

Half-space problem of weak evaporation and condensation of a binary mixture of vapors

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Abstract. Half-space problem of weak evaporation and condensation of a binary mixture of vapors is investigated on the basis of the BGK-type Boltzmann model. By a systematic asymptotic analysis, it is shown that the steady evaporation and condensation takes place only if the parameters that characterize the state of the condensed phase and that of the vapors at a far distance satisfy one relation for the condensation case and three relations for the evaporation case. As an application, the resulting relations are used as the boundary conditions of the Euler system in the study of the two-surface problem of a vapor mixture for small Knudsen numbers. This system has two branches of solutions, which causes two different solutions for the same physical situation in the continuum limit. This result is discussed in connection with the ghost effect.

INTRODUCTION

Half-space problem of evaporation and condensation is one of the most fundamental boundary-value problems in kinetic theory and has been intensively studied (see, for example, [1, 2] and the references therein). One of the important aspects of the problem is that the information about the conditions for the steady evaporation and condensation flows provides the fluid-dynamic equations with the appropriate boundary conditions; that is, it allows us to complete the macroscopic description of vapor flows with the phase change at ordinary pressure.

According to the studies so far, the conditions for the steady flows depend qualitatively on whether evaporation or condensation takes place for a pure vapor and for a mixture of a vapor and a noncondensable gas. (There is also a qualitative difference between subsonic and supersonic condensations.) In the present paper, we will first show the corresponding results for a binary mixture of vapors in the case of weak evaporation and condensation. By limiting ourselves to weak evaporation and condensation, we can take an analytical approach [3] to obtain the conditions explicitly for both evaporation and condensation cases and to clarify a qualitative difference between them. The solution discussed here is closely related to the suction boundary layer [4] in the conventional gasdynamics.

Next, we will study the two-surface problem of a binary mixture of vapors for small Knudsen numbers by the Euler set of equations with the conditions above being the boundary conditions. We show that the Euler system has two branches of solutions in general, so that two different solutions may occur for the same physical situation in the continuum limit. This result is discussed in connection with the ghost effect [1, 5] at the end of the paper.

PROBLEM AND FORMULATION

We consider a binary mixture of vapors, say vapor A and B, in a half-space in contact with their plane condensed phase. The condensed phase is kept at a uniform temperature T_w , and its surface (or the interface) is located at $X_1 = 0$, where X_i is the rectangular coordinate system. The mixture of vapors occupies the region $X_1 > 0$ and is in the equilibrium state characterized by the pressure p_∞ , temperature T_∞ , concentration (molecular number fraction) X_∞^A of species A, and flow velocity $(v_\infty, 0, 0)$ at a far distance from the interface. The Mach number M_∞ of the flow at a far distance, which is defined by $M_\infty = |v_\infty|(\frac{5}{3}kT_\infty/m_\infty)^{-1/2}$, is supposed to be small. Here k is the Boltzmann constant, m^α ($\alpha = A, B$) is the mass of a molecule of species α , and $m_\infty = m^A X_\infty^A + m^B X_\infty^B$ with $X_\infty^B = 1 - X_\infty^A$. We will investigate the steady behavior of the vapors on the basis of kinetic theory, mainly aiming at deriving the relation among the parameters that characterize the state of the condensed phase and that of the vapors at a far distance in order that the steady

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE 13 JUL 2005		2. REPORT TYPE N/A		3. DATES COVERED -	
4. TITLE AND SUBTITLE Half-space problem of weak evaporation and condensation of a binary mixture of vapors				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Department of Aeronautics and Astronautics and Advanced Research Institute of Fluid Science and Engineering, Graduate School of Engineering, Kyoto University, Kyoto 606-8501, Japan				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release, distribution unlimited					
13. SUPPLEMENTARY NOTES See also ADM001792, International Symposium on Rarefied Gas Dynamics (24th) Held in Monopoli (Bari), Italy on 10-16 July 2004.					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 6	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

evaporation or condensation takes place. We assume that (i) the behavior of the vapors is described by the Garzó-Santos-Brey equation [6] (GSB model) and that (ii) the velocity distribution of the molecules of species α ($\alpha = A, B$) outgoing from the interface is the half-Maxwellian at rest with pressure p_w^α and temperature T_w . Here p_w^α is the partial pressure of species α in the mixture saturated in contact with the condensed phase at temperature T_w . Hereinafter, we call p_w^α the *saturation pressure* of species α and $p_w \equiv p_w^A + p_w^B$ the *total saturation pressure*, respectively. In the sequel, the Greek letters α and β are symbolically used to represent the species of vapors, i.e., $\alpha, \beta = A, B$.

Let us denote by $c_*\zeta$ (or $c_*\zeta_i$) the molecular velocity and by $(n_w^A c_*^{-3/2}) f^\alpha$ the velocity distribution function of molecules of species α , where $n_w^A = p_w^A/kT_w$ and $c_* = (2kT_w/m^A)^{1/2}$. Let $x_1 = X_1/(\sqrt{\pi}/2)\ell$ be the dimensionless space coordinate, where ℓ is the mean-free path of a molecule in vapor A that is in the equilibrium state at rest with pressure p_w^A and temperature T_w . Then the present problem is written as a half-space boundary-value problem for f^α :

$$\zeta_1 \frac{\partial f^\alpha}{\partial x_1} = K^{A\alpha} \hat{n}^A (f^{A\alpha} - f^\alpha) + K^{B\alpha} \hat{n}^B (f^{B\alpha} - f^\alpha), \quad (1)$$

$$f^\alpha = \hat{p}_w^\alpha (\pi/\hat{m}^\alpha)^{-3/2} \exp(-\hat{m}^\alpha \zeta_i^2) \quad (\zeta_1 > 0), \quad \text{at } x_1 = 0, \quad (2a)$$

$$f^\alpha = (\hat{p}_\infty^\alpha/\hat{T}_\infty) (\pi\hat{T}_\infty/\hat{m}^\alpha)^{-3/2} \exp(-(\hat{m}^\alpha/\hat{T}_\infty)[(\zeta_1 - \hat{v}_\infty)^2 + \zeta_2^2 + \zeta_3^2]), \quad \text{at } x_1 \rightarrow \infty, \quad (2b)$$

where $\hat{m}^\alpha = m^\alpha/m^A$, $\hat{p}_w^\alpha = p_w^\alpha/p_w^A$, $\hat{p}_\infty^\alpha = p_\infty X_\infty^\alpha/p_w^A$, $\hat{T}_\infty = T_\infty/T_w$, and $\hat{v}_\infty = v_\infty/c_*$. Here $f^{\beta\alpha}$ is a given function of ζ with dimensionless macroscopic quantities and $\hat{m}^B$ being parameters (see [6]). $K^{\beta\alpha}$ are those constants related to collision frequencies that are symmetric in α and β and are normalized as $K^{AA} = 1$. In the present paper, we denote by $n_w^A \hat{n}^\alpha$, $T_w \hat{T}^\alpha$, and $(c_* \hat{v}, 0, 0)$ the molecular number density, temperature, and flow velocity of species α , and by $n_w^A \hat{n}$, $T_w \hat{T}$, and $(c_* \hat{w}, 0, 0)$ their counterpart of the mixture, whose definitions are omitted here (see [6]). The notations $p_w^A \hat{p}^\alpha$ for the pressure of species α , $p_w^A \hat{p}$ for the pressure of the mixture, X^α for the concentration of species α , and $(c_* \hat{w}, 0, 0)$ for the average velocity of the mixture based on the particle flux are also used. They are related to the number densities, temperatures, and flow velocities as $\hat{p}^\alpha = \hat{n}^\alpha \hat{T}^\alpha$, $\hat{p} = \hat{n} \hat{T}$, $X^\alpha = \hat{n}^\alpha/\hat{n}$, and $\hat{w} = X^A \hat{v}^A + X^B \hat{v}^B$.

ASYMPTOTIC ANALYSIS — SLOWLY VARYING SOLUTION

We carry out an asymptotic analysis of the boundary-value problem (1) – (2b) in the situation where the evaporation or condensation takes place only weakly. Following the convention, we denote by ε the small parameter of the problem: $\varepsilon = |\hat{v}_\infty|$. Since there is no geometrical characteristic length, the solution apparently has the length scale of variation of the order of the mean-free path ℓ [the length scale of $O(1)$ in the dimensionless coordinate x_1]. Nevertheless, we consider a solution having another length scale of $O(\ell/\varepsilon)$ motivated by the suction boundary layer in the classical gasdynamics (see [3, 4] and Chaps. 4 and 5 in [1]). We call such a solution a slowly varying solution, following [1].

Putting aside the boundary condition, we first consider the solution of Eq. (1) such that $\partial f^\alpha/\partial x_1 = O(f^\alpha \varepsilon)$ and derive fluid-dynamic-type equations that describe its behavior. To do this, we rescale Eq. (1) by introducing a new space coordinate $y = x_1 \varepsilon$ and seek the solution in a power series of ε , i.e., $f^\alpha = f_{(0)}^\alpha + f_{(1)}^\alpha \varepsilon + \dots$. Correspondingly, we expand the macroscopic quantities as $h = h_{(0)} + h_{(1)} \varepsilon + \dots$, where $h = \hat{n}, \hat{n}^A, \hat{n}^B, \hat{v}, \hat{v}^A, \hat{v}^B$, etc. Here we assume $\hat{v}_{(0)} = \hat{v}_{(0)}^\alpha = 0$ because of the physical situation under consideration. By substituting these expansions into the rescaled Eq. (1) and by equating the terms of the same order of ε , we can obtain the coefficient functions $f_{(n)}^\alpha$ ($n = 0, 1, 2, \dots$). There are two things to remark at this stage. The first is that $f_{(0)}^\alpha$ is a local equilibrium distribution at rest:

$$f_{(0)}^\alpha = \hat{n}_{(0)}^\alpha (\pi\hat{T}_{(0)}/\hat{m}^\alpha)^{-3/2} \exp(-\hat{m}^\alpha \zeta_i^2/\hat{T}_{(0)}). \quad (3)$$

Thus the temperature is common to species at the lowest order. The second is that the conservation laws must hold at each order of ε , from which the fluid-dynamic-type equations are obtained. Here we show the equations up to $O(\varepsilon^2)$. The discussions in the next section will be done on the basis of these equations.

$$\frac{d\hat{p}_{(0)}}{dy} = 0, \quad \frac{d}{dy}(\hat{n}_{(0)} \hat{w}_{(1)}) = 0, \quad (4a)$$

$$\hat{D}_{AB} \frac{d}{dy} \left(\hat{T}_{(0)} \frac{dX_{(0)}^A}{dy} \right) = \hat{n}_{(0)} \hat{w}_{(1)} \frac{dX_{(0)}^A}{dy}, \quad \frac{d}{dy} \left(\hat{\lambda}_{(0)} \hat{T}_{(0)} \frac{d\hat{T}_{(0)}}{dy} \right) = \frac{5}{2} \hat{n}_{(0)} \hat{w}_{(1)} \frac{d\hat{T}_{(0)}}{dy}, \quad (4b)$$

$$\frac{d\hat{p}_{(1)}}{dy} = 0, \quad \frac{d}{dy}(\hat{n}_{(0)}\hat{w}_{(2)} + \hat{n}_{(1)}\hat{w}_{(1)}) = 0, \quad (5a)$$

$$\hat{D}_{AB} \frac{d}{dy} \left(\hat{T}_{(1)} \frac{dX_{(0)}^A}{dy} + \hat{T}_{(0)} \frac{dX_{(1)}^A}{dy} \right) = (\hat{n}_{(0)}\hat{w}_{(2)} + \hat{n}_{(1)}\hat{w}_{(1)}) \frac{dX_{(0)}^A}{dy} + \hat{n}_{(0)}\hat{w}_{(1)} \frac{dX_{(1)}^A}{dy}, \quad (5b)$$

$$\frac{d}{dy} \left((\hat{\lambda}_{(1)}\hat{T}_{(0)} + \hat{\lambda}_{(0)}\hat{T}_{(1)}) \frac{d\hat{T}_{(0)}}{dy} + \hat{\lambda}_{(0)}\hat{T}_{(0)} \frac{d\hat{T}_{(1)}}{dy} \right) = \frac{5}{2}(\hat{n}_{(0)}\hat{w}_{(2)} + \hat{n}_{(1)}\hat{w}_{(1)}) \frac{d\hat{T}_{(0)}}{dy} + \frac{5}{2}\hat{n}_{(0)}\hat{w}_{(1)} \frac{d\hat{T}_{(1)}}{dy}, \quad (5c)$$

where

$$\hat{\lambda} = \frac{5}{4} \sum_{\alpha=A,B} \frac{X^\alpha}{\hat{m}^\alpha K^\alpha}, \quad K^\alpha = \sum_{\beta=A,B} K^{\beta\alpha} X^\beta, \quad \hat{D}_{AB} = \frac{1 + \hat{m}^B}{2\hat{m}^B K^{BA}}, \quad \hat{p}_{(0)} = \hat{n}_{(0)}\hat{T}_{(0)}, \quad \hat{p}_{(1)} = \hat{n}_{(1)}\hat{T}_{(0)} + \hat{n}_{(0)}\hat{T}_{(1)}, \quad (6)$$

and $\hat{\lambda} = \hat{\lambda}_{(0)} + \hat{\lambda}_{(1)}\varepsilon + \dots$ and $K^\alpha = K_{(0)}^\alpha + K_{(1)}^\alpha\varepsilon + \dots$. Equations (4a) and (4b) form a closed set of equations for $\hat{p}_{(0)}$, $X_{(0)}^A$, $\hat{T}_{(0)}$, and $\hat{w}_{(1)}$. Equations (5a) – (5c) form a closed set of linear equations for $\hat{p}_{(1)}$, $X_{(1)}^A$, $\hat{T}_{(1)}$, and $\hat{w}_{(2)}$.

Next we consider the boundary condition at the interface for Eqs. (4a) – (5c). At the leading order, as is obvious from Eq. (3), $f_{(0)}^\alpha$ can satisfy the kinetic boundary condition (2a) by setting $\hat{p}_{(0)}^A = 1$, $\hat{p}_{(0)}^B = \hat{p}_w^B$, and $\hat{T}_{(0)} = 1$. Hence the following condition is obtained as the boundary condition for Eqs. (4a) and (4b):

$$\hat{p}_{(0)} = \hat{p}_w \equiv 1 + \hat{p}_w^B, \quad X_{(0)}^A = X_w^A \equiv 1/(1 + \hat{p}_w^B), \quad \hat{T}_{(0)} = 1, \quad \text{at } y = 0. \quad (7)$$

At the higher orders, however, the slowly varying solution is seen not to match Eq. (2a) because of the flow of $O(\varepsilon)$. Thus we introduce a correction to the solution in the layer with the thickness of a few mean-free paths adjacent to the interface. Such a correction, which we denote by f_K^α , has the original length scale of variation, i.e., $\partial f_K^\alpha / \partial x_1 = O(f_K^\alpha)$. We seek the correction f_K^α in a power series of ε : $f_K^\alpha = f_{K1}^\alpha\varepsilon + f_{K2}^\alpha\varepsilon^2 + \dots$. Substitution of the expansion $f^\alpha = f_{(0)}^\alpha + [f_{(1)}^\alpha + f_{K1}^\alpha]\varepsilon + \dots$ into Eqs. (1) – (2b) gives a half-space boundary-value problem of the linearized GSB equation for f_{Kn}^α ($n = 1, 2, \dots$); and the analysis of the problem for f_{K1}^α finally leads to the following relation [7]:

$$\hat{p}_{(1)} = -a_V \hat{p}_w \hat{w}_{(1)} - a_T \frac{d\hat{T}_{(0)}}{dy} - \left(a_X - \frac{(1 - \hat{m}^B)\hat{D}_{AB} a_V}{\hat{m}_w} \right) \frac{dX_{(0)}^A}{dy}, \quad \text{at } y = 0, \quad (8)$$

$$X_{(1)}^A = -X_w^A \left[\frac{\hat{p}_{(1)}}{\hat{p}_w} + a_V^A \hat{w}_{(1)} + a_T^A \frac{1}{\hat{p}_w} \frac{d\hat{T}_{(0)}}{dy} + \left(a_X^A - \frac{(1 - \hat{m}^B)\hat{D}_{AB} a_V^A}{\hat{m}_w} \right) \frac{1}{\hat{p}_w} \frac{dX_{(0)}^A}{dy} \right], \quad \text{at } y = 0, \quad (9)$$

$$\hat{T}_{(1)} = -c_V \hat{w}_{(1)} - c_T \frac{1}{\hat{p}_w} \frac{d\hat{T}_{(0)}}{dy} - \left(c_X - \frac{(1 - \hat{m}^B)\hat{D}_{AB} c_V}{\hat{m}_w} \right) \frac{1}{\hat{p}_w} \frac{dX_{(0)}^A}{dy}, \quad \text{at } y = 0, \quad (10)$$

where $\hat{m}_w = X_w^A + \hat{m}^B X_w^B$ and $X_w^B = 1 - X_w^A$. Here a_V , a_T , a_X , a_V^A , a_T^A , a_X^A , c_V , c_T , and c_X are constants depending on X_w^A as well as m^B/m^A , K^{BB} , and K^{AB} . Equations (8) – (10) are the boundary conditions at the interface for Eqs. (5a) – (5c).

CONDENSATION AND EVAPORATION SOLUTIONS

We start with observing the properties of the solution of Eqs. (4a) and (4b) with (7) and the condition at infinity:

$$\hat{p} \rightarrow \hat{p}_\infty \equiv \hat{p}_\infty^A + \hat{p}_\infty^B, \quad X^A \rightarrow X_\infty^A \equiv \hat{p}_\infty^A / \hat{p}_\infty, \quad \hat{T} \rightarrow \hat{T}_\infty, \quad \hat{w} \rightarrow \pm\varepsilon, \quad \text{as } y \rightarrow \infty. \quad (11)$$

Here, in the last condition, the plus sign is for the evaporation case and the minus sign is for the condensation case.

First it is readily seen from the first equation of (4a) that $\hat{p}_{(0)}$ is a constant. Thus from the first conditions in Eqs. (7) and (11) the difference of $\hat{p}_\infty$ from $\hat{p}_w$ is of $O(\varepsilon)$:

$$\hat{p}_\infty - \hat{p}_w = O(\varepsilon). \quad (12)$$

Second, from the second equation of (4a), $\hat{n}_{(0)}\hat{w}_{(1)}$ is also a constant. Thus the integration of Eq. (4b) gives

$$\hat{T}_{(0)} \frac{dX_{(0)}^A}{dy} = \frac{dX_{(0)}^A}{dy} \Big|_{y=0} \exp \left(\frac{\hat{n}_{(0)}\hat{w}_{(1)}}{\hat{D}_{AB}} \int_0^y \frac{dy}{\hat{T}_{(0)}} \right), \quad \hat{\lambda}_{(0)}\hat{T}_{(0)} \frac{d\hat{T}_{(0)}}{dy} = \hat{\lambda}_w \frac{d\hat{T}_{(0)}}{dy} \Big|_{y=0} \exp \left(\frac{5}{2} \hat{n}_{(0)}\hat{w}_{(1)} \int_0^y \frac{dy}{\hat{\lambda}_{(0)}\hat{T}_{(0)}} \right), \quad (13)$$

where $\hat{\lambda}_w$ is defined by the first equation of (6) with X^α replaced by X_w^α . Equation (13) shows that $\hat{T}_{(0)}$ and $X_{(0)}^A$ are monotonic. Since $\hat{D}_{AB}$, $\hat{\lambda}_{(0)}$, and $\hat{T}_{(0)}$ are positive, it also shows that when $\hat{n}_{(0)}\hat{w}_{(1)} > 0$, i.e., the case of evaporation, $\hat{X}_{(0)}^A$ and $\hat{T}_{(0)}$ are constants. Otherwise they diverge at infinity. Thus from the second and third conditions in Eqs. (7) and (11) the difference of X_∞^A from X_w^A and that of $\hat{T}_\infty$ from 1 are also of $O(\varepsilon)$:

$$\hat{T}_\infty - 1 = O(\varepsilon), \quad X_\infty^A - X_w^A = O(\varepsilon), \quad \text{for } \hat{n}_{(0)}\hat{w}_{(1)} > 0. \quad (14)$$

On the contrary, when $\hat{n}_{(0)}\hat{w}_{(1)} < 0$, i.e., the case of condensation, such restrictions do not occur. In fact, the (implicit) expressions of $\hat{T}_{(0)}$ and $X_{(0)}^A$ are obtained for any values of $\hat{T}_\infty$ and X_∞^A by using the monotonicity (see Sec. 5.7 in [1]).

To summarize, there are three restrictions for the evaporation, while there is only one for the condensation. These give those relations among the parameters at the interface and at a far distance that must hold in order that the steady evaporation or condensation takes place. The relations are derived below by the use of Eqs. (5a) – (5c) with (8) – (11).

The case of condensation In this case, there is only one restriction, which is on the pressure. To see it more closely, we consider the equation and boundary condition for $\hat{p}_{(1)}$ [the first equation of (5a) and (8)]. Obviously $\hat{p}_{(1)}$ is a constant, and thus from the conditions (8) and (11) we have the relation

$$(\hat{p}_\infty - \hat{p}_w)\varepsilon^{-1} = -a_V \hat{p}_w \hat{w}_{(1)} \Big|_{y=0} - a_T \frac{d\hat{T}_{(0)}}{dy} \Big|_{y=0} - \left(a_X - \frac{(1 - \hat{m}^B)\hat{D}_{AB}a_V}{\hat{m}_w} \right) \frac{dX_{(0)}^A}{dy} \Big|_{y=0}. \quad (15)$$

In the meantime, $d\hat{T}_{(0)}/dy$ and $dX_{(0)}^A/dy$ at $y = 0$ can be expressed in terms of the parameters at infinity if one integrates Eq. (4b) once and takes into account the condition at infinity. Substitution of the resulting into Eq. (15) gives

$$\frac{p_\infty}{p_w} = 1 - \left[-a_V + \frac{5}{2} \frac{a_T}{\hat{\lambda}_w} \left(\frac{T_\infty}{T_w} - 1 \right) + \left(\frac{a_X}{\hat{D}_{AB}} - \frac{(1 - m^B/m^A)a_V}{m_w/m^A} \right) (X_\infty^A - X_w^A) \right] \left(\frac{T_w}{T_\infty} \right) \frac{|v_\infty|}{c_*}. \quad (16)$$

Equation (16) is the condition for the steady condensation.

The case of evaporation In this case, there are three restrictions: the restrictions on the pressure, concentration, and temperature [Eqs. (12) and (14)]. The relation for the pressure is readily obtained from Eq. (15) as

$$(\hat{p}_\infty - \hat{p}_w)\varepsilon^{-1} = -a_V \hat{p}_w \hat{w}_{(1)} \Big|_{y=0}, \quad (17)$$

because $X_{(0)}^A$ and $\hat{T}_{(0)}$ are constants. The relations for the concentration and temperature are to be obtained from Eqs. (5b) and (5c) with constant $X_{(0)}^A$ and $\hat{T}_{(0)}$ and conditions (9) – (11). The discussion on Eqs. (5b) and (5c) parallel to the first half of the present section shows that $\hat{T}_{(1)}$ and $X_{(1)}^A$ are constants. Thus, from Eqs. (9) – (11), the following relations for the concentration and temperature are obtained:

$$(X_\infty^A - X_w^A)\varepsilon^{-1} = -X_w^A(a_V^A - a_V)\hat{w}_{(1)} \Big|_{y=0}, \quad (\hat{T}_\infty - 1)\varepsilon^{-1} = -c_V \hat{w}_{(1)} \Big|_{y=0}. \quad (18)$$

Equations (17) and (18) can be transformed into the following relations:

$$\frac{p_\infty}{p_w} = 1 - a_V \frac{v_\infty}{c_*}, \quad \frac{X_\infty^A}{X_w^A} = 1 - (a_V^A - a_V) \frac{v_\infty}{c_*}, \quad \frac{T_\infty}{T_w} = 1 - c_V \frac{v_\infty}{c_*}. \quad (19)$$

These are the conditions for the steady evaporation.

APPLICATION TO THE TWO-SURFACE PROBLEM

Consider a mixture of vapors A and B between two parallel plane condensed phases. One of them is kept at temperature T_I while the other is kept at temperature T_{II} . The interface of the former is located at $X_1 = 0$ and that of the latter is at $X_1 = D$. The *saturation pressure* of species α at temperature T_I (or T_{II}) is p_I^α (or p_{II}^α), and the corresponding *total saturation pressure* is denoted by $p_I \equiv p_I^A + p_I^B$ (or $p_{II} \equiv p_{II}^A + p_{II}^B$). We investigate the steady behavior of the vapors

for small Kn. Here Kn is the Knudsen number defined by $\text{Kn} = \ell/D$ with ℓ being the mean-free-path of a molecule in a pure vapor of species A that is in the equilibrium state at rest with pressure p_I^A and temperature T_I .

According to [5], when $|p_{II}/p_I - 1| \gg \text{Kn}$, the overall behavior of the vapors is described by the Euler set of equations and its boundary condition is determined by the analysis of the half-space problem. We consider this situation and further assume that the difference of the *total saturation pressures* is still small, i.e., $\text{Kn} \ll |p_{II}/p_I - 1| \ll 1$. Then the evaporation and condensation are so weak that the boundary condition is given by Eqs. (16) and (19). Here the quantities with subscript ∞ are regarded as the values that the solution of the Euler set takes at the interfaces. Since the solution of one-dimensional Euler set is uniform, each of them takes a common value at both interfaces. The subscript w is to be properly set as I or II depending on the situation (i.e., at which interface the evaporation takes place). Suppose that the evaporation takes place at the interface I. Then the condensation takes place at the interface II ($v_\infty > 0$). By eliminating p_∞ , T_∞ , and X_∞^A from Eqs. (16) and (19), one obtains the following relation:

$$\begin{aligned} \left(1 - a_V^I \frac{|v_\infty|}{c_*^I}\right) \frac{p_I}{p_{II}} = 1 - \left(-a_V^{II} + \frac{5}{2} \frac{a_T^{II}}{\lambda_{II}} \left[\left(1 - c_V^I \frac{|v_\infty|}{c_*^I}\right) \frac{T_I}{T_{II}} - 1\right] \right. \\ \left. + \left(\frac{a_X^{II}}{\tilde{D}_{AB}} - \frac{(1 - m^B/m^A)a_V^{II}}{m_{II}/m^A}\right) \left[\left(1 - (a_V^{AI} - a_V^I) \frac{|v_\infty|}{c_*^I}\right) - \frac{X_{II}^A}{X_I^A}\right] X_I^A\right) \left(\frac{T_I}{T_{II}}\right)^{1/2} \frac{|v_\infty|}{c_*^I}. \quad (20) \end{aligned}$$

Here we added sub- and superscripts I and II to the quantities to indicate whether they are evaluated at the interface I or II. The corresponding relation in the case of condensation at the interface I ($v_\infty < 0$) is obtained by the exchange between sub- and superscripts I and II. Because of the difference between the two cases, there are two branches of solutions for the Euler system in general. We will come back this issue later. Once $|v_\infty|/c_*^I$ is obtained from Eq. (20), p_∞ , T_∞ , and X_∞^A are obtained from Eq. (19). In this way, the solution of the Euler system is obtained.

Example We give an example of the solution of the Euler set (Euler solution, hereinafter) obtained by the process mentioned above. We consider the case $p_I/p_I^A = 3$, $p_{II}/p_I^A = 2.8$, $p_{II}^A/p_I^A = 1.8$, and $T_{II}/T_I = 1$. The parameters of the molecular model are set as $m^B/m^A = 0.5$ and $K^{BB} = K^{AB} = 1$. The *total saturation pressure* is higher at the interface I ($p_I > p_{II}$), and the evaporation takes place at the interface I. For the present parameter setting, $X_I^A = 1/3 = 0.3333$ and $X_{II}^A = 1.8/2.8 = 0.6429$, and the coefficients a_V^I , a_V^{II} , a_T^I , c_V^I , a_X^I , and a_V^{AI} are given as $a_V^I = 1.7139$, $a_V^{AI} X_I^A = 0.7045$, $c_V^I = 0.3451$, $a_V^{II} = 1.9069$, $a_T^I = -0.6484$, and $a_X^{II} = 0.8421$. These are obtained numerically by solving the half-space problems occurring in the analysis of the problem for f_{K1}^α . With these data, one can solve Eq. (20) and obtain $v/c_*^I = 0.02012$. From Eq. (19) the other quantities are obtained as $p/p_I^A = 2.8965$, $T/T_I = 0.9931$, and $X^A = 0.3306$ (or $p^A/p_I^A = 0.9575$). Here we omit the subscript ∞ . Figure 1(a) shows a comparison with the solution of the GSB equation (GSB solution, hereinafter). As Kn decreases, a uniform region appears in the GSB solution and extends from the evaporation side (I) to the condensation side (II). The Euler solution predicts the values that the GSB solution takes in the uniform region correctly. For small Kn, the GSB solution changes steeply near the condensation interface, where the deviation from the Euler solution remains large. This region is the suction boundary layer in the classical gasdynamics, which corresponds to the slowly varying solution discussed in this paper. If the slowly varying solution is added to the Euler solution, the prediction becomes quite well even in the region. Figure 1(b) demonstrates it.

The limit of $p_{II} \rightarrow p_I$ The fact that there are two branches of solutions of the Euler system leads to a possibility

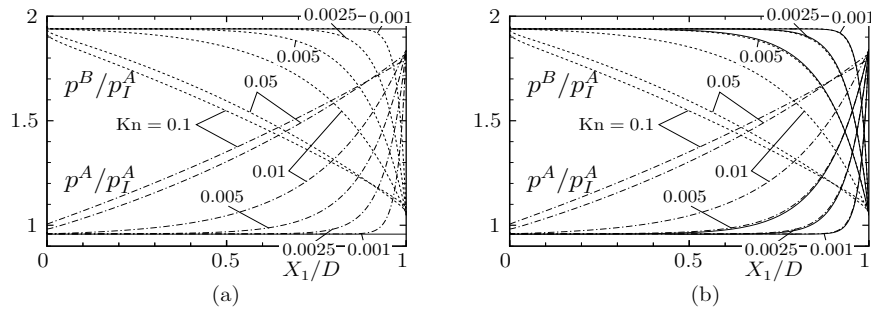


FIGURE 1. Profiles of the partial pressures p^A and p^B in the case of $p_I/p_I^A = 3$, $p_{II}/p_I^A = 2.8$, $p_{II}^A/p_I^A = 1.8$, and $T_{II}/T_I = 1$ ($m^B/m^A = 0.5$ and $K^{BB} = K^{AB} = 1$). The dashed line and dot-dashed line indicate the GSB solution for small Kn. The solid line indicates the Euler solution in (a) and that with a correction of the slowly varying solution up to $O(\epsilon)$ in (b).

of two distinct solutions for the same physical situation. We will discuss it here. Let us consider the behavior of the mixture in the limit $p_{II} \rightarrow p_I$. Because of the scaling $\text{Kn} \ll |p_{II}/p_I - 1|$, the continuum limit $\text{Kn} = 0_+$ is considered. Note that evaporation is expected to take place at the interface I if $p_I > p_{II}$ and at the interface II if $p_I < p_{II}$. Thus, the solution for $p_{II} = p_I - 0$ is to be obtained from Eq. (20) while that for $p_{II} = p_I + 0$ is to be obtained from Eq. (20) with the exchange between sub- and superscripts I and II. This difference gives two distinct solutions for the same physical situation when $p_{II} = p_I$. Figure 2 shows an example. The parameters of the molecular model are the same as before.

The present analysis reveals that there are two distinct solutions in the continuum limit when $p_I = p_{II}$. However, there is another class of solutions for the same physical situation. The Euler system, on which the above discussion is based, was derived under the assumption that $\text{Kn} \ll |p_{II}/p_I - 1|$. If one considers another scaling $\text{Kn} = O(|p_{II}/p_I - 1|)$ and takes the continuum limit, the behavior of the vapors is described not by the Euler system but by the convection-diffusion-type system. The details are discussed in [5]. According to the reference, there are innumerable solutions of the convection-diffusion-type system in the continuum limit. The family of the solutions gives a smooth transition from one to the other branch of solutions of the Euler system by changing a “hidden” parameter in the convection-diffusion-type system. This parameter is related to the infinitesimal difference of the *total saturation pressure* between the interfaces. This subject forms a new category of the ghost effect.

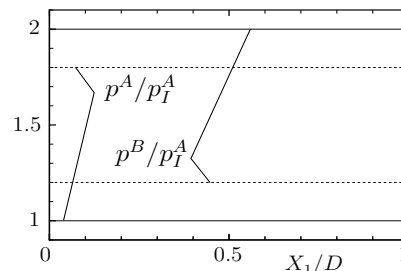


FIGURE 2. Partial pressures p^A and p^B of the Euler solution in the continuum limit in the case of $p_I/p_I^A = p_{II}/p_I^A = 3$, $p_{II}^A/p_I^A = 1.8$, and $T_{II}/T_I = 1$. The solid line indicates the solution for $p_{II} = p_I - 0$ and the dashed line that for $p_{II} = p_I + 0$.

CONCLUDING REMARKS

We investigated a half-space problem of a binary mixture of vapors bounded by their plane condensed phase on the basis of the GSB equation. Under the assumption of weak evaporation and condensation, we considered the slowly varying solution and derived a fluid-dynamic-type system, from the latter of which the relations among the parameters that allow the steady evaporation and condensation were deduced. The relations were used as the boundary conditions for the Euler system in the study of the two-surface problem of a binary mixture of vapors for small Knudsen numbers.

In the present work, the GSB model is adopted. Exactly the same result is obtained for other BGK-type models [8] with a proper choice of parameters, because these models give the same transport coefficients and the same linearized equation as the GSB model. The extension to the Boltzmann equation for Maxwell’s molecules is straightforward. However, it is not so for the other molecular models. The difference comes from the fact that the thermal diffusion effect vanishes in the fluid-dynamic-type system exceptionally for Maxwell’s molecules (and the BGK-type models). When the effect is present, the temperature and concentration profiles are not always monotonic, so that the discussion in the fourth section does not work. The extension to such a case will be treated in a forthcoming paper.

ACKNOWLEDGMENTS

The author thanks Professor François Golse for his suggestion that initiated this work. He also thanks Professor Kazuo Aoki for his encouragement throughout this work. This work is supported by the Grants-in-Aid for Scientific Research (Nos. 14350047 and 15760108) from the JSPS and by the COE program of the Japanese Ministry of Education, Culture, Sports, Science and Technology for Research and Education on Complex Functional Mechanical Systems.

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