

# Low-Frequency EM Field Penetration Through Magnetic and Conducting Cylindrical Shields

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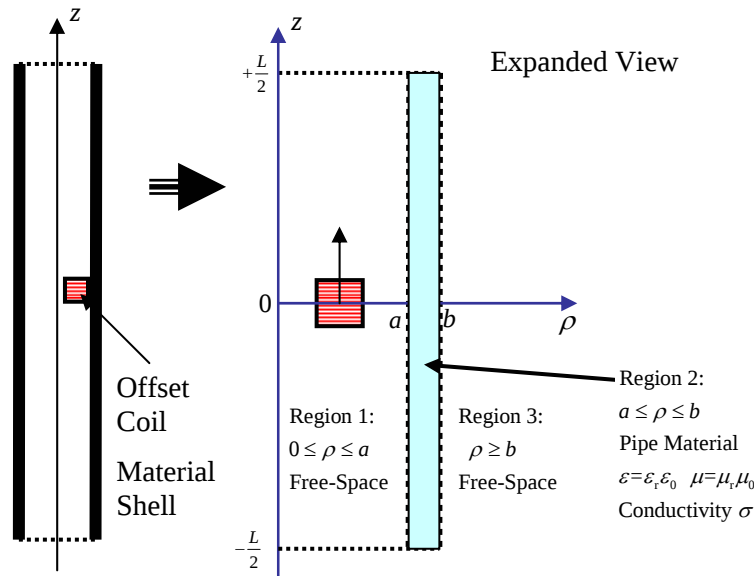
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## INTRODUCTION

Computation of low-frequency field penetration through magnetic and/or conducting materials is important for quantifying electromagnetic compatibility issues in devices and facilities, as well as prediction of field signatures external to ships and submarines due to internal electrical machinery. A computational procedure is described for accurately predicting the penetration of low-frequency fields through a cylindrical structure, possibly arranged in layers. The internal source being shielded is a large multi-turn coil having arbitrary location within the cylindrical shell. Field computation is formulated using multi-region cylindrical harmonic expansions with enforcement of continuity on tangential field components at each material interface. Example field intensities and shielding are computed for a steel pipe at frequencies of 1 Hz and 1 kHz. The relative effectiveness of induced magnetic shielding and eddy-current shielding is considered.

## FORMULATION

Computation of fields both within and external to a cylindrical shield is formulated using multi-region cylindrical harmonic expansions. Consider the single layer shell depicted in Figure 1. The field source is an offset, 500-turn coil whose quasi-static fields can be computed using superposition of the closed form elliptic integral expressions for single-turn fields derived in Smythe's classic text on *Static and Dynamic Electricity*.



**Figure 1** Cylindrical Shield With Expanded View.

| Report Documentation Page                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                    |                                     | Form Approved<br>OMB No. 0704-0188          |                                    |                                    |
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| 1. REPORT DATE<br><b>01 JAN 2005</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                    | 2. REPORT TYPE<br><b>N/A</b>        |                                             | 3. DATES COVERED<br><b>-</b>       |                                    |
| 4. TITLE AND SUBTITLE<br><b>Low-Frequency EM Field Penetration Through Magnetic and Conducting Cylindrical Shields</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                    |                                     | 5a. CONTRACT NUMBER                         |                                    |                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 5b. GRANT NUMBER                            |                                    |                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 5c. PROGRAM ELEMENT NUMBER                  |                                    |                                    |
| 6. AUTHOR(S)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                    |                                     | 5d. PROJECT NUMBER                          |                                    |                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 5e. TASK NUMBER                             |                                    |                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 5f. WORK UNIT NUMBER                        |                                    |                                    |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br><b>Naval Postgraduate School, Monterey, CA 93943</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                    |                                     | 8. PERFORMING ORGANIZATION<br>REPORT NUMBER |                                    |                                    |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                    |                                     | 10. SPONSOR/MONITOR'S ACRONYM(S)            |                                    |                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 11. SPONSOR/MONITOR'S REPORT<br>NUMBER(S)   |                                    |                                    |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT<br><b>Approved for public release, distribution unlimited</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                    |                                     |                                             |                                    |                                    |
| 13. SUPPLEMENTARY NOTES<br><b>See also ADM001846, Applied Computational Electromagnetics Society 2005 Journal, Newsletter, and Conference., The original document contains color images.</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                    |                                     |                                             |                                    |                                    |
| 14. ABSTRACT                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                    |                                     |                                             |                                    |                                    |
| 15. SUBJECT TERMS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                    |                                     |                                             |                                    |                                    |
| 16. SECURITY CLASSIFICATION OF:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                    |                                     | 17. LIMITATION OF<br>ABSTRACT<br><b>UU</b>  | 18. NUMBER<br>OF PAGES<br><b>4</b> | 19a. NAME OF<br>RESPONSIBLE PERSON |
| a. REPORT<br><b>unclassified</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | b. ABSTRACT<br><b>unclassified</b> | c. THIS PAGE<br><b>unclassified</b> |                                             |                                    |                                    |

The coil fields interact with the conducting (and perhaps magnetic) material of the shell to provide reflected fields in Region 1, standing and traveling waves in Region 2 (along with induced eddy currents) and outbound penetrating fields in Region 3.

Region 1 TE Expansions ( $H_{Cz}, H_{C\rho}, H_{C\phi}, E_{C\rho}, E_{C\phi}$  are offset coil fields)

$$\begin{aligned}
H_z(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} a_{mn} J_m(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} + H_{Cz}(\rho, \phi, z) \\
H_\rho(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} a_{mn} \frac{-jk_{zn}}{\gamma_{0n}} J_m'(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} + H_{C\rho}(\rho, \phi, z) \\
H_\phi(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} a_{mn} \frac{mk_{zn}}{\gamma_{0n}^2 \rho} J_m(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} + H_{C\phi}(\rho, \phi, z) \\
E_\rho(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} a_{mn} \frac{m\omega\mu_0}{\gamma_{0n}^2 \rho} J_m(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} + E_{C\rho}(\rho, \phi, z) \\
E_\phi(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} a_{mn} \frac{j\omega\mu_0}{\gamma_{0n}} J_m'(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} + E_{C\phi}(\rho, \phi, z)
\end{aligned}$$

Region 2 TE Expansions

$$\begin{aligned}
H_z(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} [b_{mn} J_m(\gamma_{2n}\rho) + c_{mn} H_m^{(2)}(\gamma_{2n}\rho)] e^{-jk_{zn}z} e^{jm\phi} \\
H_\rho(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} \frac{-jk_{zn}}{\gamma_{2n}} [b_{mn} J_m'(\gamma_{2n}\rho) + c_{mn} H_m^{(2)'}(\gamma_{2n}\rho)] e^{-jk_{zn}z} e^{jm\phi} \\
H_\phi(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} \frac{mk_{zn}}{\gamma_{2n}^2 \rho} [b_{mn} J_m(\gamma_{2n}\rho) + c_{mn} H_m^{(2)}(\gamma_{2n}\rho)] e^{-jk_{zn}z} e^{jm\phi} \\
E_\rho(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} \frac{m\omega\mu_r\mu_0}{\gamma_{2n}^2 \rho} [b_{mn} J_m(\gamma_{2n}\rho) + c_{mn} H_m^{(2)}(\gamma_{2n}\rho)] e^{-jk_{zn}z} e^{jm\phi} \\
E_\phi(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} \frac{j\omega\mu_r\mu_0}{\gamma_{2n}} [b_{mn} J_m'(\gamma_{2n}\rho) + c_{mn} H_m^{(2)'}(\gamma_{2n}\rho)] e^{-jk_{zn}z} e^{jm\phi}
\end{aligned}$$

Region 3 TE Expansions

$$\begin{aligned}
H_z(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} d_{mn} H_m^{(2)}(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} \\
H_\rho(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} d_{mn} \frac{-jk_{zn}}{\gamma_{0n}} H_m^{(2)'}(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} \\
H_\phi(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} d_{mn} \frac{mk_{zn}}{\gamma_{0n}^2 \rho} H_m^{(2)}(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} \\
E_\rho(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} d_{mn} \frac{m\omega\mu_0}{\gamma_{0n}^2 \rho} H_m^{(2)}(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi} \\
E_\phi(\rho, \phi, z) &= \sum_{m=-M}^M \sum_{n=0}^{N-1} d_{mn} \frac{j\omega\mu_0}{\gamma_{0n}} H_m^{(2)'}(\gamma_{0n}\rho) e^{-jk_{zn}z} e^{jm\phi}
\end{aligned}$$

The radial eigenvalues in these equations are given by:

Free-Space Regions 1 and 3:  $k_0^2 = \omega^2 \mu_0 \epsilon_0$

$$\gamma_{0n} = \sqrt{k_0^2 - k_{zn}^2} = \begin{cases} \sqrt{k_0^2 - k_{zn}^2} & \text{if } k_{zn} < k_0 \\ -j\sqrt{k_{zn}^2 - k_0^2} & \text{if } k_{zn} > k_0 \end{cases}$$

Material Region 2:  $k_2^2 = \omega^2 \mu_2 \epsilon_2 - j\omega \mu_2 \sigma_2 = \omega^2 \mu_2 \epsilon_c$  where  $\epsilon_c = \epsilon_2 - j \frac{\sigma_2}{\omega}$

$$\gamma_{2n} = \sqrt{k_2^2 - k_{zn}^2} = \sqrt{\omega^2 \mu_2 \epsilon_c - k_{zn}^2 - j\omega \mu_2 \sigma_2} = \alpha - j\beta \quad (4\text{th quadrant})$$

Solve for four sets of coefficients by matching tangential  $H_z$  and  $E_\phi$  for each  $(m,n)$  mode

at  $\rho = a$  and  $\rho = b$ .

$$\begin{aligned} -a_{mn} J_m(\gamma_{0n} a) + b_{mn} J_m(\gamma_{2n} a) + c_{mn} H_m^{(2)}(\gamma_{2n} a) &= F \{H_{Cz}(a, \phi, z)\}_{m,n} \\ -a_{mn} \frac{j\omega \mu_0}{\gamma_{0n}} J_m'(\gamma_{0n} a) + b_{mn} \frac{j\omega \mu_r \mu_0}{\gamma_{2n}} J_m'(\gamma_{2n} a) + c_{mn} \frac{j\omega \mu_r \mu_0}{\gamma_{2n}} H_m^{(2)'}(\gamma_{2n} a) &= F \{E_{C\phi}(a, \phi, z)\}_{m,n} \\ b_{mn} J_m(\gamma_{2n} b) + c_{mn} H_m^{(2)}(\gamma_{2n} b) - d_{mn} H_m^{(2)}(\gamma_{0n} b) &= 0 \\ b_{mn} \frac{j\omega \mu_r \mu_0}{\gamma_{2n}} J_m'(\gamma_{2n} b) + c_{mn} \frac{j\omega \mu_r \mu_0}{\gamma_{2n}} H_m^{(2)'}(\gamma_{2n} b) - d_{mn} \frac{j\omega \mu_0}{\gamma_{0n}} H_m^{(2)'}(\gamma_{0n} b) &= 0 \end{aligned}$$

where  $F \{g(a, \phi, z)\}_{m,n} = G(m, n) = \text{FFT}_\phi \{ \text{IFFT}_z(g(a, \phi, z)) \}$

Construction of field components using coefficients is performed using the inverse operations

$$g(a, \phi, z) = F^{-1} \{G(m, n)\} = \text{IFFT}_\phi \{ \text{FFT}_z(G(m, n)) \}$$

To avoid Hankel function overflows at low frequencies due to small arguments ( $\approx < 10^{-3}$ )

we use normalized coefficients for  $n = 1$  ( $k_{z1} = 0$ ):  $\tilde{c}_{m0} = c_{m0} H_m^{(2)'}(\gamma_{20} a)$  and  $\tilde{d}_{m0} = d_{m0} H_m^{(2)'}(\gamma_{30} b)$ .

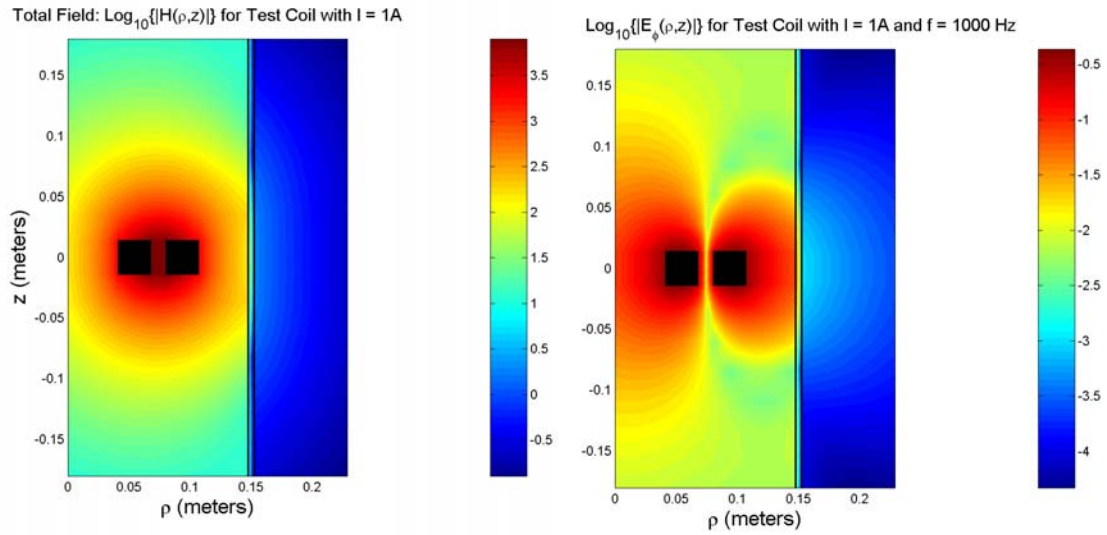
## EXAMPLE COMPUTATIONS

Example field intensities are shown in Figure 2 for the case of a 3/16" thick steel pipe at a frequency of 1000 Hz. The strong shielding shown is a result of both the induced magnetization in the pipe and the induced eddy currents due to the conductivity of the steel. The magnetization component is relatively frequency independent, while eddy currents, and their resultant shielding, are approximately proportional to frequency. The reduction of shielding due to diminished eddy currents is illustrated in Figure 3 for the case of 1 Hz.

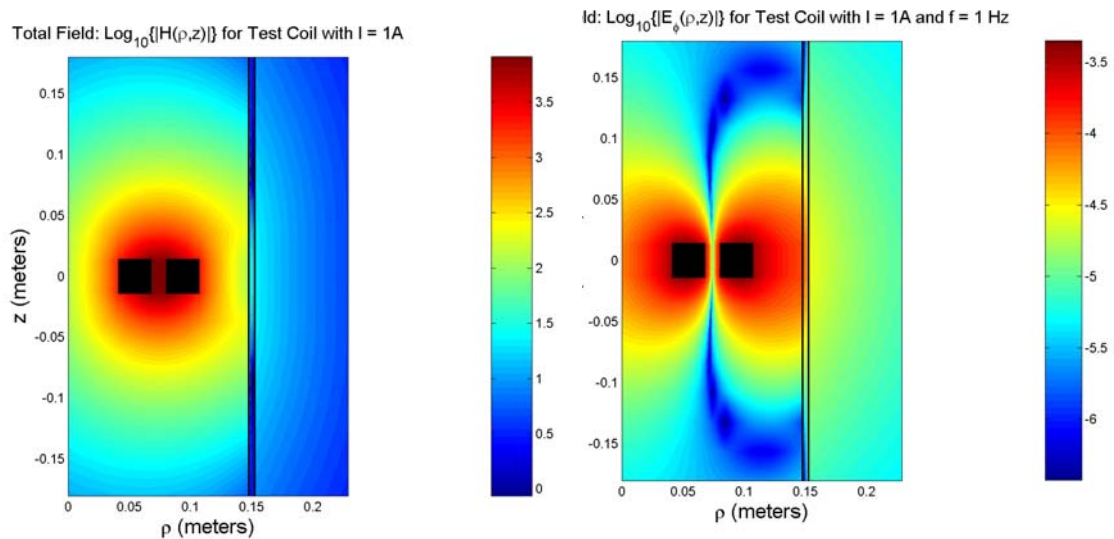
Shielding effectiveness is the dB ratio of field strength in the shielded region *without* the shield present divided by that with the shield in place,

$$S_{dB} = 20 \log_{10} \left\{ \frac{|\text{Field Due to Coil Without Shield}|}{|\text{Field With Shield Present}|} \right\}$$

At  $f=1000\text{Hz}$ , the 3/16" steel pipe provides  $S_{dB} \approx 37\text{dB}$  for both H- and E-fields. At  $f=1\text{Hz}$  the shielding effectiveness drops to about 12 dB for the H-field and 7 dB for the E-field at a radial distance of 0.2m from the pipe exterior wall nearest to the coil.



**Figure 2** Magnetic and Electric Field Intensities for Steel Cylinder at  $f=1000\text{Hz}$ .



**Figure 3** Magnetic and Electric Field Intensities for Steel Cylinder at  $f=1\text{Hz}$ .

## CONCLUSIONS

Computation of fields due to a 500 turn coil positioned arbitrarily within a pipe constructed of conducting and/or magnetic material is performed using multi-region cylindrical harmonic modal expansions. Results involving steel and aluminum indicate the relative significance of shielding due to induced magnetization and eddy-current generation. Extension to an arbitrarily oriented coil is being investigated. The ultimate goal is to design multilayered structures composed of conducting and magnetic materials for a specified frequency range. Such shields will be designed under constraints of layer number, thickness and weight using an evolutionary optimization algorithm.