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13. ABSTRACT (Maximum 200 Words) This is a final report on theoretical research on operation of magnetrons, crossed-field amplifiers, and other related crossed-field devices. The methods used were analytical and numerical. The analytical methods used were mathematical techniques designed to separate the problem into smaller parts, wherein each part could be solved analytically. Then these parts could be fitted together in a numerical routine, which would obtain numerical solutions to this problem. This report covers work done on the initiation stage of the A6 and work done on the saturation stage of a typical non-relativistic crossed-field amplifier. In the latter case, a closed form solution was obtained for the field variables in the region around one of the resonances. Expressions for the conversion, reflection, and transmission coefficients at this resonance were obtained. Other work related to soliton solutions in Type II second-harmonic generation and their stability have been reported on also, as well as methods for the quantitative estimations of variational approximations.				
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Theoretical Studies in Plasmas: Crossed-Field Devices & Ionospheric Plasmas
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OBJECTIVES

- A. We shall study the operation of relativistic, electromagnetic, cylindrical crossed-field devices, particularly the A6 magnetron in the initiation stage. Our first studies may not include periodic effects, but later studies will include periodic effects. We shall (i) obtain results for the prediction of growth rates and operating characteristics; and (ii) seek to detail and understand how each of the relativistic, and electromagnetic, and cylindrical effects affect the resulting density profiles and other operating characteristics.
- B. We shall return to the nonrelativistic, electrostatic, planar #T266, and study its operation in the saturation stage. We shall (i) obtain results for the operating characteristics; and (ii) compare with known experimental characteristics.
- C. We shall study the operation of relativistic, electromagnetic, cylindrical crossed-field devices, particularly the A6 magnetron in the saturation stage. We shall (i) obtain results for the operating characteristics; and (ii) seek to detail and understand how the relativistic, and electromagnetic, and cylindrical effects affect the resulting density profiles and other operating characteristics in this stage.
- D. We shall continue our studies of plasma interactions, linear and nonlinear, in the ionosphere. The specific objectives are: (i) description of the global linear solution for an rf electromagnetic wave injected into the ionosphere, including in the region where the cutoffs of the P-mode and the Z-mode cross; and (ii) description of electromagnetic intensity enhancements observed by HAARP when the radar beam is parallel to the Earth's ambient magnetic field.

STATUS OF EFFORT

A. This item has been completed and manuscripts have been published [P1,P3], detailing with our results for the cylindrical, non-periodic, fully relativistic case, and for voltages between $300Kvs$ and $500Kvs$. These papers are a summary of the equations in the relativistic case, and have presented numerical solutions of the important quantities, in the range of published experiments. There it is also noted two particular problems to the relativistic case. First, one cannot have cylindrically symmetric solutions in the DC (background) due to Maxwell's equations. As pointed out in that paper, as the radial current increases, there develops an azimuthal DC magnetic field, which has the eventual effect of creating longitudinal variations in the physical quantities. Thus in order to avoid a "3D problem", one must restrict the mathematical treatment to that of a longitudinally "thin" device. Even with that approximation, there was still a problem with determining consistent periodic solutions. For large currents, one could expect an rf multi-mode solution, which would then drive the DC background density away from perfect cylindrical symmetry. To treat this case, it was found that one would have to work with a background containing periodic variations. As a consequence of this, strictly speaking, a two-dimension model of a relativistic, electromagnetic, crossed-field device is not valid, unless such a device is infinitesimally thin. Then for the background to be cylindrically symmetric, it would have to be operating in a low current mode. How thin is "thin" for computational purposes, has not been addressed.

On Item A.(ii), it was determined that an understanding of the saturation stage would be of more importance, since our earlier results had indicated that it was the saturation stage which would most likely determine any "pulse shortening". We intend to return later to the interpretation of how these effects affect the operation of a device.

B. Saturation stage of magnetrons

1. Work on the WKB theory of the saturated stage of magnetron operation has been completed. A manuscript on this theory had been published in the proceedings of the "Frontiers of Nonlinear Physics" conference [C1]. The following are the major results from that manuscript. We now understand that there are five basic rf modes in the nonrelativistic magnetron, and not just the two potential modes, as has been supposed before. The three additional modes have been identified as a short wavelength drift mode and a short wavelength cyclotron mode (with two degrees of freedom), whose particle analog are the Slater orbits. Each of these three new modes have fast vertical oscillations, and will therefore generally propagate independent of the others, except when any two cross at a resonance. The operation mode of the device seems to require the short wavelength drift mode to cross the two potential modes in the region of the edge of the sheath. The excitation of the drift cyclotron mode seems to be responsible for an ultra-low noise operating mode of a magnetron. In the ultra-low noise operating state of the Varian CFA #T266, this resonance was deep inside the sheath and near the cathode.
2. To go beyond a WKB theory of the stationary stage, it is necessary to use something like a matrix WKB perturbation theory to reduce this fifth-order problem to a system which could be analytically analyzed and also numerically integrated. However, one must understand that working with fifth-order matrices is no simple matter, particularly if one is attempting to carry it out algebraically and analytically. Furthermore, this problem has a multiscale structure similar to that of a boundary layer problem, except that it is an internal resonance layer instead. Then the fact that the total system is a fifth-order set of linear ordinary differential equations (ODE), significantly complicated the solution process, even though at the resonance layer, one could reduce the system to third-order. There are other complications to this problem which are still not fully understood. Our first approach was to attempt to use matrix methods combined with a WKB perturbation theory. In this case, one expands about a set of five eigenvectors, which in the WKB limit, would be the WKB solution-vectors. Using these eigenvectors as a basis, one expands the general solution as a sum over the basis vectors, seeking to determine the equations for the amplitudes of these basis vectors. This approach does work fine as long as one is away from any resonance. However, near any

resonance, the WKB eigenvectors become generally either degenerate or singular. As a consequence of this, although the theory of this mathematical approach is sound, the resulting complexities have resulted in no satisfactory solution for this problem with this approach.

3. Thus matrix methods combined with WKB perturbations were not successful. However, another approach was taken wherein one would use WKB perturbation theory on each individual equation, which in this case proved to be a workable approach. Expanding the perturbed physical quantities in the five rf modes, one could then obtain an analytical solution in the region of the diocotron resonance. From this analytical solution we were able to obtain the conversion, reflection and transmission coefficients when any one of these modes passed through the diocotron resonance. This work has recently been published in the May issue of Physics of Plasmas [P4]. As an single example of the consequences of these results, let us assume that there were no fast drift modes between the cathode and the diocotron resonance. Then it follows that above the diocotron resonance there will always exist a fast drift mode, which will have been mode-converted out of the slow modes. At the present moment we do not understand how these fast modes affect the operation of a device. We do note that their vertical structure oscillates very rapid, so rapid that one should not expect to see them in PIC code results. We also do not know to what extent the normal turbulence level in a device would tend to average these modes. (The turbulence level is itself an unknown.) We do know that this is a new and unstudied phenomena in these devices.
4. Note that in B.3) above, one is only obtaining analytical expressions for the rf solutions in the region of the resonance. The real problem will be to numerically solve and couple the rf and dc solutions self consistently, inside and outside of any resonance region. For this to happen, we will develop the require numerical techniques for obtaining the WKB solution (in the region of validity) for the fast modes, and when we are not near a resonance, and independent of the fast modes, numerically integrate the two slow modes. At any resonance we will require techniques for integrating through the resonance and matching to the individual modes on the other side. This is work in progress.

C. This action awaits the results of Item B.

D. Ionospheric interactions

1. We have developed a Fortran code to determine the linear solutions of the EM wave in the ionosphere, at all heights, for a given electron density profile, and for given frequencies. That code is still incomplete. The difficulty in the coding lies in the logic necessary to determine when to change from the WKB method to the ODE method, and vice versa, for each and all possible combinations of the 4 modes present. Folded into this has to be the proper sorting of the modes above and below each resonance and cutoff, which further complicates the problem. Work on this section had to be suspended in favor of the above magnetron work.
2. However, the results in the above effort can be transferred over to the problem of the saturation stage of the magnetron. Although this problem has more modes, being a 5th-order system of ODE's, the resonance structure is much simpler than that in the ionospheric problem. In particular, in the magnetron problem, i) the fast modes never cross, ii) there can never be more than three locations where the modes cross, and iii) at any crossing, there will always be exactly one fast mode and two slow modes crossing. We do note that once the code is working for the saturation stage problem, then we would expect to be able to port sections of this code over to the ionospheric problem.
3. On the precursor problem (an rf wave propagating down through the ionosphere, taking a model of the ionosphere to be a box of fixed density, and the rf wave as a boxed sine wave), Mr. Galen Kaup has been making good progress. A literature survey brought out the fact that all previous work on precursors, have been in a frequency regime wherein one had strong variations in the dielectric constant. On the other hand, this problem is one wherein the operating frequency is so high that the dielectric constant is weak. However, one also is propagating the beam sufficiently far so that dispersion does have sufficient time to form a reasonably strong precursor. The current state of the work is that we have everything done, except for the evaluation of two constants of integration. The

integrals that define these constants involve multiscales, and therein lies the difficulty in evaluating them. However, work is progressing and they will soon become evaluated.

ACCOMPLISHMENTS/NEW FINDINGS

1. Our major results on the HPM problem are discussed above under Item B) above. We note here a comment and a generalization which arise out of those results.
 - a. A comparison of our model of the A6 with PIC code simulations have found the two results to be qualitatively similar. In particular there are distinct similarities in the structure of the spokes found in each case.
 - b. One of the major difficulties in basic research today is the fact that problems of current interest are often multiscale problems. Such problems require new methods of attack. Computational methods applied to multiscale problems too often become limited due to time and memory requirements. One method of attack we have found useful has been the use of a judicious combination of analytics and numerics. This is also the major reason why we have had success in modeling cross-field devices. In this approach, analytics are used to separate the problem into its different scales of length and time. Then the resulting equations can be successfully number-crunched by using only relatively low accuracy (and thereby rapid) software routines. Of course, it is also very important that the scales be separated out by their order of importances. The success of this approach does depend on correctly identifying the importances of the various scales.
2. We have been extending our understanding of the realm of nonlinear materials and interactions which could support solitons [P2]. Another nonlinear optical interaction to be found in nonlinear optical materials is called "Type II second-harmonic generation". This interaction is a parametric interaction, whereby two different modes (polarizations) at the same frequency could combine and create another mode at a higher frequency, the second harmonic. In general such a system will contain dispersion effects and Kerr-like nonlinearities as well. A model of this interaction has been created and the possible solitons which could exist in this model have been detailed. An interesting feature of solitons in this system is that many of the soliton solutions were found to be unstable. Furthermore, the decay mode of the unstable solitons was almost always a decay into some breather state. In other words, the majority of the stable nonlinear waves in this system appear to be breathers, not solitons. Thus this system has some unique properties which could provide unique interaction mechanisms for optical logic devices. The downside is that at the present moment, no studies of breather interactions, such as colliding breathers, have ever been done. Thus their potential use is currently unknown. Of the instabilities observed in this system, we note that a surprisingly different type of instability had occasionally been observed, which was essentially a chaotic-type of instability. In this instability, as one numerically integrated forward in time, using the initial conditions of the exact soliton solution plus a small random perturbation, one would not observe anything happening for a while. It would appear to be a stable solution. But suddenly one would observe a rapid growth in the shorter wavelengths cumulating with the soliton suddenly "shattering" into short wavelength fragments and pieces. Overall this is a very interesting system from the mathematical point of view, with new types of interactions occurring as well as containing a variety of different soliton and breather solutions.
3. Considering the instabilities observed in 2.) above, we also undertook, and have now completed, a study with Dr. V. Gerdjikov on how to predict these instabilities theoretically [S2]. The difficulty lies in the fact that the general system is twelfth-order, so one then needs to work with a twelfth-order linear ODE system. But this is an eigenvalue problem with a spectra parameter. Thus this is also equivalent to a twelfth-order inverse scattering problem. Then the stability of the solitons in 2.) above could be studied by using inverse scattering techniques. Now, inverse scattering problems beyond second-order are more complicated and are poorly understood in general. Thus we took on to give a full but simple description of how such higher order scattering (eigenvalue) systems need to be approached and studied, giving the theory and also discussing the numerical methods needed to locate any instability. One of the consequences of this is that systems larger than second-order will always have more than one "Evan's function".

4. Consider the Nonlinear Schrödinger equation (NLS). The important part of its solutions are the solitons. Each soliton has four key parameters: amplitude, phase, width and position. On the other hand, the number of degrees of freedom that any solution of the NLS possesses are many more degrees of freedom than this: four times a continuum. So if one only needed to describe NLS solitons and their interactions, then the NLS really has many more degrees of freedom than are needed. Then why should all this extra baggage be carried around by "soliton theory"? A simple answer is that these other degrees of freedom can cause a soliton to interact with the environment. Nevertheless, reduction in the number of degrees of freedom simplifies any problem. One way that this can be done is with a variational approximation based on the Euler-Lagrange equations. However what has been missing in the past has been some means to evaluate the validity of such an approximation. Such an evaluation can now be done quantitatively and is quite simple in concept. This is described in a manuscript that has been submitted for publication to Phys. Lett. A [S3].
5. Recently we have shown how to modify the well-known Inverse Scattering Transform (IST), in order to solve a long-standing problem in shallow water waves: the general solution of the Camassa-Holm equation, under the most general initial conditions. A manuscript on this is to appear in Stud. Appl. Math. [S1].
6. A very important nonlinear optical interaction is degenerate two-photon propagation (DTPP), because it simply takes two photons from the same beam, and combines them into a single photon, of twice the original frequency. Thus this can be a very fast interaction, basically since the photons used are from the same laser beam and are therefore always coherent. And this interaction can generate high intensity laser beams. Based on the above recent results with the Camassa-Holm equation, we now know how to solve the full DTPP equations. This work is being carried out in collaboration with Dr. Heinz Steudel, of Berlin, Germany. Manuscripts are currently being prepared which will describe this work.

PERSONNEL SUPPORTED

* Faculty: D.J. Kaup

* Other:

- + Prof. Subash Antani (Edgewood College, Madison, Wisc., consultant, nonlinear interactions in the ionosphere.)
- + Prof. Roy Choudhury (University of Central Florida, consultant, nonlinear interactions)
- + Dr. J.P. Gu (Institute for Simulation and Training, University of Central Florida, Orlando, FL, Subcontract to UCF for code writing and computer computations)
- + Ms. Irina Polandova (Employee, code writing and computer computations)
- + Mr. Galen T. Kaup (Research Assistant, mathematician)
- + Mr. Thomas K. Vogel (Research Assistant, mathematician)
- + Dr. V Gerjikov (Sofia, Bulgaria; consultant)
- + Dr. Heinz Steudel (Humboldt University, Berlin, Germany; consultant)

PUBLICATIONS

* SUBMITTED

* Books/Book Chapters

* Journals

- S1. *The Time Evolution of the Scattering Coefficients for the Camassa-Holm Equation* D.J. Kaup (to appear in Studies in Applied Mathematics)
- S2. *On linear stability of solitons in media with cubic and quadratic nonlinearities*, V. S. Gerdjikov and D. J. Kaup [submitted to J. Math. Phys.]
- S3. *Quantitative Measurement of Variational Approximations* D.J. Kaup and T.K. Vogel. (submitted to Phys. Lett. A)

* Conferences

* ACCEPTED

* Books/Book Chapters

* Journals

- P1. *Modeling an A6 Relativistic Magnetron with Analytics and Numerics*, Physica Scripta **T107**, 39-48 (2004).
- P2. *Three-wave solitons and continuous waves in media with competing quadratic and cubic nonlinearities*, Min Chen, D.J. Kaup, and Boris Malomed, Phys. Rev. E **69**, 056605 (2004).
- P3. *Theoretical Modeling of an A6 Relativistic Magnetron*, Physics of Plasmas **11**, 3151-64 (2004).
- P4. *Drift Resonance in High Density Non-neutral Plasmas*, D.J. Kaup, Phys. Plasmas **13**, 053113 (2006).

* Conferences

- C1. *Resonances in High Density Non-neutral Plasmas*, D. J. Kaup, "Frontiers of Nonlinear Physics, Proceedings of the Second International Conference", Nizhny Novgorod - St. Petersburg, Russia, 2004, edited by Alexander Litvak (Nizhny Novgorod, Russia, Institute of Applied Physics RAS, 2005), pp. 308-20.

* Participation/Presentations At Meetings, Conferences, Seminars, Etc

- + *Virtual Solitons*, Nonlinear Waves Seminar, Duke University, Raleigh, NC, April 18, 2002.
- + *Mathematical Foundations for Modeling and Simulation*, (with Gwendolyn Walton, Brian Goldiez, and Ronald Hofer), SPIE AeroSense 2003, Orlando, FL, April 22, 2002.
- + *Second Harmonic Generation as an Inverse Problem*, "SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives", Landau Institute for Theoretical Physics, Chernogolovka, Russia, August 20, 2002.
- + *Second Harmonic Generation as an Inverse Problem*, (with H. Steudel) "Southeastern Sectional AMS Meeting", University of Central Florida, Orlando, FL, Nov. 9, 2002.
- + *Discrete NLS Solitons in an Optical Network*, (with Lisa S. Cohen) "Southeastern Sectional AMS Meeting", University of Central Florida, Orlando, FL, Nov. 10, 2002.
- + *Modeling of a Relativistic Magnetron*, APS Plasma Physics Meeting, Orlando FL, Nov. 13, 2002.
- + *Introduction to Solitons*, Mathematics Dept. Colloquium, Purdue University, West Lafayette, Ind.; Jan. 14, 2003.
- + *Introduction to Solitons*, Mathematics Dept. Colloquium, University of South Florida, Tampa, FL; March 28, 2003.
- + *Discrete NLS solitons in an Optical Lattice*, Third IMACS Conference on "Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory", University of Georgia, Athens GA, April 7, 2003.
- + *Bifurcations and Competing Coherent Structures in Optical Systems*, (with Roy Choudhury and Nail Akhmediev) Third IMACS Conference on "Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory", University of Georgia, Athens GA, April 8, 2003.
- + *Painlevé's Analysis of Multicomponent Systems - An Algorithmic Method*, (with Roy Choudhury) Third IMACS Conference on "Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory", University of Georgia, Athens GA, April 8, 2003.
- + *International Turbulence Workshop*, Orlando, FL, May 19-23, 2003.
- + *ONR "Internal Solitons and their Impacts" Workshop*, College of William & Mary, Williamsburg, VA, July 24-25, 2003.
- + *Initiation of a Relativistic Magnetron*, International Topical Conference on Plasma Physics, ITCPP 2003: Complex Plasmas in the New Millennium, 8 September 2003, Fira, Santorini Island, Greece.
- + *Three-wave Solitons in Media with Competing Quadratic and Cubic Nonlinearities*, Mathematics Dept. Colloquium, University of Central Florida, Orlando, FL; March 23, 2004.
- + *Modeling Magnetrons*, PIC/High-Order PIC/Magnetron Workshop, Kirtland AFB, Albuquerque, NM; 3 May, 2004.
- + *Three-wave solitons and continuous waves in media with competing quadratic and cubic nonlinearities*, LANL, Los Alamos, NM; 10 May, 2004.

- + *Three-wave solitons and continuous waves in media with competing quadratic and cubic nonlinearities*, Max-Born-Institut For Nonlinear Optics, Berlin, Germany, 23 June, 2004.
- + *Nonneutral plasma resonances in crossed-fields*, "Frontiers of Nonlinear Physics", Nizhny Novgorod - St.Petersburg, Russia, 8 July, 2004.
- + *Variational methods as applied to discrete solitons / embedded solitons*, "Workshop on Mathematical Ideas in Nonlinear Optics", University of Edinburgh, Edinburgh, Scotland, 20 July, 2004.
- + *Nonneutral plasma resonances in crossed-fields*, "AFOSR Workshop on Nonlinear Optics", Tucson, AZ, 10 Sept., 2004.
- + *Variational Methods in Nonlinear Waves*, "FOURTH IMACS INTERNATIONAL CONFERENCE on NONLINEAR EVOLUTION EQUATIONS AND WAVE PHENOMENA: COMPUTATION AND THEORY", University of GA, Athens GA, April 11, 2005.
- + *Resonances in high density nonneutral plasmas*, "Amer. Phys. Soc. April Meeting", Tampa, FL, 17 April, 2005.
- + *Evolution of the scattering data for the Camassa-Holm Equation*, "Conference on Nonlinear Waves, Integrable Systems and their Applications", Colo. Sprgs., CO, June 5, 2005.
- + *Evolution of the scattering data for the Camassa-Holm Equation*, "Seventh International Conference on Geometry, Integrability and Quantization", Varna, Bulgaria, June 9, 2005.
- + *Evolution of the scattering data for the Camassa-Holm Equation*, "FPU+50 : NONLINEAR WAVES 50 YEARS AFTER FERMI-PASTA-ULAM..", Rouen, France, June 21, 2005.
- + *Lattice solitons and optical networks*, "Conference on Differential & Difference Equations and Applications", Melbourne, FL, Aug. 2, 2005.
- + *Drift Resonances in High Density Electron Plasmas*, "Nonlinear Optics Workshop", U of AZ, Tucson, AZ, Oct. 5, 2005.
- + *Drift Resonance in High Density Nonneutral Plasmas*, APS Plasma Physics Meeting, Denver, CO, Oct. 25, 2005.
- + *Using crowd-control simulations to predict the behavior of motile organisms*, J.E. FAUTH, D.J. KAUP, L. MALONE, Annual meeting of the "Society of Integrative & Comparative Biologists" (SICB), Orlando, FL, Jan. 8, 2006.
- + *Applying Crowd Control Models to Biological Systems*, (with J.F. Fauth, Rex Oleson II, Linda Malone, and Tom Clark), Dept. of Biology Colloquium, UCF, Orlando, FL, Jan 23, 2006.
- + *Quantitative Estimation of the Validity of Variational Approximations* (with Thomas K. Vogel), AMS Sectional meeting, Notre Dame University, South Bend, IN, April 8, 2006.

* Consultative And Advisory Functions To Other Laboratories And Agencies

- + Advisor to the Directed Energy Laboratory, Kirkland AFB.

* Transitions

NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES

None

HONORS/AWARDS

None