on an original model for multi-object auctions presented by Roth and Sotomayor. Adapted to the setting of this research, the original forward English auction mechanism is turned into a reverse sealed-bid second-price mechanism which in addition to considering each applicant's reservation wage for each vacant position also considers reservation fitness scores, representing the minimum qualification level acceptable for each position. The procedure involves matching applicants to open positions in the organization. First, it is determined which applicants are eligible for the different positions. Then, the employer announces their maximum WTP for each announced position. Based on this information, employees submit their minimum wage, or WTA, for the positions they are eligible to apply for. Then the auctioneer identifies each employee's favorite, and tries to match each employee with one of his favorites. For over-demanded positions the price is lowered until a match is obtained for all employees, relative to the valuations each applicant has submitted in his application. In detail, for each position the process starts at the employer's maximum WTP, and the wage is lowered until only one applicant remains for each position. At this point, all matches are final. Theoretically, this model would fit the process of assigning military officers to vacant positions within the Norwegian Armed Forces. If this model holds through experiments, it would have the ability to improve the current assignment process, more so than any other known mechanism. This involves both increasing the probability of filling hard-to-fill positions as well as improving the match between each position and each individual employee

B. CONCLUSIONS

No known mechanism takes complete care of the problem at hand in this research. There are several complications making auctions and matching individually insufficient for this setting. However, the hybrid solution that is outlined will have the ability to resolve some of the main considerations. Most essential, it removes the incentive for applicants to bid higher than their reservation wage, both in the case where fitness scores are not encompassed, and when fitness scores are included. At the same time, by increasing the range of salary for the positions involved in this process, this does increase the probability of filling hard-to-fill positions. This procedure may not be a minimum cost solution. However, it may be the minimum cost, truth revealing procedure. The process starts at the level indicated by the employer's maximum WTP, and decreases for those positions with over-demand. The process is terminated at the point where each applicant is matched to exactly one of the positions on his rank order list, as long as the number of open positions is equal to or greater than the number of applicants. Implemented for positions that are hard-to-fill in the military organization of the Norwegian Armed Forces, this procedure would have the potential to improve the quality of the current process.

C. RECOMMENDATIONS FOR FURTHER RESEARCH

There are several aspects of the problem at hand that are not covered in this thesis. First of all, the theoretical mechanism that is recommended in this thesis ought to be examined through tests and/or experiments, to see if it works in a practical setup. If the model holds, and the Norwegian Armed Forces considered implementing the mechanism, this would involve further analysis of some strategic questions, including the following. These questions represent challenges that should be considered prior to implementing a new mechanism in the context of this setting. Failure to do so could result in unexpected behavior among the participants, and even failure for the whole model.

- Are there positions in the organization of the Norwegian military that over time have experienced more problems than others with lack of qualified applicants, and as such are harder to fill than others?
- Should all positions be offered by the recommended process, or just some positions?
- If only a portion of the positions are offered by this mechanism, which criteria should then be used to decide which positions to be treated this way?
- Should there be limitations, allowing each employee to submit bids for only a certain number of jobs per cycle? And how should the procedure be laid out if some of the positions are offered by this mechanism, in order to avoid strategic behavior?
- What would be the optimal decision with regards to maximum wages and minimum qualifications? Given distribution of employee WTA and qualifications, what would be optimal for the employer/bid-taker?

APPENDIX. THE SUB-OPTIMALITY OF TRADITIONAL AUCTION BIDDING STRATEGIES WHEN WINNERS ARE CHOSEN BASED ON BID-TAKER BENEFITS AS WELL AS COSTS[99]

Equilibrium bidding strategies in traditional first- and second-price auctions are well established. The purpose of this appendix, however, is to illustrate that these established strategies are no longer optimal when the winner of an auction is not the bidder with the highest bid (or lowest bid in a reverse auction), but is instead determined by comparing bidders using a measure that combines bids with other exogenous bidder characteristics.

In particular, we will consider a FPSB reverse auction in which the winning bidder, instead of being the bidder with the lowest wage bid, is the bidder who offers the employer the greatest value or surplus (given by quality/fitness minus cost/wage). In this scenario, we will show that a bidder with a high quality or fitness level has an incentive to increase his wage bid above the level that he would optimally bid in a traditional reverse auction in which the winner was simply the bidder with the lowest wage bid. Obviously, this incentive to submit higher wage bids can raise the salary costs of the employer.

Suppose that two potential employees are bidding for a single position. Each employee has a reservation wage for the position (the minimum wage he is willing to accept) and a fitness level for the position.

Each employee's reservation wage is drawn from the uniform distribution between zero and some upper bound W. In other words, $w_i \sim U[0,W]$ where w_i is the reservation wage for employee i (i = 1 or 2). Each employee's reservation wage is known only to the employee himself, however the distribution of reservation wages is common knowledge.

[99] The concept in this Appendix is developed by Associate Professor Pete Coughlan, NPS.

Each employee's fitness level is drawn from the uniform distribution between zero and some upper bound F. In other words, $f_i \sim U[0,F]$ where f_i is the fitness level for employee i (i = 1 or 2). Each employee's fitness level is known only to the employee and the potential employer (i.e. not to the other employee), however the distribution of fitness levels is common knowledge.

If employee i is given the position, the employee's surplus is equal to b_i-w_i and the employer's surplus is equal to f_i-b_i, where b_i is employee i's wage bid in the auction. In order for both the employee and employer to receive positive surplus, we must have $f_i > b_i > w_i$. For this to be a commonly achievable outcome, we will assume that the fitness levels have a higher upper bound than the reservation wages or, in other words, that F > W.

In a traditional FPSB reverse auction, in which the winning bidder is the bidder with the lowest bid, the equilibrium bidding strategy is for bidder i with reservation wage w_i to bid ½(W+w_i). To prove this, let us assume that bidder 2 is bidding according to this strategy, and we will show that it is optimal for bidder 1 to do the same (the reverse will hold as well).

With bidder 2 bidding ½(W+w_2), bidder 1's expected utility given reservation wage w_1 and bid b is given by:

$$
\begin{aligned}
EU_1(w_1, b) &= (\text{probability 1 wins}) \times (\text{surplus if 1 wins}) \\
&= \text{Prob}\left(\tfrac{1}{2}(W + w_2) > b\right) \times (b - w_1) \\
&= \text{Prob}(w_2 > 2b - W) \times (b - w_1) \\
&= (1 - F(2b - W)) \times (b - w_1) \\
&= \frac{2W - 2b}{W} \times (b - w_1) \\
&= 2(b - w_1) - \frac{2b}{W}(b - w_1) \\
&= 2b - 2w_1 - \frac{2b^2}{W} + \frac{2bw_1}{W}
\end{aligned}
$$

To identify bidder 1's utility maximizing strategy, we need to differentiate $EU_1(w_1,b)$ with respect to b and set this equal to zero.

$$\frac{\partial EU_1(w_1,b)}{\partial b} = \frac{\partial}{\partial b}\left(2b - 2w_1 - \frac{2b^2}{W} + \frac{2bw_1}{W} \right)$$
$$= 2 - \frac{4b}{W} + \frac{2w_1}{W}$$
$$\frac{\partial EU_1(w_1,b)}{\partial b} = 0 \Leftrightarrow 2 - \frac{4b}{W} + \frac{2w_1}{W} = 0$$
$$\Leftrightarrow 2W - 4b + 2w_1 = 0$$
$$\Leftrightarrow W - 2b + w_1 = 0$$
$$\Leftrightarrow b = \tfrac{1}{2}(W + w_1)$$

Thus, bidder i's optimal strategy in a traditional FPSB reverse auction is to bid $\frac{1}{2}(W+w_i)$.

Now suppose that the winner of the auction (and the employee who is given the position) is the bidder who provides the employer the greatest surplus, in other words, the winning bidder has the maximum f_i-b_i. This, of course, is precisely what the employer hopes to maximize through the auction.

We will show that, in this modified FPSB reverse auction, it is no longer optimal for a bidder with reservation wage w_i to bid $\frac{1}{2}(W+w_i)$. In particular, we will show that, if other bidders are following the traditional bidding strategy, a bidder with a high fitness level has an incentive to raise his bid above $\frac{1}{2}(W+w_i)$.

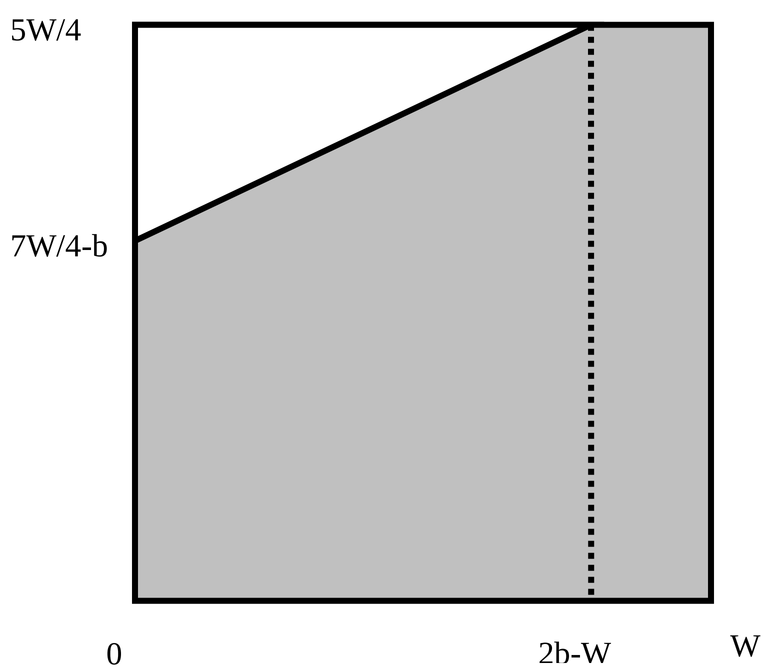
It suffices to show that this incentive to deviate from the original equilibrium exists for some possible reservation wage and fitness score and for some feasible values of W and F. Therefore, let us consider the scenario in which bidder 1 has the minimum achievable reservation wage, so that w_1=0. Note that, in this case, the traditional equilibrium strategy for bidder 1 would be to bid ½W. Let us also suppose that bidder 1 has the maximum achievable fitness level so that f_1=F. Lastly, let us further simplify our analysis by assuming F = $\tfrac{5}{4}$W. Under these conditions, we will calculate what bidder

1's optimal bidding strategy would be if bidder 2 were following the original strategy of bidding ½(W+w_2), and we will show that bidder 1 has an incentive to bid above his original equilibrium bid of ½W.

In this case, bidder 1's expected utility given reservation wage w_1 and bid b is given by:

$$\begin{aligned} EU_1(w_1, b) &= (\text{probability 1 wins}) \times (\text{surplus if 1 wins}) \\ &= \text{Prob}\left(f_2 - \tfrac{1}{2}(W + w_2) < F - b\right) \times (b - w_1) \\ &= \text{Prob}\left(f_2 - \tfrac{1}{2}(W + w_2) < \tfrac{5}{4}W - b\right) \times b \\ &= \text{Prob}\left(f_2 < \tfrac{1}{2}w_2 - b + \tfrac{7}{4}W\right) \times b \end{aligned}$$

Because f_2 is distributed uniformly between 0 and F while w_2 is distributed uniformly between 0 and W, we can illustrate the value of Prob($f_2 < \tfrac{1}{2}w_2 - b + \tfrac{7}{4}W$) graphically. Under four assumptions we will address below, the combinations of f_2 and w_2 which satisfy the inequality $f_2 < \tfrac{1}{2}w_2 - b + \tfrac{7}{4}W$ lie in the shaded region illustrated below.



Note that the diagonal line in the graph above is given by the equation $f_2 = \frac{1}{2}w_2 - b + \frac{7}{4}W$, and thus the feasible combinations of f_2 and w_2 below the line represent the region for which $f_2 < \frac{1}{2}w_2 - b + \frac{7}{4}W$.

The four assumptions necessary for the above diagram to accurately represent the appropriate region satisfying the inequality $f_2 < \frac{1}{2}w_2 - b + \frac{7}{4}W$ are as follows:

(1) $0 \leq 2b\text{-}W$
(2) $2b\text{-}W \leq W$
(3) $7W/4\text{-}b \geq 0$
(4) $7W/4\text{-}b \leq 5W/4$

Note that assumption (2) holds if and only if $b \leq W$ whereas assumption (3) holds if and only if $b \leq 7W/4$. Therefore assumption (3) will hold whenever assumption (2) holds (i.e. whenever $b \leq W$). Also note that assumptions (1) and (4) each hold if and only if $b \geq ½W$. Thus, the four assumptions above actually reduce to two assumptions:

(A) $b \leq W$
(B) $b \geq ½W$

These two assumptions place requirements on employee 1's bid relative to the maximum achievable reservation wage W. The discussion that follows will hold only for values of b that satisfy both of these assumptions. As we will see from the optimal bidding strategy derived below, these assumptions are indeed both satisfied in our scenario. For now, however, let us simply comment on the reasonableness of these assumptions.

Assumption (A) simply states that the optimal bid b must be less than the maximum reservation wage W. Because the value of W is common knowledge, the employer need never pay a wage above W, so there would never be any reason to bid above W. Moreover, the traditional equilibrium strategy has bidder 1 bidding $b = ½W < W$. Thus, assumption (A) can immediately be revealed as quite general.

Assumption (B) requires that employee 1 in our scenario bid at or above ½W. Note that the traditional equilibrium strategy of bidding ½W satisfies this constraint. Moreover, in our scenario, employee 1 has the highest possible fitness score of F, increasing his chances of winning the auction for any given bid b, and thus reducing his incentive to lower his bid relative to the traditional equilibrium strategy (this is precisely what is illustrated below). Thus, assumption (B) is also quite general.

Now that we are satisfied that the graph above reasonably depicts the combinations of f_2 and w_2 which satisfy the inequality $f_2 < \frac{1}{2}w_2 - b + \frac{7}{4}W$, we can calculate the value of Prob($f_2 < \frac{1}{2}w_2 - b + \frac{7}{4}W$), which is equal to the area of the shaded region in the graph above divided by the area of the entire rectangle. Alternatively, the value of Prob($f_2 < \frac{1}{2}w_2 - b + \frac{7}{4}W$) is equal to 1 minus the area of the un-shaded triangle in the graph above divided by the area of the entire rectangle. Thus, we have:

$$\begin{aligned}
\text{Prob}\left(f_2 < \tfrac{1}{2}w_2 - b + \tfrac{7}{4}W\right) &= 1 - \frac{\text{Area of Unshaded Triangle}}{\text{Area of Entire Rectangle}} \\
&= 1 - \frac{\tfrac{1}{2}\left(b - \tfrac{1}{2}W\right)\left(2b - W\right)}{\tfrac{5}{4}W^2} \\
&= 1 - \frac{(2b - W)(2b - W)}{5W^2} \\
&= 1 - \frac{4b^2 - 4bW + W^2}{5W^2} \\
&= \frac{4}{5} - \frac{4b^2}{5W^2} + \frac{4b}{5W}
\end{aligned}$$

Plugging this into our expected utility equation, we have:

$$\begin{aligned}
EU_1(w_1, b) &= \text{Prob}\left(f_2 < \tfrac{1}{2}w_2 - b + \tfrac{7}{4}W\right) \times b \\
&= \left(\frac{4}{5} - \frac{4b^2}{5W^2} + \frac{4b}{5W}\right)b \\
&= \frac{4b}{5} - \frac{4b^3}{5W^2} + \frac{4b^2}{5W}
\end{aligned}$$

To identify bidder 1's utility maximizing strategy, we need to differentiate $EU_1(w_1,b)$ with respect to b and set this equal to zero.

$$\frac{\partial EU_1(w_1,b)}{\partial b} = \frac{\partial}{\partial b}\left(\frac{4b}{5} - \frac{4b^3}{5W^2} + \frac{4b^2}{5W}\right)$$

$$= \frac{4}{5} - \frac{12b^2}{5W^2} + \frac{8b}{5W}$$

$$\frac{\partial EU_1(w_1,b)}{\partial b} = 0 \Leftrightarrow \frac{4}{5} - \frac{12b^2}{5W^2} + \frac{8b}{5W} = 0$$

$$\Leftrightarrow 4W^2 - 12b^2 + 8Wb = 0$$

$$\Leftrightarrow 3b^2 - 2Wb - W^2 = 0$$

$$\Leftrightarrow 3b^2 - 2Wb = W^2$$

$$\Leftrightarrow b^2 - \tfrac{2}{3}Wb = \tfrac{1}{3}W^2$$

$$\Leftrightarrow b^2 - \tfrac{2}{3}Wb + \tfrac{1}{9}W^2 = \tfrac{4}{9}W^2$$

$$\Leftrightarrow \left(b - \tfrac{1}{3}W\right)^2 = \tfrac{4}{9}W^2$$

$$\Leftrightarrow b - \tfrac{1}{3}W = \tfrac{2}{3}W$$

$$\Leftrightarrow b = W$$

Thus, if bidder 2 sticks to the original equilibrium strategy of bidding $\frac{1}{2}(W+w_2)$, bidder 1's optimal strategy in this scenario is to bid W, which is twice his original equilibrium bid of ½W.

In conclusion, we have demonstrated that the equilibrium bidding strategy in a traditional FPSB reverse auction is no longer an equilibrium strategy in the modified auction in which the winner is determined by calculating employer surplus rather than minimum cost. Moreover, we have shown that bidders with high fitness levels in such a modified auction have an incentive to bid higher than they would bid in a traditional FPSB reverse auction.

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