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Time-Adaptive Sampling of a Chemical Hazard Area

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Overview

- Background
- Problem statement and assumptions
- Methodology
- Illustrations
- Conclusions
- Future Work



Background

- Chemical and biological WMDs are a current threat to the United States
 - 2001 U.S.: anthrax attacks
 - 1998 Iraq: “cocktail” of weapons killed 5,000+
 - 1995 Tokyo: sarin nerve gas, killed 12, injured thousands
- Terrorist groups are willing to use asymmetric measures
 - Easy manufacturing, storing, and transportation appeal to terrorists



Problem Dynamics

- A chemical agent weapon is released over a fixed operational site
 - Entire site enters the highest level of MOPP
 - Contamination from secondary vapors is the main concern
- Reduce mission oriented protective posture (MOPP)
 - MOPP is cumbersome
 - High levels of MOPP can reduce work efficiency



Problem Dynamics

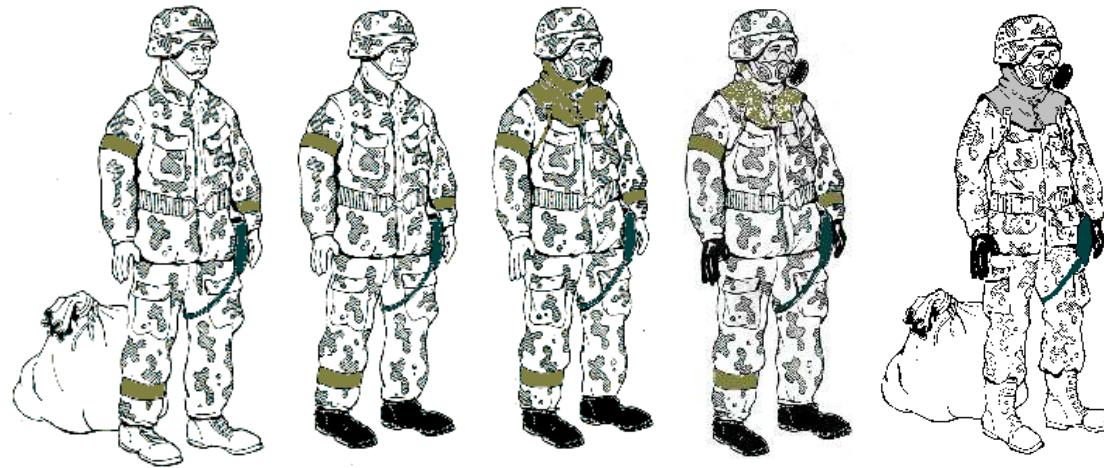


Figure 1. Mission-oriented protective postures.

- MOPP 0, MOPP 1, MOPP 2, MOPP 3, MOPP 4, MOPP Alpha
- Progressively add gear for increased safety



Problem Statement

- Develop an optimal sampling strategy
 - Route a search crew
 - * Reach as many locations as possible (to identify maximum number of areas below the vapor concentration threshold)
 - * Time constraint
- Provide a framework for future work
 - Using sensor data
 - Predicting future hazard areas



Model Assumptions

- Rectangular region with a finite number of “critical” areas
- Single crew that samples vapor concentrations
- Static, deterministic, and symmetric travel times
- Travel at constant velocity with zero delays
- Fixed amount of time allotted for the search



Model Assumptions

- Chemical agent/characteristics are known
- Only one instrument reading is required, consuming a fixed amount of time
- Known fixed threshold indicating contamination/no contamination
- Secondary vapor concentrations evolve spatially



Optimization Model

- Model the site and its critical areas as a network
- Develop a technique for optimally searching the site
- Desired outcome: Identify areas where secondary vapor levels have decreased (below the fixed vapor concentration level v^*) so MOPP can be safely reduced at those locations



Network Model

Consider the following notional site

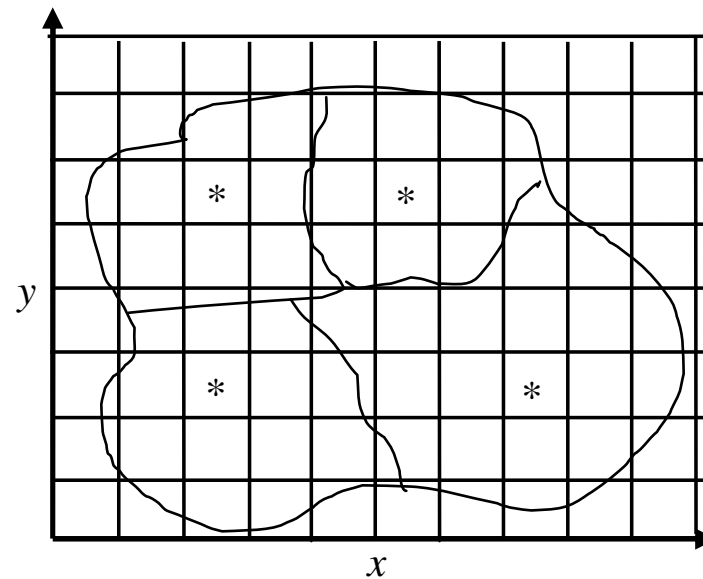


Figure 2. Graphical depiction of areas on an installation.



Network Model

Definitions:

- $G = (\mathcal{N}, \mathcal{A})$ describes the graph with:
 - $\mathcal{N} \equiv \{1, 2, \dots, N\}$, where N is the number of critical areas
 - $\mathcal{A} \equiv$ set of arcs (i, j) for $i, j \in \mathcal{N}$



Network Model

- $\mathcal{N}_i \equiv$ set of nodes adjacent to node i
- $t_{i,j} \equiv$ constant time required to travel from node i to node j
 - $t_{i,j} > 0, \forall (i,j) \in \mathcal{A}$
 - $t_{i,j} = t_{j,i}$
- $v_j(t) \equiv$ nonnegative vapor concentration at node $j \in \mathcal{N}$ at time t
- $r_j(t) \equiv$ binary reward received from searching node j at time t



Network Model

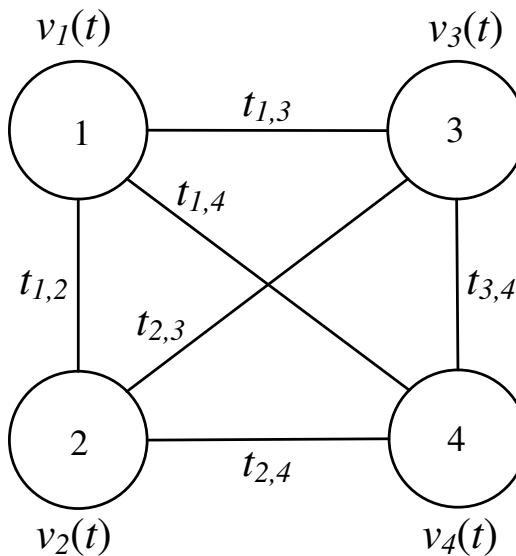


Figure 3. Example of the network representation for a 4-node site.

- $\mathcal{N} = \{1, 2, 3, 4\}$
- $\mathcal{A} = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)\}$



Well-known Network Models

- Shortest Path Problem (SPP)
 - Path from source to sink
 - Not all nodes must be reached
- Knapsack Problem
 - Maximize a value with a constraint on the resource
 - Order does not matter



Well-known Network Models

- Travelling Salesperson Problem (TSP)
 - Minimize tour length
 - Must reach every city
 - Start and end at the origin



Methodology

We consider four distinct cases:

- Static and deterministic vapor concentrations
- Static and stochastic vapor concentrations
- Dynamic and deterministic vapor concentrations
- **Dynamic and stochastic vapor concentrations**



Dynamic and Deterministic

- Deterministic:
 - Assume vapor level concentration at each node can be calculated deterministically
- Dynamic:
 - Vapor levels depend on time, $v_j(t)$, for all $j \in \mathcal{N}, t \geq 0$



Dynamic and Deterministic

Objective: Maximize reward:

$$\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_j(t) x_{i,j}$$

- Time constraint
- Backtracking *is* allowed
- Vapor concentrations are dynamic $\Rightarrow$ rewards are dynamic
- Possibly not all nodes will be reached



Algorithm

Initialization:

$$\mathcal{N} = \{1, 2, \dots, N\}; \mathcal{N}_i = \{j : i \rightarrow j\};$$

$$\mathcal{R} = \emptyset; \psi = \emptyset;$$

$$t \leftarrow t_0;$$

$$i = 1;$$

Calculate current vapor level at node i, $v_i(t)$

If $v_i(t) < v^*$

$$r_i(t) \leftarrow 1;$$

$$\mathcal{R} \leftarrow \{i\}; \psi \leftarrow \psi \cup \{i\};$$

Else

$$r_i(t) \leftarrow 0;$$

$$\psi \leftarrow \psi \cup \{i\};$$

End



Algorithm

Step 1

Calculate $v_j(t + t_{i,j}) \forall j \in \mathcal{N}_i$

If $v_j(t + t_{i,j}) < v^*$

$r_j(t + t_{i,j}) \leftarrow 1;$

Else

$r_j(t + t_{i,j}) \leftarrow 0;$

End



Algorithm

Step 2

For each $j \in \mathcal{N}$ such that $r_j = 1$

Choose j such that $v_j(t + t_{i,j}) = \arg \min_{j \in \mathcal{N}_i} \{v^* - v_j(t + t_{i,j})\}$

$\mathcal{R} \leftarrow \mathcal{R} \cup \{j\};$

$\psi \leftarrow \psi \cup \{j\};$

End

If $r_j(t + t_{i,j}) = 0 \ \forall \ j \in \mathcal{N}_i$

Choose j such that $t_{i,j} = \min_j \{t_{i,j}\} \forall j \in \mathcal{N}_i$

$\psi \leftarrow \psi \cup \{j\};$

End

$t \leftarrow t + t_{i,j};$



Algorithm

Step 3

If $t \geq T$

STOP

Else

$i \leftarrow j;$

Return to Step 1

End



Dynamic/Deterministic

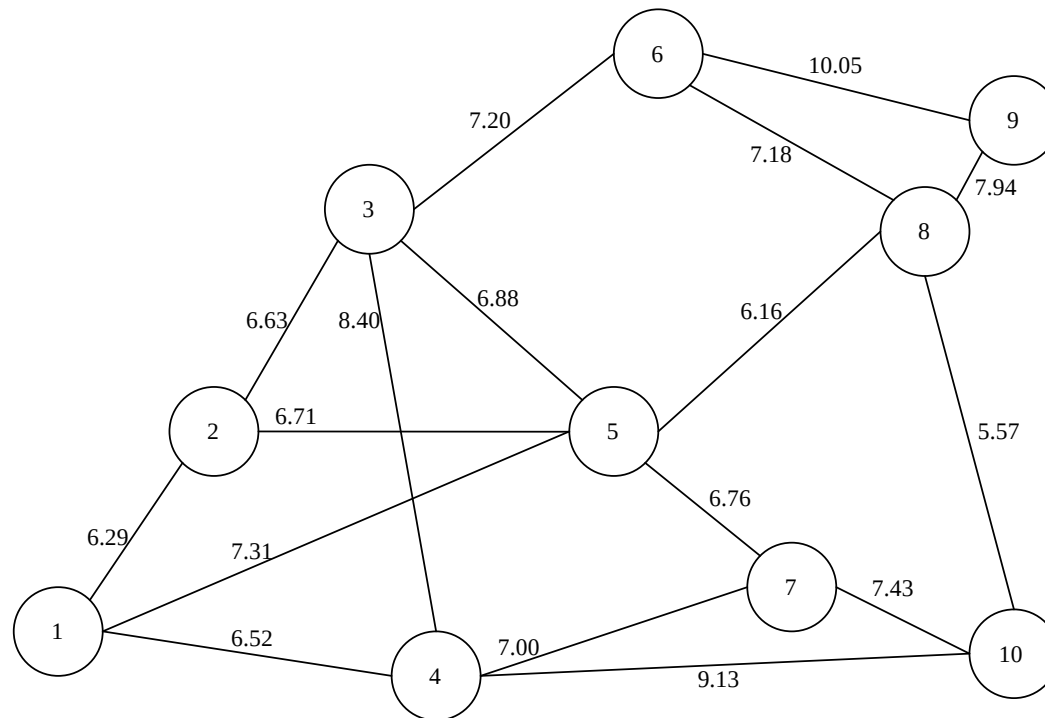
Result: iterative process yields a time-adaptive policy

- Future decisions depend on arrival times at nodes
- Vapor concentrations (rewards) drive the solution



Dynamic/Deterministic

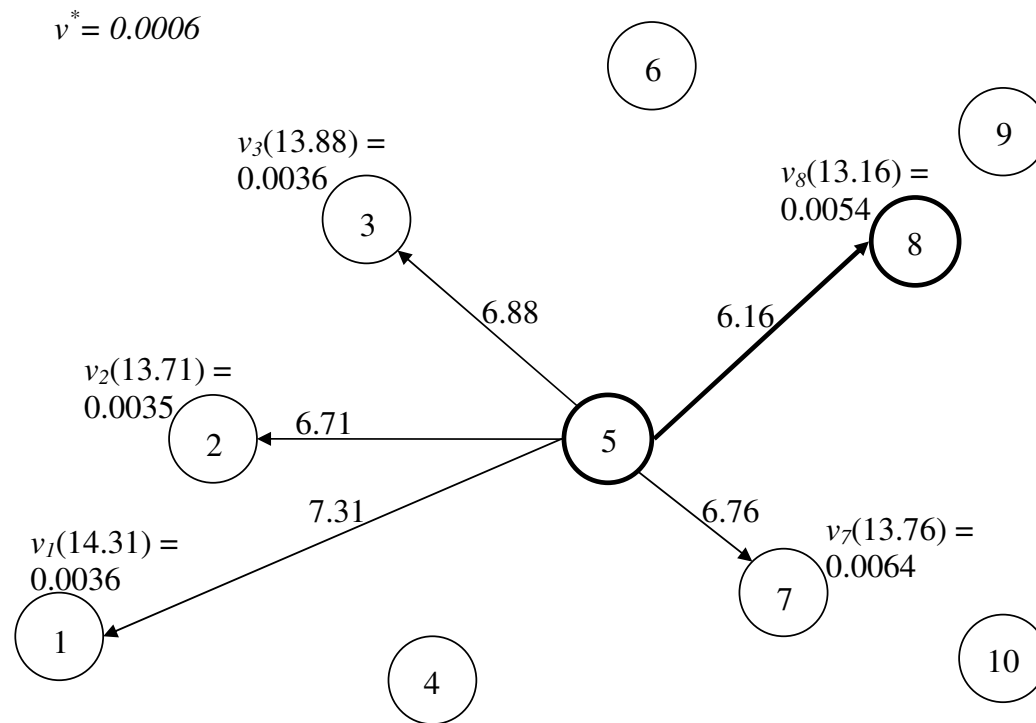
Network Configuration





Dynamic/Deterministic

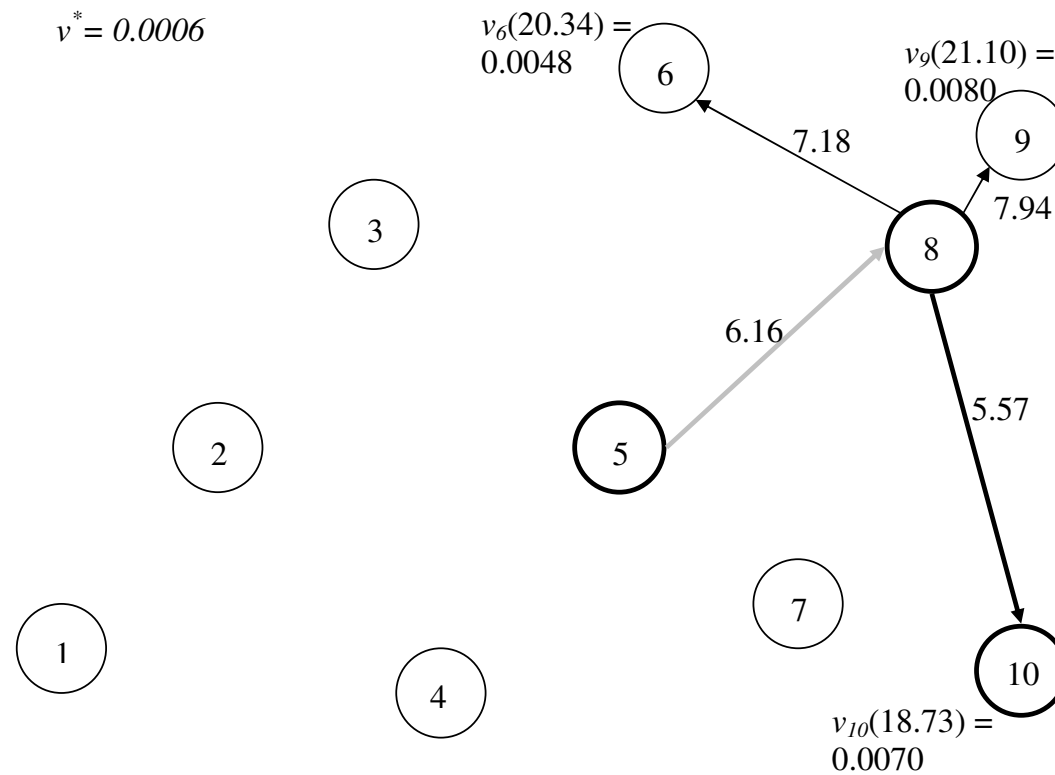
Example: Iteration 1: $t_0 = 7$





Dynamic/Deterministic

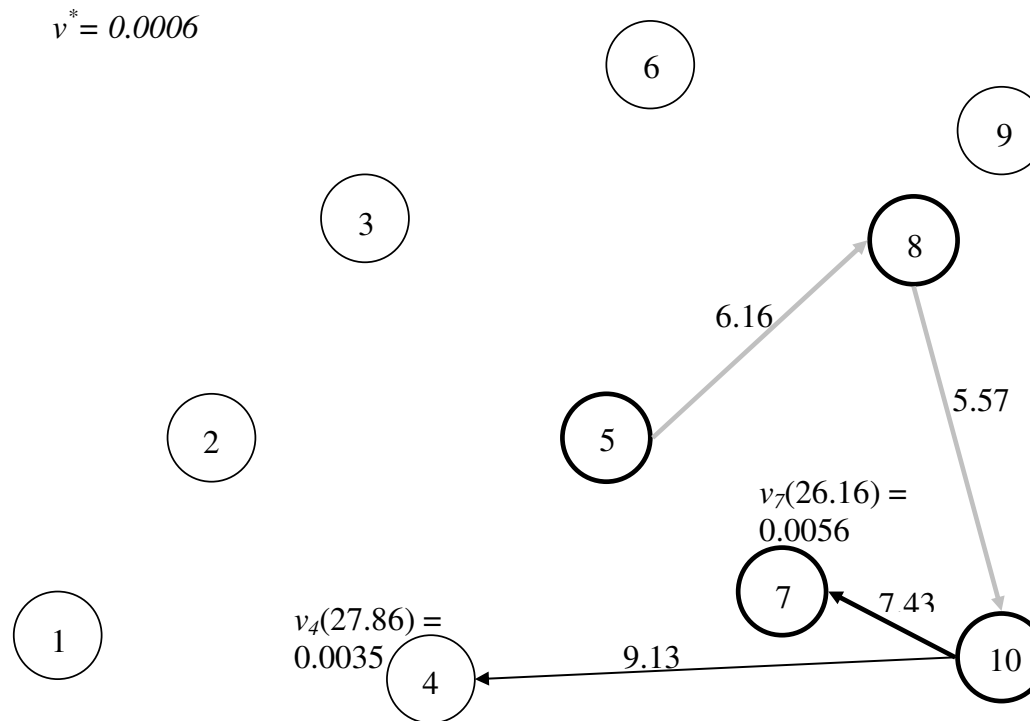
Iteration 2:





Dynamic/Deterministic

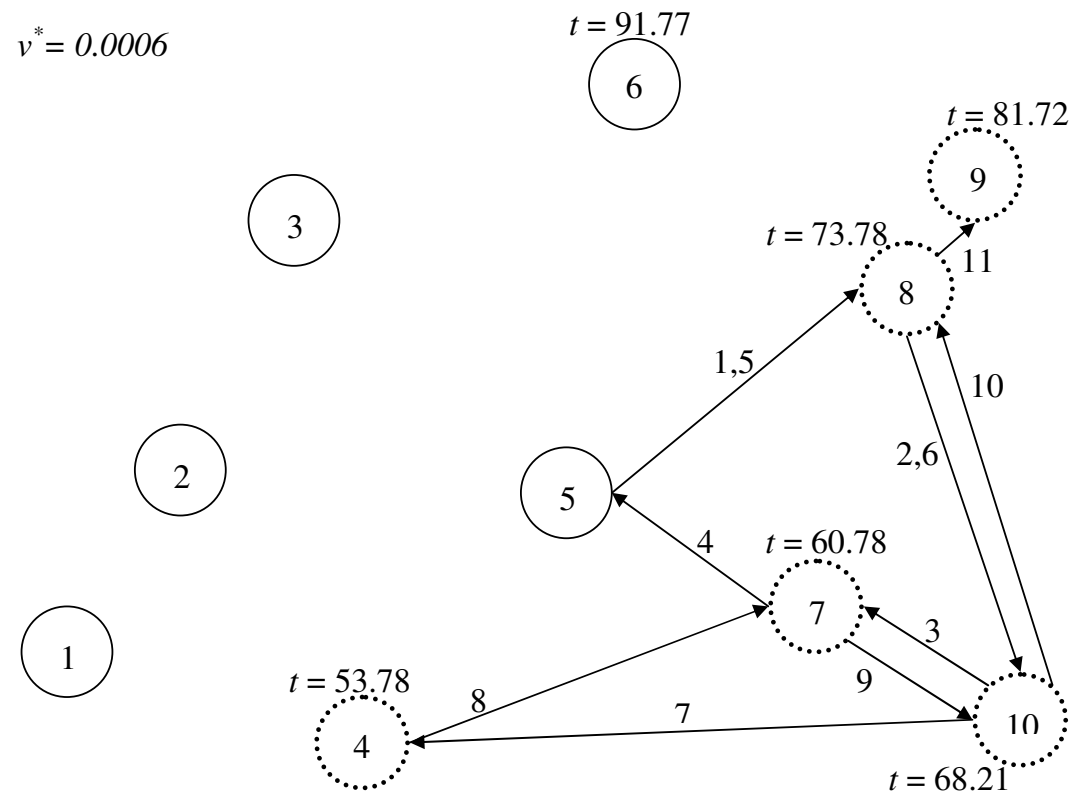
Iteration 3:





Dynamic/Deterministic

Final Solution:





Dynamic/Deterministic

Table 1. Vapor concentrations and rewards for nodes in ψ .

<i>Node</i>	<i>t</i> (min)	$v_j(t)$	$r_j(t)$
5	7.00	0.000650	0
8	13.16	0.005400	0
10	18.73	0.007000	0
7	26.16	0.005600	0
5	32.92	0.002800	0
8	39.08	0.002200	0
10	44.65	0.001400	0
4	53.78	0.000410	1
7	60.78	0.000330	1
10	68.21	0.000190	1
8	73.78	0.000110	1
9	81.72	0.000059	1



Dynamic/Deterministic

Table 2. Vapor concentrations and rewards at termination.

<i>Node</i>	$v_j(81.72)$	$r_j(\tau^*)$
1	0.000023	1
2	0.000025	1
3	0.000030	1
4	0.000034	1
5	0.000040	1
6	0.000042	1
7	0.000052	1
8	0.000054	1
9	0.000059	1
10	0.000057	1



Dynamic and Stochastic

- Time-variant probability distribution for each node (e.g., $V_j(t) \sim \exp(\mu_j(t))$ for all $j \in \mathcal{N}$)
- Objective: Maximize reward - The number of areas searched where the vapor concentration has *most likely* decreased below v^*
 - Reward is dynamic and computed from the expected value
 - If $E[V_j(t)] < v^*$, $r_j(t) = 1$, otherwise $r_j(t) = 0$.



Algorithm

Initialization:

$$\mathcal{N} = \{1, 2, \dots, N\}; \mathcal{N}_i = \{j : i \rightarrow j\};$$

$$\mathcal{R} = \emptyset; \psi = \emptyset;$$

$$t \leftarrow t_0;$$

$$i = 1;$$

Obtain realization of vapor level $v_i(t)$

If $v_i(t) < v^*$

$$r_i(t) \leftarrow 1;$$

$$\mathcal{R} \leftarrow \mathcal{R} \cup \{i\}; \psi \leftarrow \psi \cup \{i\};$$

Else

$$r_i(t) \leftarrow 0;$$

$$\psi \leftarrow \psi \cup \{i\};$$

End



Algorithm

Step 1

Calculate $\pi_j(t + t_{i,j}) \equiv P\{V_j(t + t_{i,j}) < v^*\} \forall j \in \mathcal{N}_i$

Step 2

Choose j such that $\pi_j(t) = \max_{j \in \mathcal{N}_i} P\{V_j(t + t_{i,j}) < v^*\}$

Obtain instrument reading at this node.

If $v_j(t + t_{i,j}) < v^*$

$r_j(t) \leftarrow 1; \mathcal{R} \leftarrow \mathcal{R} \cup \{j\}; \psi \leftarrow \psi \cup \{j\};$

$t \leftarrow t + t_{i,j};$

Else

$r_j(t) \leftarrow 0;$

$\psi \leftarrow \psi \cup \{j\};$

$t \leftarrow t + t_{i,j};$

End



Algorithm

Step 3

If $t \geq T$

STOP

Else

$i \leftarrow j;$

Return to Step 1

End



Dynamic/Stochastic

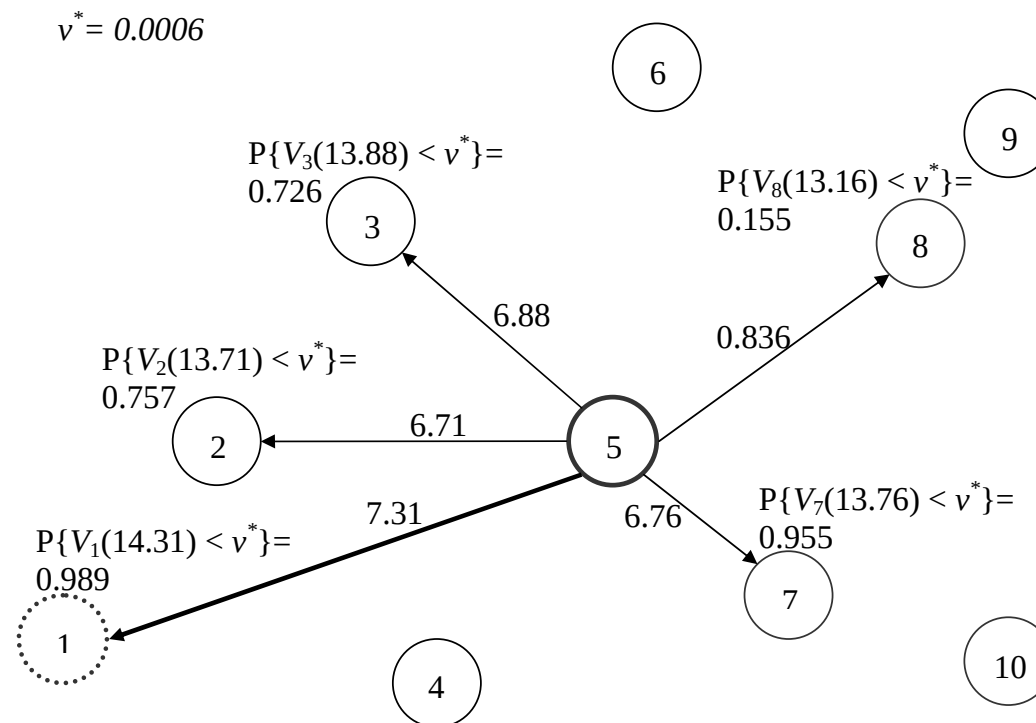
Result: iterative process yields a time-adaptive policy

- Future decisions depend on arrival times
- *Probability* a vapor concentration is below the threshold v^* drives the solution



Dynamic/Stochastic

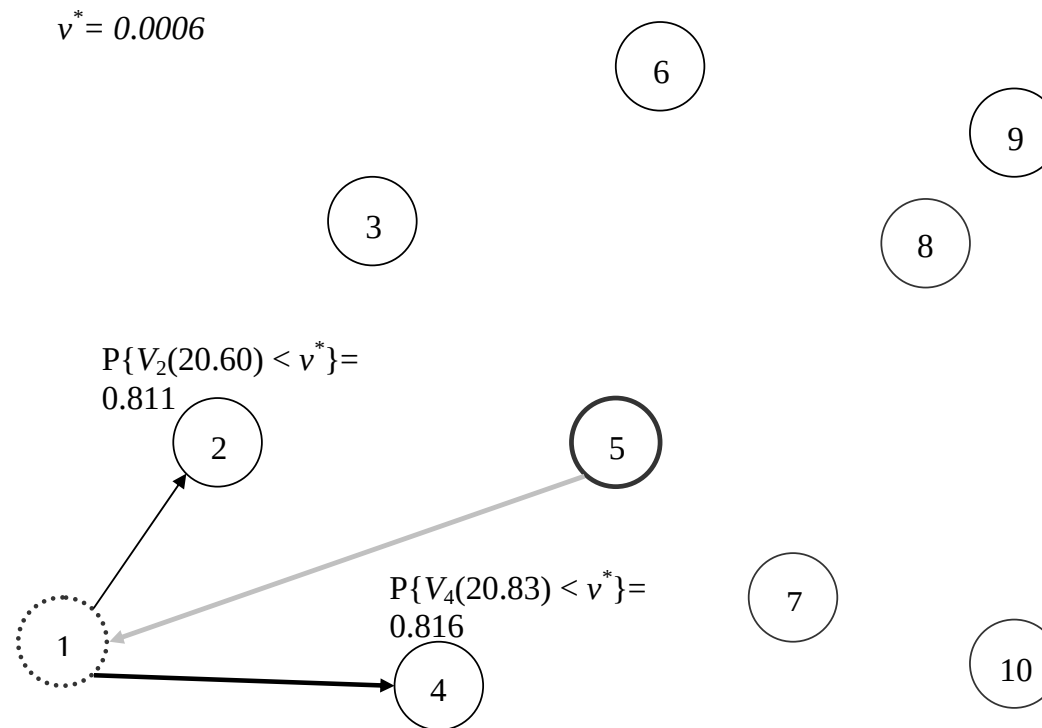
Example: Iteration 1:





Dynamic/Stochastic

Iteration 2:





Dynamic/Stochastic

Table 3: Vapor concentrations for nodes in ψ ($v^* = 6.0 \times 10^{-4}$).

<i>Node</i>	<i>t</i> (min)	$v_j(t)$	$r_j(t)$
5	7	0.0011	0
1	14.31	0.000012	1
4	20.83	0.000106	1
10	29.96	0.000172	1
7	37.39	0.000103	1
5	44.15	0.000113	1
1	51.46	0.000172	1
4	57.98	0.000179	1
10	67.11	0.000609	1
7	74.54	0.000693	1
5	81.3	0.000803	1
1	88.61	0.000001	1



Summary/Conclusions

- Ignoring dynamics may lead to under- or over-estimation of the number of safe areas (in these examples)
- Spatiotemporal characteristics are critical in developing the sampling strategy
- Want to minimize Type II error (i.e., accept H_0 that area is safe given it is not)
- Data was assumed to exist for illustrative purposes, however...
- Real problem presents significant data requirements



Future Work

- Relax assumptions
 - Consider non-deterministic travel times
 - Multiple search crews
 - Estimate probability distributions
- Incorporate real-time information
 - Real-time concentration readings from sensors
 - Road closures/openings
 - Weather changes (e.g., wind velocity, temperature, humidity, etc.)



Questions?



Backups



Backup: A-D Equation

The following parameters must be known to employ advection diffusion equation to compute v_j :

$x, y, z \equiv$ coordinates in the direction of the mean wind, horizontal cross-wind, and upwards vertical direction.

$k_x, k_y, k_z \equiv$ eddy diffusivities in $\text{m}^2\text{sec}^{-1}$

$q \equiv$ the total mass release in kg

$h \equiv$ instantaneous gas release height above the ground in m

$u \equiv$ wind velocity in m/sec



Backup: A-D Equation

$$v_j = \frac{q}{8\pi^{\frac{3}{2}}(k_x k_y k_z)^{1/2} t_0^{3/2}} \exp \left[-\frac{(x - ut_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} \right] \times \\ \left(\exp \left[-\frac{(z - h)^2}{4k_z t_0} \right] + \exp \left[-\frac{(z + h)^2}{4k_z t_0} \right] \right). \quad (1)$$

Ref: Kathirgamanathan, P., McKibbin, R., and R.I. McLachlan (2003). Source release-rate estimation of atmospheric pollution from a non-steady point source - Part 1: Source at a known location. *Res. Lett. Inf. Math. Sci.*, **5**, 71-84.



Backup: A-D Equation

Equation 1 can be simplified to

$$v_j = \frac{q}{8\pi^{\frac{3}{2}}(k_x k_y k_z)^{1/2} t_0^{3/2}} \exp \left[-\frac{(x - ut_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} \right] \times \left(2 \exp \left[-\frac{h^2}{4k_z t_0} \right] \right), \quad (2)$$

since $z = 0$ for our numerical illustrations.



Backup: Initial Rate Parameters

Table 6. Rate parameters chosen for the exponential distributions used for example 2.

<i>Node</i>	μ_j	$E[V_j](\times 10^{-4})$	r_j
1	6666.67	1.50	1
2	1538.46	6.50	0
3	1322.75	7.56	0
4	1574.80	6.35	0
5	1754.39	5.70	1
6	10000.00	1.00	1
7	4347.83	2.30	1
8	2222.22	4.50	1
9	909.09	11.00	0
10	7692.31	1.30	1



Case 1: Static and Deterministic

- Deterministic:
 - Assume vapor level concentration at each node is calculated via a deterministic formula immediately after the attack
- Static:
 - Assume for each $j \in \mathcal{N}$, v_j does not evolve over time



Case 1: Static/Deterministic

Objective: Minimize time required to reach as many areas as possible to obtain the maximum reward (i.e., maximum number of areas not requiring protective gear).

- Time constraint implies it is possible that not all areas will be sampled
- No backtracking unless necessary (i.e., there is no reward for returning to an area)
- No subtours ($\mathcal{S} \subset \mathcal{N} \equiv$ set of all possible subtours)



Case 1: Static/Deterministic

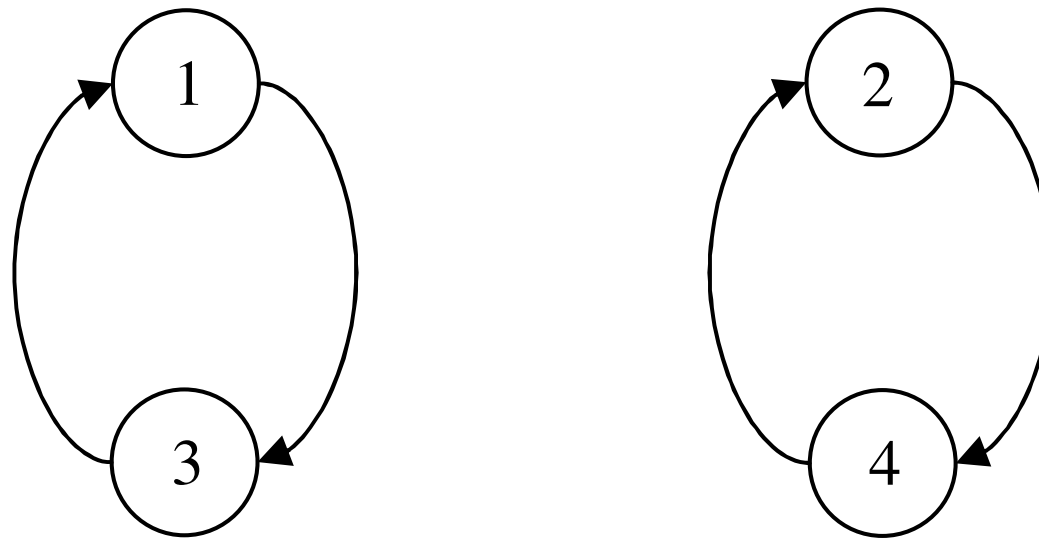


Figure 4. Example of subtour in a 4-node site.



Case 1: Static/Deterministic

$$\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_j x_{i,j}(\text{opt}), \quad \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j}$$

subject to

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j} < T$$

$$\sum_{j \in \mathcal{N}} x_{s,j} = 1 \text{ for } s \in \mathcal{N}$$

$$\sum_{i \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } j = 1, \dots, N; j \neq i$$

$$\sum_{j \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } i = 1, \dots, N; i \neq j$$

$$x_{i,j} + x_{j,i} \leq 1 \text{ for all } (i,j) \in \mathcal{A}$$

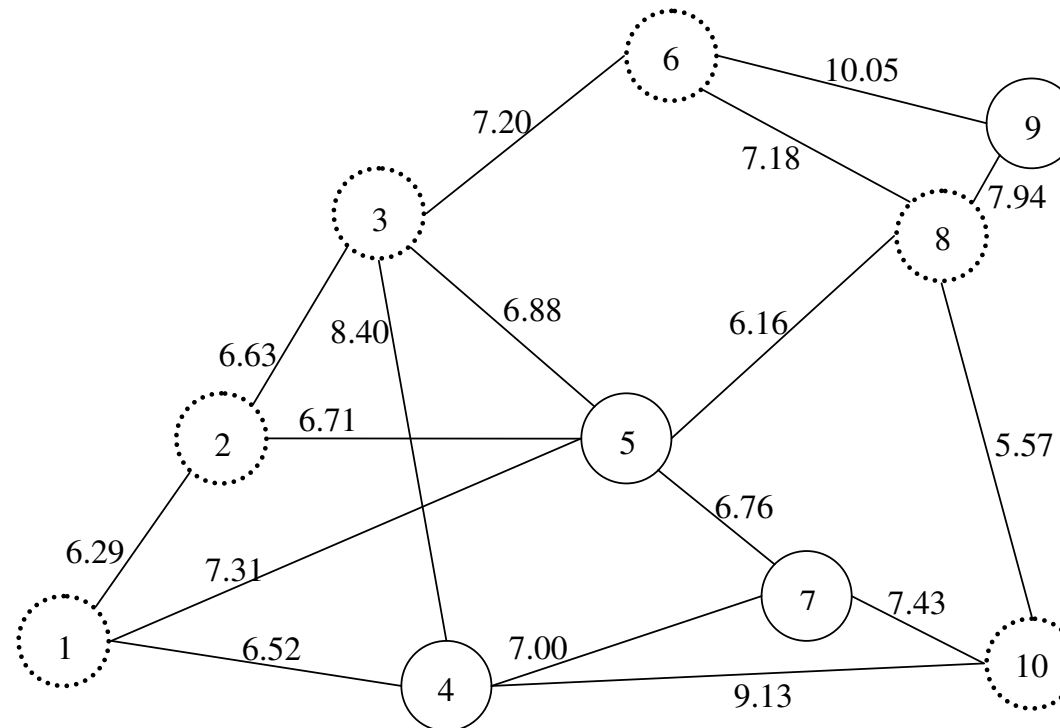
$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} x_{i,j} \leq |\mathcal{S}| - 1 \text{ for } \mathcal{S} \in \mathcal{N}, 2 \leq |\mathcal{S}| \leq N - 1$$

$$x_{i,j} \in \{0, 1\}$$



Case 1: Static/Deterministic

Example 1: Consider the following 10-node network

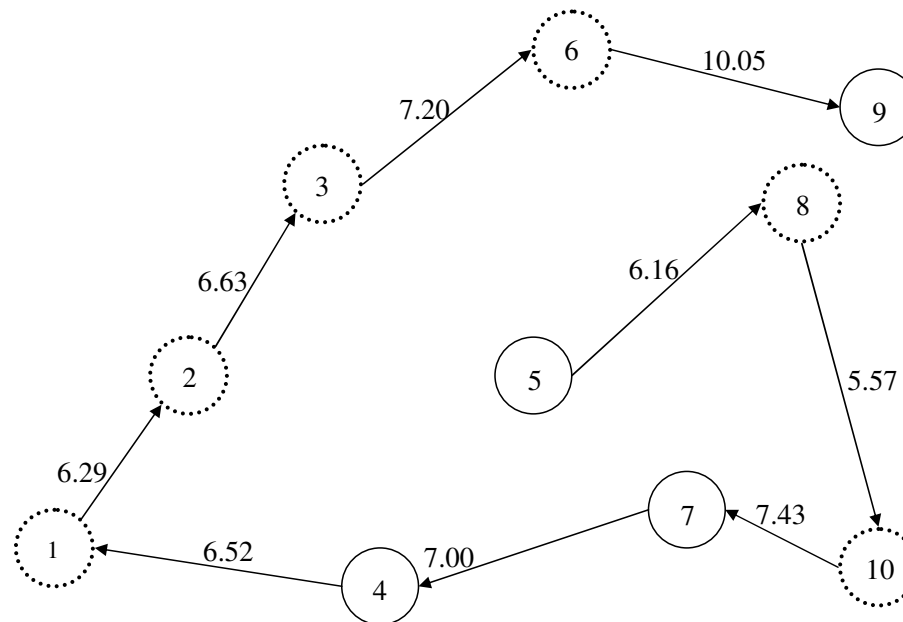


$$r_j = 1 \text{ for } j = 1, 2, 3, 6, 8, 10; \quad r_j = 0 \text{ for } j = 4, 5, 7, 9$$



Case 1: Static/Deterministic

Solution:



- Total time of search: $\tau^* = 62.85$ minutes
- Optimal path: $\psi = [5, 8, 10, 7, 4, 1, 2, 3, 6, 9]$
Total reward: $r^* = 6$ from nodes 1, 2, 3, 6, 8, 10



Case 2: Static and Stochastic

- Stochastic: Assume vapor level concentration at each node is a random variable V_j , for all $j \in \mathcal{N}$, with an associated probability distribution
- Static: $P\{V_j \leq v^*\}$ does not change with time, nor does $E[V_j] \forall j \in \mathcal{N}$



Case 2: Static/Stochastic

Objective: Minimize time required to reach as many areas as possible to obtain the maximum reward

- Same formulation as Case 1
- Rewards are found from expected vapor concentrations
 - E.g., $V_j \sim \exp(\mu_j)$ for all $j \in \mathcal{N}$
 - $E[V_j] = \frac{1}{\mu_j}$
 - If $E[V_j] < v^*$, $r_j = 1$, otherwise $r_j = 0$.



Case 2: Static/Stochastic

$$\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_j x_{i,j}(\text{opt}), \quad \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j}$$

subject to

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j} < T$$

$$\sum_{j \in \mathcal{N}} x_{s,j} = 1 \text{ for } s \in \mathcal{N}$$

$$\sum_{i \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } j = 1, \dots, N; j \neq i$$

$$\sum_{j \in \mathcal{N}} x_{i,j} \leq 1 \text{ for } i = 1, \dots, N; i \neq j$$

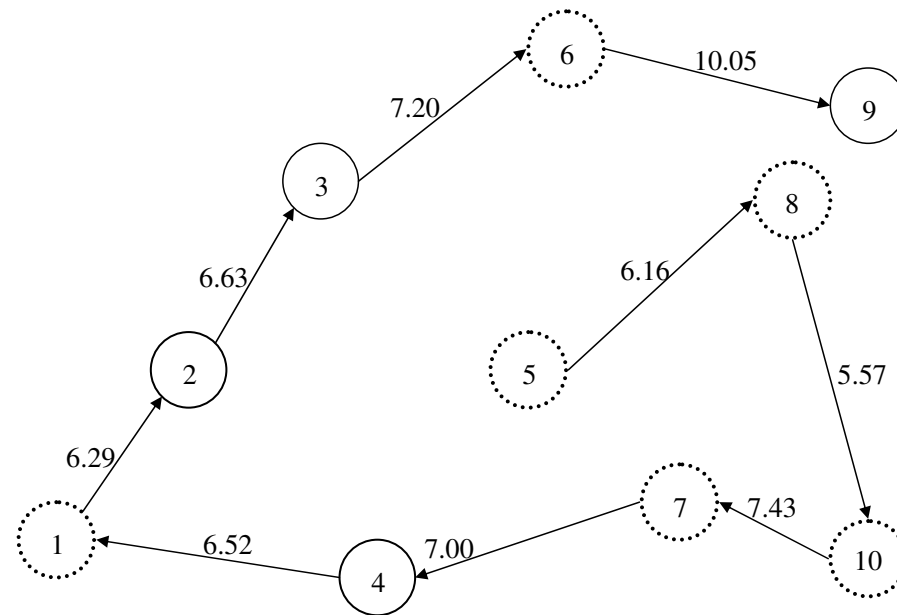
$$x_{i,j} + x_{j,i} \leq 1 \text{ for all } (i,j) \in \mathcal{A}$$

$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} x_{i,j} \leq |\mathcal{S}| - 1 \text{ for } \mathcal{S} \in \mathcal{N}, 2 \leq |\mathcal{S}| \leq N - 1$$

$$x_{i,j} \in \{0, 1\}$$



Case 2: Static/Stochastic



- Total time of search: 62.85 minutes
- Optimal path: $\psi = [5, 8, 10, 7, 4, 1, 2, 3, 6, 9]$
- Total reward: $r^* = 6$ from nodes 1, 5, 6, 7, 8, 10



Comparison: Deterministic Results

Table 4. Comparison of solutions to the static/deterministic and dynamic/deterministic examples.

<i>Static</i>		<i>Dynamic</i>	
<i>Node</i>	r_j	<i>Node</i>	$r_j(\tau)$
5	0	5	0
8	1	8	0
10	1	10	0
7	0	7	0
4	0	5	0
1	1	8	0
2	1	10	0
3	1	4	1
6	1	7	1
9	0	10	1
		8	1
		9	1
<i>Total Time (min)</i>	r^*	<i>Total Time (min)</i>	$r^*(\tau^*)$
62.85	6	81.72	5



Comparison: Deterministic Results

- Static/deterministic vapor concentration case
 - Search each node exactly once
 - Less amount of time
- Dynamic/deterministic vapor concentration case
 - Searches only critical nodes
 - Utilizes time allotted
 - Total reward value accounts for dynamic nature of concentrations



Comparison: Deterministic Results

Main result of comparisons: Incorporating temporal evolution reduces risk of overestimating/underestimating the number of areas safely operating without protective gear.

- Solution 1: Static
 - 60% of the areas are determined to be safe
 - 33% of those will become unsafe at later times
- Solution 2: Dynamic
 - 50% of areas are determined to be safe
 - 3 of these were previously unsafe



Comparison: Stochastic Results

Table 5. Comparison of solutions to the static/stochastic and dynamic/stochastic examples.

<i>Static</i>		<i>Dynamic</i>	
<i>Node</i>	r_j	<i>Node</i>	$r_j(\tau)$
5	1	5	0
8	1	1	0
10	1	5	0
7	1	7	1
4	0	10	1
1	1	8	1
2	0	6	1
3	0	3	1
6	1	5	0
9	0	1	0
		4	0
		10	1
<i>Total Time (min)</i>	r^*	<i>Total Time (min)</i>	$r^*(\tau^*)$
62.85	6	88.61	5



Comparison: Stochastic Cases

- Static/stochastic vapor concentration case
 - Search each node exactly once
 - Less amount of time
 - Reward based on *expected* vapor concentrations
- Dynamic/stochastic vapor concentration case
 - Search is driven by probability a node will be less than the threshold v^*
 - Rewards determined from *expected* values and rate parameters are time-dependent



Comparison: Stochastic Cases

Main result: Reduce risk of overestimating/underestimating safe areas in dynamic case. Stochastic elements account for randomness of the real problem.

- Solution 1:
 - 60% of areas are determined to be safe
 - Following this path declares safe areas prematurely
 - 2 of the areas would likely not be safe at later times



Comparison: Stochastic Cases

- Solution 2:
 - 50% of areas are determined to be safe
 - 1 of the unsafe nodes in the previous case becomes safe at a later time