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14. ABSTRACT The research achievements of the project have been: 1.1 Control of conservative systems In the papers [17, 18] we presented a framework for developing necessary and sufficient conditions for controllability in a class of conservative systems with drift. Systems preserving a smooth measure on the phase space, such as Hamiltonian systems of classical dynamics or incompressible flows of fluid dynamics attract a lot of interest in control theory. In [17] we describe some work on the notion of controllability in systems that are measure-preserving and possess drift. Relationship between controllability, a fundamental concept in control theory, and the concepts of integrability and ergodicity, fundamental in dynamical systems theory is addressed. The basic idea is that studying recurrence (or ergodic) properties of trajectories of the drift is key to establishing necessary and sufficient conditions for controllability in such systems.					
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Final Report: Nonlinear Dynamics and Ergodic Theory Methods in Control AFOSR grant F49620-03-1-0096.

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January 1, 2003 to December 31, 2005.

Abstract

The objective of our research was to develop new control theory and dynamical systems/ergodic theory tools for the problems of active control of complex systems and control of mixing and apply the developed theoretical tools to problems of active control of mixing and particle control.

Year 1

At the PI's group, 1 graduate student (Umesh Vaidya) was working on the problem of control of discrete-time, conservative systems [33, 34] and quantum control [32]. Another student (Zoran Levnajić) worked on problems in visualization of dynamical systems [11]. A postdoc (Dmitri Vainchtein) continued working on the problem of flow control using tools from dynamical systems theory and vortex dynamics [38, 36] and pursued a problem in control of nanoparticle separation [37]. The PI worked on ergodic theory methods for control of systems with drift [17, 18] and optimization of mixing in a paper that will appear in the journal *Nature* [31]. The PI also worked on extending the framework for model validation of Random Dynamical Systems in the framework of the Koopman operator developed in [23] to dynamical systems treatment of uncertainty analysis [26]. There was an active exchange of information with researchers from the United Technologies Research Center on uncertainty analysis, bluff body combustor flow control [15] micromixing and jet noise.

Year 2

We have continued the work on control of complex nonlinear systems and pursued applications to control of mixing, control of nanoscale processes and control of microscale mixing. At the PI's group, 1 graduate student (Umesh Vaidya) was working on the problem of control of discrete-time, conservative systems [35]. Another student (Zoran Levnajić) worked on problems in visualization of dynamical systems. A postdoc (Dmitri Vainchtein) continued working on the problem of flow control using tools from dynamical systems theory and vortex dynamics [40] extending the previous study sponsored by AFOSR in [29] and pursued a problem in control using resonances in conservative systems [39]. The PI worked on developing methods for model reduction [21] extending the previous study sponsored by AFOSR [24], and optimization of mixing [14]. There was an active exchange of information with researchers from the United Technologies Research Center on uncertainty analysis in the context of seed DARPA funding administered by AFOSR [19, 42].

Year 3

We have continued the work on control of complex nonlinear systems and pursued applications to control of mixing, control of nanoscale processes and control of microscale mixing. We have pursued an investigation of a system of coupled oscillators that exhibits conformational transitions upon targeted, small external perturbations [22]. We have worked on operator theoretic methods in uncertainty analysis of conservative systems [9]. We have developed graph-theoretic techniques for analysis of dynamical systems on graphs [20, 43]. We have developed techniques for control of reactions in fluid flows in [8]. We have studied effects of resonances in three-dimensional fluid flows in [41] and completed the study of visualization of higher-dimensional dynamical systems in [12]. There was an active exchange of information with researchers from

1 Accomplishments

The research achievements of the project have been:

1.1 Control of conservative systems

In the papers [17, 18] we presented a framework for developing necessary and sufficient conditions for controllability in a class of conservative systems with drift. Systems preserving a smooth measure on the phase space, such as Hamiltonian systems of classical dynamics or incompressible flows of fluid dynamics attract a lot of interest in control theory. In [17] we describe some work on the notion of controllability in systems that are measure-preserving and possess drift. Relationship between controllability, a fundamental concept in control theory, and the concepts of integrability and ergodicity, fundamental in dynamical systems theory is addressed. The basic idea is that studying recurrence (or ergodic) properties of trajectories of the drift is key to establishing necessary and sufficient conditions for controllability in such systems. The benefit of this approach is that controllability proofs contain a constructive procedure for control. Control of Hamiltonian systems with drift is investigated for the case when the drift is integrable. Transformation of the system to action-angle coordinates is used to describe the ergodic partition of the drift. This is in turn used to obtain conditions for controllability of such systems. The key idea is that control must be capable of moving the system transverse to any set in the ergodic partition of the drift Hamiltonian vector field. Using this, additional results on controllability of more general systems are obtained.

In the paper [34] we study the controllability question for a class of discrete time nonlinear systems which arise as a discretization of a continuous time integrable Hamiltonian systems. We give necessary and sufficient condition for global controllability of these discrete time nonlinear systems under the assumption that system satisfies weak regularity condition. We also show that under these regularity condition the system is almost everywhere controllable. The result in this paper are an extension of results in [33].

1.2 Visualization of dynamical systems using harmonic analysis methods

A method for visualization of dynamical systems based on harmonic analysis was pursued in [11]. We considered a discrete-time dynamical system

$$\begin{aligned} x_{i+1} &= T(x_i), \\ y_i &= f(x_i), \end{aligned} \tag{1}$$

where $i \in \mathbb{Z}$, $x_i \in M$, $T : M \rightarrow M$ measurable and f a smooth real function on a compact Riemannian manifold M endowed with the Borel sigma algebra. We call the function f^* the time average of a function f under T if

$$f^*(x) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} f(T^i x)$$

almost everywhere (a.e.) with respect to the measure μ on M . The time average f^* is a function of the initial state x . This function can be used to visualize invariant sets [27]. Harmonic averages of the form

$$f_\omega^* = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=0}^{n-1} e^{i2\pi j\omega} f(T^j(x)),$$

can be used to visualize resonances in the system (invariant sets for higher iterates of the map). Such visualization for $n = 2, 3, 4, 5, 6$ iterate of the Standard Map, given by

$$T \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} x_t + y_t + \epsilon \sin(2\pi x_t) \pmod{1} \\ y_t + \epsilon \sin(2\pi x_t) \end{pmatrix},$$

is shown in figures on the next page.

1.3 Controlled capture into resonances

In the paper [37] we propose a method to use *capture into resonance* to control the behavior of a certain class of dynamical systems. In many dynamical systems the coupling between the unperturbed system and weak periodic perturbations (wave) can be reduced to a purely resonant interaction occurring in the vicinity of a certain surface in the phase space. While resonance interaction can change invariants of the unperturbed system (e.g. energy), it is random in nature, and, consequently, is rather inefficient as a mechanism of regular transport. We propose a method to structure the resonance interaction with little additional cost. When the nominal dynamics brings the system close to a resonance surface we apply a short control pulse to force the capture of a phase point into the resonance with the wave. A captured point is transported by the wave across the energy levels. We apply the second pulse to release a phase point from the resonance when the desired energy level is achieved. As a model problem we consider dynamics of a charged *nanoparticle* in an electromagnetic field.

1.4 Uniform, resonant chaotic mixing in fluid flows

In the paper [31] which will appear in the journal *Nature*, we pursued experimental confirmation of the theory developed in [16] sponsored in part by AFOSR. Laminar flows can produce particle trajectories that are chaotic, with nearby tracers separating exponentially in time. For time-periodic, two-dimensional and steady, three-dimensional (3D) flows, enhancements in mixing due to chaotic advection are typically limited by impenetrable transport barriers that form at the boundaries between ordered and chaotic mixing regions. However, for time-dependent, 3D flows, it has been proposed theoretically [16] that completely uniform mixing is possible via a resonant mechanism called singularity-induced diffusion (SID), even if the time-dependent and 3D perturbations are infinitesimally small. It is important to establish the conditions for which uniform mixing is possible and whether or not those conditions are met in flows that typically occur in nature. In the paper [31] we present experimental and numerical studies of mixing in a laminar, weakly 3D, weakly time-periodic vortex flow. An oscillating horizontal vortex chain is generated magnetohydrodynamically; the flow is weakly 3D due to a secondary flow forced spontaneously by Ekman pumping, a mechanism common in vortical flows with rigid boundaries. As predicted, completely uniform mixing is found, only for oscillation periods close to typical circulation times. In figure 4 we present experimental evidence of fast mixing of dye in the experimental apparatus.

1.5 Spectral Properties of Dynamical Systems, Model Reduction and Decompositions

Since we now understand that - barring a "blinding new technology" - the power of computers that will be available in the foreseeable future will not allow us to compute the details of physical interactions in many of the current problems in biological and physical sciences, such as molecular conformation or turbulence, the problem of *model reduction* has percolated to the top of the pile of open problems in Applied Mathematics. The number of different approaches in this direction is large, with some of the work relying on decompositions commonly used in probability theory - such as the Proper Orthogonal Decomposition (POD) (or Karhunen-Loeve, or Singular Value Decomposition), and other projection methods such as the Mori-Zwanzig formalism and optimal prediction, the formalism that involves replacing higher-order nonlinear terms with stochastic processes, scale-separation and averaging methods, balanced truncation methods developed for linear control systems, operator-theoretic projection methods and coarse time-stepping methods.

In the paper [21] we discuss two issues important for model reduction that are directly related to the asymptotic properties of the dynamics. The first is the relationship of the spectral properties of the dynamics on the attractor of the original, high-dimensional dynamical system with the properties and possibilities for model reduction. We review some specifics of the spectral theory of dynamical systems - in the form developed in [24]. We apply this theory to obtain a new type of the decomposition, that combines spectral and POD

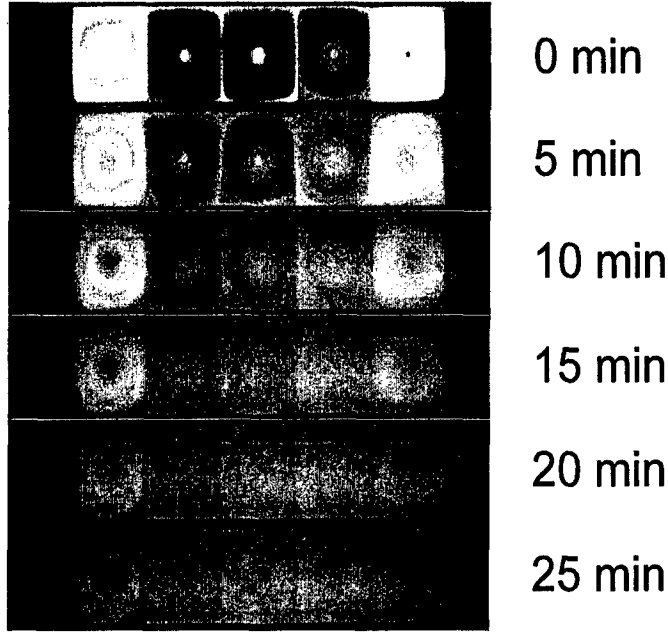


Figure 1: Time evolution of mixing in an electrohydrodynamically driven flow.

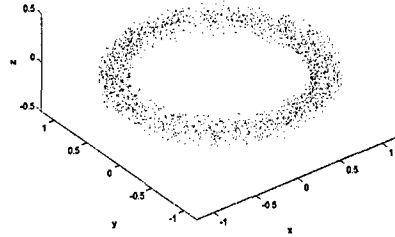


Figure 2: Representation of a dynamical process as a "random" cloud around a circle (contrast with POD representation that would approximate the data with a two-dimensional plane).

decomposition. The second topic we discuss is that of model validation, where the original, possibly high-dimensional dynamics and the dynamics of the reduced model - that can be deterministic or stochastic - are compared in some norm.

The key observation is that the dynamics on the attractor can be split into an almost periodic part and a part that has continuous spectrum. The almost periodic part of the Koopman operator leads naturally to the definition of the almost periodic mean of the process. The rest of the field has continuous spectrum. Thus every process is represented as a "noisy, m -dimensional cloud" around a deterministic dynamics on an s -dimensional torus, as depicted in figure 2.

1.6 Control of conservative systems: Controlled capture into resonances

In near-integrable Hamiltonian systems small "perturbations" can be used to reduce the cost of control, (see e.g. [35]). One of classical control objectives for near-integrable systems is to move a phase point from one invariant

manifold of the underlying integrable system to another, for example, to change the energy of a particle.

In a variety of near-integrable Hamiltonian dynamical systems progress in understanding the dynamics of the system can be achieved by reducing the coupling between the unperturbed system and weak periodic perturbations (waves) to purely resonant interactions occurring in the vicinity of a certain surface in the phase space. A wide range of applications of this technique includes energy exchange between coupled oscillators, mixing in fluids, Josephson junctions and dynamics of charged particles in electromagnetic fields. Theory of the most prominent resonance phenomena, *scattering on resonance* and *capture into resonance* was developed in Neishtadt's work. Although resonance interactions can substantially change adiabatic invariants, they are random in nature, and thus are rather inefficient as a control mechanism.

In the paper [40], we propose a method to structure the resonance interaction with little additional cost. As a model problem we consider the dynamics of charged particles in a uniform magnetic field and a weak electrostatic wave. When the internal dynamics brings particles close to a resonant surface a short control pulse forces the capture of selected particles into the resonance with the wave. Captured particles are transported by the wave across the energy levels. The second pulse is applied to release particles from the resonance when the desired energy level is achieved. We show that the proposed mechanism is very sensitive to the accuracy of positioning of the capturing pulse and thus can be used to affect a specific part of the resulting particle distribution function. The obtained results may be interesting not just for wave-particle interactions, but for a variety of problems where resonant interaction is important, and in practical problems such as particle separation. As a model problem we considered dynamics of a charged *nanoparticle* in an electromagnetic field.

The sensitivity of the capture to the match between the locations of the pulse and the resonance suggest possible applications of the control via capture. One of the possible applications is to separate particles of two different types that differ by mass only: m_1 and m_2 . We performed a set of simulations with mass ratio $m_2/m_1 = 1.05$. For the type "2" particles the pulse, that is synchronized with the type 1, is applied at a "wrong" moment. Consequently, only a few (of order of 3%) of the type "2" particles are captured due not to a control pulse, but the natural dynamics. Histograms of the final distributions of the values of an adiabatic invariant are shown in Fig. 3a and Fig. 3b for particles with of the first and second type, respectively. We aimed for the target value of the adiabatic invariant of 0.65. The distribution in Fig. 3a clearly has a peak near the target value. The distribution in Fig. 3b has a peak value of 1.97 near the original value of 2.0. Similarly, particles can be separated based on the initial discrepancy in energy or coordinates.

1.7 Visualization of resonances in dynamical systems using harmonic analysis methods

To be able to detect resonances in complex systems, whose importance in control has been emphasized in the preceeding paragraph, analysis of the method for visualization of dynamical systems based on harmonic analysis that we reported on previously was pursued. We considered a discrete-time dynamical system

$$\begin{aligned} x_{i+1} &= T(x_i), \\ y_i &= f(x_i), \end{aligned} \tag{2}$$

where $i \in \mathbb{Z}$, $x_i \in M$, $T : M \rightarrow M$ measurable and f a smooth real function on a compact Riemannian manifold M endowed with the Borel sigma algebra. We call the function f^* the time average of a function f under T if

$$f^*(x) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} f(T^i x)$$

almost everywhere (a.e.) with respect to the measure μ on M . The time average f^* is a function of the initial state x . This function can be used to visualize invariant sets. Harmonic averages of the form

$$f_\omega^* = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=0}^{n-1} e^{i2\pi j\omega} f(T^j(x)),$$

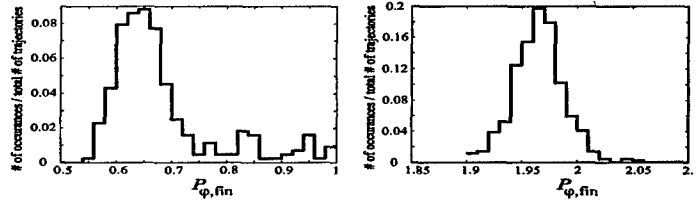


Figure 3: Histograms of the distribution of the final values of the adiabatic invariant.

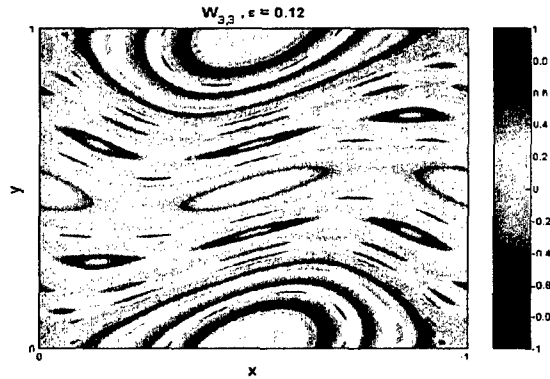


Figure 4: Phase portrait of the standard map obtained using a member of wavelet basis.

can be used to visualize resonances in the system (invariant sets for higher iterates of the map). In this period we have shown that the convergence rates of harmonic averages vary in regular regions as $1/n$, where n is the number of iterates and in chaotic regions as $1/\sqrt{n}$. These predictions were confirmed using numerical simulations. We tested wavelet basis for computation of multi-scale features of dynamical system phase portraits. Such a computation, when f is chosen to be a member of a wavelet basis, is shown in figure 4, where the detailed structure of resonances in the so-called standard map is shown).

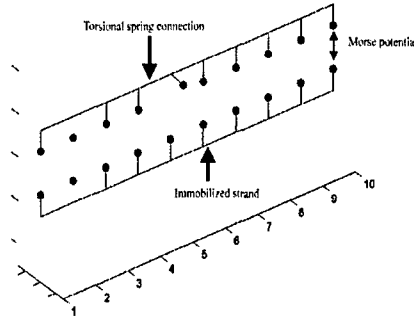


Figure 5: A schematic representation of the system.

1.8 Coupled oscillators

In [22] we showed that the reduced time between different conformations of a class of coupled pendulum models is due to features of the dynamics of these models that are the consequence of their structural (graph-theoretic) features. These properties are shared by a class of networked systems with strong local interconnections and long-range nonlinear forces that thus exhibit flexibility and robustness at the same time. The class of coupled oscillators that exhibits similar properties to those of the below described model are models of thermoacoustic instabilities.

Consider a simple model of a class of pendulum oscillator models that exhibit a strong circular backbone structure. Attached to the backbone there are side chains that are represented as a single mass on a pendulum attached to the backbone (see figure 5). The side chains are allowed to move in the plane orthogonal to the backbone and feel force due to the Morse potential interaction with the side chains of the other strand. The side chains are coupled to nearest neighbors by torsional spring forces. The attractive-repulsive force is derived from the Morse potential and it does not depend on the position of the side chain with respect to its nearest neighbors. Such models have been used for example for modelling of the coarse-grained dynamics of the DNA molecule [5, 30] and minimalist models of protein folding [7, 2]. The system described above has two global energy minima. In fact, the phase space of a single component of the system, when a side chain is isolated from its neighbors on the same strand is shown in figure 6 in the form of the contour plot of its Hamiltonian. This phase space, that is parametrized by the angle θ , and the associated angular velocity v , has two stable equilibria, positioned symmetrically with respect to $\theta = 0$. There are also two unstable equilibria, at $\theta = 0$ and $\theta = \pi$. Upon coupling of N such components, the state that occurs when all of them reside in the left equilibrium state is one of the two global energy minima, the other one occurring when all of the side chains reside in the right equilibrium state. A remarkable feature of this model system is capable of strongly amplifying localized disturbances and converting them into large-scale coherent motion. Consider the sequence of snapshots (a)-(h) in figure 7 obtained by simulating the system of 30 side chains. At the initial time, (a) one of the free side chains is forced into the repulsive region of the Morse potential. The energy so acquired, first gets distributed to other side chains sequentially on both sides of the localized perturbation (b,c,d). However, when the number of side-chains in motion is large enough (e), they self-organize and perform synchronized rotational motion around the backbone of the strand (f). Finally they perform a flip to the other side of the strand (g), thus changing the system's conformation. They then oscillate in the potential well on the other side of the strand (h) until another sufficiently strong local disturbance induces a new change of conformation. The essentials of the mechanism of transition can be understood by utilizing the dynamical systems theory of release from resonance and subsequent capture [28]. In the paper [39], we proposed a method to structure the resonance

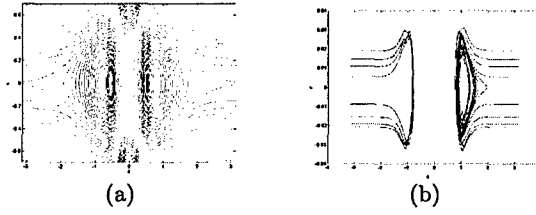


Figure 6: Left: The phase space of the Morse potential. Right: A trajectory of the whole system projected onto (θ, v) plane.

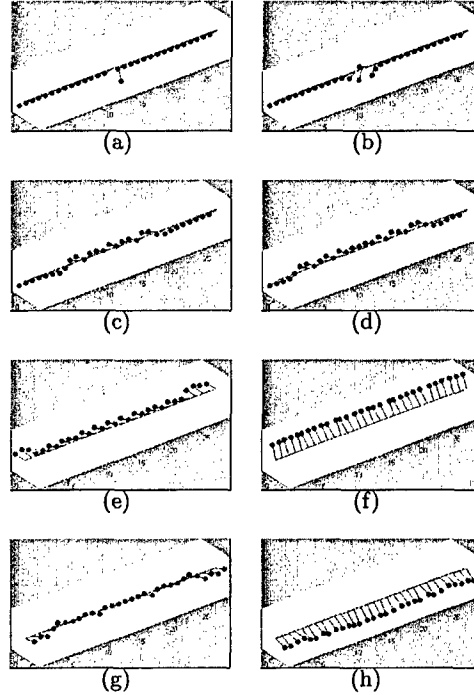


Figure 7: Sequence of system configurations in transition between the two global energy minima.

interaction with little additional cost. As a model problem we considered the dynamics of charged particles in a uniform magnetic field and a weak electrostatic wave. When the internal dynamics brings particles close to a resonant surface a short control pulse forces the capture of selected particles into the resonance with the wave. Captured particles are transported by the wave across the energy levels. The second pulse is applied to release particles from the resonance when the desired energy level is achieved. We show that the proposed mechanism is very sensitive to the accuracy of positioning of the capturing pulse and thus can be used to affect a specific part of the resulting particle distribution function. The obtained results are of interest not just for wave-particle interactions, but for a variety of problems where resonant interaction is important, such as the coupled oscillator model described above, that has a big (1-1-1...-1) resonance. As a model problem we considered dynamics of a charged *nanoparticle* in an electromagnetic field.

It is interesting that the dynamics of the model of coupled oscillators described above has a simplified description in a specific set of coordinates that can be revealed using graph-theoretic considerations described in the next section. Consider the average angle of the system defined by $\theta = (1/N) \cdot \sum_{k=1}^N \theta_k$. To witness this, it is instructive to compare the single side-chain phase space with the trajectory of the whole system projected onto (θ, v) plane, shown on the right in figure 6.

The system shows very interesting behaviour in this projection. It essentially follows the contours of the constant Hamiltonian around the global energy minima, with small kicks taking it off one of the contours onto another. This indicates that the phenomenon is low-dimensional and thus model reduction is possible.

1.9 Geometrical methods and graph theory

Geometrical methods in the theory of dynamical systems have a long and distinguished history [1, 6]. Asymptotic-in-time dynamics is encoded in the concept of attractors. Dynamics close to attractors is governed by its local stable and unstable directions. More generally, many aspects of system dynamics are encoded in the so-called stable and unstable foliations of a system [13]. While these concepts have been very successful for treatment of low-dimensional systems, they are far less developed for large-scale interconnected systems. However, the low-dimensional concepts and the theory of dynamical systems on graphs can be coupled fruitfully: graph decompositions allow for splitting of a large-dimensional system in low-dimensional strongly connected components on which geometrical methods indicated above can be used, as we have shown in [20, 43].

We start with the following definitions establishing the mathematical framework for the analysis of *dynamical systems on graphs*: Let (X, μ, T^t, P) be a dynamical system and $\mathcal{N} = 1, \dots, N \subset \mathbb{Z}$ where $X = \bigotimes_{i \in \mathcal{N}} X_i$, each X_i a compact subset of $\mathbb{R}$. Let $\Pi_i : X \rightarrow X_i$ be the usual projection to the i -th component. In addition, $\mu = \bigotimes_{i \in \mathcal{N}} \mu_i$ where μ_i is a probabilistic measure on X_i and $T^t(x, p) : X \times P \rightarrow X$ is either a discrete-time family induced by a map (in which case $t \in \mathbb{Z}$) or a flow of a system of first-order autonomous ordinary differential equations. The parameters p are defined on $P = \bigotimes_{i \in \mathcal{M}} P_i$, where each P_i a compact subset of $\mathbb{R}$. The change in x_i is given by a function $f_i(x)$, where $x \in X$. This, of course is the i th component of either the vector field $f(x) = d/dt|_{t=0}(T^t(x, p))$ or $T^1(x, p) - x$, in the case $t \in \mathbb{Z}$. The Jacobian matrix is defined as

$$J(x, p) = Df(x, p),$$

or, in components,

$$J_{ji}(x, p) = \frac{\partial f_i}{\partial x_j}(x, p),$$

where j represents the column and i the row of the matrix.

We define the following matrix $M(x, p)$ from $J(x, p)$:

$$\begin{aligned} M_{ji}(x, p) &= \frac{1}{l_i} \text{ if } |J_{ji}(x, p)| \neq 0, \\ M_{ji}(x, p) &= 0 \text{ otherwise.} \end{aligned} \tag{3}$$

where l_i is the number of non-zero entries in row i . Note that an arbitrary state-dependent matrix $A(x)$ could be transformed to a stochastic matrix this way. Many different systems will have the same matrix M and thus the transformation does not differentiate all the features of systems. However, some important features of the system are encoded by properties of M . For example, once we make the transition from a high-dimensional nonlinear dynamical system $\dot{x} = f(x)$ to the matrix M , the simplest of tools from spectral graph theory can already be helpful: the depth-first decomposition can be used to transform the system into a hierarchical form that substantially simplifies model computation and considerations of uncertainty [20, 43]. In figure 8 we show the graph representation of the coupled oscillator network introduced above. It is clear that the top two "averaged" states play the dominant role in the dynamics, confirming the dynamical systems considerations stated above from the graph-theoretic perspective. The dynamics is efficiently represented by two states and thus the dimensionality reduction achieved is very large, and also allows for simplified understanding of uncertainty propagation in such systems.

1.10 Uncertainty analysis

In [9] we studied the effect of uncertainty, using random perturbations, on area preserving maps of $\mathbb{R}^2$ to itself. We focused on the standard map and a discrete Duffing oscillator as specific examples. We related the level

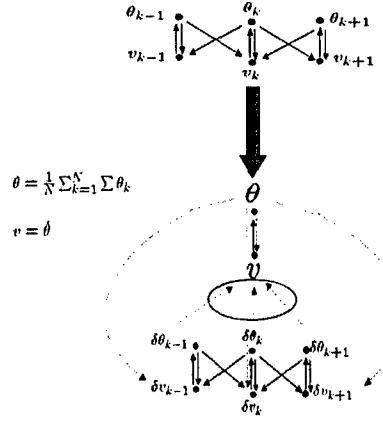


Figure 8: Graph-theoretic decomposition of the coupled oscillator network model.

of uncertainty to the large scale features in the dynamics in a precise way. We also studied the effect of such perturbations on bifurcations in such maps. The main tools used for these investigations are a study of the eigenfunction and eigenvalue structure of the associated Perron-Frobenius operator along with set oriented methods for the numerical computations.

In [25] we developed the theory of uncertainty propagation in nonlinear dynamical systems starting from a monolithic, phase-space approach. In that paper we were interested in defining and calculating the effect of probabilistic uncertainty in input parameters and initial conditions on the output of a dynamical system that may be far from linearity, has multiple steady states and exhibits nonlinear behavior such as bifurcations that depend on the uncertain parameters of the system. We developed an approach that involves defining uncertainty propagation in the system through the invariant measures of the system and defining uncertainty as a worst-case distance from a delta measure in the space of output measures:

Definition 1 Let $|\cdot|$ denote a metric (or a pseudo-metric) on the space $\mathcal{M}$ of probability measures on $\mathbb{R}$. For any measure $\varpi \in \mathcal{M}$ the uncertainty of ϖ induced by $|\cdot|$ is defined by

$$u(\varpi) = \min_{z \in \mathbb{R}} |\varpi - \delta_z|,$$

where δ_z is the Dirac delta distribution at z .

A possible pseudo-metric is defined through use of cumulative distribution functions F_ϖ , defining

$$|\varpi_1 - \varpi_2| = \|F_{\varpi_1} - F_{\varpi_2}\|_1,$$

where $\|\cdot\|_1$ is the L^1 norm. This is well defined for measures with compactly supported P_ϖ .

The uncertainty measure u has distinct advantages over variance in describing uncertainty in non-Gaussian probability distributions that commonly occur in nonlinear dynamical systems. Related, but nevertheless quite different concepts have been considered in the reliability literature [4], where uncertainty is defined in terms of a distance between cumulative distribution functions, and in the atmospheric sciences [10], where uncertainty (called there predictability) is defined as the relative entropy between uncertain and certain measures.

2 Personnel supported:

Faculty: Igor Mezić, **Postdoctoral fellow:** Dmitri Vainchtein, **Partially supported graduate students:** Umesh Vaidya, Thomas John, Zoran Levnajic, Andre Valente.

3 Interactions/transitions:

3.1 Year 1

3.1.1 Academic interactions/Transitions

The PI and other members of the group gave a number of invited lectures on the topics of research described here, for example at Northwestern University, Caltech, SIAM Conference on Applications of Dynamical Systems, UCLA Institute for Pure and Applied Mathematics etc.

3.1.2 Industrial interactions

There was an interaction with UTRC's Control and Dynamics group and Combustion Research and Flow Technology on topics in flow control and uncertainty analysis

3.1.3 Transitions

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Control of combustion instabilities presented in [15]. 2. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Mark Myers. *Result:* Planning and technical contributions to the uncertainty analysis program.

3.2 Year 2

3.2.1 Academic interactions/Transitions

The PI and other members of the group gave a number of invited lectures on the topics of research described here, for example at University of Chicago, Stanford University, AIMS Conference on Mathematical Sciences etc..

3.2.2 Industrial interactions

There was an interaction with UTRC's Control and Dynamics group on topics in flow control and uncertainty analysis

3.2.3 Transitions

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Planning and technical contributions to the uncertainty analysis program sponsored by DARPA and administered by AFOSR.

3.3 Year 3

The PI and other members of the group gave a number of invited lectures on the topics of research described here, for example at MIT, Caltech, Snowbird SIAM Conference on Applications of Dynamical Systems etc.

There was an interaction with UTRC's Control and Dynamics group on topics in control and uncertainty analysis of thermoacoustic instabilities.

3.3.1 Transitions

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Successful jet engine test of thermoacoustic instability control concept derived from coupled oscillator models.

4 Honors/Awards

The PI gave an invited plenary lecture at SIAM Control Theory Meeting, New Orleans, 2005,

The PI became gave an invited plenary lecture at Dynamics Days Europe 2005, Palma de Mallorca, Spain.

The PI became an Editor for Journal of Applied Mechanics.

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