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Name of Principal Author and all other author(s):
Steve Wilcox, Ph.D.
Pete Bulanow

Principal Author's Organization and address:
Northrop Grumman Information Technology
2100 Washington Boulevard
Arlington, VA 22204

Phone: 703-312-2511

Fax: 703-312-2780

Email: steve.wilcox@ngc.com

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Evaluating Statistical Assumptions When Analyzing Deterministic Simulation Results Using a Design of Experiments

**. Steve Wilcox, Ph.D.
Pete Bulanow**

**Northrop Grumman
Information Technology Sector
June 2005**



- **Background**
 - **Value Added Analysis process using Vector-In-Commander (VIC) to assess trades**
 - **Standard practice of employing fractional factorial experimental designs plus Analysis of Variance (ANOVA)**
 - **Non-monotonic relationships between parameters and VIC outcomes**
- **Issue**
 - **Statistical assumptions underlying ANOVA in this context**
- **Questions**
 - **Are these assumptions valid?**
 - **How to test them and assess impact?**
- **Analysis methodology**
 - **Graphical evidence**
 - **Independent replications of a small study whose parameters are known to have an expected value of zero**
- **Finding**
 - **Evidence that the ANOVA assumptions are not met**
 - **P-values from ANOVA are understated**

The Value Added Analysis Process

- Supporting the Center for Army Analysis (CAA)
- Uses the Vector-in-Commander (VIC) Corps-level combat simulation model
- Objective:
 - Estimate the incremental contribution of system trades to combat effectiveness
 - Perform a cost-benefit analysis to determine the actual 'value-added' of the systems of interest.

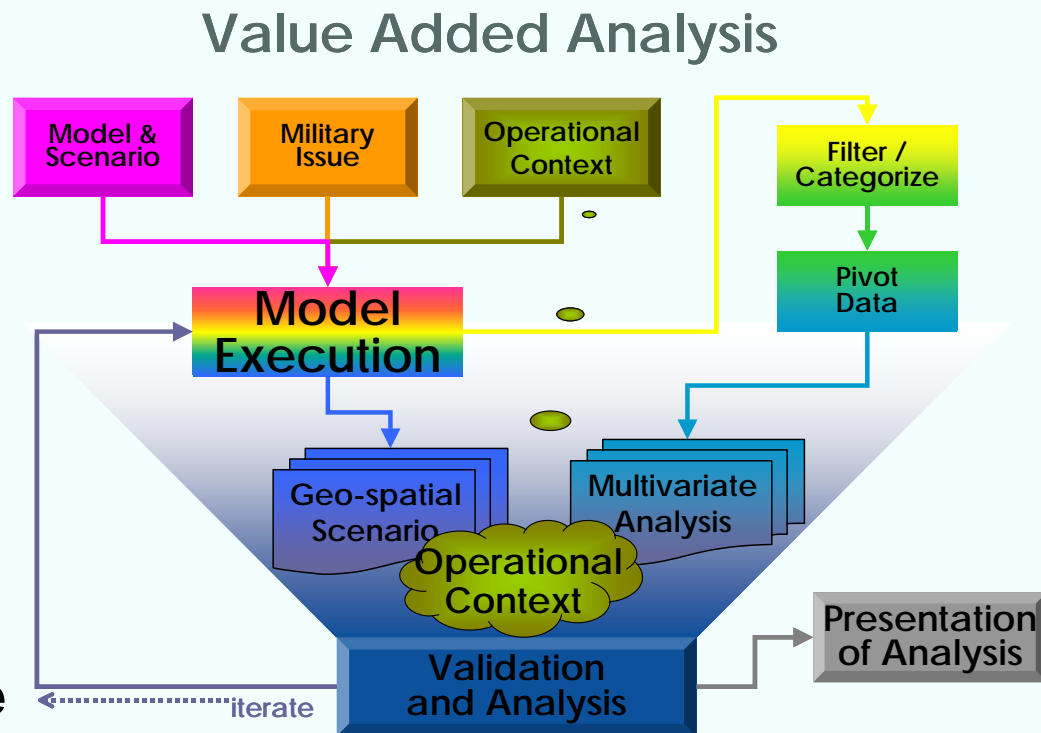


Figure 1. The VAA Process

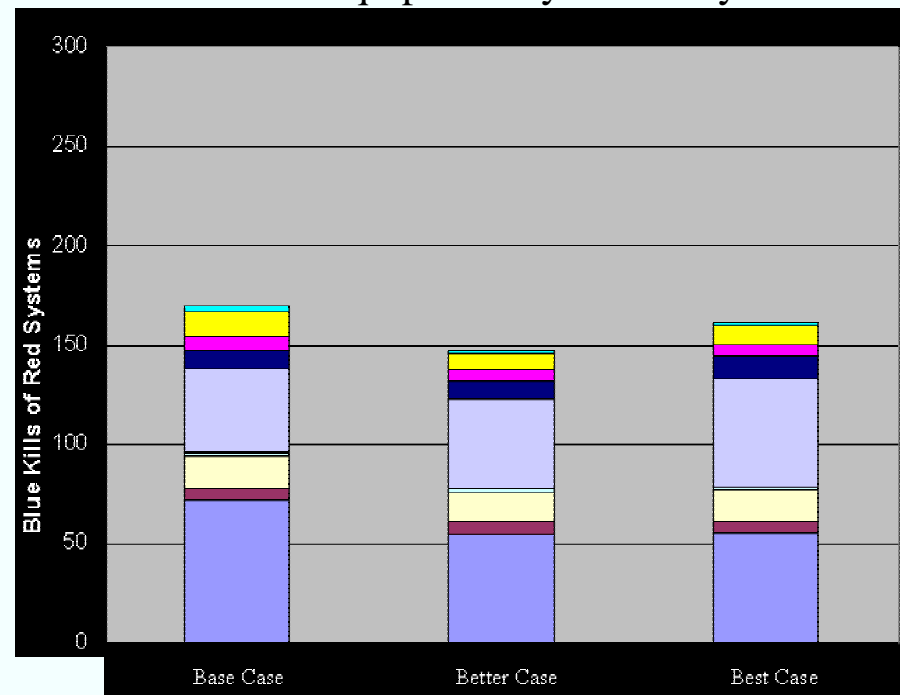
- Previous methodology was a typical Design of Experiments (DOE) approach
- Now a perturbation methodology induces stochastic behavior in VIC



Non-Monotonicity in Deterministic Combat Models

- **A.K.A. Sensitivity to initial conditions and parameter values**
 - Extensively noted in toy models of combat
 - The RAND model (Dewar, et al, 1991)
 - Also noted in VIC
 - Saeger & Hinch (2001)
 - Geoff Hawkins (1984) with VECTOR-2
- Impedes analysis of equipment trades

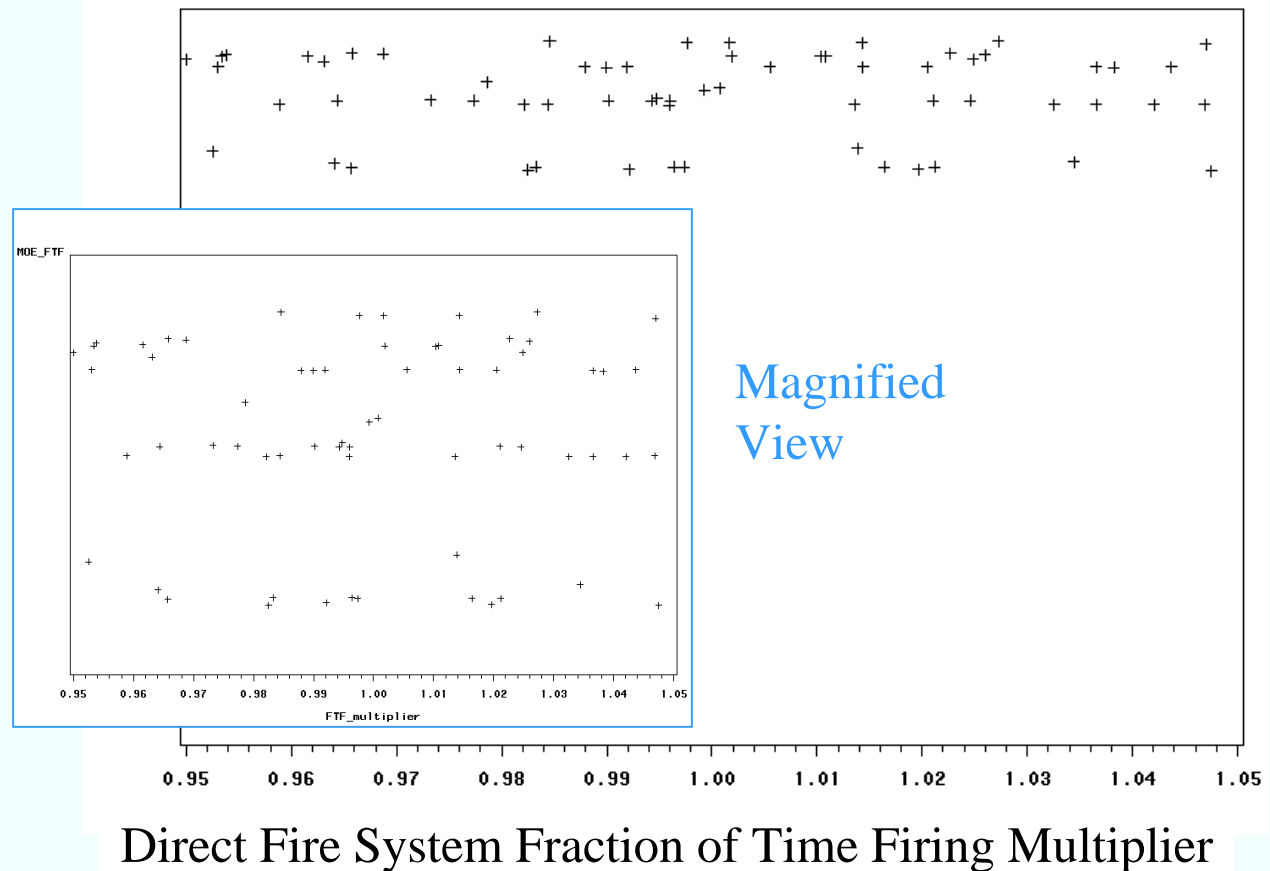
Kills of Selected Equipment by a Blue System of Interest



Parametric Sensitivity

- The direct fire system (DF Sys) fraction of time firing (FTF) is multiplied by a number randomly selected from the interval (0.95, 1.05)
- Blue kills vary non-monotonically
 - and significantly
- If the analyst had chosen a very slightly different number, the MOE would be different

Kills by Blue of High-Value Equipment

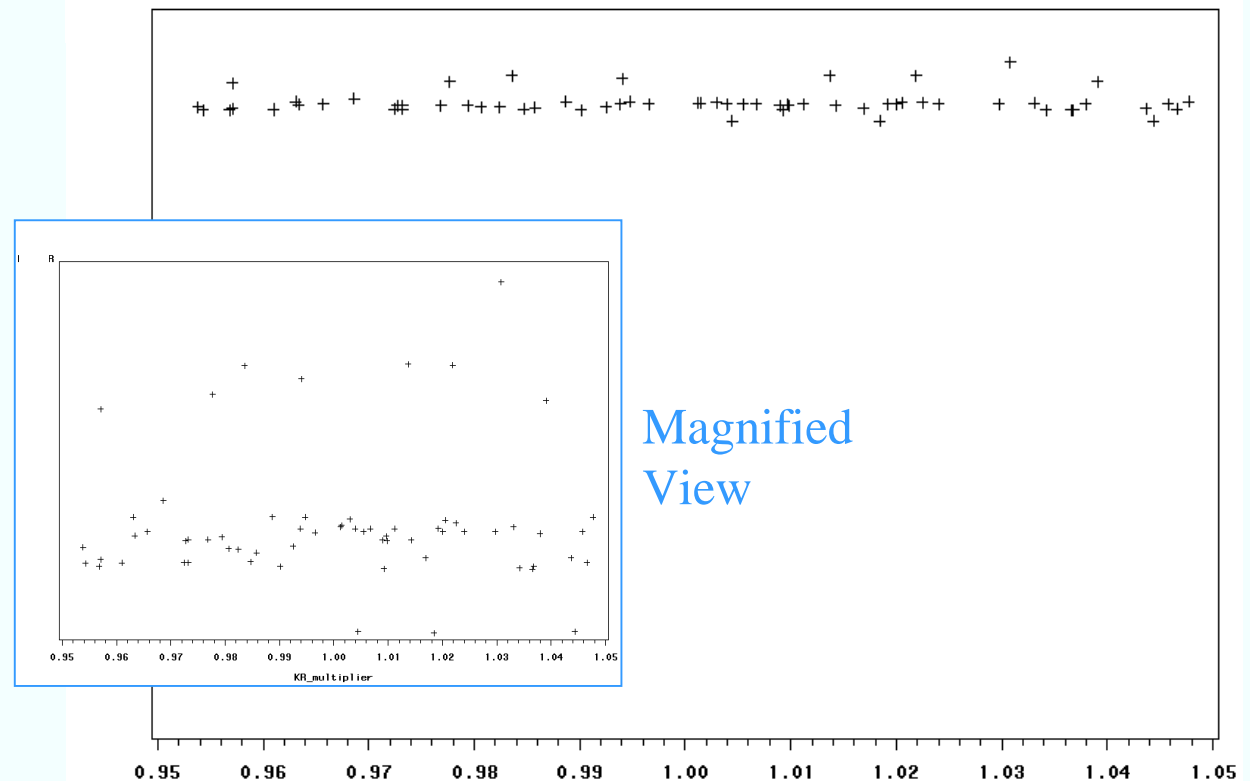




Another Example

- The rotary wing system (RW Sys) “kill rate” (KR) is multiplied by a number randomly selected from the interval (0.95, 1.05)
- Over this small interval, the true relationship is relatively weak
 - Non-systematic variation is much more important than the systematic relationship

Kills by Blue of High-Value Equipment

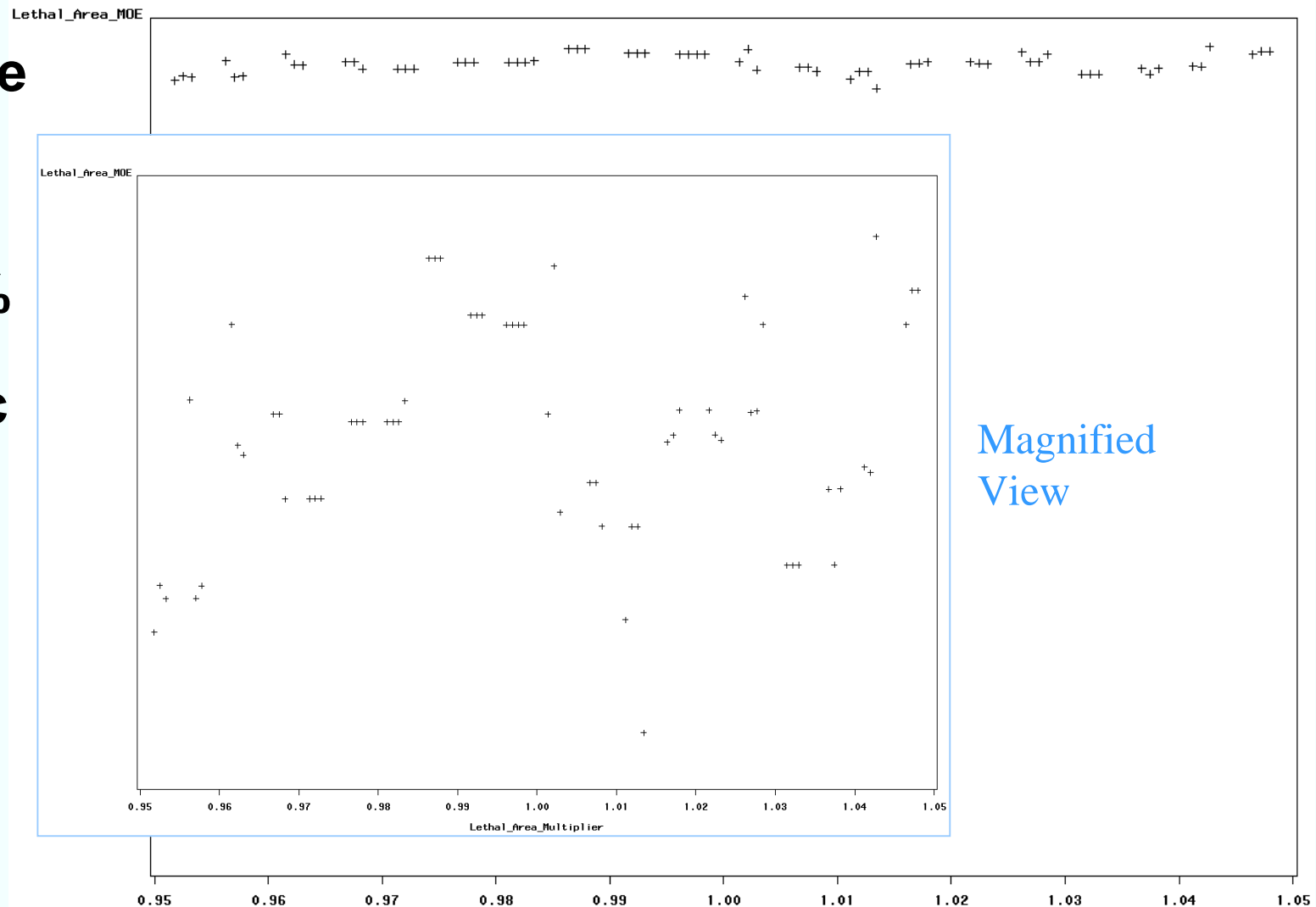


Rotary Wing System Kill Rate Multiplier



Slightly Varying Artillery Lethal Area

- Varying the artillery munition lethal area within $\pm 5\%$ has non-monotonic effects on the MOE
- Notice the flat spots



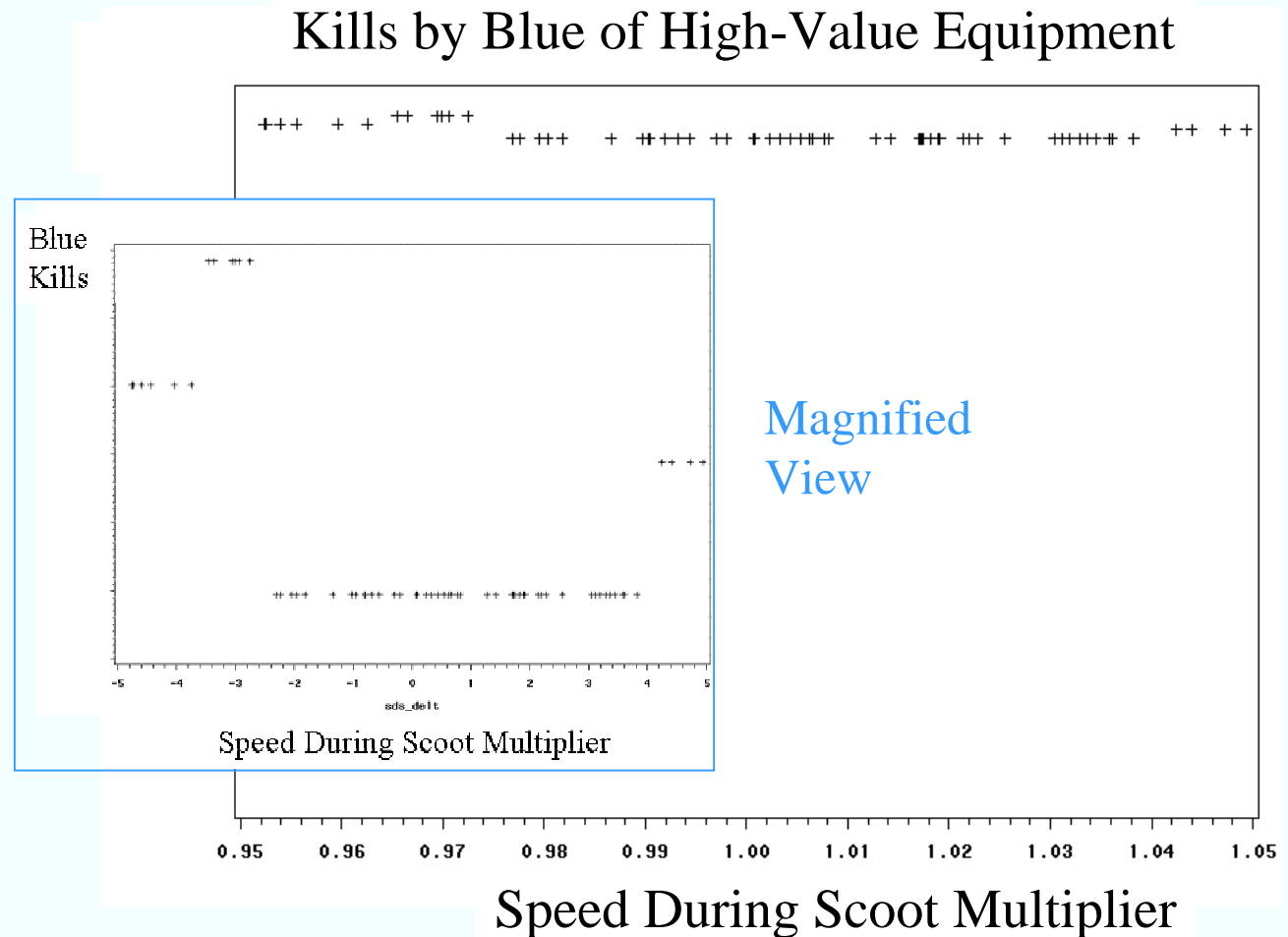
Magnified
View

Lethal Area (LA) Multiplier



The Amount of Sensitivity is Not the Same for all Parameters

- Varying speed during scoot $\pm 5\%$
- Only four clusters appear in the multivariate data
 - When viewed in total or by victim equipment group





Statistical Assumptions in the Design of Experiments

- A typical DOE is to run a number of combinations of experimental settings
 - And then analyze the MOEs using analysis of variance
- Different parameter settings are superimposed for each run
- ANOVA makes the assumption that error terms are **mean-zero** and **statistically independent**

Run	Sys1	Sys2	Sys3	Sys4	Sys5	Sys6	Sys7
1	1	1	1	1	1	1	1
2	1	1	1	-1	1	-1	-1
3	1	1	-1	1	-1	-1	-1
4	1	1	-1	-1	-1	1	1
5	1	-1	1	1	-1	-1	1
6	1	-1	1	-1	-1	1	-1
7	1	-1	-1	1	1	1	-1
8	1	-1	-1	-1	1	-1	1
9	-1	1	1	1	-1	1	-1
10	-1	1	1	-1	-1	-1	1
11	-1	1	-1	1	1	-1	1
12	-1	1	-1	-1	1	1	-1
13	-1	-1	1	1	1	-1	-1
14	-1	-1	1	-1	1	1	1
15	-1	-1	-1	1	-1	1	1
16	-1	-1	-1	-1	-1	-1	-1

$$y_i = \alpha + \sum_{k=1}^m \beta_k x_{ki} + \varepsilon_i$$

$$E[\varepsilon_i | \mathbf{x}] = 0 \quad i = 1, \dots, n$$

$$\text{Cov}[\varepsilon_i, \varepsilon_k] = 0 \quad i = 1, \dots, n; \quad k = 1, \dots, m$$

Effect of Superimposing Parameter Settings

- A perturbation to the rotary wing system kill rate (KR) is superimposed on that of direct fire system fraction of time firing (FTF)

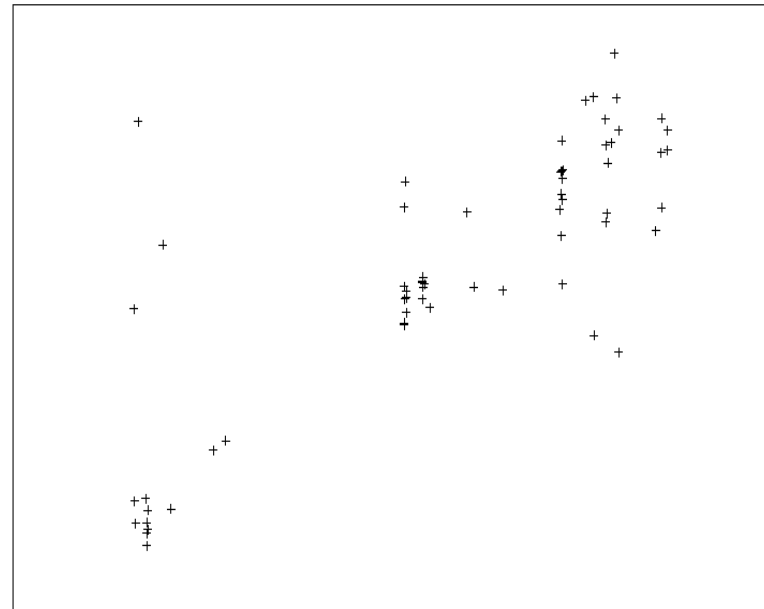
1. Perturb FTF

2. Perturb FTF and KR

- If the first run had a high MOE, the second one was also likely to be high

- This is a “memory effect”
- The artifacts from the first perturbation are “remembered” even though another perturbation is applied.

Y
Kills by Blue after further adding a perturbation of the rotary wing system kill rate



X
Kills by Blue after perturbing the direct fire fraction of time firing

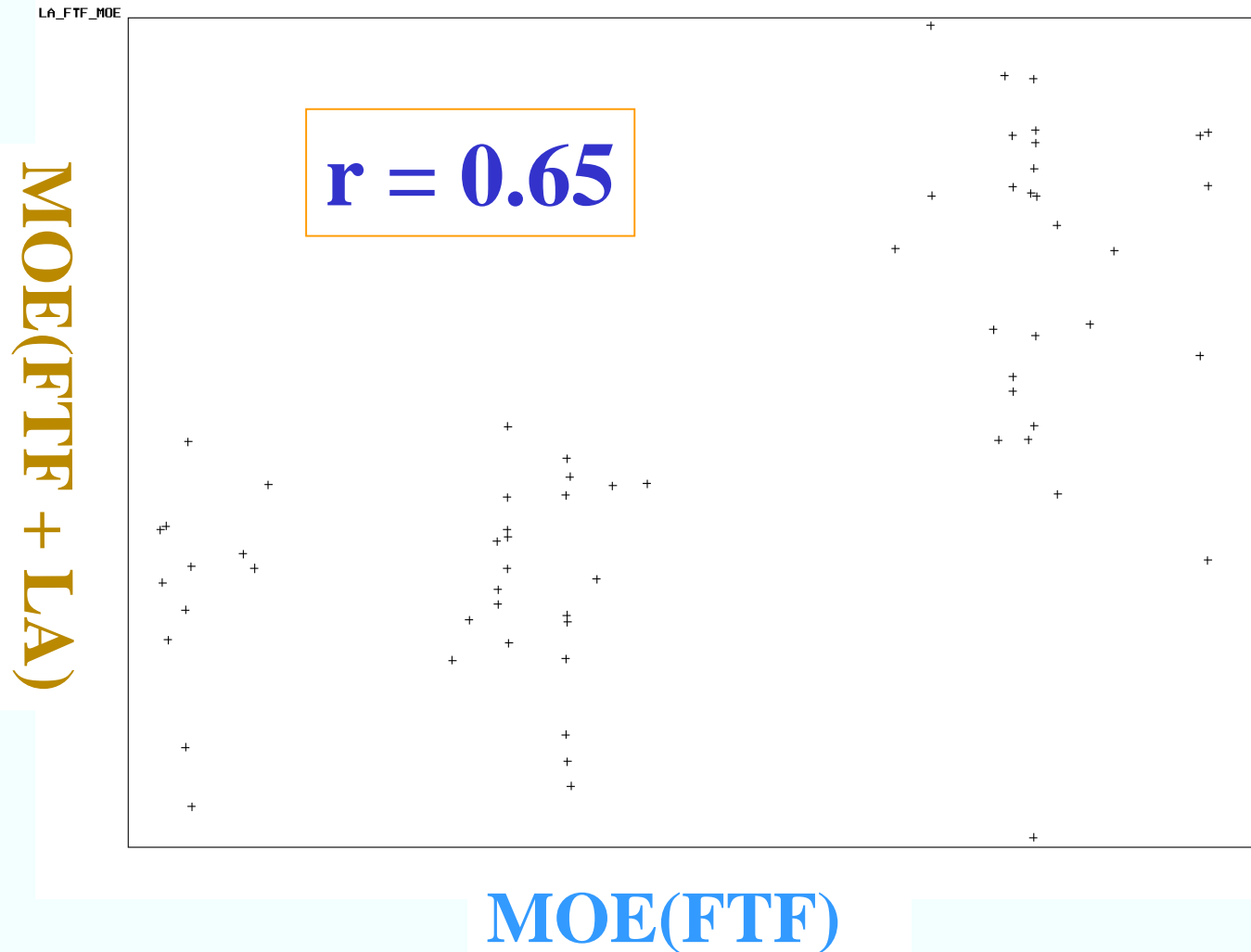
Run#	FTF	Kills(FTF)	KR	Kills(FTF+KR)
1	FTF#1	X1	KR#1	Y1
2	FTF#2	X2	KR#2	Y2
3	FTF#3	X3	KR#3	Y3
...

(Seen Before)

(Combined Perturbation)

The “Memory Effect” As Correlation

- The variability of X and Y are essentially noise in the narrow parameter ranges employed.
- But the noise is “correlated” when a perturbation in one parameter is added to the perturbation of another



The Relationship Between DOEs and ANOVA Assumptions

An example
DOE:

Run	FTF Option	KR Option
0	base	base
1	improved	base
2	base	improved
3	improved	improved



Run	x1	x2
0	0	0
1	1	0
2	0	1
3	1	1

When slightly different performance parameter values are chosen (as by different analysts or rounding), we have:

	Try #1		Try #2		Try #3		...
Run(i)	FTF Value	KR Value	FTF Value	KR Value	FTF Value	KR Value	
0	base	base	base	base	base	base	
1	FTF#1	base	FTF#2	base	FTF#3	base	
2	base	KR#1	base	KR#2	base	KR#3	
3	FTF#1	KR#1	FTF#2	KR#2	FTF#3	KR#3	

Ultimately the model being estimated is

$$MOE_i = \alpha + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_i$$

But ε_3 is correlated with ε_1 and ε_2 due to the memory effect.

This is a deviation from the ANOVA assumptions



A Model Study That Should Have No Findings

- A small study with an expectation of zero findings is independently performed 64 times
- How often is “statistical significance” achieved?
- The true values of the parameters being estimated are zero
 - The deviations from the base case are small ($\pm 5\%$) and are centered on the base case values
 - The parameter settings are independently varied (as if a different analyst were assigned each time)

Run Number	FTF	KR	LA
1	-1	-1	-1
2	-1	-1	1
3	-1	1	-1
4	-1	1	1
5	1	-1	-1
6	1	-1	1
7	1	1	-1
8	1	1	1



More on the Model Study

- 64 independent repetitions of a 3-factor factorial design
- Factors
 - Fraction of time firing for a direct fire weapon system
 - “Kill rate” for a rotary wing system, and
 - An artillery munition lethal area
- MOE is kills by Blue’s high-value equipment of Red’s high-value equipment
- Analyzed using a 3-factor main effects unbalanced ANOVA (least squares regression) model
 - One observation is missing in a number of repetitions
 - More than one error d.f. for robustness of findings
- ANOVA results are analyzed statistically



Results For the ANOVA Overall F Statistic

Dependent Variable	Average P-Value	P-Value of F (mean=0.5)	Number Statistically Significant	t statistic (versus 5% level of significance)
Blue Kills Red Important Equipment	0.28	2.31E-09	9	2.069*
Red Kills Blue Important Equipment	0.27	1.00E-09	15	3.454**

- The chance of Type I error exceeds the alpha of 5%
- P-Values from ANOVA will not actually have the nominal statistical significance level
- Similar results were found for individual ANOVA model parameters



Conclusions and Way Forward

- **P-values from ANOVA applied to DOE data from deterministic combat simulations may not mean what one expects**
 - **Due to the “memory effect” seen in VIC**
- **The way forward is to employ perturbation to randomize the error terms in the ANOVA model**
 - **See Bulanow, et al 2004 MORSS presentation.**
 - **This erases the “memory effect” and assures error term independence for all practical purposes.**



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