

# Spatial Division Multiplexing of Space Time Block Codes for Single Carrier Block Transmission

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**Abstract:** This paper presents a new Spatial Division Multiplexing (SDM) based scheme combined with Space Time Block Coding (STBC) for Cyclic Prefix based Single Carrier Block Transmission (CP-SCBT). CP-SCBT technique converts the linear convolution of the frequency selective fading channel to the circular convolution. The optimum detection scheme based on the Minimum Mean Square Errors (MMSE) criterion can be easily solved with FFT/IFFT after simple pre-processing in the receiver. The Bit Error Ratio (BER) performances over different channels are assessed by simulations.

## 1. Introduction

Since its introduction of by Alamouti in [1], space-time block code has attracted considerable attention because of many attractive features. The Alamouti STBC scheme in [1] assumes a flat-fading channel. The STBC is used in symbol level. Recently, several schemes have been proposed to extend the Alamouti STBC to frequency selective fading channels. The STBC is encoded in block level such as FDE-STBC and TR-STBC [3]. S. Rouquette-Leveil proposed a Spatial Division Multiplexing based scheme combined with STBC [6]. But this scheme only can be used in flat-fading channels.

Block transmissions (e.g., [2][4][5]) such as cyclic prefix based single carrier block transmission (CP-SCBT) and orthogonal frequency division multiplexing (OFDM) are very effective techniques to combat frequency selective channels. Due to the use of FFT/IFFT operations the receiver complexity is kept significantly below the complexity of conventional single carrier system with time domain equalizers. We propose a new SDM of STBC scheme for CP-SCBT which can be used in frequency selective fading channels.

The notation adopted in this paper conforms to the following convention. Vectors are column vectors and are denoted in lower case bold:  $\mathbf{x}$ . Matrices are upper cases bold:  $\mathbf{A}$ .  $\mathbf{I}_N$  denotes the identity matrix of size  $N \times N$ .  $\mathbf{e}_i$  denotes the  $i$ -th unit vector.  $\mathbf{A}_{k,m}$  denoted the  $(k, m)$ -th entry of a matrix  $\mathbf{A}$ .  $(\cdot)^*$ ,  $(\cdot)^T$ ,  $(\cdot)^H$  denotes conjugate, transpose, and Hermitian transpose, respectively.  $\text{diag}\{\mathbf{a}\}$  stands for a diagonal matrix with the vector  $\mathbf{a}$  on its diagonal.  $\otimes$  denotes Kronecker product.

## 2. Space-Time Joint Transmit for CP-SCBT

Fig.1 shows the space-time joint transmitter of the proposed spatial division multiplexing of space time block codes for CP-SCBT. We assume that the number of transmit antennas is  $N=4$ , and the number of receive antennas is  $M$ . The transmit antennas are split into two groups. Two streams can be transmitted simultaneously, and each stream is transmitted over two antennas and two data blocks according to the block-level Alamouti coding.

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The ST encoder takes four consecutive blocks  $\mathbf{x}$  of length  $N_D$  as input, and it generates the following output  $2N_D \times 4$  ST block-coded matrix:

$$\begin{bmatrix} \mathbf{s}_1^{(0)} & \mathbf{s}_2^{(0)} & \mathbf{s}_3^{(0)} & \mathbf{s}_4^{(0)} \\ \mathbf{s}_1^{(1)} & \mathbf{s}_2^{(1)} & \mathbf{s}_3^{(1)} & \mathbf{s}_4^{(1)} \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{x}^{(0)} & \mathbf{x}^{(1)} & \mathbf{x}^{(2)} & \mathbf{x}^{(3)} \\ -\mathbf{P}\mathbf{x}^{*(1)} & \mathbf{P}\mathbf{x}^{*(0)} & -\mathbf{P}\mathbf{x}^{*(3)} & \mathbf{P}\mathbf{x}^{*(2)} \end{bmatrix} \begin{matrix} \rightarrow \text{space} \\ \downarrow \text{time} \end{matrix}, \quad (1)$$

where  $\mathbf{P}$  is a permutation matrix that is drawn from a set of permutation matrices  $\{\mathbf{P}_{N_D}^{(k)}\}_{k=0}^{N_D-1}$ , with  $N_D$  denoting the dimensionality  $N_D \times N_D$ . Each  $\mathbf{P}_{N_D}^{(k)}$  performs a reverse cyclic shift (that depends on  $k$ ) when applied to a  $N_D \times 1$  vector  $\mathbf{a} = [\mathbf{a}(0) \ \mathbf{a}(1) \ \cdots \ \mathbf{a}(N_D-1)]^T$ . Specifically, the  $p$ -th entry of  $\mathbf{P}_{N_D}^{(k)}\mathbf{a}$  is  $[\mathbf{P}_{N_D}^{(k)}\mathbf{a}]_p = \mathbf{a}((N_D - p + k - 1) \bmod N_D)$ .

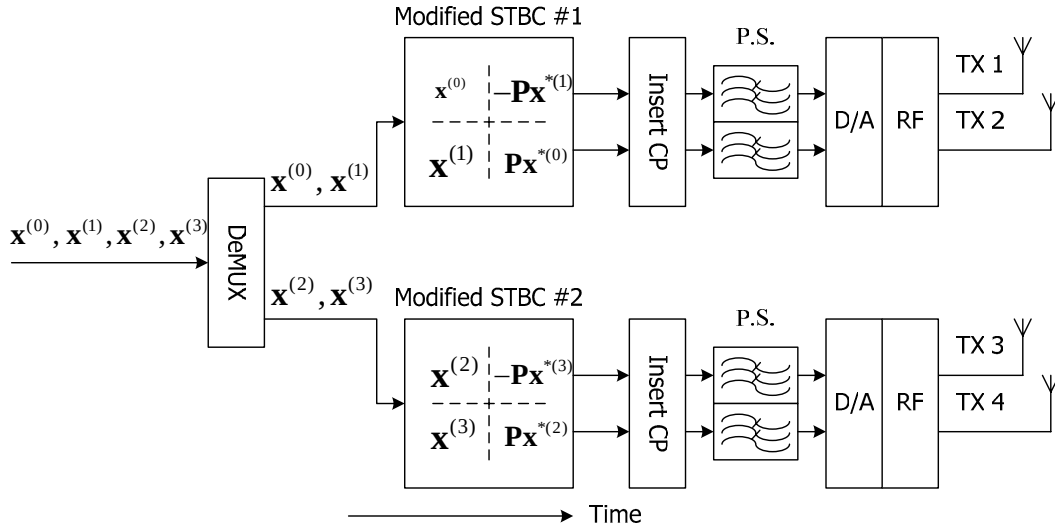


Fig.1. SDM of STBC for CP-SCBT

### 3. MMSE Space-Time Joint Detection for 4 TX Antennas

We assume that the receiver achieved the perfect synchronization and perfect channel estimation. After removing the CP, the received signal vector in  $m$ -th antenna in  $j$ -th data block can be expressed as:

$$\mathbf{r}_m^{(j)} = \sum_{n=1}^4 \mathbf{H}_{m,n}^{(j)} \mathbf{s}_n^{(j)} + \mathbf{z}_m^{(j)}, \quad m = 1, 2, \dots, M; \quad n = 1, 2, \dots, 4; \quad j = 0, 1 \quad (2)$$

The different received signals can be concatenated vertically,

$$\mathbf{r} = [\mathbf{r}_1^{T(0)} \ \cdots \ \mathbf{r}_M^{T(0)} \ \mathbf{r}_1^{T(1)} \ \cdots \ \mathbf{r}_M^{T(1)}]^T. \quad (3)$$

If the matrix  $\mathbf{C}$  is a circulant matrix, pre- and post-multiplying  $\mathbf{C}$  by  $\mathbf{P}$  yields [7]:

$$\mathbf{P}\mathbf{C}^*\mathbf{P} = \mathbf{C}^H, \quad (4)$$

$$\mathbf{C}^H\mathbf{C} = \mathbf{C}\mathbf{C}^H. \quad (5)$$

We assume that the channels are fixed over two consecutive blocks, i.e.,  $\mathbf{H}_{m,n}^{(0)} = \mathbf{H}_{m,n}^{(1)} \triangleq \mathbf{H}_{m,n}$ . Using (4) and (5), we get

$$\underbrace{\begin{bmatrix} \mathbf{r}_1^{(0)} \\ \vdots \\ \mathbf{r}_M^{(0)} \\ \mathbf{Pr}_1^{*(1)} \\ \vdots \\ \mathbf{Pr}_M^{*(1)} \end{bmatrix}}_{\tilde{\mathbf{r}}} = \underbrace{\begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \mathbf{H}_{1,3} & \mathbf{H}_{1,4} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_{M,1} & \mathbf{H}_{M,2} & \mathbf{H}_{M,3} & \mathbf{H}_{M,4} \\ \mathbf{H}_{1,2}^H & -\mathbf{H}_{1,1}^H & \mathbf{H}_{1,4}^H & -\mathbf{H}_{1,3}^H \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_{M,2}^H & -\mathbf{H}_{M,1}^H & \mathbf{H}_{M,4}^H & -\mathbf{H}_{M,3}^H \end{bmatrix}}_{\tilde{\mathbf{H}}} \underbrace{\begin{bmatrix} \mathbf{x}^{(0)} \\ \mathbf{x}^{(1)} \\ \mathbf{x}^{(2)} \\ \mathbf{x}^{(3)} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \mathbf{z}_1^{(0)} \\ \vdots \\ \mathbf{z}_M^{(0)} \\ \mathbf{Pz}_M^{*(1)} \\ \vdots \\ \mathbf{Pz}_M^{*(1)} \end{bmatrix}}_{\mathbf{z}}. \quad (6)$$

Using vector and matrix notation we obtain

$$\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\mathbf{x} + \mathbf{z}. \quad (7)$$

Assuming that the transmit signal vector  $\mathbf{x}$  is zero mean and statistically independent and independent with noise vector  $\mathbf{z}$ , and the vector  $\mathbf{z}$  is the zero mean complex additive white Gaussian noise with the variance of  $\mathbf{z}$  is  $\sigma_z^2$ , we get the optimum detection of transmitted data blocks  $\mathbf{x}$  according to the MMSE criterion:

$$\hat{\mathbf{x}} = \left( \sigma_z^2 \mathbf{I}_{4N_D} + \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \right)^{-1} \tilde{\mathbf{H}}^H \tilde{\mathbf{r}} = \mathbf{W}^{-1} \tilde{\mathbf{H}}^H \tilde{\mathbf{r}} \quad (8)$$

where  $\mathbf{W} \triangleq \sigma_z^2 \mathbf{I}_{4N_D} + \tilde{\mathbf{H}}^H \tilde{\mathbf{H}}$ .

Since  $\mathbf{H}_{m,n}$ , ( $m=1,2,\dots,M, n=1,2,\dots,4$ ) are circulant matrices, using the relation of unitary matrices, we can get the result of  $\mathbf{W}^{-1}$ :

$$\mathbf{W}^{-1} = \left( \left( \underbrace{\mathbf{W}_{1,1} \mathbf{W}_{3,3} - \mathbf{W}_{1,3}^H \mathbf{W}_{1,3} - \mathbf{W}_{2,3}^H \mathbf{W}_{2,3}}_{\mathbf{U}_1} \right)^{-1} \otimes \mathbf{I}_4 \right) \underbrace{\begin{bmatrix} \mathbf{W}_{3,3} & 0 & -\mathbf{W}_{1,3} & \mathbf{W}_{2,3}^H \\ 0 & \mathbf{W}_{3,3} & -\mathbf{W}_{2,3} & -\mathbf{W}_{1,3}^H \\ -\mathbf{W}_{1,3}^H & -\mathbf{W}_{2,3}^H & \mathbf{W}_{1,1} & 0 \\ \mathbf{W}_{2,3} & -\mathbf{W}_{1,3} & 0 & \mathbf{W}_{1,1} \end{bmatrix}}_{\mathbf{U}_2}, \quad (9)$$

or more compactly

$$\mathbf{W}^{-1} = (\mathbf{U}_1^{-1} \otimes \mathbf{I}_4) \cdot \mathbf{U}_2, \quad (10)$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{1,1} & \mathbf{0}_{N_D} & \mathbf{W}_{1,3} & -\mathbf{W}_{2,3}^H \\ \mathbf{0}_{N_D} & \mathbf{W}_{1,1} & \mathbf{W}_{2,3} & \mathbf{W}_{1,3}^H \\ \mathbf{W}_{1,3}^H & \mathbf{W}_{2,3}^H & \mathbf{W}_{3,3} & \mathbf{0}_{N_D} \\ -\mathbf{W}_{2,3} & \mathbf{W}_{1,3} & \mathbf{0}_{N_D} & \mathbf{W}_{3,3} \end{bmatrix}.$$

Since  $\mathbf{U}_1$  is still a circulant matrix, the inverse of  $\mathbf{U}_1$  has a fast solution using FFT/IFFT [7]:

$$\mathbf{U}_1^{-1} = \mathbf{Q}^* \left( \underbrace{\text{diag} \{ \sqrt{N_D} \mathbf{Q}(\mathbf{U}_1 \mathbf{e}_1) \}}_{\boldsymbol{\Lambda}_1} \right)^{-1} \mathbf{Q} = \mathbf{Q}^* \boldsymbol{\Lambda}_1^{-1} \mathbf{Q} \quad (11)$$

where  $\mathbf{Q}$  is the normalized DFT matrix,  $\mathbf{Q}^*$  is the normalized IDFT matrix. We found that the diagonal elements of  $\boldsymbol{\Lambda}_1$  are all real values, so  $\boldsymbol{\Lambda}_1^{-1}$  only need the real division. We rewrite (8) into

$$\hat{\mathbf{x}} = (\mathbf{Q}^* \boldsymbol{\Lambda}_1^{-1} \mathbf{Q} \otimes \mathbf{I}_4) \mathbf{U}_2 \tilde{\mathbf{H}}^H \tilde{\mathbf{r}}. \quad (12)$$

Our proposed detector is shown in Fig.2.

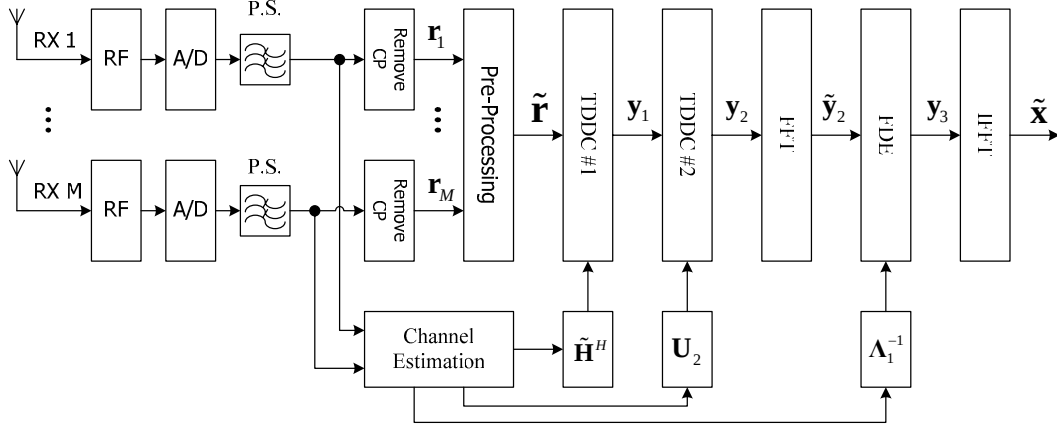


Fig.2. MMSE ST Joint Detector for MIMO (TX antennas:  $N=4$ , RX antennas:  $M$ )

| Item                 | Parameter | Item               | Parameter                                                                      |
|----------------------|-----------|--------------------|--------------------------------------------------------------------------------|
| Carrier frequency    | 2.45GHz   | Channel estimation | A root-of-unity sequence of length 32, LS method                               |
| Bandwidth            | 1.28MHz   | Modulation         | 16QAM                                                                          |
| Channel environment  | COST 207  | Turbo code         | (11, 13), information bits: 7677, code rate: 1/2, iteration number: 6, Log-MAP |
| TX and RX antennas   | $N=M=4$   | Inner interleaver  | interleaver size: 7677, S-random interleaver with $S=61$                       |
| Cyclic prefix length | 16        | Outer interleaver  | matrix interleaver, interleaver size: 30720                                    |

Table 1. Simulation parameters

#### 4. Simulation Results

In this section, we demonstrate the performance of the proposed scheme in mobile multipath fading channels. Table 1 shows the simulation parameters. The BER versus SNR @ per RX results of the proposed MIMO system are presented in Fig. 3.

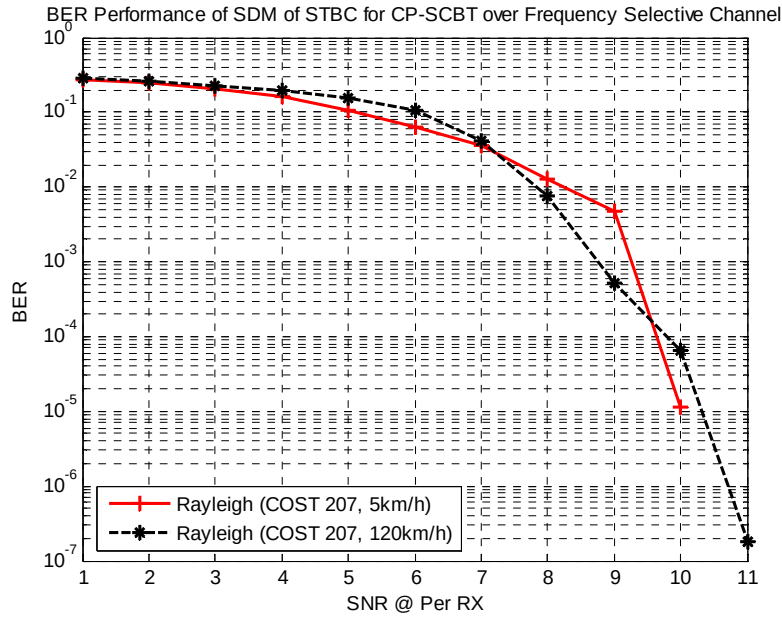


Fig. 3. BER performance of the proposed MIMO system

## 5. Conclusion

Combining SDM of STBC and CP-SCBT, we propose a new SDM of STBC scheme for CP-SCBT which can be used in frequency selective fading channels. We also present a low complexity MMSE ST joint detection. Considering CP and pilots inserted in time slots, the spectrum efficiency is about 3 bps/Hz in our simulation example.

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