A TUTORIAL AND WORKBOOK ON THE STATISTICAL ANALYSIS OF DATA AND QUALITY ASSURANCE FOR THE SHIPBUILDING INDUSTRY

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16. Abstract

The Tutorial/Workbook was developed in conjunction with a set of four (4) videotapes to convey the need and applicability of statistical quality control concepts to the shipbuilding industry. The workbook can be used alone or in conjunction with the videotapes which are available through the Audio Visual Material Available for Shipyard Training (AVMAST) catalog. Inquiries should be made to: Transportation Research Institute, University of Michigan, 2901 Baxter Road, Ann Arbor, Michigan 48109. Attn: AVMAST Coordinator. The titles of the video tapes are: (1) The Deming Philosophy of Modern Management, (2) Statistical Control Charts, (3) Statistical Techniques for Discrete Random Experiments, (4) Statistical Techniques for Continuous Random Experiments.

The Tutorial/Workbook is the result of a project developed by the Ship Production Committee's Education and Training Panel (SP-9) under the auspices of the University of Michigan Transportation Research Institute, Marine Systems Division. Funding and support for the project was through the National Shipbuilding and Research Program of the U. S. Department of Transportation by direction of the Maritime Administration.

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Dedicated To The Memory

Of

David C. Sonderman

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PREFACE

This workbook is a result of a growing awareness on the part of the Ship Production Committee's SP-9 Education Panel that both the Deming philosophy of modern management and statistics should be used more extensively in the United States shipbuilding industry. The workbook was written as a supplement to four educational videotapes on: the Deming philosophy of management, the statistical control chart, statistics for discrete random experiments, and statistics for continuous random experiments. Although the workbook is self-contained, it is recommended that the four educational m~dules be viewed prior to reading the workbook. The educational modules present the subject matter in a less mathematical way and with a slightly different viewpoint. They should, therefore, provide a basic introduction which will assist you in reading the workbook.

The workbook is designed to reach a wide variety of people with a range of educational backgrounds. For the person who is interested in only an overview of the subject matter, the authors recommend that only Chapter 1 be studied. For those who wish to learn the basics of statistics and to understand the theory behind the statistical control chart, it is recommended that the entire workbook be studied.

ACKNOWLEDGMENTS

The authors of this workbook would like to thank all of those people from the shipbuilding industry who patiently gave their time to provide input to the development of both the educational modules and the workbook. In particular we wish to thank the members of the advisory panel: Mr. C.G. Higgins of Peterson Builders, Inc., Mr. D. Rananchuk of Bethlehem Steel Shipyard, Mr. J. Berger and Mr. K. Witmer of Bath Iron Works, and Mr. R. Baseler of General Dynamics Quincy Shipbuilding Division.

1. INCENTIVES FOR LEARNING STATISTICS - THE DEMING APPROACH TO MODERN MANAGEMENT AND THE STATISTICAL CONTROL CHART

1.1 The Deming Philosophy of Modern Management

In 1950, at the invitation of the Japanese Union of Science and Engineering, Dr. W. Edwards Deming introduced to Japanese industrialists his approach to modern management. Since that time Dr. Deming has demonstrated repeatedly that his philosophy works and that all organizations, both large and small, can benefit from the approach. As a result Dr. Deming is being referred to by many individuals as the father of the third wave of the industrial revolution. The first wave involved the use of factories and modern machinery (e.g., Eli Whitney); the second wave dealt with mass production (e.g., Henry Ford); and the third wave is characterized by the use of statistical methods to improve quality (e.g., W. Edwards Deming). In his series of videotapes and in his textbook [11 it is stated repeatedly that quality is the responsibility of everyone in the organization and that quality must be led by management. A specific approach for implementing the Deming philosophy of modern management is described in a recent series of videotapes by Dr. Myron Trybus [2]. "Management's Five Deadly Diseases" and "Roadmap for Change," two videotapes produced by the Encyclopedia Britannica Educational Corporation [3][4], provide an excellent introduction to the Deming approach to modern management and give a case study on its implementation. These tapes should be viewed by individuals at every level of management and by every worker.

In the Deming approach managers must take the position that an improvement in quality will lead to an increase in productivity and to a decrease in costs. This position is directly opposite to traditional thinking, in which managers assumed that higher quality could be achieved only by decreasing productivity and by increasing costs.

In a modern quality assurance program it is assumed that the customer is the ultimate judge of quality and that the hidden cost of poor quality must be taken into account. When a company manufactures a product of poor quality, one of two situations is encountered:

- o The customers don't complain and they go elsewhere, or,
- o The product comes back but the custaners don't.

Managers who fail to take into account the hidden cost of poor quality are considered by Dr. Deming to have one of the five major diseases of management. The quality of the process, which includes product design, materials, and production systems, can be evaluated more clearly by management. Furthermore, with leadership by management, the quality of the process can be controlled and improved. This can be achieved only if everyone in the organization understands the system; if everyone becomes an inspector; and if everyone down the line is treated as a customer. Management must lead the way to improve the system and to achieve the ultimate objective of producing a product without any final inspection.

Any organization, including for-profit organizations which either manufacture a product or provide a service and those which operate on a non-profit basis, can be viewed as an extremely complicated system that consists of a huge number of operating subsystems. Each subsystem operates with inherent variability and therefore contributes to the inherent randomness of the overall organization. The Deming approach to modern management recognizes this situation and, furthermore, it recognizes that the worker is in the best position to understand a particular subsystem and to make suggestions for improvement. In order to search for improvements, both the worker and management must communicate effectively in the common language of statistics.

In the Deming philosophy modern managers must be willing to admit that 80-85 percent of all variability is a direct result of the subsystem itself and of the interactions between subsystems. Most variability is not the result of poor performance on the part of the worker. The modern approach is to assume that any adversarial relationship between management and the worker is harmful to the overall purpose of the organization: namely, to continue to stay in business by providing a better product or service at a lower cost. The lack of a "constancy of purpose" is also considered to be one of the five major diseases of management. The worker and management must communicate effectively in the language of statistics, and the communication must take place in a cooperative environment that is free of fear and punishment.

In a modern quality assurance program, the role of management must be changed to that of improving the system. Deming suggests that traditional "management by objectives" be changed to that of "management by walking around" (MBWA). From the chief executive officer down to the production supervisor, MBWA must become the way of doing business. This approach requires major changes in the attitudes of everyone. Corporate executives will be required to develop a corporate goal statement in which quality is emphasized and nunerical goals related to growth and profit are de-emphasized. The corporate goal statement should become a document that all people in the organization can support with enthusiasm.

Those who practice MBWA will be responsible for locating and eliminating all barriers to quality and productivity. One of these barriers is the production quota. In the Dening approach all production quotas are considered to be detrimental to the task of improving quality and productivity. Similarly, the imposition of higher quality standards (e.g., zero defects) without any change in the system is considered to be unacceptable practice. Artificial quality standards, production quotas, and performance evaluation based on annual dividends and profit, all cr eate an environment in which the worker and management are afraid to do the job. Fear is considered to be a major barrier to improved quality and productivity. In MBWA, the manager?s job is to identify all situations which create fear and to change the system to eliminate fear. Creativity, responsibility, risk taking, and honest communication are all reduced by fear and, as a result, fear is considered to have a negative impact on quality. In the past, managers who gained experience at several companies over a short period of time were considered to have upward mobility. This practice created managers who were unfamiliar with the sytem they were assigned to manage. It also enhanced the practice of management by objectives with annual merit rating and an emphasis on short-term profits. Mobility, annual merit rating, and emphasis on short-term profits are considered by Dr. Deming to be three more of the five major diseases of management.

Another major barrier to improved quality and productivity is found in the traditional practice of purchasing from suppliers who provide the lowest bid. In the Deming philosophy modern managers must make a sincere effort to reduce the nunber of suppliers and to do business with only those who can demonstrate a dedication to quality. In this approach the purchasing agent becomes directly involved and specifications to suppliers are written to emphasize quality.

Production quotas and fear are two factors that interfere **With** pride of workmanship. In the Deming philosophy modern managers must identify all factors that have a negative effect on pride of workmanship. These factors must be eliminated and an environment must be created to enhance pride of workmanship.

Dr. Deming's approach is characterized by the following 14 points:

1. Create constancy of pur pase toward improvement of product and service, with a plan to become competitive and to stay in business.

2. Adopt the new philosophy. We are in a new economic age. We can no longer live with commonly accepted levels of delays, mistakes, defective materials, and defective workmanship.

3. Cease dependence on mass inspection. Require, instead, statistical evidence that quality is built in, to eliminate need. for inspection on a mass basis. Purchasing managers have a new job, and must learn it.

4. End the practice of awarding business on the basis of price tag. Instead, depend on meaningful measures of

quality, along with price. Eliminate suppliers that cannot qualify with statistical evidence of quality.

5. Find problems. It is management's job to work continually on the systan (design, incoming materials, composition of material. , maintenance, improvement of machine, training, supervision, retraining).

6. Institute modern methods of training on the job.

7. Institute modern methods of supervision of production workers. The responsibility of foremen must be changed from sheer numbers to quality. Improvement of quality will automatically improve productivity. Management must prepare to take immediate action on reports fran foremen concerning barriers such as inheri ted defects, machines not maintained, poor tools, fuzzy operational definitions.

8. Drive out fear, so that everyone may work effectively for the company.

9. Break down barriers between departments. People in research, design, sales, and production must work as a team, to foresee problems of production that may be encountered with various materials and specifications.

10. Eliminate numerical goals, posters, and slogans for the work force, asking for new levels of productivity without providing methods.

11. Eliminate work standards that prescribe numerical quotas.

12. Remove barriers that stand between the hourly worker and his right to pride of workmanship.

13. Institute a vigorous program of education and retraining.

14. Create a structure in top management that will push every day on the above 13 points.

The Deming approach to modern management is being recognized as a positive step in the future of the U.S. shipbuilding industry. This workbook is a result of that recognition. It is designed to teach statistics with examples frcm the shipbuilding industry and to use the principles of statistics to explain the concept of a statistical control chart.

The ultimate objective of this workbook is to give those individuals who plan to use statistical methods in the shipbuilding industry a practical level of understanding of a widely used graphical technique called the statistical control chart. The authors of this workbook believe firmly that a sound level of understanding of the statistical control chart can be developed only after the student is exposed to an extensive introduction to basic concepts in statistical methods. For this reason the workbook is devoted almost entirely to basic concepts of statistical analysis. After the student develops an understanding of basic statistics, the control chart can then be understood without any difficulty as a straightforward application of statistical methods.

In order to provide the incentive for learning basic statistics and, for those individuals who are interested in a basic introduction to statistical control charts, the concept of a control chart is discussed in this section of the workbook in a non-mathematical way.

1.2 Statistical Control Charts

In a broad sense a statistical control chart is a graphical technique for monitoring, periodically, an operating system in the shipyard to determine, quickly, if something out of the ordinary has taken place. It is a tool that provides basic information to someone who has the authority to take corrective action when information is received to indicate that something is going wrong. The control chart, when used properly, gives the decision maker the signal that something is going wrong. It should be understood that the control chart, when used in a modern and progressive quality assurance program, is not a technique for taking punitive action against a worker. The statistical control chart is only a technique for monitoring an operating system and to give some decision maker (hopefully the worker) a signal to take corrective action. The corrective action may include an adjustment to the operating system; the institution of a special training program; a complete change in the operating system; a heart-to-heart discussion with vendors; etc. It should also be emphasized that a statistical control chart can be used in a variety of situations, which include accounting, billing, secretarial services, production systems, engineering, shipping, incoming supplies, welding, cutting, pipe construction, electrical systems, fabrication, painting, etc. The list of applications will increase as more and more individuals from different departments are exposed to the technique. You should keep in mind as you are exposed to the concept of a statistical control chart that numerous types of control charts are used in a progressive program to apply statistical methods. In this introduction you will only be exposed to a small sample of the overall picture. Without a sound knowledge of basic statistics, you will not understand the details of the control chart. You should, however, develop an understanding of the significance of the control chart and *you* should begin to see its areas of application.

In order to illustrate the basic concept of a control chart, we begin with an example:

Suppose we have the following operating system and that we wish to monitor its behavior.

The operating system

At exactly 12 noon each working day you eat at the shipyard cafeteria. Each day with your main course you order a side dish of peas and carrots and, since You 1ike peas but are partial to carrots, you decide to monitor the average nuuber of carrots over a five-day period.

The basic characteristics of the operating system

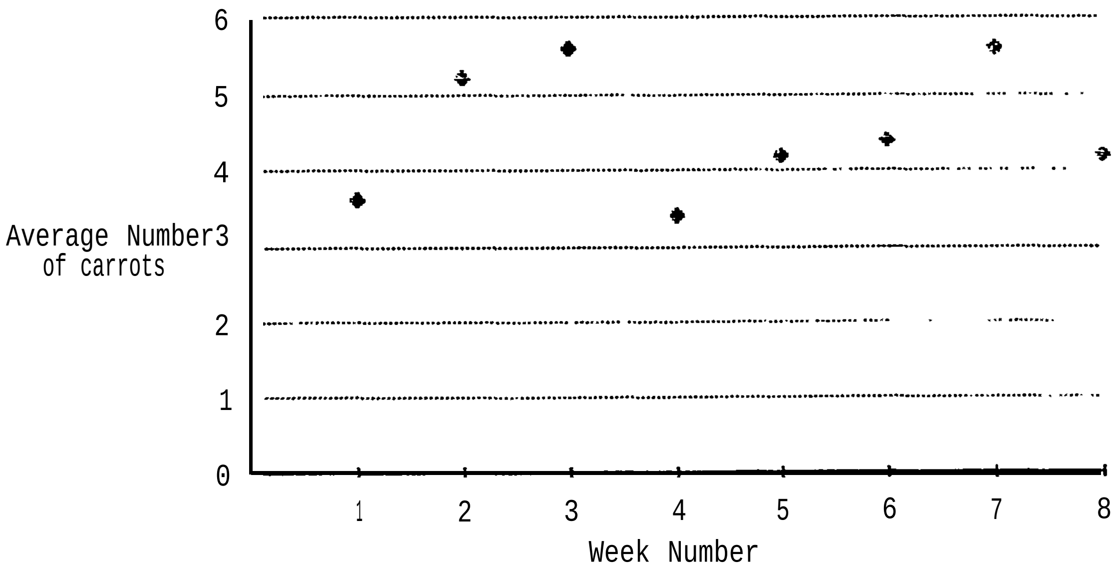
First of all you imagine the scoop of peas and carrots you receive each day as a random experiment where the variable of interest is the number of carrots. Each day this experiment is repeated under essentially the same conditions and has a variable outcome, namely, the number of carrots. Each day you might appear somewhat strange to your colleagues as you count the nunber of carrots, but you nevertheless accept the challenge. The random variation in the nunber of carrots is scmething you have to live with-it’s part of the system--you must learn to cope with the situation.

The monitoring system

Each day you count the number of carrots and, after a five-day period on Friday after lunch, you determine the average number of carrots. After the first eight weeks you see the results which are given in Figure 1-1.

This example represents a typical operating system which is being monitored by a graphical display. The characteristics of the operating system are:

1. A random outcome (number of carrots)



Average Number of Carrots - 8 Weeks
FIGURE 1-1

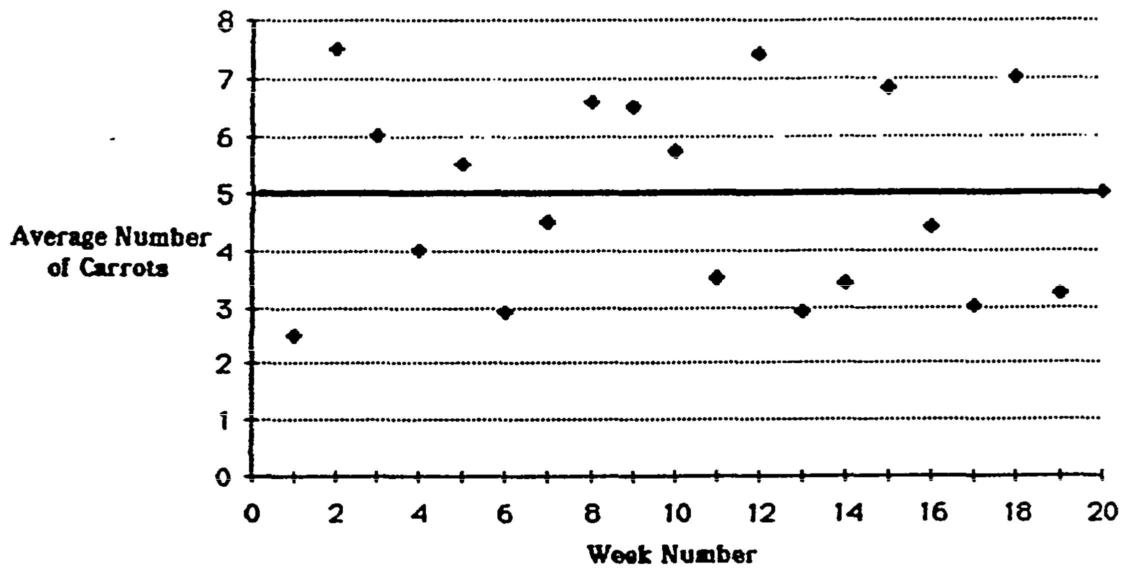
2. Repeatability under essentially the same conditions

3. No control over the random outcome, and

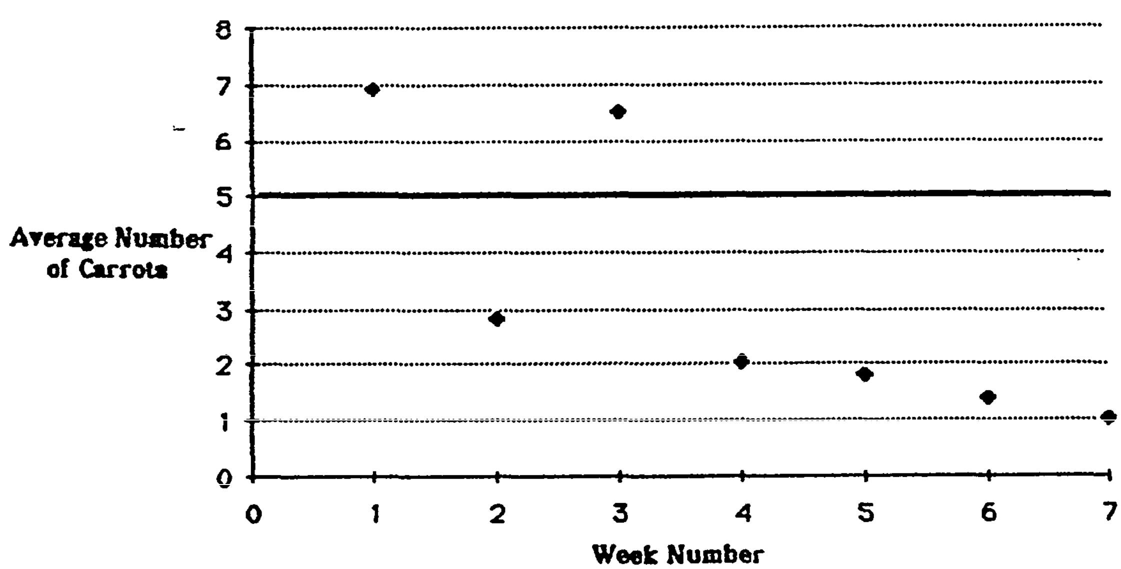
4. A variable (the average number of carrots) to be computed and plotted at specific points in time (e.g., end of each week).

If the operating system is in a "state of statistical control" you would expect the average nunber of carrots to bounce around some line that you could draw on your graphical display. For example, if you observed this system over a long period of time, say 20 weeks, and if it showed a pattern similar to the one in Figure 1-2, you might compute the average of the 20 average values and use that value as the sol id line shown in Figure 1-2. By this exercise you have established a base line for a system which appears to be operating in a state of statistical control.

Now assume that you are satisfied with your first 20 weeks of analysis and, using the base line you have established, you decide to monitor the system starting all over again with the end of next week as week nunber one. Suppose that after the next seven weeks you observe the situation shown in Figure 1-3. For the first three or four weeks the systan appears to be bouncing around the way it did before. In weeks 5, 6, and 7, however, something strange is happening. If you like carrots yoIR' graphic display is sending you a signal that you might not like. From your point of view the signal is suggesting that sane corrective action is necessary. Perhaps you might talk to the chef or you may have to train the person who is doing the scooping to work in a different manner. Then again, at this point you might argue that what happened over weeks 5, 6, and 7 is perfectly natural and you should do nothing. You then say, "Wouldn't it be great if, in addition to the center 1ine, I could draw on the graph two other lines--one above the



Average Number of Carrots - 20 Weeks
FIGURE 1-2



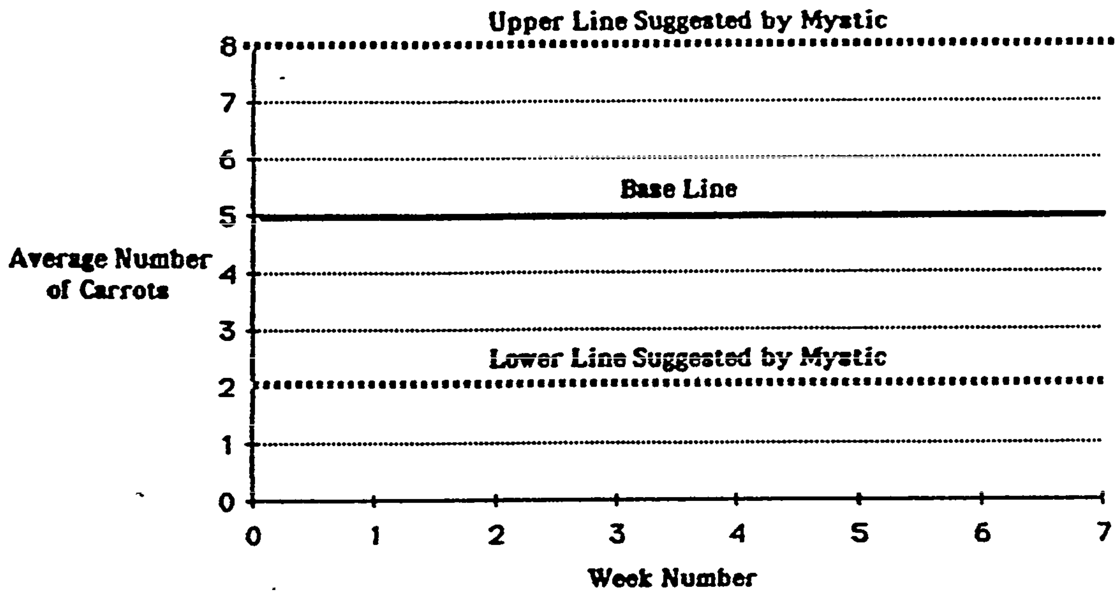
Average Number of Carrots - 7 Weeks
FIGURE 1-3

center line and one below the center line in such a way that when the operating system is in control any average value has a very high chance of falling between the upper line and the lower line." At this point you consult your local mystic, who suggests that you can meet your objective by using the lines shown in Figure 1-4. You now replot your average values for the seven weeks on this new graphic display and produce the results shown in Figure 1-5. At this point with your new graphic display you now have a more powerful tool to monitor the system. You can now argue that because of the way the chart was constructed, the averages observed at the end of weeks 5, 6, and 7 are rare events and suggest strongly that something is wrong:

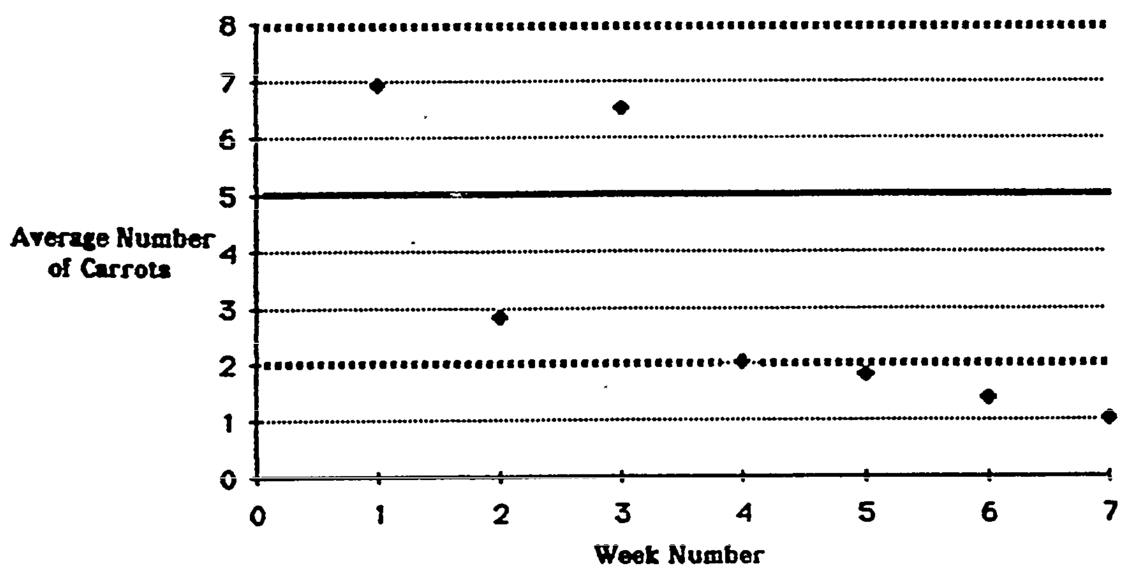
The example we have just considered is typical of the situation that is encountered in the use of statistical control charts. In general terms, a statistical control chart can now be defined. The general characteristics of a statistical control chart are shown in Figure I-6 and are as follows:

1. The Center Line (CL) (The base line in the carrot example)
2. The Upper Control Limit (UCL)
3. The Lower Control Limit (LCL)
4. Time Point or Subgroup Number on the horizontal axis
5. The value of a Variable to be plotted on the vertical axis and to be computed at different points in time or for different Subgroups.

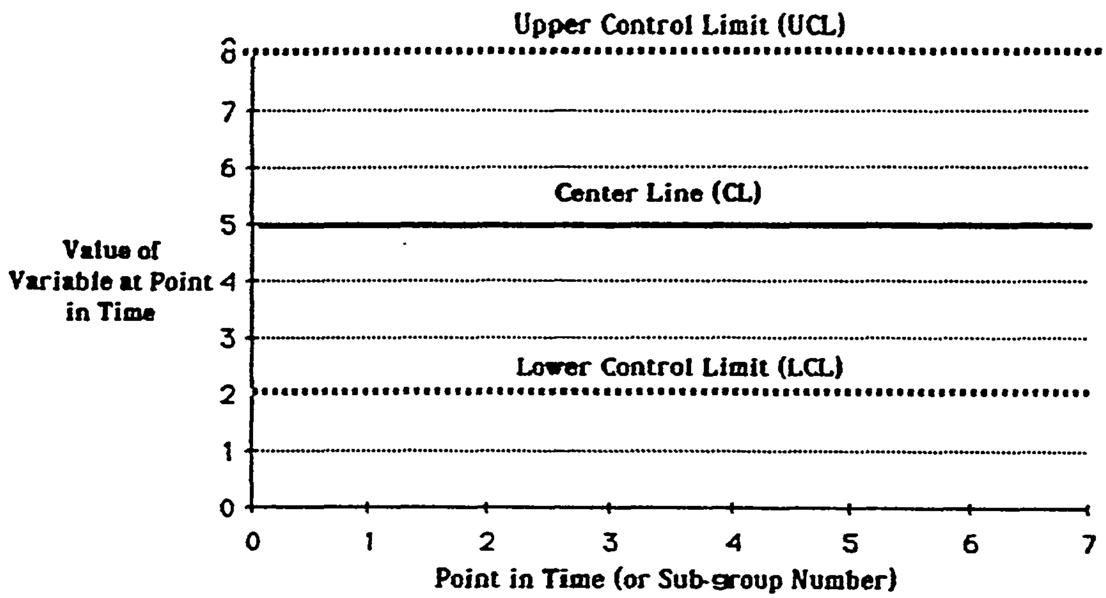
In the example with peas and carrots the average nutier of carrots for the five-day period was selected as the variable. We could have computed some other value (e.g., total nuuber of carrots over the five-day period or the difference between the largest number of carrots over



Upper Line and Lower Line
FIGURE 1-4



Average Number of Carrots - 7 Weeks
FIGURE 1-5



Layout of General Control Chart
FIGURE 1-6

the five-day period and the smallest number of carrots). We could have used the ratio of the total number of carrots to the total number of peas (counting the number of peas might cause sane problems). Also we could have made our computations at the end of each three-day period, four-day period, etc.

In the development of a control chart it is common practice to use a solid line for the center line and dashed lines for the upper and lower control limits. It is also common practice to choose the UCL and LCL in such a manner that, when the process is in a state of statistical control, each computed value has at least a 99 percent chance of f alling between the UCL and the LCL. In other words, there is a one percent chance or less that a computed value will fall outside the control limits. In this manner, if a computed value fells outside the control limits (above or below), we can say that a rare event has occurred. A good knowledge of statistics is needed in order to understand how the CL, UCL, and LCL are computed for a given experimental situation. It is not important at this point to understand how they are computed.

Now consider a second example:

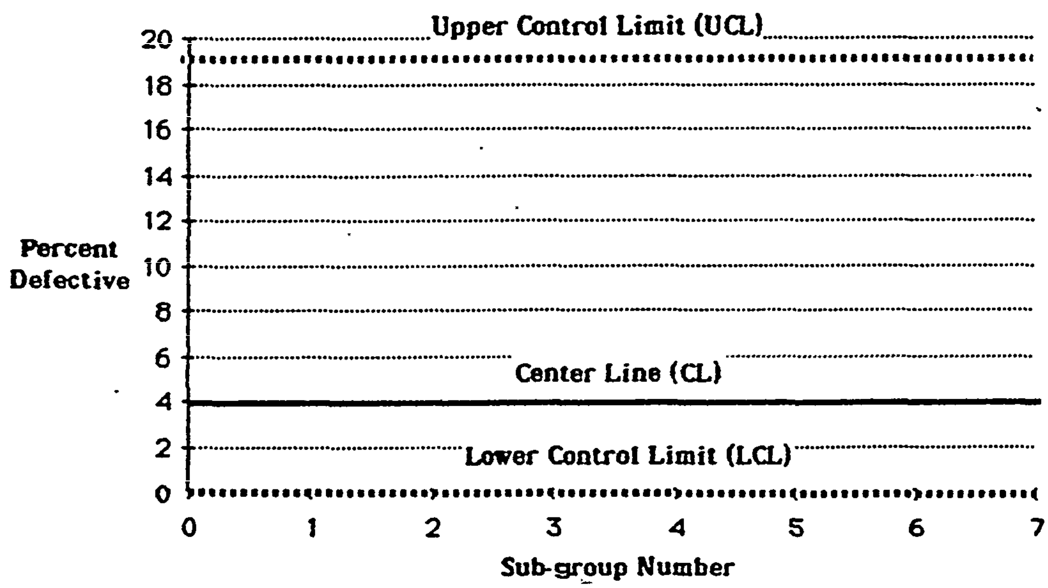
Steel plate of specified dimensions arrives fran sane vendor at the loading dock of the shipyard. At the loading dock, arriving steel plate is placed in stacks and, as each one is removed frcm the stack, it is classified as either good or defective. For each group of **15**, a control chart is then developed to monitor the percent defective.

In this example it is reasonable to alter the control chart and to plot the sub-group nunber on the horizontal axis instead of time. The statistical control chart for this situation would be similar to the one shown in Figure 1-7.

As 15 items are examined and the percent defective is determined, the value would be plotted as shown in Figure 1-7. The statistical control chart could then become the basis for monitoring the steel and for detecting a change in the quality of incoming steel. Again, in this example other variables could have been considered for the vertical axis of the control chart. For example, we could have used the total nunber of def ectives or the fraction defective. The use of a different variable would result in a different control chart. The horizontal axis would remain the same but the CL, UCL, and LCL would change. We could have completely changed the example by assuning that each plate could be examined to determine the nunber of significant defects (e. g., 0, 1 , 2, 3,...). The variable of interest for the 15 items could then become the average nunber of defects or perhaps the total. nunber of defects.

The examples present ed thus far (including the carrots) represent situations in which the variable of int crest is discrete or countable (e. g., number of defects, nunber of carrots, etc.). The control charts for these types of variables are referred to as control charts for attributes. Although a wide variety of control charts can be used with attribute data, the two most commonly used are referred to as the p chart and the c chart.

The p chart is a statistical control chart which has percent defective as the variable of interest on the vertical axis. The horizontal



Control Chart for Percent Defective

FIGURE 1-7

axis is usually the time point at which the percent defective is computed from the sample. This type of control chart is used in the following situation:

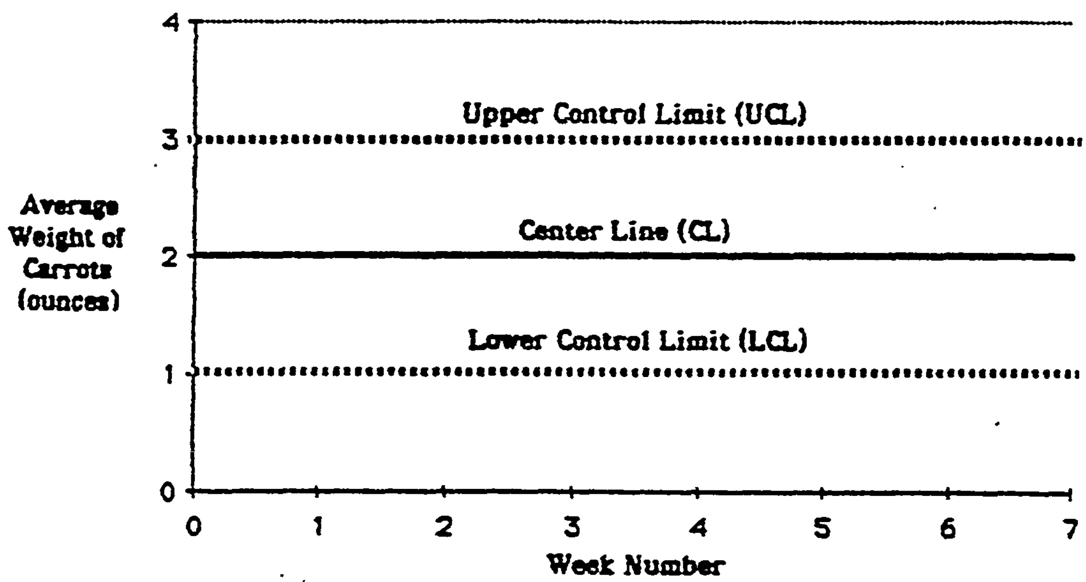
Items (k of them) are examined one at a time in a situation where each item can be classified as either good or defective. For example, in a repetitive welding operation 50 consecutive welds (k=50) from one welder are examined and each is classified as either good or defective. The total number of defective welds is then determined and divided by 50 to obtain the fraction defective. This value is then multiplied by 100 to obtain the percent defective. The percent defective is then plotted on the control chart.

The c chart is a statistical control chart which has total nunber of defects per inspection unit as the variable of interest on the vertical axis. The horizontal axis is the time at which the inspection unit is examined and the total nunber of defects is calculated. In this application the inspection unit may be one item or several items. For example, if 15 units of steel plate are examined and if the number of defects per plate is counted and then totaled, we may consider the 15 units of steel plate as the subgroup.

In addition to control charts for attribute data there is also a wide variety of charting techniques for experiment al situations in which some variable is measured rather than counted. For example, in our situation with peas and carrots, on each day you might separate the peas and carrots and then, with an accurate scale, weigh the carrots. In this situation you are dealing with a variable that is called infinitely divisible. It can take on, at least theoretically, any value in a range of continuous values. It should be obvious, however, that a variable of this type could be used to monitor the statistical regularity of the operating system. For example, suppose that each day the carrots are weighed and at the end of the week, after five consecutive days, the average weight is determined. This value could very easily be plotted on a chart such as the one shown in Figure 1-8. Again, as in the situation with attribute data, we could draw in the CL, the UCL, and the LCL and then monitor the system. If the lines are chosen to have a very high chance of finding an average weight between the UCL and LCL, a point outside these lines would send us a signal that something is going wrong. At this point sane action maybe required. We could also argue that if the process is in a state of statistical control, you might expect the average weight of carrots to bounce around the center 1ine in some random fashion where you would not expect to see increasing or decreasing trends or too many sequential values above or below the CL.

Numerous variables can also be plotted in the situation where sane variable is measured. In the example with peas and carrots, we could have computed for each five-day period the minimun weight for the week and then used this value on the vertical axis of our chart. Similarly we could have computed the maximum weight over the five-day period. The maximun weight minus the minimum weight (the range) over the five-day period could also have been used *as* the variable to be monitored. The average and range charts are used extensively in quality assurance for measured variables.

In order to illustrate the types of variables that could be measured and monitored with statistical control charts in the shipbuild-ingindustry, the following examples are presented:



Average Weight of Carrots by Week Number
FIGURE 1-8

1. Prior to welding steel plate are butted together as shown in Figure 1-9 and, using a feeler gauge, the maximum gap size is measured. This situation is repeated five times and the average maximum gap is computed.

2. A pre-cut header of specified dimensions is supplied by a vendor and is inserted between two beams as shown in Figure 1-10. Prior to inserting the header in place the maximum overall length is measured. The situation is repeated eight times and the range of the lengths is computed.

3. An automatic burning machine repetitively cuts a triangular steel plate as shown in Figure 1-11. After each plate is cut dimension b shown in Figure 1-11 is measured and then the difference between the as-cut measurement and the design measurement (30 inches) is computed. After the production of ten triangular pieces the average difference is computed.

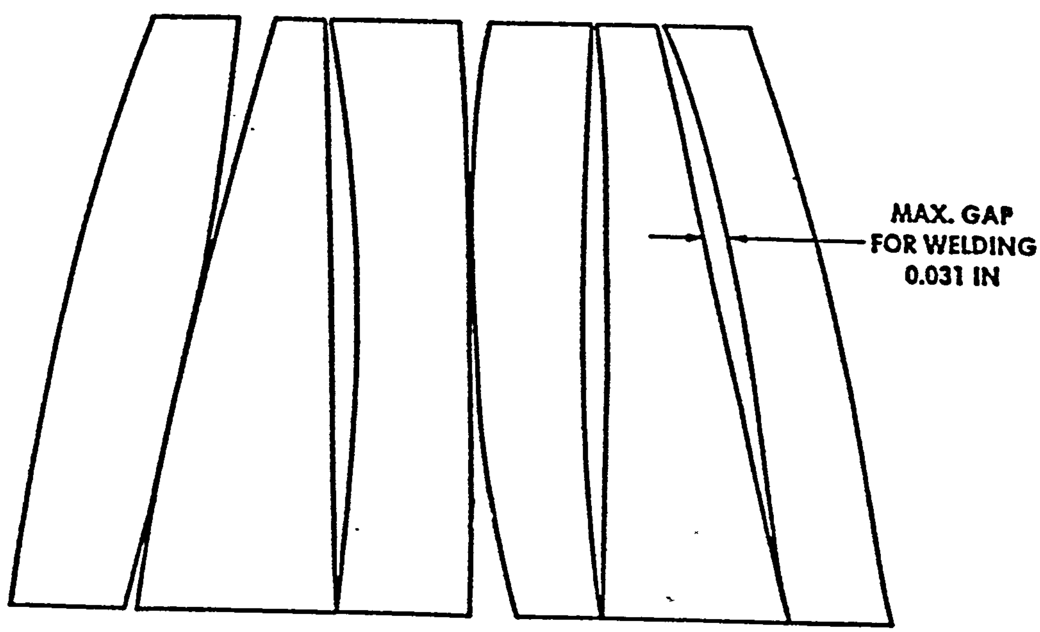
4. In a paint spraying operation the line pressure to the paint sprayer is determined at one minute intervals. At the end of each five minute interval the average line pressure is computed.

5. In a specific section of the shipyard steel plate is used in some repetitive operation. Each plate is weighed and the average weight for ten consecutive items is computed.

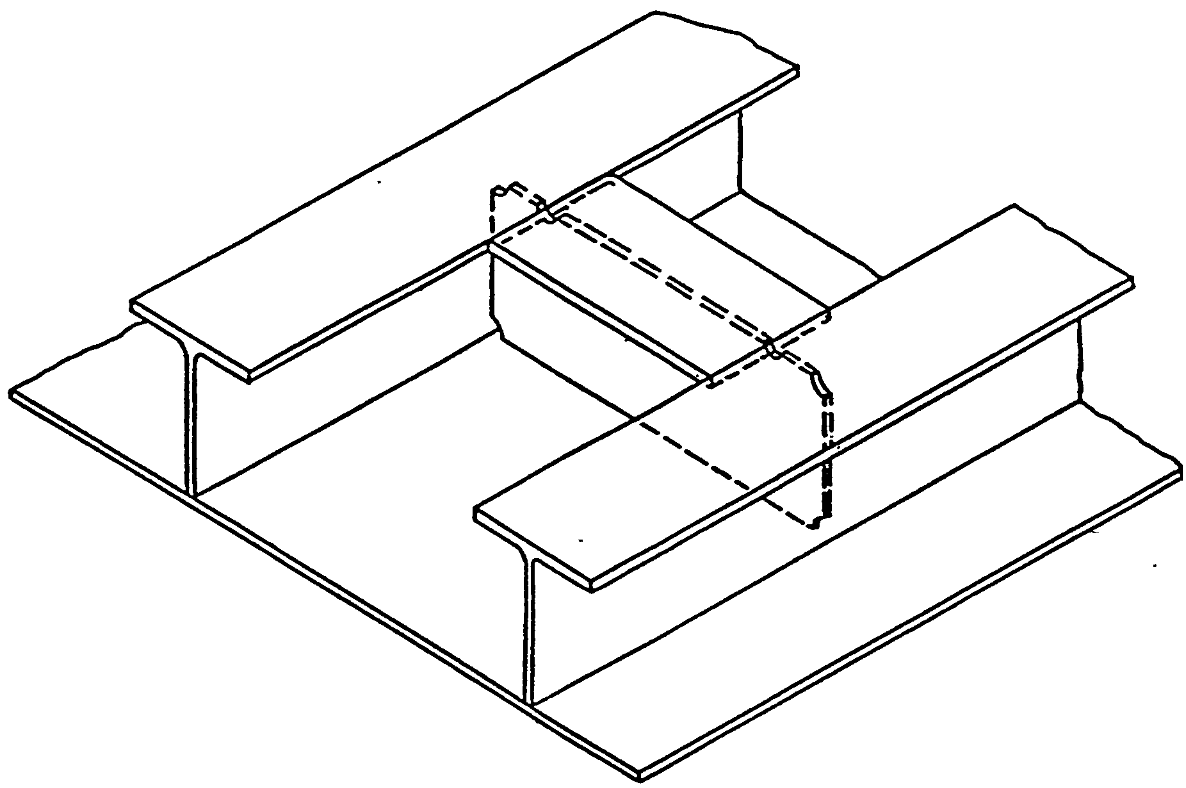
1.3 The Use of A Control Chart - Inherent Variability and Assignable Causes

Once a statistical control chart has been constructed and placed into operation it must be interpreted periodically in order to determine if something is going wrong with the process. Any quality characteristic that is being monitored by a control chart will exhibit inherent variability that cannot be either identified specifically or controlled. The inherent variability is the result of a multitude of factors that are generally related to one of three causes: people, materials, and equipment. Inherent variability is part of the system. The control limits on a control chart define the boundaries for inherent variability, and the process should continue to operate between these

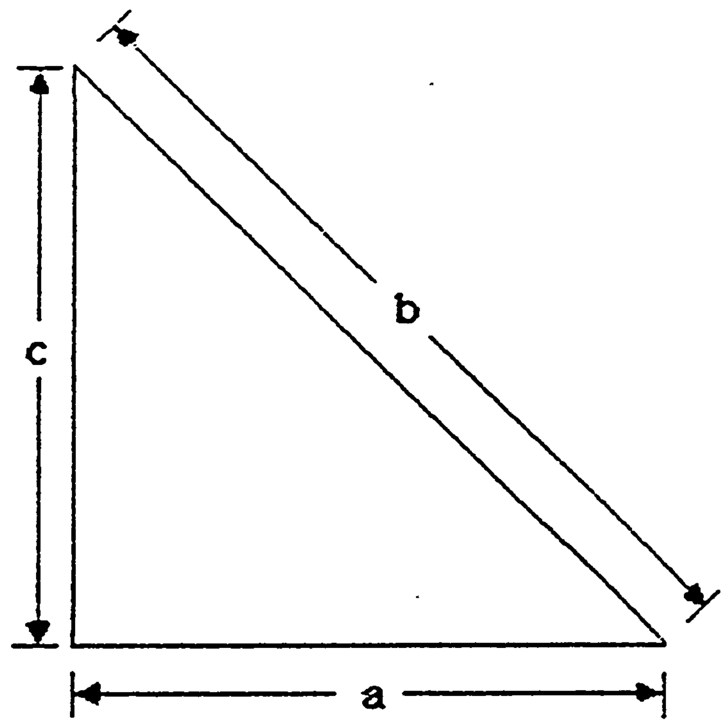
DECK PLATE FITTING



Maximum Gap in Adjacent Steel Plate
Figure 1-9



Typical Deck Header Installation
Figure 1-10



Engineering Drawing Dimensions

a = 24 inches
b = 30 inches
c = 18 inches

Triangular Section of Steel Plate
Figure 1-11

boundaries in a perfectly natural way. Any change in the process which causes it to deviate significantly from its natural manner of variability is referred to as an assignable cause. Assignable causes should be located and action should be taken to return the process to a state of statistical control (natural variability). The purpose of the control chart is to identify occurrences of assignable causes.

Without any knowledge of the specifics of a control chart, one can develop an understanding of the use of a control chart to identify assignable causes. For example, if we refer back to our example with peas and carrots and if we consider the situation where we are monitoring the average weight of the carrots at the end of each week, the following general rules could be established.

1. If the average weight at the end of any week falls outside the control limits look for an assignable cause.

2. If the average weight for each of eight consecutive weeks is below the Center Line look for an assignable cause.

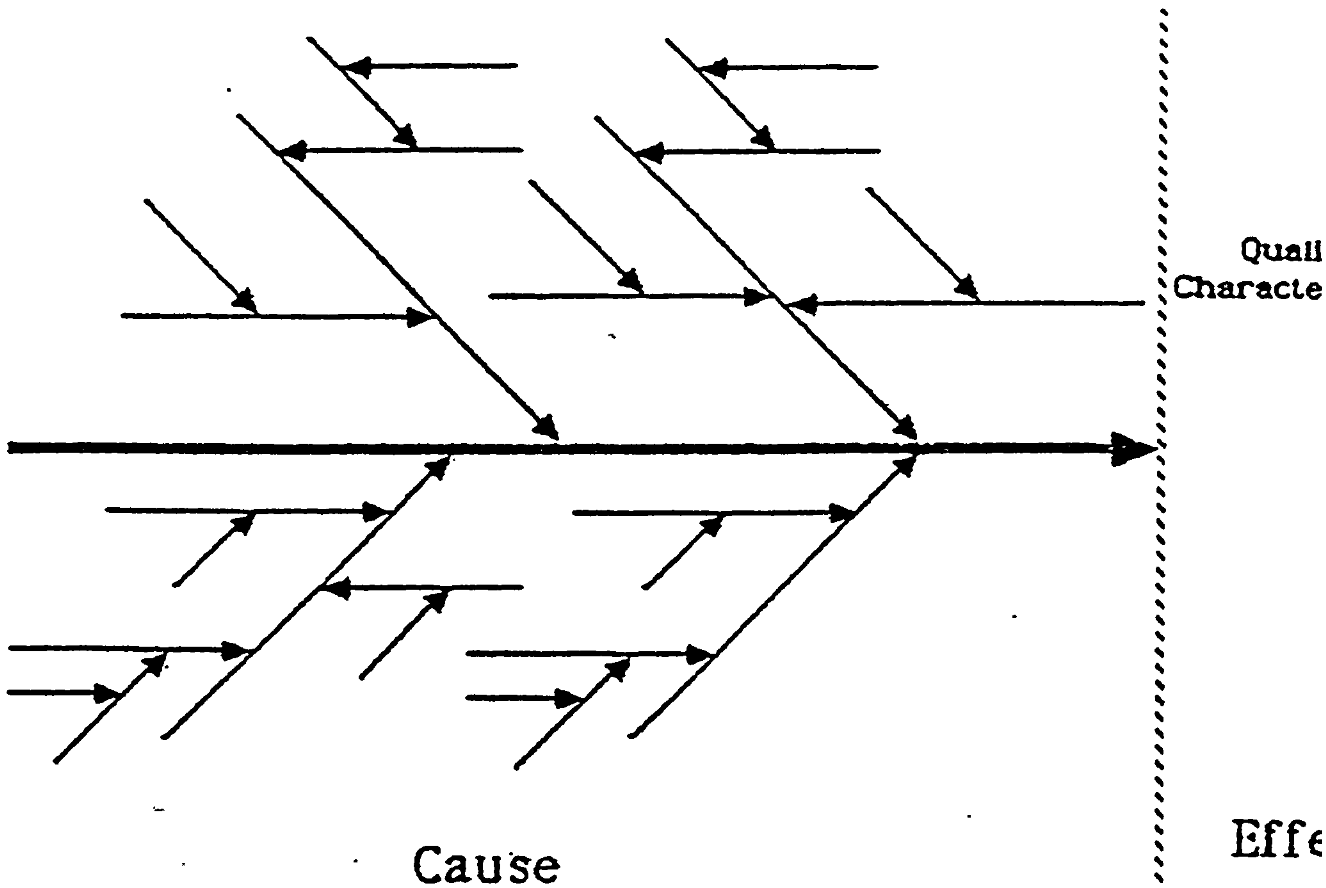
3. If the average weight for each of eight consecutive weeks falls above the Center Line look for an assignable cause and

4. If the average weight for each of four consecutive weeks shows either an increasing or decreasing trend look for an assignable cause.

For each type of control chart that could be developed a specific set of rules could be established for the identification of assignable causes. In this way, unless we have reason to believe that some assignable cause has appeared, the process should be allowed to operate in a natural manner. It should be emphasized that any adjustment to a process that is operating in a state of statistical control will lead to increased variability.

Other types of charting methods called Fishbone Charts or Cause and Effect Charts are helpful in the search for assignable causes. These methods are graphical procedures for identifying those factors (causes) which may influence some quality characteristic (effect). They are called fish-bone charts because of the way they appear subsequent to construction. In the fishbone chart the quality characteristic of interest is listed to the right of a bold arrow as shown in Figure 1-12. Stems to the main arrow are then constructed where each stem is characterized by a group of factors such as person, materials, and equipment. Twigs are then placed on each stem to identify in more detail those factors in each group which may have an effect on the quality characteristic. Twigs may then be placed on each twig until all factors have been identified. The actual construction of the fishbone chart forces you to think about the quality characteristic and, in this manner, it provides a useful exercise to identify problems. Quality circles can be used effectively in the construction of a fishbone chart . where the chart becomes the basis for meaningful discussion. A clear understanding of the process leads to a detailed fishbone chart.

As an example on the construction of a fishbone chart, consider a spray painting operation in which the quality characteristic of interest is the number of bubbles in 100ft2 of painted surface. In a preliminary attempt to develop a fishbone chart by a group of people who are familiar with the operation, the following major factors might be identified: Environmental Conditions, Equipment, Surface .



Cause and Effect (Fishbone) Chart

Figure 1-12

Characteristics, and Workers. In each major factor important sub-factors could be identified and discussed. The procedure would continue until the process is well understood and diagremmed as a fishbone chart. See Figure 1-13 for a preliminary attempt to construct a fishbone chart for this example.

The fishbone chart should be used in any initial examination of a process to determine significant factors that should be measured or counted through the use of a control chart. Other charts are also useful in this stage of analyzing a process. For example, the Pareto Chart provides the analyst with a means of identifying those factors which may produce the most significant improvement in quality. In this manner, with limited funds and personnel, significant improvements may be realized very quickly. The Pareto Chart is based on the Pareto Law which states, in essence, that if many factors are involved in an end result a very few will be found to contribute a completely disproportionate share of the total. This law has been found to be true in many situations in quality assurance. A Pareto Chart is simply a graphical procedure for ranking those factors which affect sane quality characteristic so that the analyst can identify those which contribute most significantly to the end result.

As an example on the development of a Pareto Chart, consider your own home and its consumption of electriCal energy. If you analyzed the monthly consumption of energy (in kilowatt hours) for your home, you might identify the following factors which contribute to the total monthly consumption.

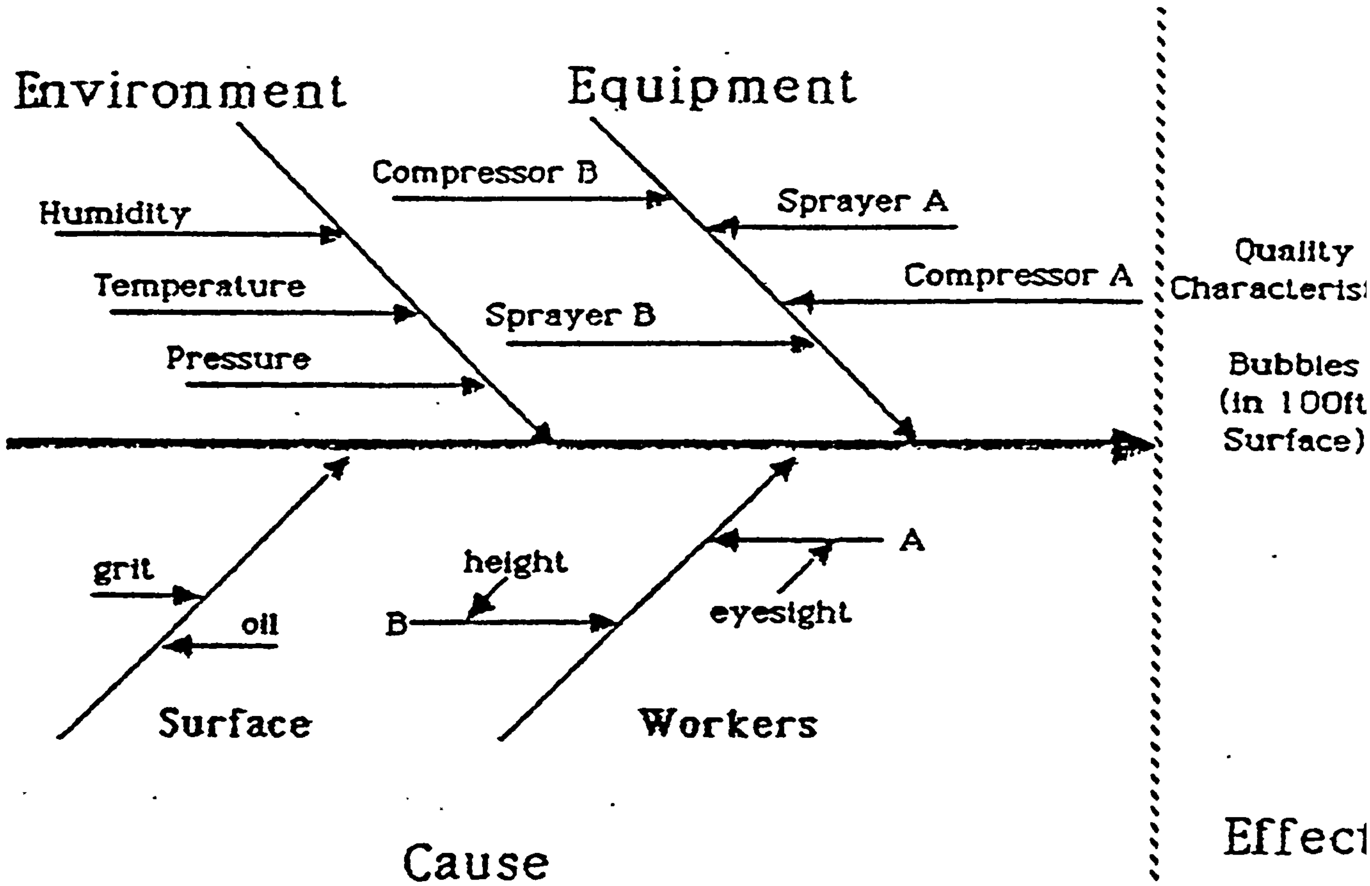

Environment
Equipment
Compressor B
Humidity
Sprayer A
Quality Characteris
Temperature
Compressor A
Sprayer B
Pressure
Bubbles (in 100ft Surface)
A
grit
height
oil
B
eyesight
Surface
Workers
Cause
Effec

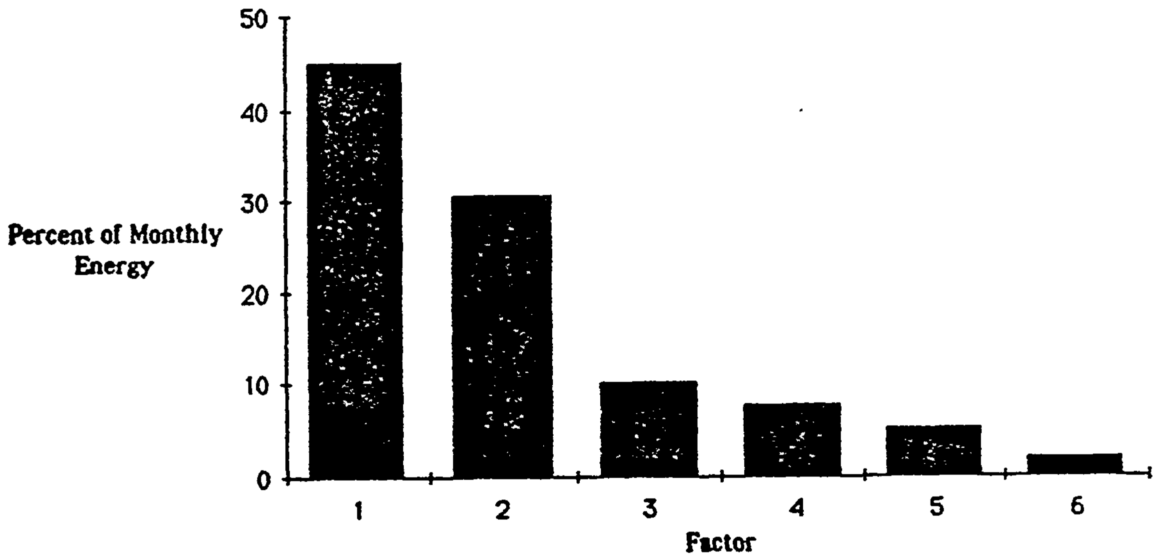
Figure 1-13

1. Hot Water Tank
2. Refrigerator
3. Clothes Dryer
4. Washing Machine
5. Small Appliances and
6. Lights

If you collected data on monthly energy consumption, it is very likely that each factor would contribute to monthly energy as follows:

Fact or	Percent of Monthly Energy
1. Hot Water Tank	45.0
2. Refrigerator	30.5
3. Clothes Dryer	10.0
4. Washing Machine	7.5
5. Small Appliances	5.0
6. Lights	2.0

This situation, in graphical form, is shown in Figure 1-14. This figure gives the basic elements of a Pareto Chart where, using a bar graph, the variable of interest is plotted in the vertical direction and the factors are 1isted in decreasing importance in the horizontal direction. In most situations the Pareto Chart will show clearly those factors which contribute significantly to the total. In this case the hot water tank and the refrigerator are the factors to be considered for major reductions in energy consumption.



Pareto Chart for Monthly Energy
Figure 1-14

The Pareto Chart is also useful in comparing, in graphical form, before and after conditions. In our example we might decide to insulate the hot water tank and then determine the effect of this corrective measure. A second Pareto chart would then be constructed to examine the effect of the corrective measure.

1.4 Control Charts to Identify A State of Statistical Control

The statistical control chart can be used to determine if a process has reached a state of statistical control. In general the approach is *as* follows: 1) Data for a series of subgroups are collected; 2) The data are then used to establish control limits and the center line of the control chart; 3) For each subgroup the variable of interest is plotted on the control chart; and 4) The results are then analyzed to determine if statistical control has been reached. If the process does not exhibit a state of statistical control, then, with a fishbone chart or by sane other means, the process must be examined to locate a significant cause (or significant causes) to eliminate all but inherent variability. This basic procedure is repeated (collecting new data if necessary) until it can be determined that a state of statistical control has been reached.

The basic procedure outlined above can be used in both discrete and continuous random experiments. In the discrete case the type of chart to be used will depend on the specific situation and will be either: 1) The Fraction Defective Chart; or 2) The Total Number of Defects Chart. In the continuous case, statistical control charts *are* constructed for the Average (x) outcome and range (R) of the outcomes in each subgroup.

These two charts are then used simultaneously to determine if a state of statistical control has been reached. The discrete cases are discussed first and are then followed by a discussion of the x and R chart for continuous experiments.

The Discrete Random Experiment: Fraction Defective Chart

In this case it is assumed that data have been collected for r sub groups where each of the k items in a subgroup has been classified as either defective or non defective. The number of defective for each subgroup is then presented in tabular form as shown in Table 1-1.

Using the values shown in Table 1-1, the average value of the fraction defective is then computed as follows:

$$\bar{p} = \text{Average Value of Fraction Defective} = \frac{n_1 + n_2 + \cdots + n_r}{kr}$$

The center line and the control limits are then computed to be:

$$\text{Center Line (CL)} = \bar{p}$$

$$\text{Upper Control Limit (U CL)} = \bar{p} + \frac{3}{\sqrt{k}}\sqrt{\bar{p}(1-\bar{p})}$$

$$\text{Lower Control Limit (LCL)} = \bar{p} - \frac{3}{\sqrt{k}}\sqrt{\bar{p}(1-\bar{p})}$$

Subgroup Number	Number of Defectives in Subgroup
1	n_1
2	n_2
.	.
.	.
.	.
r	n_r

Number of Defectives in Each Subgroup

Table 1-1

For ease of computation the value of $\frac{3}{\sqrt{k}}$ for different values of k is given in Table 1-2. If the LCL is computed to be a negative value, it should be set equal to zero.

Once the control chart has been constructed the fract ion defective for each subgroup is plotted on the control chart and an assessment of the state of statistical control would then be made.

As an example of a fraction defective control chart, consider a situation where electrical switches are classified as either defective or non-defective and that 30 subgroups, each consisting of 40 switches, were examined, with the resulting number of defectives shown in Table 1-3. In this situation $\bar{p} = 0.079$ and from Table 1-2, $3/\sqrt{40} = 0.4743$. The control limits are then computed as follows:

$$CL = \bar{p} = 0.079$$

$$UCL = \bar{p} + \frac{3}{\sqrt{k}}\sqrt{\bar{p}(1-\bar{p})} = 0.207$$

$$LCL = \bar{p} - \frac{3}{\sqrt{k}}\sqrt{\bar{p}(1-\bar{p})} = 0.000$$

The control chart and a plot of the fraction defective for each subgroup is shown in Figure 1-15. Since the process appears to be in a state of statistical control the control chart would be used for monitoring the process.

Subgroup Size	Factor	Subgroup Size	Factor	Subgroup Size	Factor
1	$3/\sqrt{k}$	k	$3/\sqrt{k}$	k	$3/\sqrt{k}$
2	2.1213	22	0.6396	42	0.4629
3	1.7321	23	0.6255	43	0.4575
4	1.5000	24	0.6124	44	0.4523
5	1.3416	25	0.6000	45	0.4472
6	1.2247	26	0.5883	46	0.4423
7	1.1339	27	0.5774	47	0.4376
8	1.0607	28	0.5669	48	0.4330
9	1.0000	29	0.5571	49	0.4286
10	0.9487	30	0.5477	50	0.4243
11	0.9045	31	0.5388	51	0.4201
12	0.8660	32	0.5303	52	0.4160
13	0.8321	33	0.5222	53	0.4121
14	0.8018	34	0.5145	54	0.4082
15	0.7746	35	0.5071	55	0.4045
16	0.7500	36	0.5000	56	0.4009
17	0.7276	37	0.4932	57	0.3974
18	0.7071	38	0.4867	58	0.3939
19	0.6882	39	0.4804	59	0.3906
20	0.6708	40	0.4743	60	0.3873
21	0.6547	41	0.4685	61	0.3841

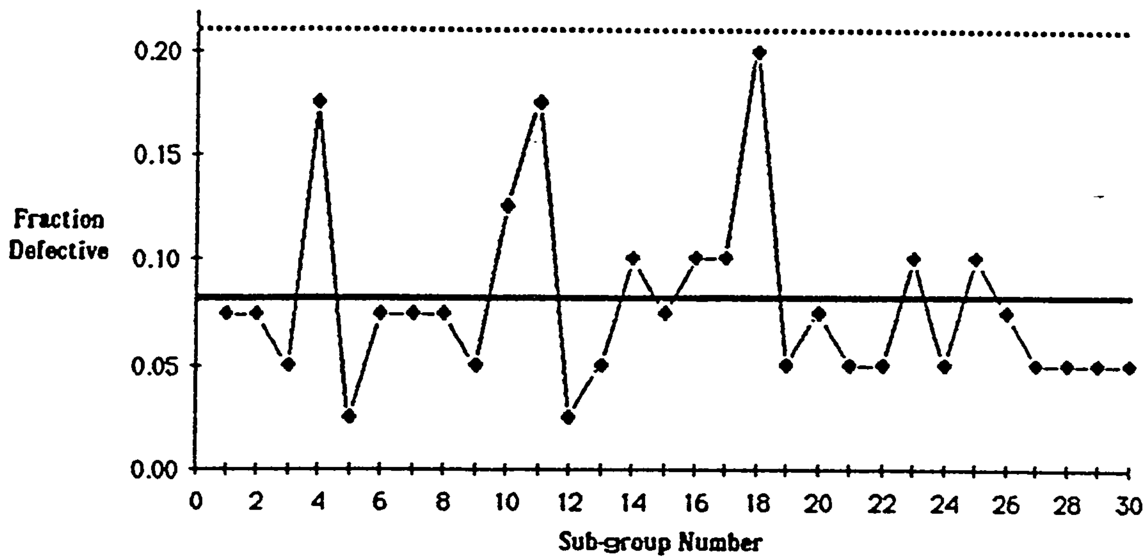
Value of $3/\sqrt{k}$ vs k

Table 1-2

Subgroup Number	Number of Defectives
1	3
2	3
3	2
4	7
5	1
6	3
7	3
8	3
9	2
10	5
11	7
12	1
13	2
14	4
15	3
16	4
17	4
18	8
19	2
20	3
21	2
22	2
23	4
24	2
25	4
26	3
27	2
28	2
29	2
30	2

Number of Defectives in 40 Electrical Switches

Table 1-3



Control Chart for Fraction Defective - Light Switches

Figure 1-15

The Discrete Random Experiment: Total Number of Defects

In this case it is assumed that data has been collected for r subgroups where, in each subgroup, there are k items. Each item will have 0, 1,2,3, . . . defects and the total number of defects for the subgroup is reported as shown in Table 1-4. Using the values in Table 1-4 the average value of the total number of defects is then computed as follows:

$$\bar{T} = \text{Average Value of Total Number of Defects} = \frac{T_1 + T_2 + \ldots + T_r}{r}$$

The center line and the control limits are then computed to be:

$$\text{Center Line (LC)} = T$$

$$\text{Upper Control Limit (UCL)} = \bar{T} + 3\sqrt{\bar{T}}$$

$$\text{Lower Control Limit (LCL)} = \bar{T} - 3\sqrt{\bar{T}}$$

If LCL is computed to be a negative value it is set equal to zero and, again, the total. number of defects for each subgroup would be plotted on the control chart to determine if the process is in a state of statistical control.

As an example of this type of control chart consider a situation in which each subgroup consists of 30 plates of steel and the total number of defects for the 30 plates was counted for each of 50 subgroups. The results are shown in Table 1-5. In this situation T = 13.88 and the center line and control limits are computed as follows:

subgroup Number	Total Number of Defects in Subgroup
1	T_1
2	T_2
3	T_3
.	.
.	.
.	.
.	.
.	.
.	.
r	T_r

Number of Defects in Each Subgroup
Table 1-4

Number	Total Number of Defects	Subgroup Number	Total Number of Defects
1	15	26	18
2	13	27	13
3	12	28	15
4	15	29	14
5	7	30	12
6	18	31	14
7	8	32	6
8	15	33	8
9	11	34	18
10	16	35	10
11	18	36	24
12	12	37	10
13	11	38	12
14	15	39	20
15	16	40	13
16	16	41	12
17	13	42	14
18	17	43	17
19	13	44	17
20	12	45	13
21	12	46	19
22	12	47	18
23	17	48	13
24	12	49	12
25	12	50	14

Total Number of Defects in 30 Plates of Steel

Table 1-5

$$CL = T = 13.88$$

$$UCL = \bar{T} + 3\sqrt{\bar{T}} = 25.06$$

$$LCL = \bar{T} - 3\sqrt{\bar{T}} = 2.70$$

The Total Number of Defects for each subgroup is plotted on the control chart in Figure 1-16. Again the control chart indicates a state of statistical control.

The Continuous Random Experiment (x Chart, R Chart)

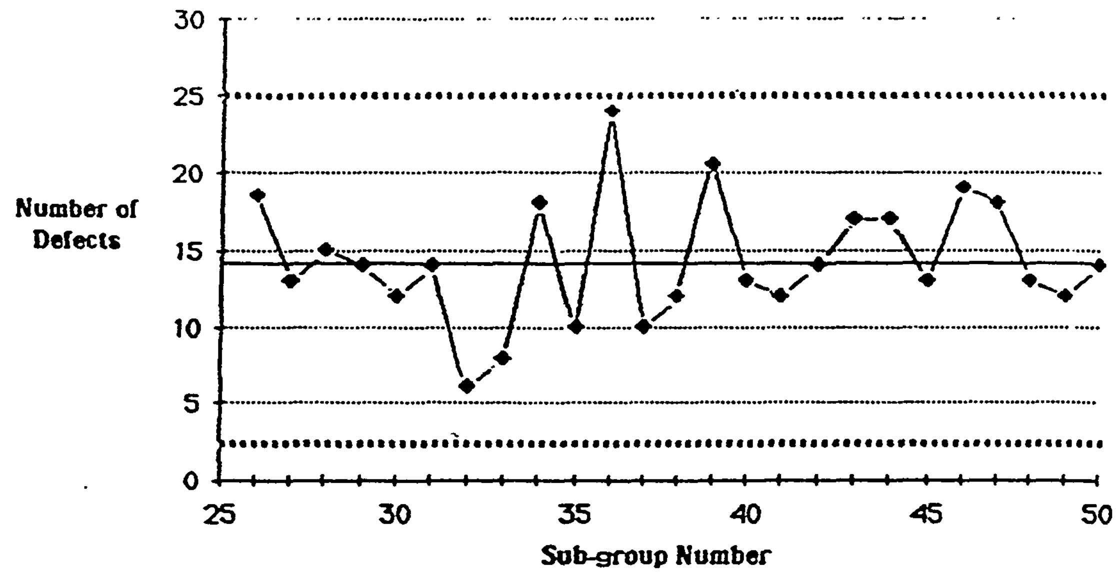
In this case it is assumed that data have been collected from the process and that k measurements of some continuous variable have. been made for each of r subgroups. For each subgroup the average value (x) and the Range (R) have been calculated and conveniently displayed as shown in Table 1-6.

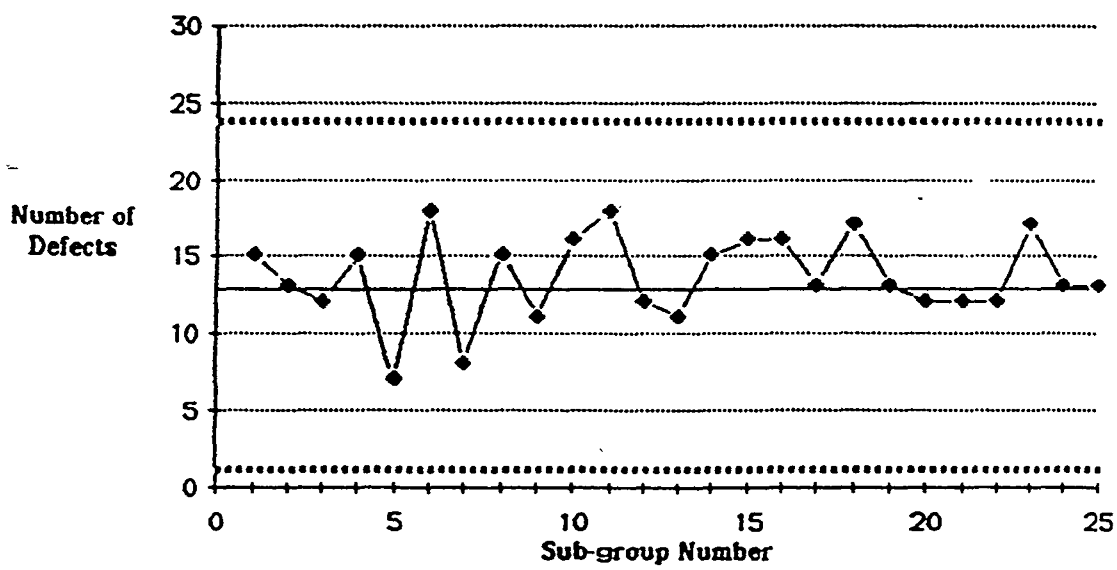
Using the notation given in Table 1-6 the following computations are made:

$$\bar{X} = \text{the average of the subgroup Averages} = \frac{\bar{X}_1 + \bar{X}_2 + \ldots + \bar{X}_r}{r}$$

and

$$\bar{R} = \text{the Average of the subgroup Ranges} = \frac{R_1 + R_2 + \ldots + R_r}{r}$$





Control Chart for Total Number of Defects
Figure 1-16

Subgroup Number	Measurements for Subgroup	Average of Subgroup $\bar{X}$	Range of Subgroup R
1	X_{11} X_{12} X_{13} ... X_{1k}	$\bar{X}_1$	R_1
2	X_{21} X_{22} X_{23} ... X_{2k}	$\bar{X}_2$	R_2
3	X_{31} X_{32} X_{33} ... X_{3k}	$\bar{X}_3$	R_3
.			
.			
.			
r	X_{r1} X_{r2} X_{r3} ... X_{rk}	$\bar{X}_r$	R_r

Subgroup Data from Process (Average and Range)

Table 1-6

These two values are then used to determine the center lines and the control limits for the x and R charts as follows:

<u>**$\bar{X}$ Chart**</u>

Center Line (CL) $= \bar{X}$

Upper Control Limit (UCL) $= \bar{X} + A_2\bar{R}$

Lower Control Limit (LCL) $= \bar{X} - A_2\bar{R}$

The value of A_2 can be determined from Table 1-7.

<u>R Chart</u>

Center Line (CL) = F

Upper Control Limit (UCL) $= D_4\bar{R}$

Lower Control Limit (LCL) $= D_3\bar{R}$

The values of D_3 and D_4 are also determined from Table 1-7.

As an example of the use of $\bar{X}$ and R charts consider the situation where a laborer is using a shovel to move material from a pile into a hopper . Before each full shovel is placed into the hopper it is weighed, with the results shown in Table 1-8. For this situation the control chart for the average weight would be established as follows: .

$$CL = \bar{X} = 50.00$$

$$UCL = \bar{X} + A_2\bar{R} = 64.165$$

$$LCL = \bar{X} - A_2\bar{R} = 35.835$$

The control chart for the range of weights would become:

$$CL = \bar{R} = 24.552$$

Subgroup Size (k)	Value of Factor A_2	D_4	D_3
2	1.880	3.267	0
3	1.023	2.267	0
4	0.729	2.282	0
5	0.577	2.115	0
6	0.483	2.004	0
7	0.419	1.924	0.076
8	0.373	1.864	0.136
9	0.337	1.816	0.184
10	0.308	1.777	0.223

Table i-7

Subgroup Number	Measurement Weight (pounds) of Material on Shovel					Average of Subgroup x	Range of Subgroup R
1	58.95	51.48	64.51	52.93	42.33	54.04	22.18
2	67.11	46.83	58.03	38.03	56.80	53.36	29.08
3	70.15	43.83	73.37	48.08	50.24	57.13	29.54
4	50.57	45.99	49.50	39.15	51.20	47.28	12.05
5	60.91	46.14	58.72	47.52	46.18	51.89	14.77
6	34.00	45.88	49.55	42.83	40.42	42.54	15.55
7	55.86	51.93	49.99	52.86	33.74	48.88	22.12
8	52.70	68.97	67.69	41.79	32.87	52.80	36.10
9	44.58	34.92	45.27	43.32	48.06	43.23	13.14
10	41.22	54.60	52.56	59.31	40.59	49.66	18.72
11	56.11	60.97	69.27	53.02	36.22	55.12	33.05
12	56.57	33.80	60.66	43.53	47.37	48.39	26.86
13	45.66	49.17	52.86	62.08	78.16	57.59	32.50
14	30.60	49.07	57.65	62.05	48.08	49.49	31.45
15	44.15	79.84	51.75	37.49	53.05	53.26	42.35
16	65.42	50.81	42.99	39.78	27.09	45.22	38.33
17	40.78	52.49	37.85	51.08	36.61	43.76	15.88
18	49.95	40.63	41.59	48.47	47.15	45.56	9.32
19	38.16	56.08	43.71	58.91	59.23	51.22	21.07
20	37.68	40.18	64.66	43.64	61.80	49.59	26.98

Weight (in pounds) of Material on Shovel

Table 1-8

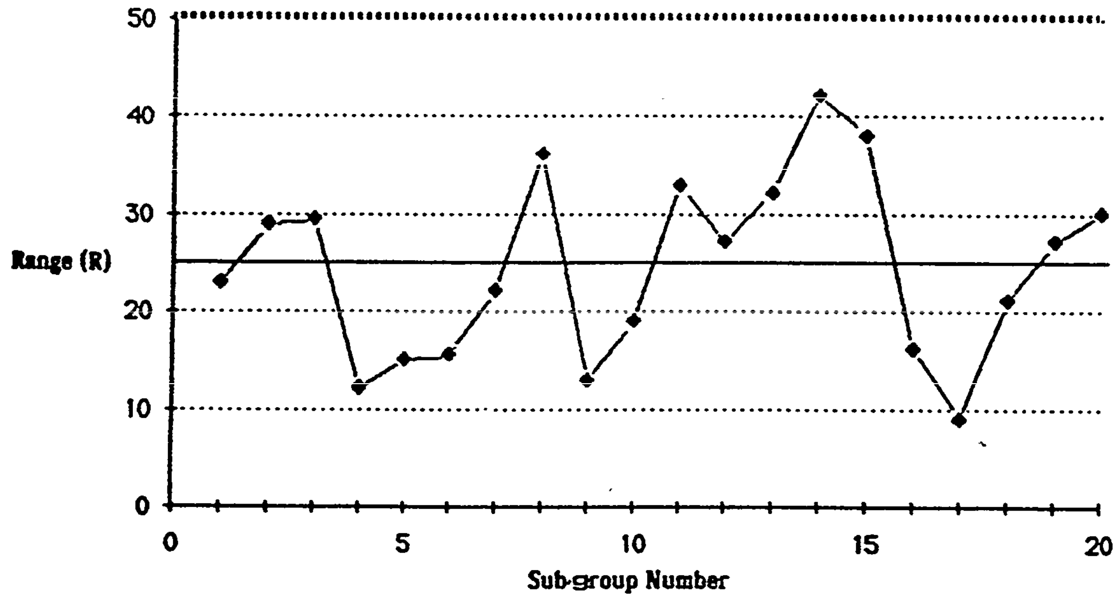
$$UCL = D_4\bar{R} = 51.927$$

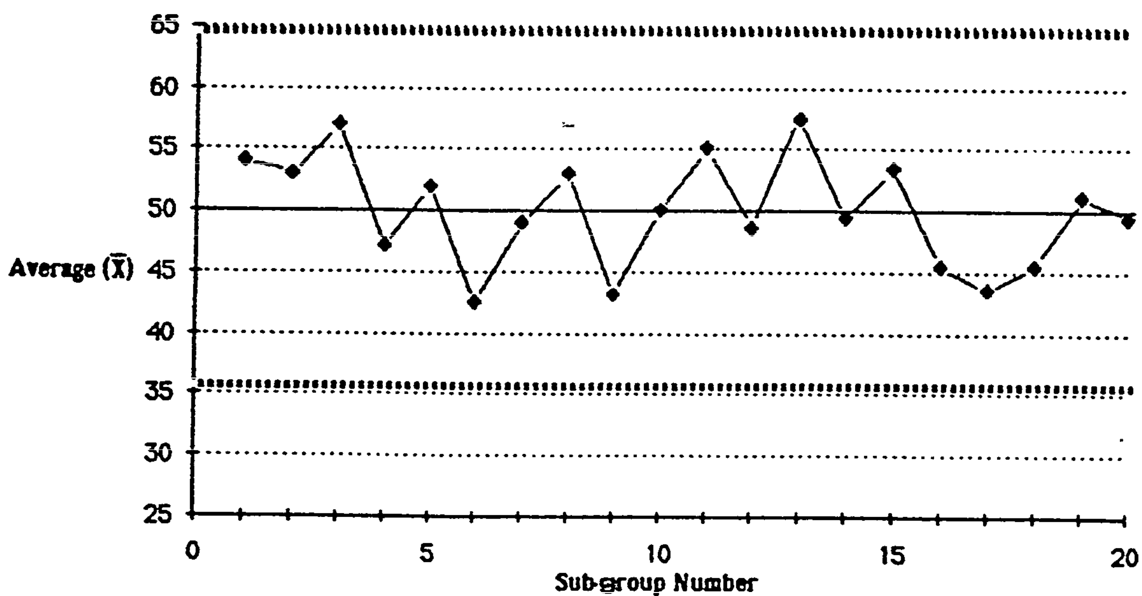
$$LCL = D_4\bar{R} = 0.000$$

The resulting X and R charts and the plotted values for each subgroup are shown in Figure 1-17. Again, it would be determined that the process is in a state of statistical control.

1.5 Description of Remaining Chapters

The purpose of the remaining chapters of this workbook is to provide, using only elementary mathematics, the statistical background necessary for those individuals who are either involved or are planning to become involved in the continued use of sound quality assurance techniques in the shipbuilding industry. In the development every attempt has been made to illustrate the statistical theory with examples taken directly from situations in the shipbuilding industry that have already been examined from the point of view statistical quality assurance. The layout of the workbook is directly related to the manner in which we conceptualize a shipyard. From our point of view we consider the shipyard as a system which consists of "a large number of operating subsystems. These subsystems may in themselves be quite large or they may be as small as one worker and one tack weld on two plates of steel. From our viewpoint each subsystem can be imagined as an experiment which can be repeated over and over again under essentially the same conditions where the outcome of the experiment changes from trial to trial in a random manner. The random nature of the *outcome* or, in other words, the unpredictable nature of the outcome, is a direct result of the subsystem itself. We are assuning that everything possible has been done





Average and Range Charts ($\bar{X}$,R)

Figure 1-17

to eliminate variability in the outcome of the experiment (the subsystem), but because of intrinsic factors we have to accept a certain degree of inherent variability. The key to understanding this inherent variability lies in the proper understanding of statistics.

As an example consider the following situation. An apprentice electrician assigned the task of taking a large spool of electrical cable and producing individual cables, each with a length of **25** feet. The apprentice electrician is provided with a specific working environment and a specific set of tools to accomplish the task of producing each cable. We consider the production of one cable of length 25 feet as a subsystem of the shipyard. We also consider this subsystem as an experiment in which the outcome is a cable which is supposed to be of length 25 feet. The experiment can obviOUS1y be repeated over and over again under essentially the same conditions and, each time the experiment is repeated, the length of the cable will change. An accurate device to measure the length of the cable would show clearly that random variation does exist in the cables that are supposed to be of 1ength 25 feet. Despite this random variation, as long as the conditions under which the experiment is repeated do not change, we are said to be in a state of statistical control. Since the output of this subsystem usually becomes the input to other subsystems and since the variability in the outcome may have physical and economic impact in these other subsystems, this variability must be understood and controlled. This can only be accomplished by using a sound statistical analysis.

Each random experiment (each subsystem of the shipyard) can be classified as either discrete or continuous, depending on the type of experimental outcome. Discrete random experiments are those in which the experimental outcome (the variable of interest) is one of a finite or countable number of possible outcomes. In the discrete random experiment there is a clearly defined list of possible outcomes exactly one of which must occur each time the experiment is executed. The number of defects in a weld of length 100 inches-is an example of a discrete random experiment.

In the continuous random experiment there is no list of possible outcomes. The experimental out come comes from a continuous range of possible outcomes. The actual length of an electrical cable which is supposed to be 25 feet long is an example of a continuous outcome, assuming the 'actual length can be measured to any degree of accuracy.

Because of our view of the shipyard as a large number of random experiments and because each random experiment can be viewed as either discrete or continuous, this workbook is structured in a specific manner. In Section 2, discrete random experiments are defined and illustrated. Continuous random experiments are then defined and illustrated in Section 3. In each situation the appropriate statistical theory is introduced. In Section 4, we sunmarize the basics of statistical control charts.

1.6 Problems

I. The amount of paint (ounces) to cover 100 ft^2 of steel Plate was measured by randomly selecting 20 painters. Each painter conducted five trials of a basic experiment in which 100 ft^2 of steel plate was painted and the amount (in ounces) was measured. The results are shown in Table 1-9. Using the x and R charts, determine if the process is in a state of statistical control.

Subgroup Number	Value of Variable 1 Ounces of Paint to Cover 100 ft^2					Value of Variable 2 Average	Range
1	30.42	22.29	19.68	19.05	20.00		
2	27.97	24.64	12.73	23.67	28.42		
	25.30	29.28	26.05	20.96	28.19		
4	32.30	23.61	20.08	32.02	24.12		
5	33.58	23.15	29.54	35.18	16.62		
6	26.69	30.91	24.88	26.70	25.07		
7	28.84	25.21	24.03	29.00	23.11		
8	32.95	24.41	21.06	19.44	25.35		
9	26.47	35.41	34.01	29.86	20.97		
10	23.82	24.55	31.98	25.97	24.76		
11	24.46	29.95	28.20	31.65	27.54		
12	24.79	15.16	26.01	18.93	28.53		
13	27.40	26.48	28.98	27.44	33.83		
14	18.18	30.15	33.82	26.19	27.71		
15	31.26	24.12	22.42	22.29	27.89		
16	25.79	31.52	21.94	19.45	36.06		
17	27.61	34.53	26.68	21.36	31.21		
18	23.07	21.82	19.62	20.33	25.08		
19	28.93	27.07	23.54	24.80	32.30		
20	15.67	28.74	19.20	25.94	20.17		

Ounces of Paint to Cover 100 Square Feet
Table 1-9

II. **In a welding operation 20 welders used the same welding machine to** weld two pieces of steel plate along a scan of length 200 inches. Each welder repeated the experiment five times and for each repetition the total length of. defective weld was. determined. The results are shown in Table 1-10. Using the x and R charts, determine if the process is in a state of statistical COntrOl.

Subgroup Number (Welder)	Value of Variable 1 Inches of Defective Weld				Value of Variable 2 Average Range
1	4.69	4.12	4.74	4.02	6.38
2	5.03	4.38	5.08	4.21	4.82
3	4.92	3.49	1.93	3.64	5.43
4	4.42	4.08	5.56	4.65	4.20
5	1.74	4.71	3.74	2.65	3.24
6	4.49	3.96	6.43	5.50	4.83
7	2.18	5.71	4.68	4.97	3.54
8	3.51	4.48	3.18	2.22	3.51
9	3.09	4.71	5.08	3.44	5.93
10	4.15	4.23	3.64	3.55.	2.90
11	4.57	2.82	5.07	4.79	4.69
12	4.38	3.72	3.59	4.07	2.69
13	4.91	3.69	3.62	2.41	5.35
14	5.07	4.79	3.55	3.75	4.21
15	2.92	5.86	2.07	2.76	4.13
16	4.20	4.20.	3.46	3.54	4.70
17	4.23-	4.38	4.91	5.14	5.22
18	2.46	3.65	5.19	3.75	3.44
19	4.67	4.89	3.62	6.42	4.83
20	4.02	4.87	3.61	2.27	3.38

Inches of Defective Weld -20 welders
Table 1-10

III. Over a period of 30 days pressure gauges were received in lots of size 30 from a supplier. Each lot was examined to determine the number of defectives. The results are shown in Table 1-11. Using a fraction defective control chart, determine if the process is in a state of statistical control.

Subgroup Number (Lot)	Number of Defective	Subgroup Number (Lot)	Number of Defectives
1	9	16	6
2	6	17	3
3	6	18	3
4	5	19	5
5	4	20	10
6	0	21	3
7	1	22	5
8	5	23	3
9	7	24	3
10	5	25	6
11	5	26	5
12	5	27	7
13	9	28	4
14	1	29	3
15	3	30	8

Number of Defectives in Each Lot of Size 30
Table 1-11

Each day for a period of 40 days 100 pieces of electrical conduit were assembled from three different parts as specified by an engineering drawing. A time standard was specified for the assembly and at the end of the day, the number of assemblies that were not within ± 15% of the time standard was determined. The results are shown in Table 1-12. Using a fraction defective. (fraction not within ± 15% of the time standard) chart, determine if the process is in a state of statistical control.

Subgroup Number	Number Outside (±15%) Time Standard	Subgroup Number	Number Outside (±15%) Time Standard
1	3	21	5
2		22	4
3	3	23	4
4		24	4
5	3	25	4
6	8	26	5
7	4	27	5
8	2	28	2
9	3	29	3
10	6	30	4
11	7	31	4
12	3	32	5
13	6	33	4
14	4	34	5
15	2	35	7
16	5	36	4
17	4	37	3
18	3	38	6
19	6	39	1
20	0	40	4

Number of Assemblies Not Meeting Time Standard
Table 1-12

V. Twenty spools of insulated electrical cable, with each spool containing 1000 ft of cable, were subjected to the same high voltage. For each cable spool a count was made of the number of points where the insulation failed. The results are shown in Table 1-13. Using a number of defects chart, determine if the process is in a state of statistical control. In this case assume that a defect is a point at which the insulation failed.

Subgroup Number (spool)	Number of Points Where Insulation Failed
1	16
2	15
3	10
4	22
5	6
6	8
7	10
8	10
9	12
10	9
11	19
12	17
13	8
14	8
15	8
16	10
17	6
18	12
19	11
20	9

Number of Insulation Failures in 1000 Ft of Cable
Table 1-13

VI. In a painting process 25 sections of aluminum plate, each consisting 'of 500 ft2, where. painted by the same person Upon completion of each plate the. number of bubble defects was counted. The results are shown in Table 1-14. Using a number of defects chart, determine if the painting process is in a state of statistical control.

Subgroup Number	Number of Bubble Defects
1	14
2	10
3	17
4	10
5	17
6	17
7	12
8	15
9	19
10	17
11	15
12	12
13	13
14	23
15	17
16	9
17	16
18	6
19	13
20	15
21	18
22	19
23	16
24	13
25	13

Number of Bubble Defects in 500 Ft^2 of Aluminum Plate
Table 1-14

2. DISCRETE RANDOM EXPERIMENTS

2.1 The Discrete Random Experiment and Examples

In this section of the workbook a technique for displaying data from a discrete random experiment is introduced. The technique, called a histogram, gives a visual display of the data and provides a basis for developing an appropriate mathematical model of the random experiment. The technique also provides a method for describing visually the behavior of sane variable associated with a series of repetitions of sane basic random experiment. After the concept of the histogram is introduced, our attention will be focused on certain calculations which are produced from a set of data to summarize the information contained in the set of data. These calculations, which are called summary measures, are:

1. The sample aver age
2. The sample variance
3. The sample standard deviation
4. The sample coefficient of variation.

As we proceed in our discussion the histogram will be used extensively to illustrate the meaning of each of the summary measures. It is important at this point to remind the reader that our ultimate goal is to understand statistically the behavior of a specific operating system in the shipyard. In order to reach this goal it is important to define exactly the operating system under study and to think of the operating system as a random experiment. Once the random experiment is defined, the techniques for analyzing the situation will follow quite naturally.

we must also emphasize the fact that in this section of the workbook only the discrete random experiment is being studied.

EXAMPLE 2-1

The Operating System

In a specific section of the shipyard two sections of steel plate 120 inches long are welded together to form a single unit and are then moved to another section of the shipyard.

The Discrete Random Experiment

We randomly select a welded unit (the output of the operating system) and, using ultrasonic testing, determine the total number of defects in the weld of length 120 inches.

The Set of Outcomes

The outcome of the experiment will be either O defects, in 120 inches, 1 defect in 120 inches, 2 defects in 120 inches, etc. Notice, theoretically, there is no upper 1imit on the number of defects. From a practical point of view, however, one does exist.

EXAMPLE 2-2

The Operating System

In a particular section of the pipe shop a spool piece is manufactured. Using motion and time study a standard has been established for the time to manufacture the unit.

The Discrete Random Experiment

At the end of the day randomly select a spool piece from the daily production of’ spool pieces and determine whether or not the actual time to manufacture the piece was within fifteen percent of the specified time standard.

The Set of Outcomes

The outcome of the experiment will be either, yes, the piece is within fifteen percent of the time standard or, no, the piece is not within fifteen percent of the time standard. In this type of discrete random experiment it is common in mathematics to define the outcome by a number. For example, we might say the outcome is O if the spool piece is within fifteen percent of the time standard and 1 if not. Such O-1 (or binary) variables are common in statistics.

EXAMPLE 2-3

The Operating System

At the loading dock of a shipyard, incoming materials are inspected and transferred to their appropriate locations.

The Discrete Random Experiment

Fran a recent shipment of 4′ x 8′ sheets of 5/1 6“ plywood (yes, wood is used in the shipbuilding industry) select one sheet of plywood and count the number of significant defects. We assume that a precise definition of a significant defect has been provided.

The Set of Outcomes

Each time the experiment is conducted, the outcome will be either, O significant defects, 1 significant defect, 2 significant defects, etc., the same set of possible outcomes as in Example 2-1.

At this point it is important to illustrate the manner in which more complicated random experiments can be developed from the repetition of some basic random experiment. Using the previous three examples as basic random experiments, more complicated discrete random experiments **will be illustrated through the following examples.**

EXAMPLE 2-4

The Operating System

The same situation described in Example 2.1

The Discrete Random Experiment

Randomly select five welded units and using ultrasonic testing, determine the number of defects per unit. Compte the total number of defects from all five units and then divide the total by five.

The Set of Outcomes

The outcome of the experiment will be either 0/5, 1/5, 2/5, etc.

EXAMPLE 2-5

The Operating System

The same situation described in Example 2-2.

The Discrete Random Experiment

At the end of the day randomly select 30 spool pieces from the daily production and' determine the total number that are not within fifteen percent of the specified time standard.

The Set of Outcomes

The outcome of the experiment will be either 0,1,2...,29, or 30.

EXAMPLE 2-6

The Operating System

The same situation described in Example 2-3.

The Discrete Random Experiment

From the shipment of plywood randomly select 20 sheets of plywood, and determine the number of significant defects per sheet. Then determine the total for all 20 sheets.

The Set of Outcomes

The set of outcomes will be either 0, 1, 2, 3, etc.

2.2 The Histogram

The six examples given in Section 2.1 can be divided into two general types: (1) the basic random experiment and (2) repetitions of the basic random experiment. Examples 2-1 through 2-3 each describe basic random experiments, while Examples 2-4 through 2-6 were constructed by repeating a given basic random experiment. It is important to emphasize at this point that a statistical analysis of data is usually conducted for basic random experiments. Once the statistical behavior of the basic random experiment is understood, mathematical statistics can then be used to describe most variables which are associated with repetitions of the basic random experiment. It should also be pointed out that in any statistical analysis of random experiments, great care must be spent in defining clearly the basic random experiment and the variable that is being studied. The analyst should also be concerned with the operating system and any problems that may arise with repeating the basic random experiment under essentially the same conditions.

We now direct our" attention to the concept of a histogram for a discrete random experiment. We assume that a random- experiment has been defined with k possible outcomes that have been identified and have been assigned numerical values $a_1, a_2, \ldots, a_k$. In Example 2-2, the Pipe Shop, k = 2 since there are only 2 out-comes. If we decided that the numbers 0 and 1 would represent yes and no respectively, then al = 0 and $a_2 = 1$.

In general we will assume that the operating system is in a state of statistical control and that we have observed data from n repetitions of the basic discrete random experiment. Assume that in the n repetitions the outcome al occurred f, times, the outcome a_2 occurred f_2 times, ..., the outcome a_k occurred f_k times. Technically speaking, k may be as large as necessary to adequately describe the experiment: We call f_i the frequency of occurrence of outcome a_i for i = 1,2, ... k and $\frac{f_i}{n}$ is called the relative frequency of occurrence of outcome a_i for i = 1,2,... k. Note that $f_1 + f_2 + \ldots + f_k = n$ and

$$\frac{f_1}{n} + \frac{f_2}{n} + \ldots + \frac{f_k}{n} = 1$$

We are now in a position to define the histogram.

Definition

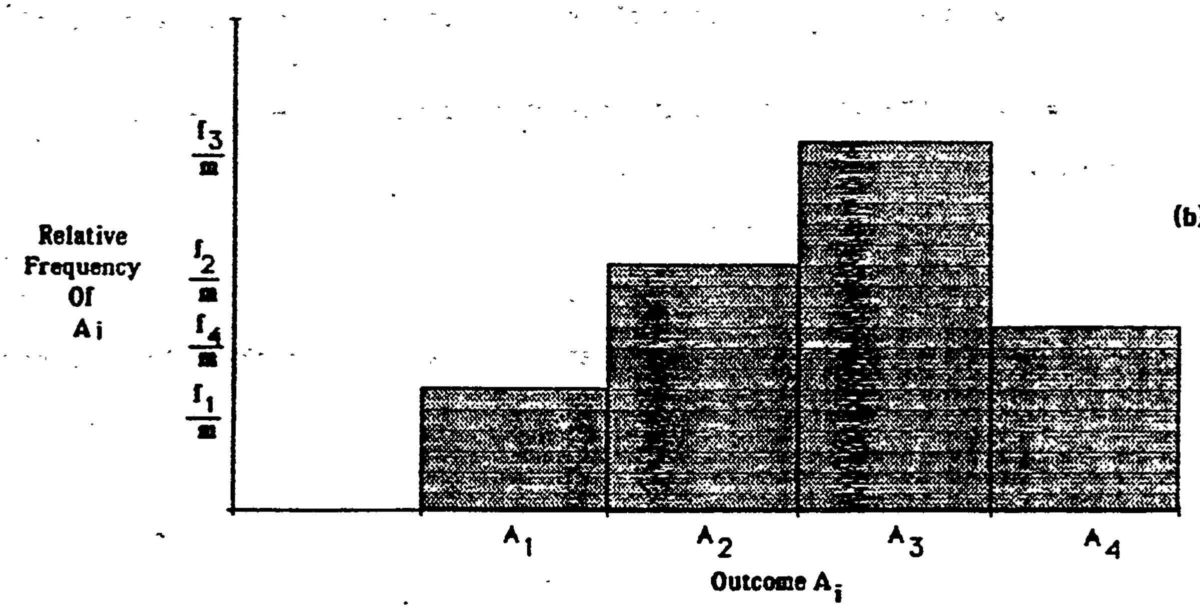
A histogram of data from a discrete random experiment is a plot of the relative frequency of occurrence of outcome a_i $(\frac{f_i}{n})$ versus a_i for $i = 1,2,...,k$.

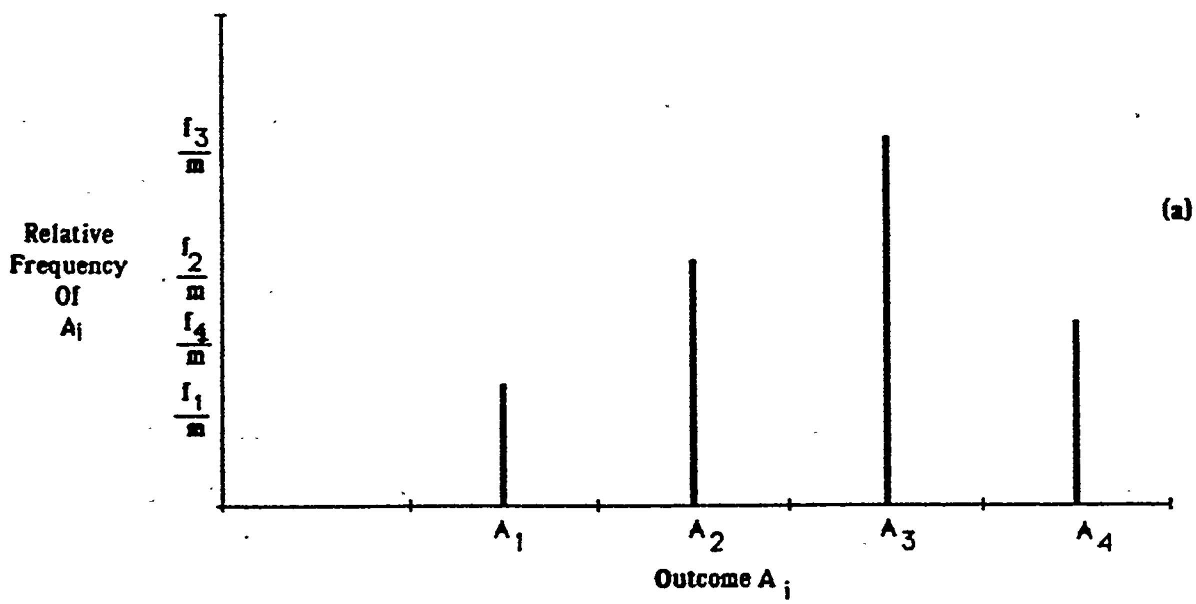
Graphically the histogram for k = 4 is shown in Figure 2-l(a). When the a_i for i = 1 ,2,..., k are sequential numerical values it is common practice to plot the histogram as shown in Figure 2>1 (b), using rectangles with vertical 1ines at the midpoint of adj scent values and a height of $\frac{f_i}{n}$ for $i = 1,2,...,k$. This is the approach that will be used in the remainder of this workbook. It proves a more graphic picture of the manner in which the data are distributed.

In an actual experimental situation a_1, a_2, a_3, a_4, f_1, f_2, f_3, f_4 and n would be numbers. For example, suppose that we take the situation described in Example 2-2 of Section 2.1, and collected data on 1000 welded units (e. g., the basic experiment was repeated 1000 times, n = 1000), with the results shown in Table 2-1. The histogram for this example is shown in Figure 2-2.

2.3 Summary Measures

The histogram of a data set is an important means to visually summarize the data. Another important means is the use of quantities known as summary measures. These numbers are essential in comparing different data sets, setting control 1imits on control charts, and in understanding statistically the behavior of the operating System. The most





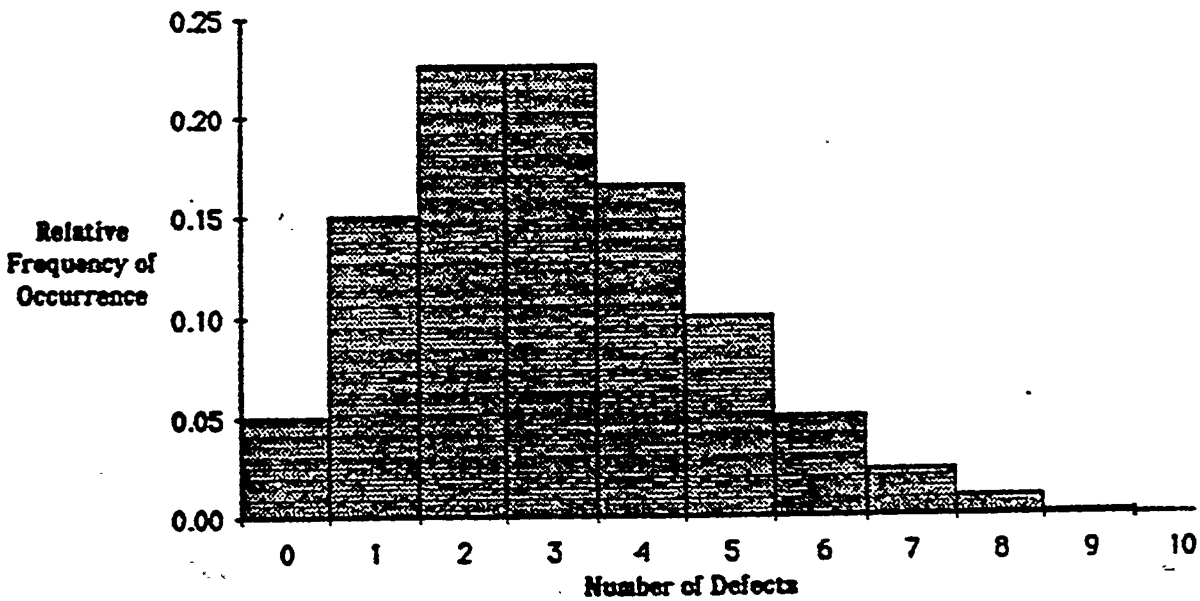
Alternative Forms of the Histogram

Figure 2-1

Number of Defects per Unit (a_i)	Frequency of Occurrence (f_i)	Relative Frequency of Occurrence ($f_i/1000$)
0	50	0.050
1	150	0.150
2	224	0.224
3	224	0.224
4	168	0.168
5	101	0.101
6	50	0.050
7	22	0.022
8	9	0.009
9	2	0.002

Summary of Data - 1000 Welded Units

Table 2-1



Histogram of Data - 1000 Welded Units

Figure 2-2

commonly used summary measures are defined as follows, with the meaning of each measure described later.

1. Sample Average (x)

$$\bar{X} = \frac{1}{n}\left(a_1f_1 + a_2f_2 + \ldots\ldots + a_kf_k\right)$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{k} a_if_i \qquad \text{(see footnote*)}$$

2. Sample Variance (S^2)

$$S^2 = \frac{1}{n}\left[(a_1f_1 - \bar{X})^2 + (a_2f_2 - \bar{X})^2 + \ldots + (a_kf_k - \bar{X})^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{k}(a_if_i - \bar{X})^2$$

Computationally, it is convenient to use an equivalent alternative definition where

$$S^2 = \frac{1}{n}\left[a_1^2f_1 + a_2^2f_2 + \ldots\ldots + a_k^2f_k - n\,\bar{X}^2\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{k} a_i^2f_i - n\,\bar{X}^2\right]$$

* If you are not familiar with this mathematical symbol you should note that $\sum_{i=1}^{k} a_if_i$ is simply a replacement for the quantity $a_1f_1 + a_2f_2 + \ldots + a_kf_k$.

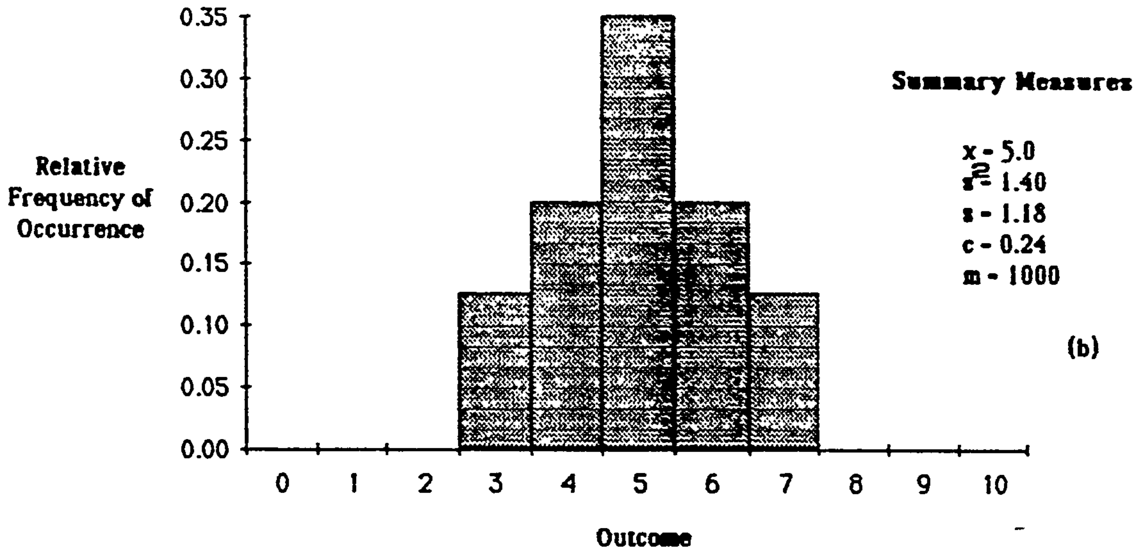
3. **Sample Standard Deviation (S)**

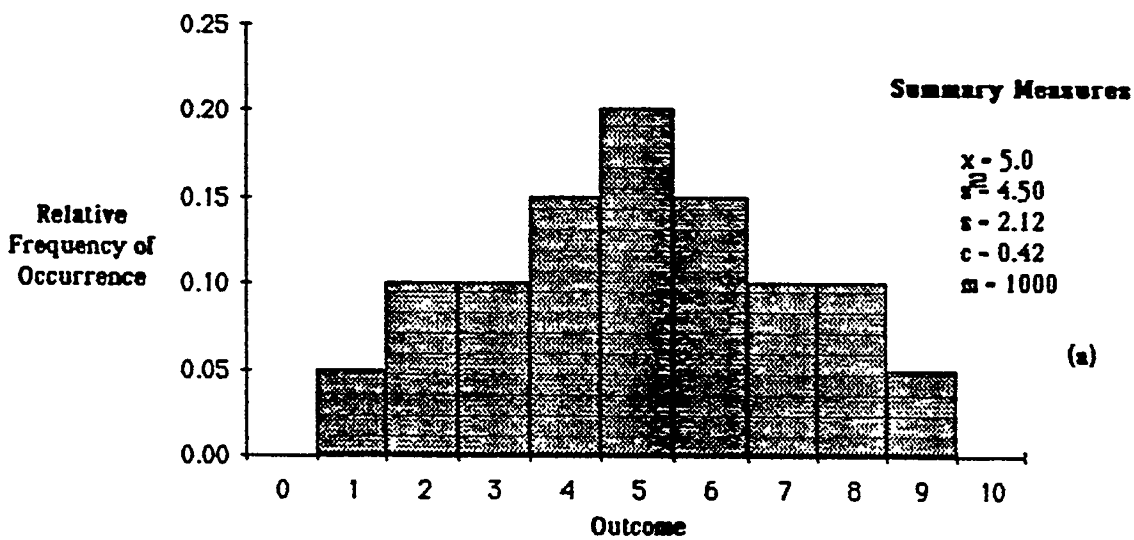
$$S = \sqrt{S^2}$$

4. Sample Coefficient of Variation (C)

$$C = \frac{S}{\bar{X}}$$

The sample average x is a measure of central location of the data. If you imagine the histogram as being made from material of uniform density, then the sample average, x would correspond to the center of gravity or the balance point of the histogram. It is important to note that the sample average is not necessarily equal to any one of the experimental outcomes $a_1, a_2, \ldots, a_k$. The sample variance, S^2, and the sample standard deviation, S, are measures of dispersion. They give the analyst a measure of the amount of variability in the data and are particularly useful in comparing two sets of data with the same sample average. The data set with higher value of S or S^2 has the higher variability. Since the sample standard deviation S has the same unit of measurement as the variable being analyzed, it is more convenient to use as a measure of variation. The significance of the sample standard deviation (or the sample variance) can be illustrated with the histogram. In Figure 2-3 two histograms are shown. In each case the histograms have the same sample average, but histogram(b) is more con-Centrated about the sample average The sample standard deviation of Figure 2-3(a) is correspondingly much higher than that of Figure 2-3(b).





Example of Two Histograms
Figure 2-3

The final summary measure, the coefficient of variation, is not used extensively in quality assurance. It is, however, both important and useful in comparing two histograms that have different sample averages because it provides, for each histogram, a measure of dispersion relative to the sample average.

2.4 Repetitions of A Discrete Random Experiment

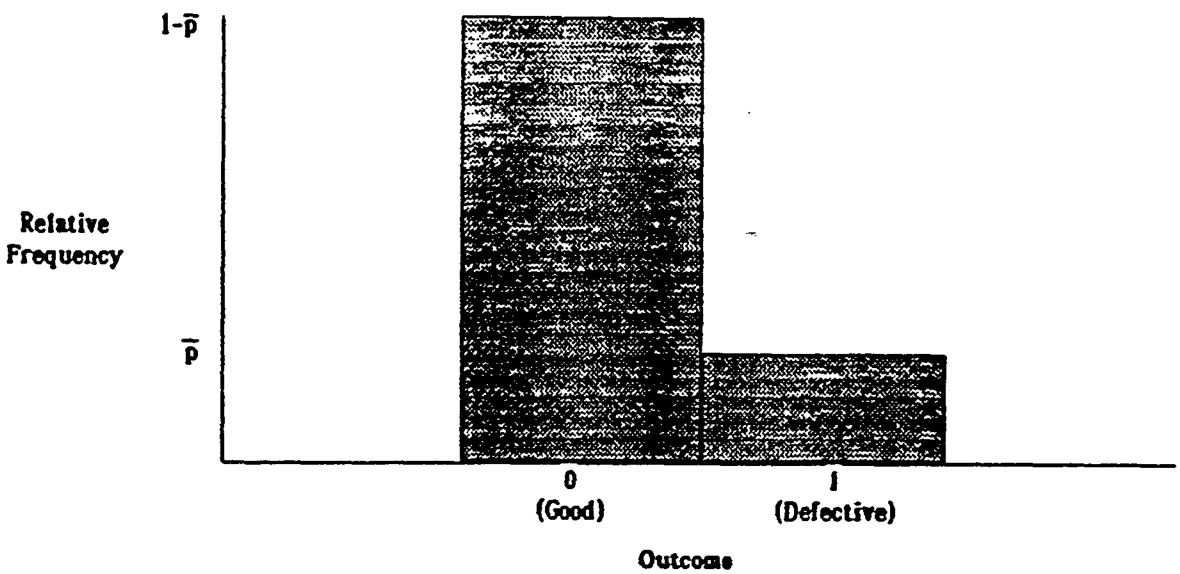
In practical applications of quality assurance the summary measures associated with the basic random experiment become the foundation for analyzing more complicated experiments that involve computations which are made from several repetitions of the basic random experiment. In this section of the workbook the histogram of the basic random experiment will be used to illustrate the statistical behavior of variables that are commonly used in the analysis of several repetitions of a random experiment. At this point the reader should review Examples 2-4 to 2-6 of Section 2.1.

In the following sections these variables are discussed for each of two important kinds of discrete random experiments.

Repetitions of A Discrete Random Experiment - Two Important Cases

Case 1 - The (0,1) Discrete Random Experiment

In this case the outcome of the basic random experiment will be either 0 (a good item) or 1 (a defective item) and the histogram of the basic random experiment will appear as shown in Figure 2-4. This type of histogram is common in quality assurance (e.g., an electrical switch



Histogram of Basic Random Experiment - Case 1
Figure 2-4

is either good or defective; a weld is either good or defective; insulation on an electric cable either breaks down at high voltage (defective) or does not (good); etc.).

When the system is in a state of statistical control and when the basic random experiment is repeated k times, three important variables are of interest: A) The total number of defective (the sum of the outcomes); B) the fraction defective (the average outcome); and C) The percent defective (100 times the fraction defective). In each of the three cases the theory of mathematical statistics can explain the behavior of the histogram of each of these variables. The results are as follows:

A. The Histogram of the total Number of Defective will have:

1. -An Average that is (kp)

2. A Variance that is $k\bar{p}(1-\bar{p})$ and

3. A Standard Deviation that is $\sqrt{k\bar{p}(1-\bar{p})}$.

B. The Histogram of the Fraction Defective will have:

1. An Average that is $\bar{p}$

2. A Variance that is $\frac{\bar{p}(1-\bar{p})}{k}$ and

3. A Standard Deviation that is $\sqrt{\frac{\bar{p}(1-\bar{p})}{k}}$.

C. The Histogram of the Percent Defective will have:

1. An Average that is 100P

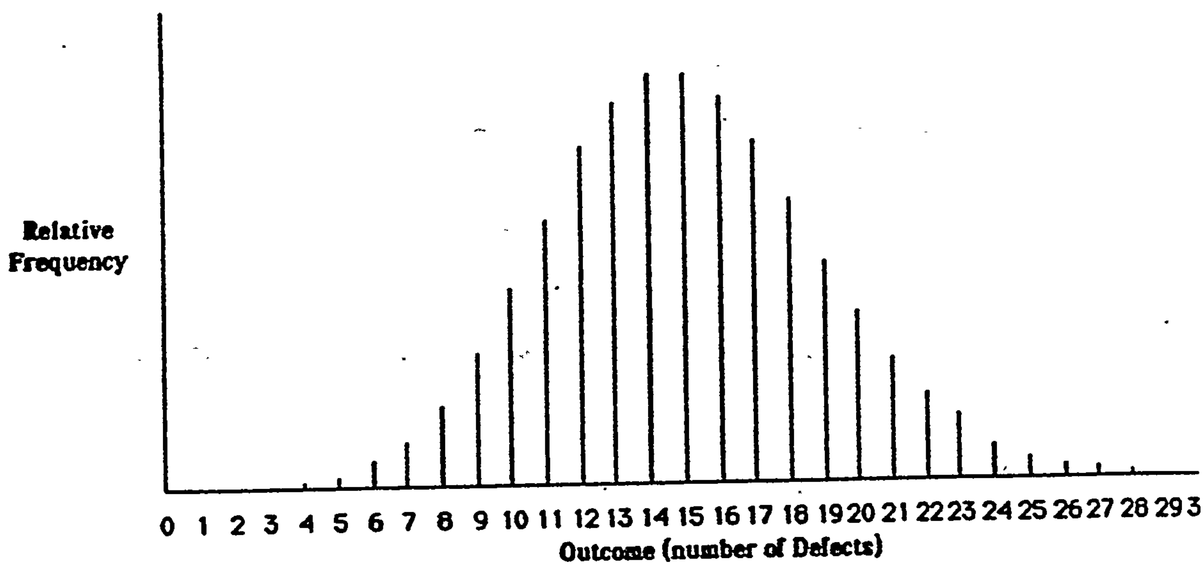
2. A Variance that is $\frac{100^2\bar{p}(1-\bar{p})}{k}$ and

3. A Standard Deviation that is $100\sqrt{\frac{\bar{p}(1-\bar{p})}{k}}$.

At this point YOU should begin to understand how the center line and control 1imits of a control chart are defined. refer to Chapter 1, Section 1.4.

Case 2 - The Discrete Random Experiment with Outcomes 0, 1,2, . . .

In this situation, the discrete basic random experiment will have a set of outcomes 0,1 ,2,3,... and the histogram of the basic random experiment will be similar to the one as shown in Figure 2-5. In this situation we are generally dealing with some typs of complicated assembly where the number of defects is being counted and it is unlikely to find a small number of defects. For example, pinhole defects in the painted surface of a large area or the number of mistakes in a large array of complicated wiring.



Histogram of Basic Random Experiment - Case 2
Figure 2-5

When the basic random experiment is repeated k times two variables are of interest: A) The total number of defects and B) The average numbar of defects. Again, for a process that is in statistical control, the theory of mathematical statistics can be used to explain the histogram of these variables as follows:

A. The Histogram of the Total Number of Defects will' have:

1. An average value that is the same as the average of the histogram of the basic random experiment multiplied by k, $k\bar{X}$.
2. A Variance that is $k\bar{X}$ and
3. A Standard Deviation that is $\sqrt{k\bar{X}}$.

B. The Histogram of the Average Number of Defects will have:

1. An average that is the same as the average of the histogram of the basic random experiment, $\bar{X}$.
2. A Variance that is $\frac{\bar{X}}{k}$ and
3. A Standard Deviation that is $\sqrt{\frac{\bar{X}}{k}}$.

2.5 <u>Problems</u>

I. An electronic assembly consists of five modules where each module can be replaced if it is found to be defective. The electronic assemblies are manufactured by a supplier and are packaged five to a carton. At the loading dock of the shipyard 50 cartons were examined and the number of defective modules in each electronic assembly was determined with the results shown in Table 2-2. Imagine each carton as a series of five repetitions of a basic random experiment where the outcome of the basic random experiment is 0 defective modules, 1 defective

Carton Number	Number of Defective Modules Electronic Assembly Number					Carton Number	Number of Defective Modules Electronic Assembly Number				
	1	2	3	4	5		1	2	3	4	5
1	0	3	2	2	0	26	2	0	1	2	0
2	0	0	2	1	4	27	3	1	1	2	2
3	0	0	2	0	1	28	1	1	1	1	0
4	0	0	1	2	0	29	1	1	0	1	0
5	0	0	2	0	0	30	2	1	0	2	0
6	1	0	0	0	2	31	1	1	1	2	1
7	2	0	1	1	1	32	1	2	1	1	0
8	1	2	3	0	3	33	0	3	1	0	0
9	1	0	1	3	0	34	1	2	0	1	2
10	4	1	0	1	2	35	1	2	1	0	3
11	0	1	2	2	1	36	1	0	1	0	1
12	0	3	2	0	1	37	0	1	2	0	1
13	1	0	0	2	1	38	0	2	1	0	1
14	1	1	0	1	2	39	0	0	1	1	2
15	1	2	0	3	1	40	1	2	2	1	1
16	0	1	1	0	3	41	1	0	0	1	2
17	1	0	1	0	4	42	0	0	2	0	0
18	3	1	0	0	1	43	2	0	1	0	1
19	1	1	2	2	1	44	2	0	1	0	2
20	0	0	1	1	1	45	0	1	1	2	0
21	3	o	o	1	2	46	2	3	1	1	1
22	0	2	2	2	2	47	1	0	2	1	1
23	0	1	1	2	1	48	0	0	3	0	2
24	3	2	4	0	1	49	1	2	0	3	1
25	1	1	3	0	1	50	2	3	1	2	2

Number of Defective Modules in Electronic Assemblies

Table 2-2

module, ... or 5 defective modules. Construct a histogram of basic random experiment and compute the four summary measures ($\bar{x}$, s^2, s and c). For each carton compute the average number of defective modules and plot a histogram of these 50 values.

II. Identical electric cables, each of length 100 ft., were subjected to a high voltage to determine the number of points in each cable where the insulation failed. As each group of our cables was tested the results were reported as shown in table 2-3. Plot a histogram of the basic random experiment and compute the four sunmary measures (~, s^2, s and c). Then plot a histogram of the Average Number of Insulation Failures and- compute the summary measures. Compare these values to the results that are supposed to occur from the results of Section 2.4.

Test Number	Number of Insulation Failures in Cable Number 1	2	3	4	Average number of Insulation Failures
1	6	3	5	2	4.00
2	1	1	5	3	2.50
3	3	5	2	2	3.00
4	3	3	3	3	3.00
5	4	2	1	3	2.50
6	1	3	3	2	2.25
7	1	2	3	2	1.75
8	1	2	3	1	1.75
9	2	4	2	4	3.00
10	1	1	2	4	2.00
11	1	3	3	5	3.00
12	1	1	4	4	2.50
13	3	3	0	2	2.00
14	0	0	1	0	0.25
15	1	1	6	3	2.75
16	4	5	3	3	3.75
17	4	1	4	2	2.75
18	0	4	0	3	1.75
19	3	3	1	4	2-75
20	1	1	2	3	1.75
21	1	3	2	5	2.75
22	3	5	0	3	2.75
23	1	2	4	3	2.50
24	1	2	3	3	2.25
25	3	3	1	5	3.00
26	1	1	2	8	3.00
27	3	4	1	1	2.25
28	0	4	2	1	1.75
2 9	1	2	2	2	1.75
30	2	1	3	0	1.50

Number of Insulation Failures in 120 Electric Cables
Table 2-3

3. CONTINUOUS RANDOM EXPERIMENTS

3.1 Introduction

In this section of the workbook a technique for displaying data from continuous random experiments is introduced. The technique gives a visual display of the data and again is called the histogram. The development of the histogram, however, is somewhat different when the data are from an operating system that can be imagined as a continuous random experiment. The four summary measures that were defined for discrete random experiments will be introduced again for the continuous case. The computations, however, are somewhat different and will be explained in detail.

To illustrate the type of continuous random experiments that are found in the shipbuilding industry, three basic random experiments are described. They are followed by three additional experiments to illustrate that repetitions of a basic random experiment are, as in the discrete case, important in the continuous case.

EXAMPLE 3-1

The Operating System

In a specific part of the shipyard two sections of steel plate 120 inches long are welded together along the 120-inch dimension to form a single unit. This unit is then transferred to another section of the shipyard.

The Continous Random Experiment

Prior to welding, sections of steel plate are butted together as shown in Figure 3-1 and a feeler gauge is used to measure the maximum gap that occurs over the entire 120 inches.

The Set of Outcomes

The set of outcomes cannot be counted. When measuring the maximum gap, there is no such thing as a first outcome, a second outcome etc. ; the outcome can be any value on a continuous scale from zero to sane realistic upper limit. The outcome scale is said to be infinitely divisible.

EXAMPLE 3-2

The Operating System

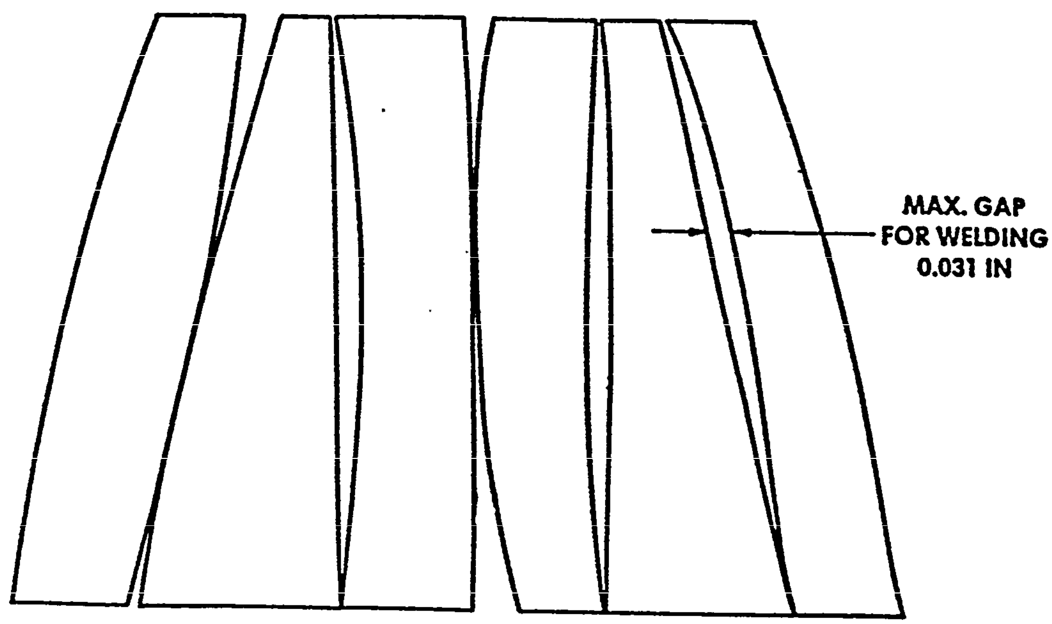
In a certain section of the shipyard pre- cut steel headers are received from an external supplier. The headers are then welded between two I beams which are attached to a steel plate. See Figure 3-2.

The Continous Random Experiment

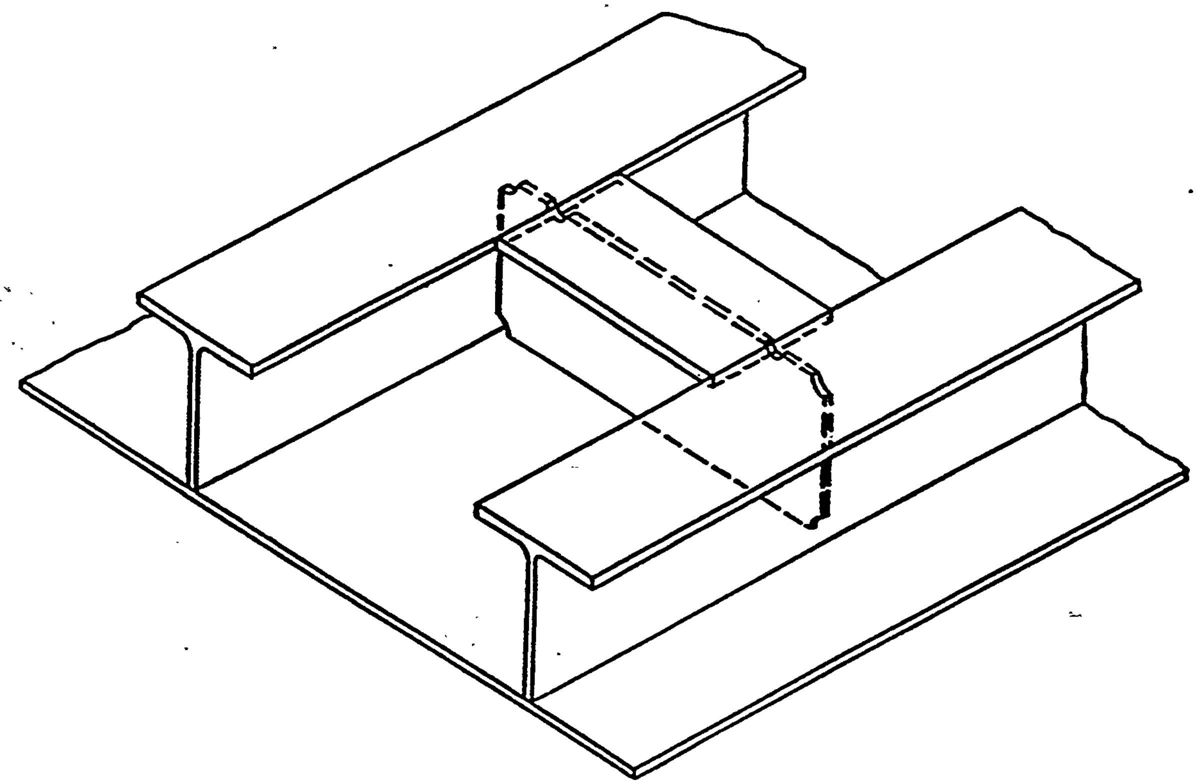
A steel header is selected at random from the available headers and its maximun overall length is measured.

The Set of Outcomes

When measuring the maximun overall length, the outcome scale is infinitely divisible, as with Example 3-1.



Maximum Gap in Adjacent Steel Plate
Figure 3-1



Typical Deck Header Installation
Figure 3-2

is selected at random and the difference between the engineering drawing dimension for length a in Figure 3-3 and the actual dimension as cut for length a is determined.

The Set of Outcomes

Again the set of outcomes is infinitely divisible.

EXAMPLE 3-4

The Operating System

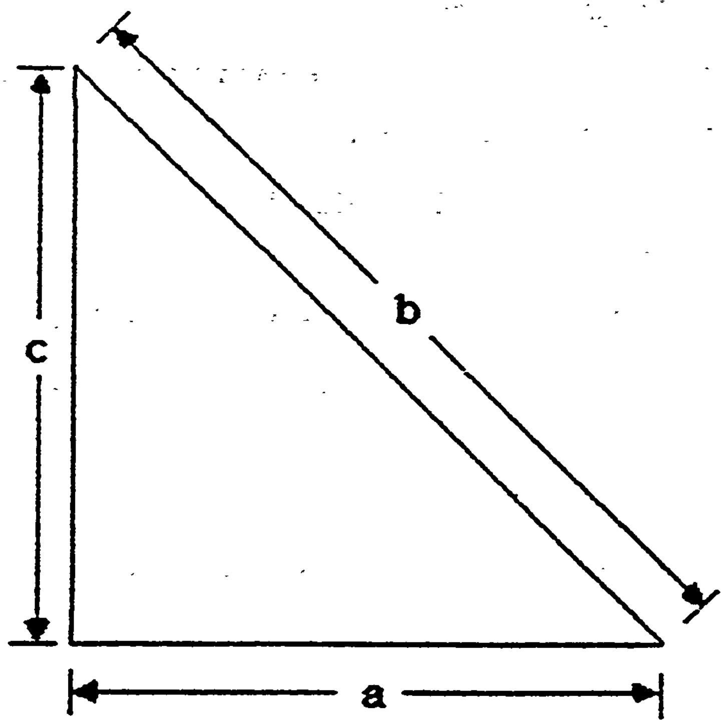
The same situation described in Example 3-1.

The Continuous Random Experiment

The basic experiment described in Example 3-1 is repeated five times (e. g., the maximum gap for each of the butted sections shown in Figure 3-1 is computed). The average of the five measurements (maximum gap size in each trial) and the range (largest-smallest) is then determined.

The Set of Outcomes

Again infinitely divisible.



Engineering Drawing Dimensions

a = 24 inches
b = 30 inches
c = 18 inches

Triangular Section of Steel Plate
Figure 3-3

EXAMPLE 3-5

The Operating System

The same situation described in Example 3-2.

The Continuous Random Experiment

The basic experiment described in Example 3-2 is repeated five times. The average and the range of the five values are determined.

The Set of Outcomes

Again infinitely divisible.

EXAMPLE 3-6

The Operating System

The same situation described in Example 3-3.

The Continuous Random Experiment

The basic experiment described in Example 3-3 is repeated eight times. The average difference and the range of the differences are computed.

The Set of Outcomes

Again infinitely divisible.

3.2 The Histogram- Continuous Data

As we saw in the treatment of discrete random experiments, a statistical analysis of data is usually conducted for the basic random experiment. In the language of statistics, understanding a basic random experiment is analogous to understanding a population. When the basic random experiment is repeated under essentially the same conditions, we are, in the language of statisticians, sampling from a population. Once the basic random experiment (or population) is understood statistically, mathematics can be used to describe those variables which are usually associated with a random sample from the population. As we did in the discrete case the histogram will be defined after the introduction of basic notation.

We assume that a basic random experiment has been clearly defined and that the experiment results in an outcome (the variable) which has been determined to be infinitely divisible (continuous or measurable). We also assume that the basic continuous random experiment has been repeated n times under essentially the same conditions in which the operating system is in a state of statistical control. The n repetitions of' the basic random experiment produce n numerical values (the data or measurements) for the variable of interest which for notational purposes are called $x_1, x_2, x_3, \ldots, x_n$. With this notation X_1 is the numerical value from the first repetition of the experiment, X_2 is the numerical value from the second repetition, etc. The construction of a histogram for continuous data requires that we know the largest and the smallest numerical value in the set of data. Therefore let M be the largest numerical value in the set of data. M is one of the X's and it is the largest. Rotationally we write $M = \text{maximum}\ (x_1, x_2, \ldots, x_n)$. Let L be the smallest numerical value in the set of data. Again, L is one of the x's and it is the smallest. Rotationally we write L = minimum (xl,x2, . ..xn). The range, R, of the data set is then defined to be the difference between M and, L or R =M-L. Before we begin With our discussion the reader should re-examine Figure 2-1 and should examine specifically the horizontal axis of the histogram for the discrete random experiment. In that case specific values for the outcomes could be identified and then relative frequencies of occurrence could be plotted at these values. This is not true in the continuous random experiment. The way the horizontal axis is constructed in the continuous random experiment is as follows:

Divide the range, R = M-L, into equally spaced intervals, called cells, in such a way that every data point falls into one and only one cell. Usually the number of cells to be used will be between 7 and 20 and will depend on the number of data points. The more data you have, the more cells you will use. This is a very subjective process that is learned by working with different amounts of data from different experimental situations.

Now, assuming that k cells have been identified, let f_i be the number of data points out of the n that have values which place them in cell i for i = 1, 2, ..., k. In other words, for cell i all f_i data points have values which are greater than some number which is the beginning of cell i, and for cell i all f_i data points have values which are less than a second number which is the end point of cell i. As we defined in the discrete case we have

f_i = number (frequency) of data points in cell i.

f_i/n = relative number (relative frequency) of data points in cell i for i = 1, 2, ..., k.

With these definitions we must have

(1) $f_1 + f_2 + \ldots + f_k = n$ and

(2) $\frac{f_1}{n} + \frac{f_2}{n} + \ldots + \frac{f_k}{n} = 1$.

We are now in 'a position to define the histogram for a continuous random experiment.

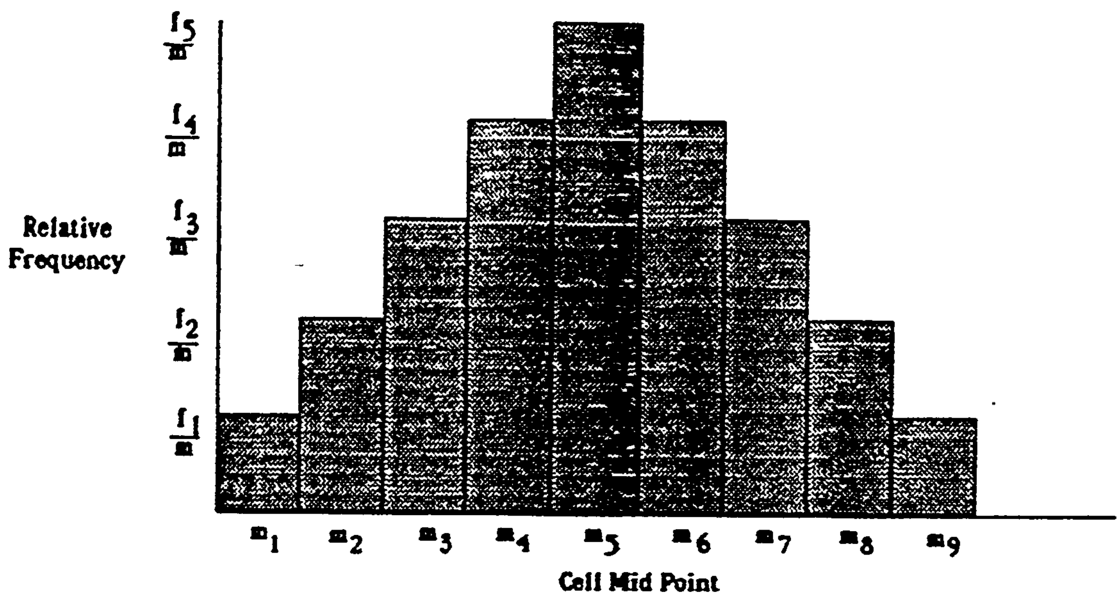
Definition:

A histogram for a continuous random experiment is a plot of the relative frequency of occurrence in cell i $(\frac{f_i}{n})$ versus cell i for i = 1, 2, ..., k.

In the construction of the histogram it is common practice to label the horizontal axis using the midpoints of each cell (say m_1, m_2, . ..~ ~) and, for cell i, to draw a horizontal line of constant height, $\frac{f_i}{n}$ over the cell width.

The general notation we have just defined and the procedure for constructing a histogram is illustrated in Figure 3-4 for a situation with nine cells.

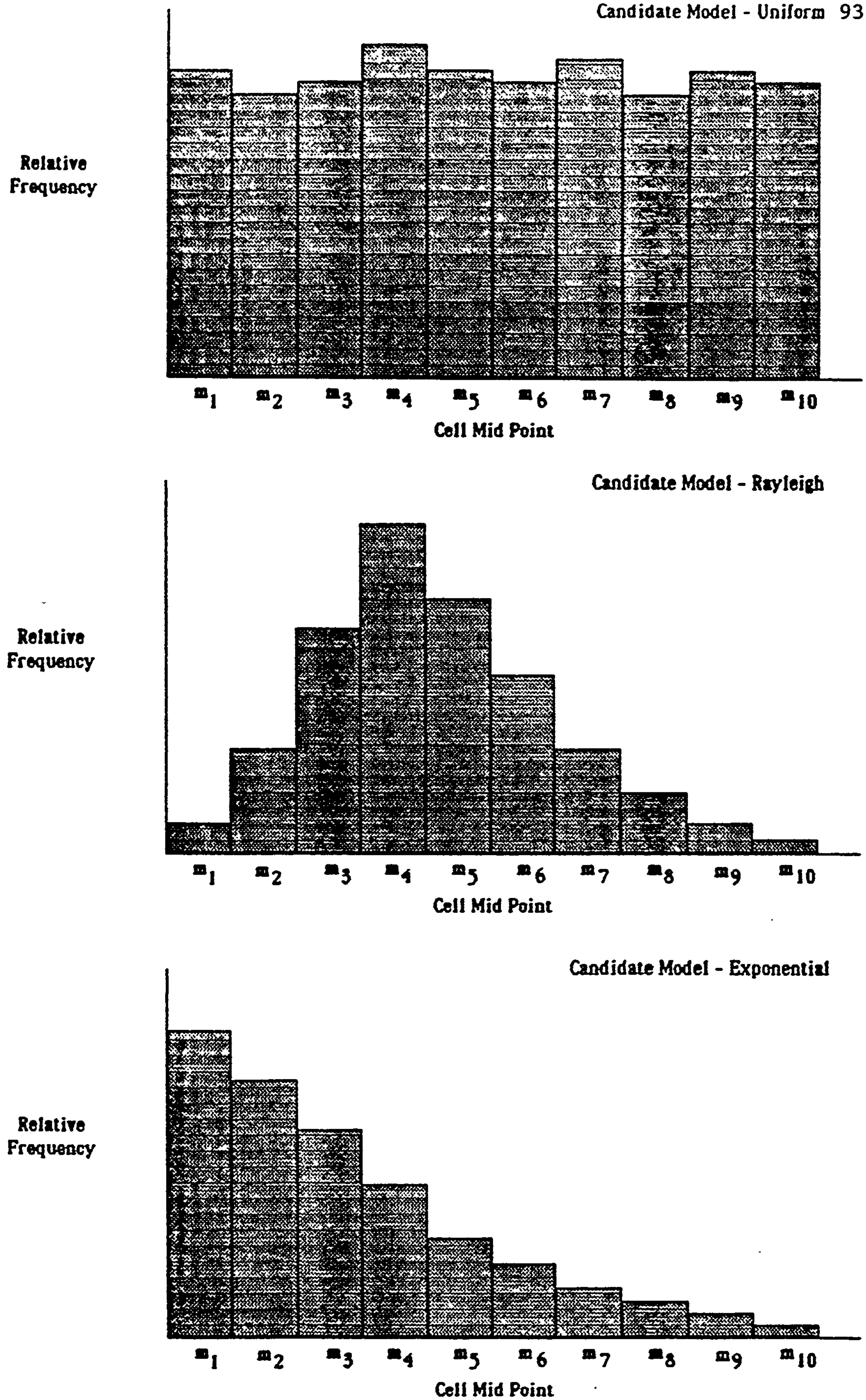
As we mentioned earlier, the histogram is constructed for the basic random experiment and gives us a visual display of the statistical behavior of the population from which we will be drawing samples (repeating the experiment). It is important to understand that the shape of the histogram is most significant. The shape of the histogram will help us to identify an appropriate e mathematical model for the basic



The General Histograms - Nine Cells
Figure 3-4

random experiment. For example, a statistician would look at the shape in Figure 3-4 and conclude that the population might be modeled by something called the normal distribution (the bell-shaped curve). Other shapes lead to other types .of models and are illustrated in Figure 3-5. Although possible mathematical models are listed for each shape in Figure 3-5, it is not important to concern ourselves with these models at this point. It should also be emphasized that the histogram of the basic random experiment is important in an examination of sane dimension to determine if tolerances are being met.

In a detailed statistical analysis of a continuous basic random experiment, a specific type of mathematical function called a density function is used as a mathematical model for the basic random experiment. For a given histogram, statistical procedures can be used to determine the appropriate mathematical model. In the detailed analysis the statistician usually makes a plot of the density function on the sane graph paper used to plot the histogram. In this manner the analyst is provided with a visual description of the extent to which the mathematical model " fits" the histogram. Since all density functions are such that the area under the function is one, the scale of the vertical axis of the histogram must be adjusted to create a histogram in which the total area of the rectangles suns to one. Refer to Figure 3-4 and note that, in general, unless the cell width is one, the sum of the rectangular areas would not be one. To make the sum of the rectangular areas equal to one, change the scale of the vertical axis by dividing each relative frequency $(\frac{f_i}{n})$ on the vertical axis by the cell width (the difference between two adjacent cell midpoints). Therefore, if you



Typical Histograms - Continuous Case
Figure 3-5

wish-to plot a mathematical density function on the same graph as the histogram, the plot should be relative frequency divided by cell width versus cell midpoint. If the cell width is W then a 1ine of constant height $\frac{f_i}{nW}$ would be plotted over cell i. By adjusting the vertical scale in this manner the shape of the histogram does not change and, at the same time, the density function can be plotted and then compared to the histogram.

Since the normal density function is used extensively in many situations in quality assurance, it is introduced at this point in the workbook.

The normal density function, denoted by f(x), is defined as fOllows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad \text{for all } x \text{ between } -\infty \text{ and } +\infty$$

The function includes two--numbers v and u whi ch are called parametrs of the density function. The shape of the function is il-lustrated in Figure 3-6. For any value of v and for any positive value of u the function is centered on the parameter M and is symmetric about v with the shape of a 'bell ." The total area under the density function is one and the function is always positive (alwaYs above the x axis). The parameter v is called the location parameter and is that point on the x axis at which the function reaches its highest value and about

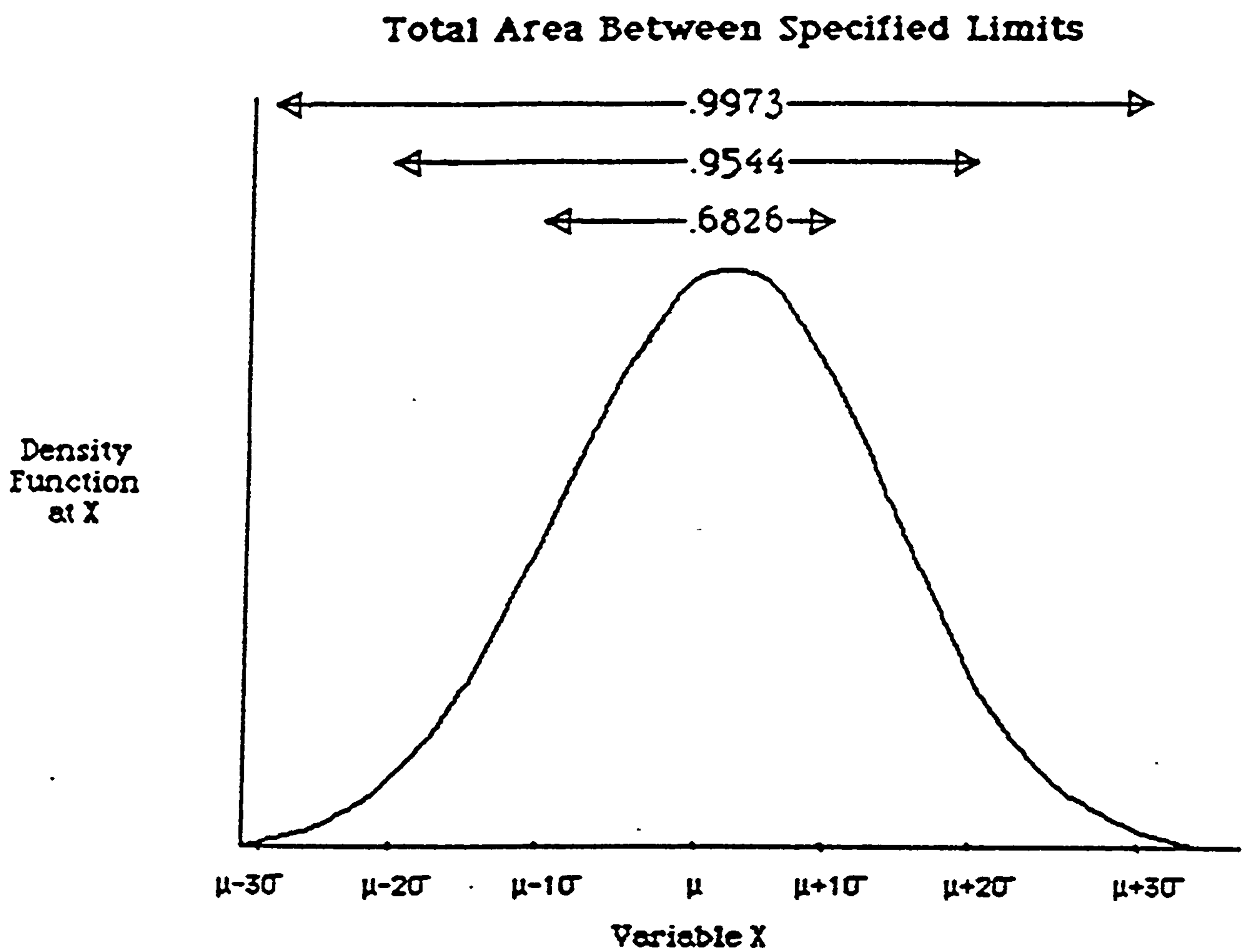

Total Area Between Specified Limits
.9973
.9544
.6826
Density
Function
at X
μ-3σ
μ-2σ
μ-1σ
μ
μ+1σ
μ+2σ
μ+3σ
Variable X

Figure 3-6

which the function is symmetric. The parameter σ is called the dispersion parameter and determines the shape of the bell. For a fixed value for μ, as σ decreases the density function becomes more and more concentrated about the parameter U. This phenomenon is illustrated in Figure 3-7. The total area under the curve between X values of V-U and μ+σ is 0.6826; between μ-2σ and μ+2σ the area is 0.9544; and between μ-3σ and μ+3σ; the area is 0.9973. This is illustrated in Figure 3-6.

In a practical situation the analyst would collect a set of data and plot a histogram for sane basic continuous random experiment. If the histogram has the bell shape an attempt would then be made to use the normal density function as an appropriate mathematical model. The summary measures of the histogram would be used to assign numerical values to M and u. Once the parameters have been assigned numerical values the density function would be defined specifically and would then be plotted on the same graph as the histogram. This would provide the analyst a picture of the degree to which the normal density function is an appropriate e model for the basic random experiment under study. In reality, a statistical test would be conducted to have a more scientific test of statistical appropriatenes.

3*3 Summary Measures - Continous Random Experiments

As we saw in the discrete case, summary measures can be defined to summarize the information content of the data. The four summary measures used in the discrete case (Sample Average X, Sample Variance S^2, Sample Standard Deviation S, and the Coefficient of Variation C) are

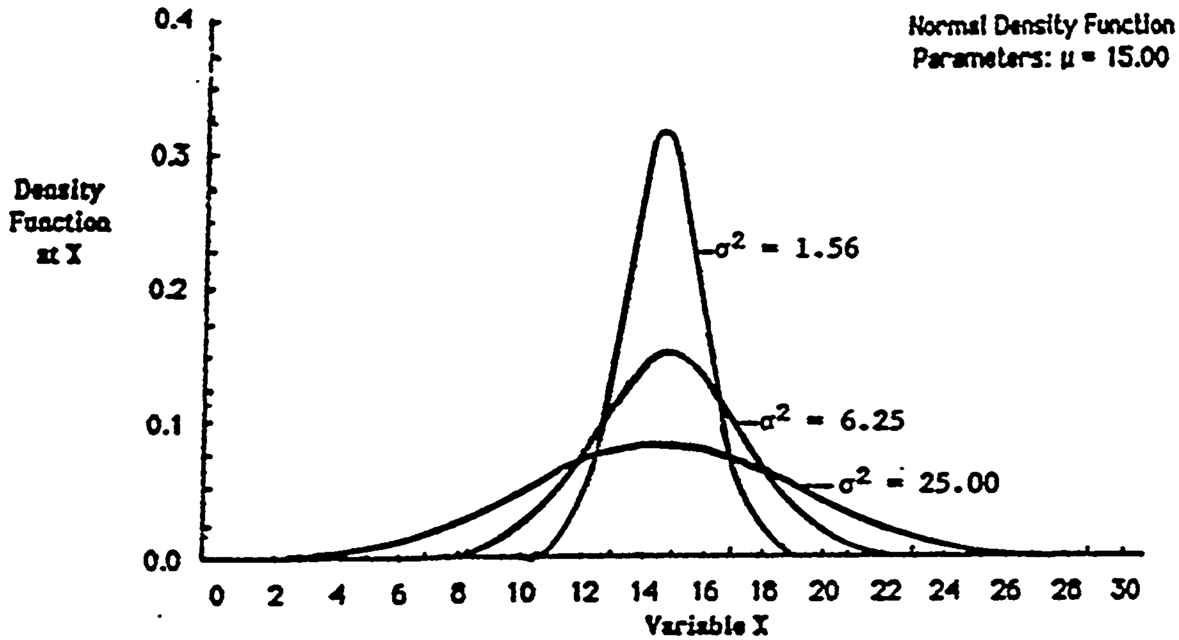

0.4
0.3
0.2
0.1
0.0
Density
Function
at X
Normal Density Function
Parameters: μ = 15.00
σ2 = 1.56
σ2 = 6.25
σ2 = 25.00
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
Variable X

Figure 3-7

also defined in this section for the continuous random experiment. They are defined for two situations which are commonly encountered in the analysis of continuous data where

(1) The actual data (x_1, x_2, ..., x_n) is available and,

(2) Only the histogram f_i/n vs m_i for i = 1, 2, ... k is available.

The procedure for computing the summary measures is different in each situation and are treated separately as follows:

Situation 1: Use of Actual Data (Xl , $X_2$9 . ..~ x_n)

Sample Average (x)

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Sample Variance (S^2)

$$S^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$$ or equivalently,

$$S^2 = \frac{1}{n} \left[\sum_{i=1}^{n} X_i^2 - n \bar{X}^2 \right]$$

Sample Standard Deviation (S)

$$S = \sqrt{S^2}$$

Sample Coefficient of Variation (C)

$$C = S/\bar{X}$$

Situation 2: **Use of Histogram (f_i/n vs m_i) for $i = 1, 2, \ldots, k$**

Sample Average ($\bar{X}$)

$$\bar{X} \cong \frac{1}{n} \sum_{i=1}^{k} f_i m_i$$

Sample Variance (S^2)

$$S^2 \cong \frac{1}{n} \left[\sum_{i=1}^{k} m_i^2 f_i - n \bar{X}^2 \right]$$

Sample Standard Deviation (S)

$$S \cong \sqrt{S^2}$$

Sample Coefficient of Variation-(C)

$$C \simeq S/\bar{X}$$

The reader should understand that when the summary measures are computed using only the histogram (Situation 2) , *an* approximation to each summary measure is being computed. The true summary measure should be computed using the actual data. The approximation, however, is good when the analyst has a large data set and is using a histogram with a large number of cells.

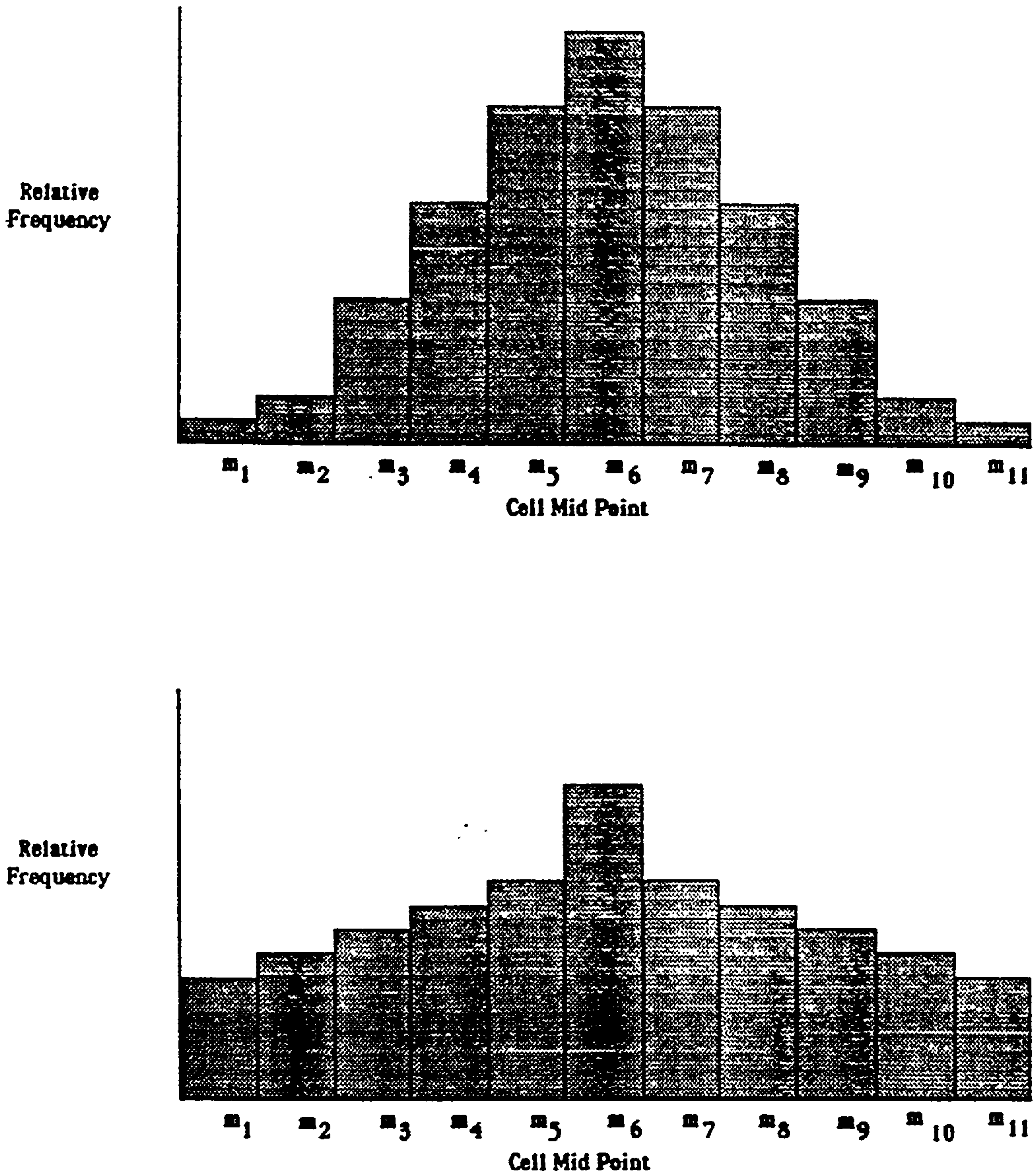
The two histograms shown in Figure 3-8 illustrate the meaning of the sample variance and the sample standard deviation (e. g., for the same sample average a lower sample standard deviation or sample variance means a higher concentrate on of the data near the sample average).

3.4 Repetition of A Continuous Random Experiment

The discussion we gave in the case of discrete random experiments will again be followed in the statistical analysis of variables which are associated with the repetition of some basic continuous random experiment. We assume that a subsystem of the shipyard has been analyzed and that the histogram and summary measures have been developed. We are now concerned with repetitions of the basic continuous random experiment and are interested in developing procedures for understanding statistically three general types of variable:

(1) The Average Outcome,

(2) The Sum of the Outcomes, and



Variability of Data - 2 Cases
Figure 3-8

(3) Variability of the Outcomes.

Again, each of these general categories is treated separately in the subsequent sections.

The Aver age Out come

In the continuous case, as it was in the discrete case, it is certainly possible to collect data on the average outcome of a specified number of repetitions of a basic experiment. The data could then be used to construct a histogram of the average outcome and to provide all of the summary measures. This approach does not have to be followed. We can determine mathematically the statistical behavior of the average outcome from a specified number of repetitions of a basic continuous random experiment. For a process that is in a state of statistical control, the mathematical results are as follows:

1. The average (X_A) of the histogram for the average outcome will be close to the average (x) of the histogram for the basic random experiment. Thus $\bar{X}_A \simeq \bar{X}$.

2. The vari ante of the histogram for the average outcome S_A^2 will be close to the variance of the histogram for the basic random experiment S^2 divided by the number of repetitions k.

$$\text{Thus } S_A^2 \simeq \frac{S^2}{k} \text{ , and}$$

3. The histogram of the average outcome will, for a 1arge number of repetitions of the basic random experiment, be symmetric

about the average and will have the bell shape. If the histogram of the basic random experiment is normal , then regardless of the number 0f repetitions, the histogram of the average outcome will be normal.

In order to illustrate theses results for the continous case consider the following example.

In the electrical shop a piece of electrical conduit four feet long is placed in a bending machine to form a right angle at the midpoint of the conduit and two equal lengths of conduit. The distance between the end points of the conduit (the hypotenuse of the 90° triangle) is then measured.

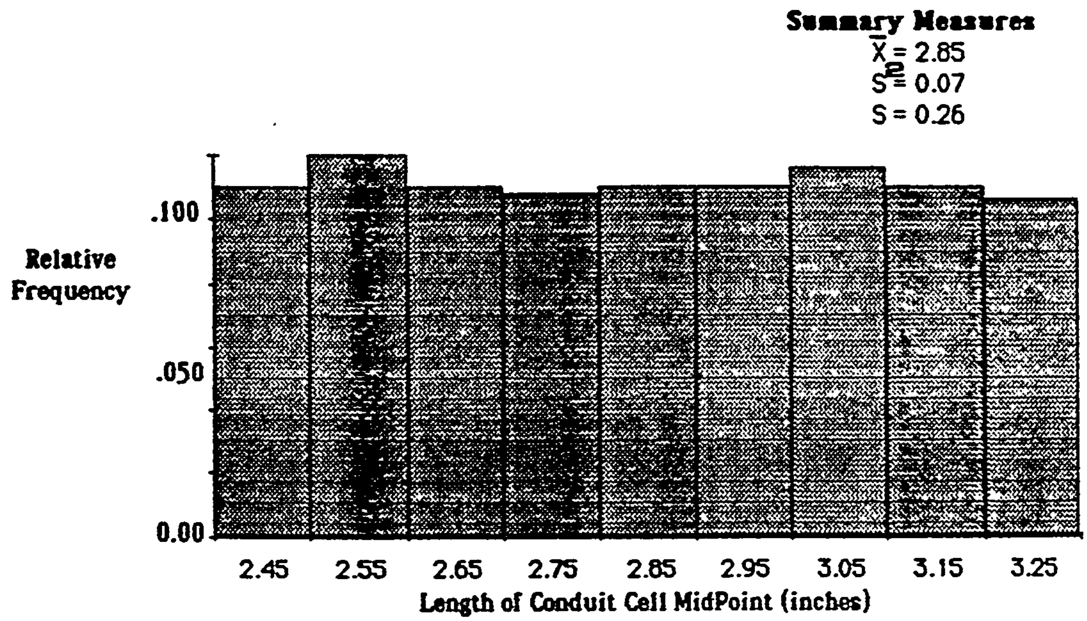
When the system was in a state of statistical control 500 completed pieces of conduit were measured with the resulting histogram and summary measures shown in Figure 3-9.

For this example the results of the central 1imit theorem for 15 repetitions and 20 repetitions of the basic experiment are shown in Figure 3-10. It is clear from Figure .3-10 that the variability deer eases as the number of repetitions increases and that the histogram is centered on the average for the histogram of the basic random experiment.

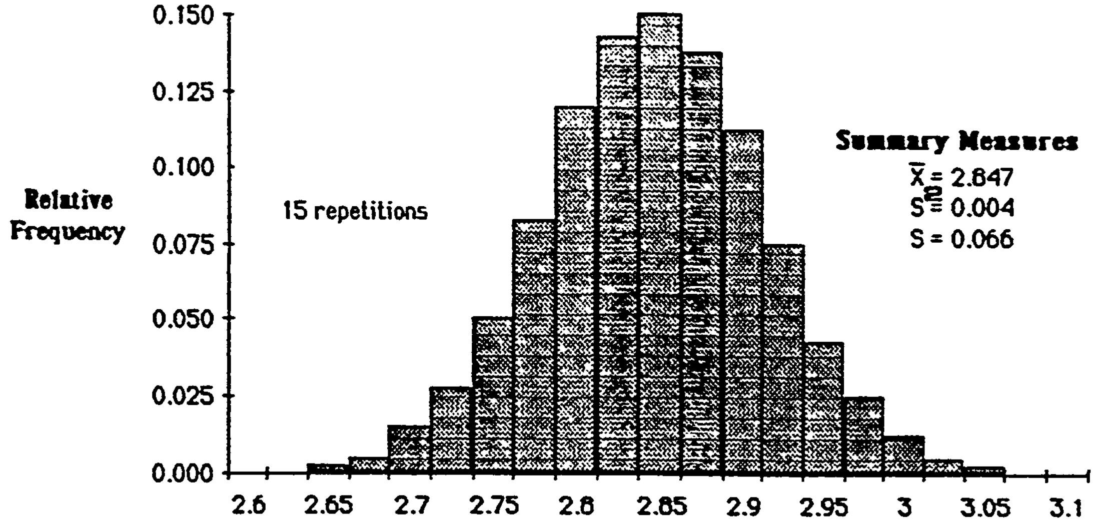
It should be emphasized again that the results are true regardless of the shape of the histogram of the basic random experiment.

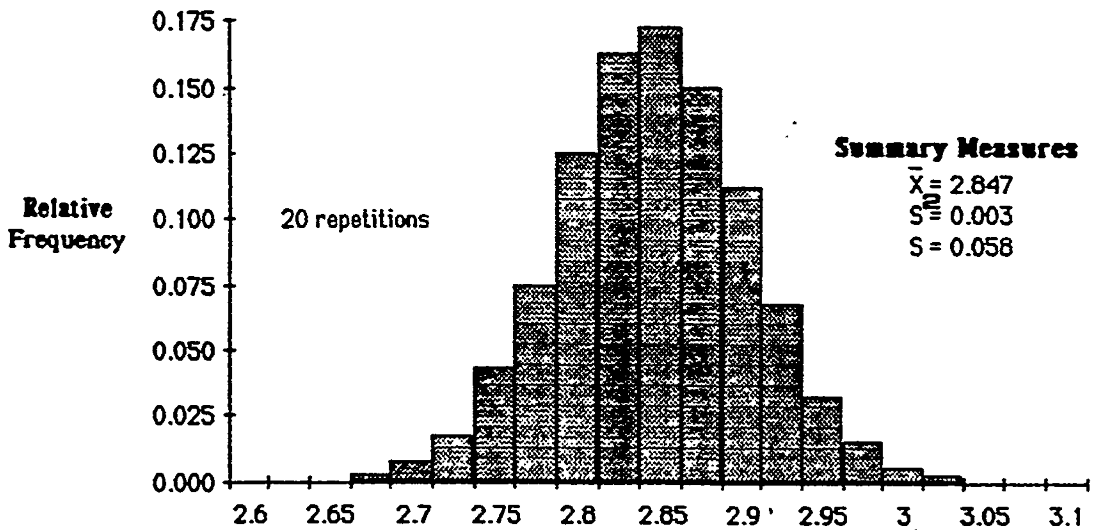
<u>The Total of The Outcomes</u>

In this situation a basic continuous random experiment is repeated a fixed number of times and the variable of interest is the sum of the



Histogram of Length of Conduit
Figure 3-9





Histograms of Average Length of Conduit

Figure 3-10

values of the individual outcomes. This is a situation frequently encountered in the shipyard industry. For example, suppose our basic random experiment is one where a welder selects a steel plate 4 ft. wide and 8 ft. long. Now suppose that we imagine a situation in which the basic random experiment is repeated two times and the two plates of steel are welded together to produce one steel plate 4 ft. wide by 16 ft. long. Our variable of interest here might be the total length of the welded unit. Without any random variation in the basic random experiment the total length of the welded unit would be exactly 16 ft. With random variation in the basic random experiment, there will also be random variation in the total length of the welded unit.

In this situation we are again saved by mathematics and can draw the following conclusions:

1. For k repetitions of the basic random experiment, the average ($\bar{X}_T$) of the histogram for the sum of the individual outcomes will be approximately k times X.
2. For k repetitions of the basic random experiment the variance (S_T^2) of the histogram for the sum of the individual outcomes will be approximately k times S^2.
3. The histogram of the sum of the outcomes will, for a large number of repetitions of the basic random experiment, be symmetric about the average (kX) and will have the shape of the normal density function. If the histogram of the basic random experiment has the shape of the normal density function then the histogram of the sum of the outcomes will be normal regardless of the number of repetitions of the basic experiment.

It is instructive at this point to take an example with specific numerical values to illustrate the benefits of knowing the statistical behavior of a particular situation.

The basic random experiment is to select a steel plate 1 ft. wide by 10 ft. long where the length is described by the histogram shown in Figure 3-11(a). The basic random experiment is repeated five times and the individual plates of steel are welded together to produce one plate 1 ft. x 50 ft. The histogram of the total length is shown in Figure 3-11(b).

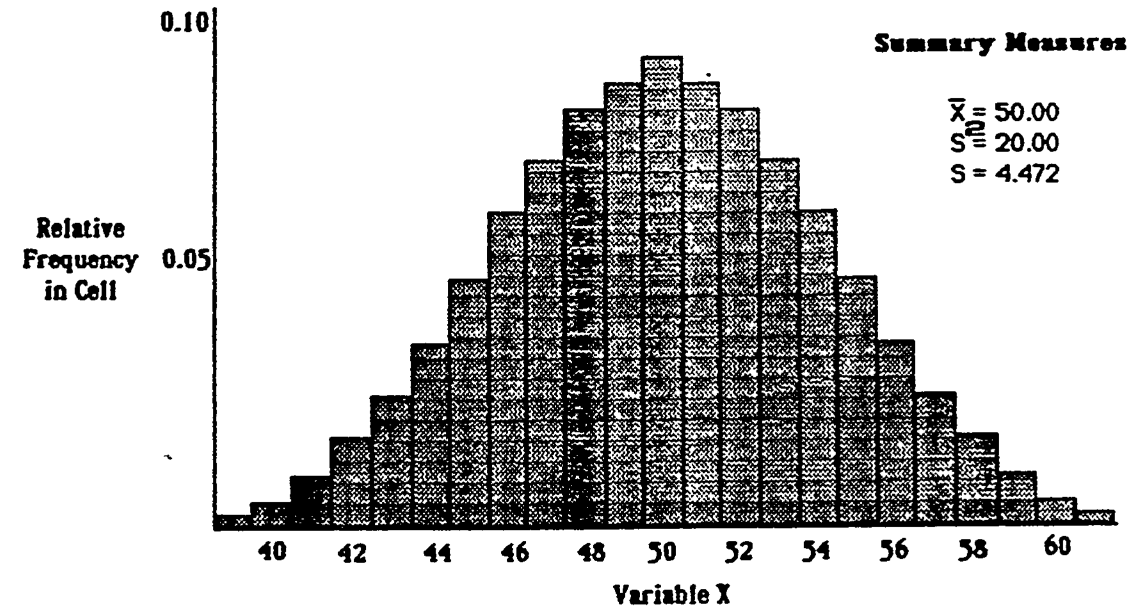
Because of the characteristics of the normal distribution we can state the following:

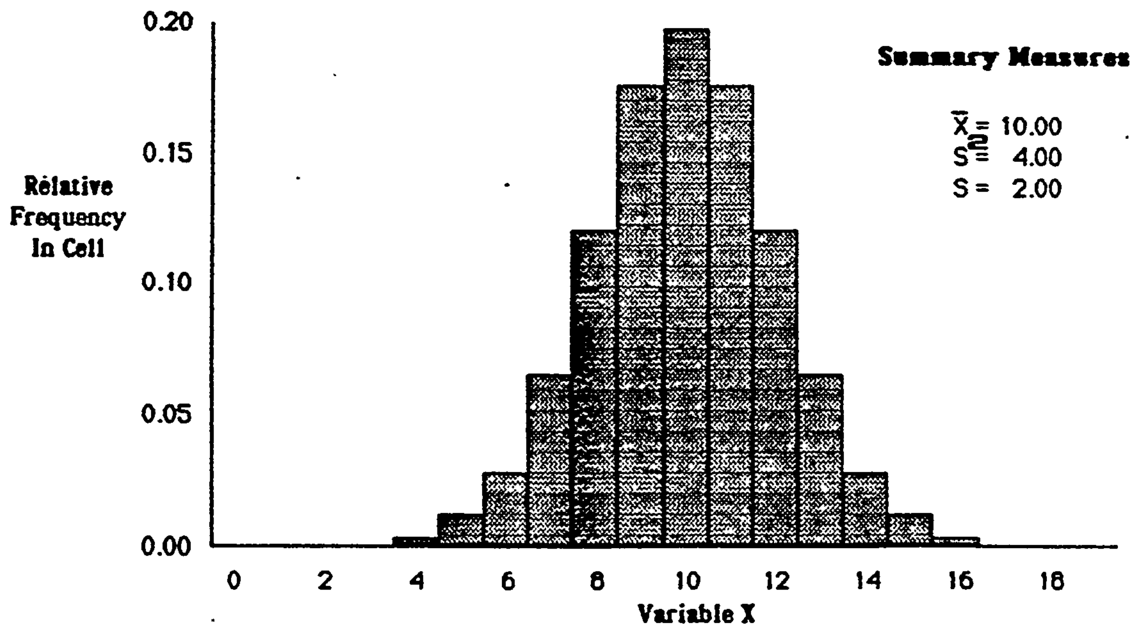
1. 68.26% of the welded units will have total length between $\bar{X} - S_T$ and $\bar{X} + S_T$ or 45.53 ft. and 54.479 ft.
2. 95.44% of the welded units will have a total length between $\bar{X} - 2S_T$ and $\bar{X} + 2S_T$ or 41.00 ft. and 58.94 ft.
3. 99.73% Of the welded units will have a total length between $\bar{X} - 3S_T$ and $\bar{X} + 3S_T$ or 36.58 ft. and 63.42 ft.

If the welded unit is to fit into a space 1 ft. by 50 ft., then we know from statement 1 that 31 .74% will be short by 4.47 ft. or less or long by 4.47 ft. or more.

<u>Variability of the Outcome in a Repeated Basic Experiment</u>

In the statistical analysis of variability in repetitions of a basic random experiment, it is common to use the range as a basic measure of variabilities. Here, the reader should understand that we are discussing the variability between repetitions of the basic random experiment. Our definitions of Variability are as follows:





Histograms for Length of Steel Plate
Figure 3-11

Range of Outcomes (R)

Let M be the largest value of the outcome in the k repetitions of the basic random experiment.

Let L .be the smallest value of the variable in the k repetitions of the basic random experiment.

The range, R, of the outcomes is then defined to be,

$$R =1 M-L .$$

When the histogram of the basic random experiment has the shape of the normal density function (the bell shape) and when the process is in statistical control, sane general statements can be made about the behavior of the range.

Range of the Outcome in A Repeated Experiment (R)

For a small number of repetitions of a basic random experiment ($k <$ 10), the range, R, can be used as a measure of variability. In the case where the basic random experiment has a histogram that has the bell shape the following general statements can be made.

1. The histogram of the range of the outcome in a repeated random experiment will have an average value which is approximately $g_k \times S$ where g_k can be determined from Table 3-1 and s is the standard deviation of the basis experiment.

2. The histogram of the range of the outcome in a repeated random experiment will have a variance which is approximately $h_k \times S^2$ where h_k can be determined from Table 3-1 and S^2 is the variance of the basic experiment.

3. The histogram of the the of the range of the outcome in a repeated random experiment will have a standard deviation which is approximately $\sqrt{h_k \times S^2}$.

Number of Repetitions of Basic Experiment	Factors for Average	Factors for Variance
n	°k	¨k
2	1.1284	0.7268
3	1.6926	0.7893
4	2.0588	0.7740
5	2.3259	0.7467
6	2.5344	0.7191
7	2.7044	0.6942
8	2.8472	0.6721
9	2.9700	0.6525
10	3.0775	0.6354

Average and Variance Factors - Range of Outcome

Table 3-1

3.5 Problems

I. A header is placed between two beans and then welded in place. For each header in a group of five, the time to complete the operation was recorded as shown in Table 3-2. Construct a histogram of the basic random experiment (the time to complete one header) and comPute the s~mary measures ($\bar{X}$, S^2, S and C). Compute the average of each subgroup and the range of each subgroup. Construct a control chart for this situation and determine if the process is in statistical control.

II. The purchasing agent of a shipyard has been asked to examine the validity of complaints about the lifetime of 100 watt electric light bulbs that are supplied by a certain manufacturer. An experiment was conducted in which four bulbs from each of 25 cartons were selected at random. The lifetime (in hours) was then determined as shown in Table 3-3. Construct a histogram of the bulb 1ifetime and compute the summary measures ($\bar{X}$, S^2, S and C). If the average 1ifetime is supposed to be 600 hours, what would you conclude?

Subgroup Number	Time (minutes) to Weld Header Number 1	2	3	4	5
1	20.3	3.3	4.5	1.1	3.1
2	25.9	8.2	4.2	7.4	5.8
3	7.1	0.1	6.0	5.2	0.9
4	0.5	3.7	5.4	11.1	5.1
5	12.4	5.5	8.5	33.9	3.0
6	0.6	2.5	6.8	3.8	18.2
7	11.9	1.6	10.6	15.0	3.3
8	28.8	21.1	0.9	0.9	4.2
9	4.0	28.5	3.8	0.3	8.6
10	5.1	7.7	7.1	3.1	25.2
11	6.6	2.5	4.2	7.7	23.7
12	2.7	15.6	15.2	7.6	0.3
13	2.0	15.5	12.5	17.5	4.8
14	32.4	2.1	9.4	5.6	3.3
15	39.6	0.9	1.9	9.5	28.2
16	0.7	3.7	36.7	5.2	1.3
17	1.6	0.9	1.5	3.6	14.3
18	16.6	12.1	29.5	23.1	10.6
19.	0.7	1.6	7.6	8.5	0.8
20	25.8.	13.9	6..7	1.0	5.7
21	0.7	19.1	8.2	3.7	30.7
22	7.0	5.8	2.9	14.4	3.2
23	2.3	15.7	7.1	4.5	1.4
24	14.6	1.7	4.5	0.2	0.1
25	0.6	4.4	8.4	4.0	15.1

Time to Weld Header in Place

Table 3-2

Subgroup Carton Number	Lifetime (hours) of Bulb Number 1	2	3	4
1	491.2	500.4	513.4	566.5
2	487.4	511.6	500.2	503.4
3	502.7	519.6	527.3	497.2
4	492.1	559.5	512.7	444.6
5	496.6	461.4	494.4	519.1
6	520.5	476.1	481.2	496.8
7	492.9	497.1	520.6	516.0
8	497.7	514.7	535.0	512.6
9	479.2	463.9	520.6	460.5
10	496.0	482.0	479.5	547.0
11	538.5	519.5	453.2	490.9
12	486.8	510.4	513.9	514.0
13	517.4	458.7	538.6	493.9
14	455.6	479.4	502.6	512.3
15	473.2	546.3	523.8	511.5
16	479.5	473.2	509.7	474.0
17	475.3	480.0	489.9	518.7
18	495.1	513.2	494.1	526.5
19	474.2	512.9	511.5	481.6
20	537.4	536.3	538.1	481.6
21	503.4	549.8	517.6	507.6
22	506.6	491.3	459.0	546.8
23	485.5	469.2	493.7	506.0
24	467.7	452.5	516.9	518.7
25	515.9	512.5	498.3	561.2

Lifetime of 100 Watt Electric Light Bulbs

Table 3-3

4. SUMMARY OF STATISTICAL CONTROL CHARTS

4.1 Introduction

If you are a manager and if you follow the Deming philosophy of modern management, then you will be involved in management by walking around (MBWA). Your new responsibilities will 'include a basic understanding of the fact that 80-85 percent of all problems are with the system. In addition, you will adhere to the following rule:

> Your new *role* is to learn the system and to change the system in such a way that both quality and productivity are improved.

In order to learn the system you will have to become actively involved in the analysis of variability and become familiar with the subject of "decision-making based on facts" where the facts are numbers that exhibit variability. This is exactly the situation you will encounter when you begin to apply the general techniques of statistics and, in particular, when you use a statistical control chart.

With the statistical control chart you will be looking at a set of numbers that exhibit variability (the facts) and, based on these numbers, you will make a decision about the process from which the numbers were drawn. You will be deciding if the process is operating with only natural variability or if the process is operating with unnatural variability. If the facts lead you to conclude that unnatural variability is present, then you will have to search for an assignable cause (or assignable causes) that produced the unnatural variability.

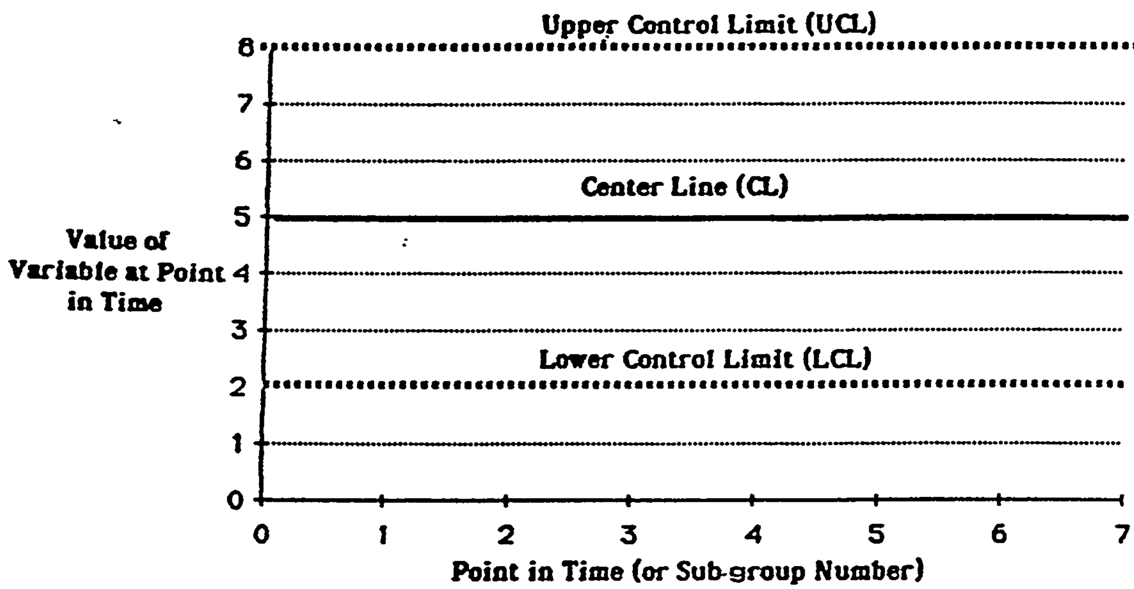
In this decision-making process your objective is to return the system to its state of natural variability and to continue the decision-making process by collecting more data (the facts) and by repeating your analysis of the facts.

A statistical control chart is a technique for providing a graphical display of the facts to assist you in making a correct decision about the state of statistical control of the process. The chart is simply a plot of the value of some variable that is computed from a series of repetitions of a basic random experiment. The chart, along with basic rules about the behavior of the chart for a process which is in statistical control, will provide you with the proper structure to reach a logical conclusion.

In the construction of a control chart the repeated experiments are called subgroups, and one value of the variable of interest is plotted for each subgroup number. Thus, the vertical axis of the control chart will be the important variable and the horizontal axis will be subgroup number or 1, 2, 3, 4, ... r, where r is the number of subgroups you have.

The chart will also have three horizontal lines called the Upper Control Limit (UCL), the Lower Control Limit (LCL), and the Center Line (CL). The general format of the Statistical Control Chart is shown in Figure 4-1.

The central tendency and the standard deviation of the variable of interest are used to determine values for the CL, the UCL, and the LCL. In general the CL is the central tendency of the important variable, the UCL is the central tendency plus three standard deviations of the



Layout of General Control Chart
Figure 4-1

variable; and the LCL is the central tendency minus three standard deviations. For all types of control charts and for a process that is in statistical control, the central tendency and the standard deviation of the important variable can be related to the central tendency and the standard devi at ion of the basic experiment. In this manner, and using some additional knowledge of probability theory and statistics, you can make the following statement.

> If the process is in a state of statistical control, then the important variable should behave in a random but natural way. If your plotted values of the important variable do not behave as they should, then you can only conclude that the process is not operating in a state of statistical control.

The basic elements of statistics were presented in Chapters two and three to introduce you to both the concept of a hi stogram and certain summary measures of a set of data that were celled the average (central tendency) and the standard deviation (dispersion). This introduction to statistics should put you in a position to understand the basics of the statistical control chart,

The manner in which the ‘statistics” of the important variable are related to the ‘statistics" of the basic experiment is a topic which requires a deeper understanding of probability and statistics than we could provide in this workbook.

4.2 Specific Types of Statistical Control Charts

Most practical decision-making situations that should be analyzed with the help of a statistical control chart can be placed into one of three categories. Each category and the important variables that are commonly used in each category are as follows:

Category I: Repeated experiments where the outcome of each trial is: 0 (good or conforming) or 1 (defective or nonconforming)

Important Variables

Total Number of Defectives

Fraction Defective

Percent Defective

Category II: Repeated experiments where the outcome of each trial is: 0, or 1, or 2, or 3, or, ... (e.g. defects)

Import ant Variable

Total Number of Defects

Average Number of Defects

Category III: Repeated experiments where the outcome of each trial is: measurable (continuous)

Important Variables

Aver age Out come

Range of Outcomes

For a process that is in a state of statistical control each of the important variables in categories I and II will have a statistical behavior (central tendency and standard deviation) that can be specified and related to the average ($\bar{X}$) and standard deviation (S) of the basic experiment. If the basic random experiment is repeated k times and if the process is in a state of statistical control, the statistical behavior of the important variables in each category should be as follows:

Category I: Total Number of Defectives

Central Tendency = $k\bar{X}$

Standard Deviation = $\sqrt{k\bar{X}(1-\bar{X})}$

Fraction Defection

Central Tendency = $\bar{X}$

Standard Deviation = $\frac{\sqrt{\bar{X}(1-\bar{X})}}{\sqrt{k}}$

Percent Defective

Central Tendency = $100\ X$

Standard Deviation = $100\frac{\sqrt{\bar{X}(1-\bar{X})}}{\sqrt{k}}$

Category II: Total. Number of Defects

Central Tendency = $k\bar{X}$

Standard Deviation = $\sqrt{k\bar{X}}$

Average Number of Defects

Central Tendency = $\bar{X}$

Standard Deviation = $\frac{1}{\sqrt{k}}\sqrt{\bar{X}}$

In the continuous case (Category III) and for a process that is in statistical control with k repetitions of the basic experiment the statistics of the two important variables are as follows:

Category III: Average Out come

Central Tendency = $\bar{X}$

Standard Deviation = $B_1\ \bar{R}$

Where $\bar{R}$ is the average of the values for the subgroup ranges and B_1 can be determined from Table 4-1.

Range of Outcomes

Central Tendency = $\bar{R}$

Subgroup Size (k)	Value of Factor	
	‘1	‘2
2	0.627	0.756
3	0.341	0.422
4	0.243	0.427
5	0.192	0.372
6	0.161	0.335
7	0.140	0.308
8	0.124	0.288
9	0.112	0.272
10	0.103	0.259

Factors for Computing Standard Deviations

Table 4-1

Standard Deviation = B_2 R

where B_2 can be determined from Table 4-1.

Now that we have listed the central tendency and the standard deviation for each of the important variables the construction of the. control chart for each variable is accomplished as follows:

Center Line = Central Tendency

Upper Control Limit = Central Tendency + 3 Times Stander d Deviation

Lower Control Limit = Central Tendency - 3 Times Standard Deviation

4.3 Rules for Statistical Control

Once the control chart has been constructed and the values of the important variable have been plotted you have to decide if the process is operating with only natural variability (statistical control) or with unnatural variability (out of Statistical Control) l

The following rules are generally accepted:

1. If a value of the important variable is plotted above the upper control limit or below the lower control 1imit look for cause of .unnatural variability.
2. If 8 consecutive values of the important variable appear above the center 1ine or below the center line look for cause of unnatural variability.
3. If 12 out of 14 consecutive values of the important variable are either above the center 1ine or below the center 1ine look for cause of unnatural variability
4. If 6 consecutive values of the important variable show either an increasing trend or a decreasing trend look for cause of unnatural variability.
5. If the values of the important variable show cyc1ical or periodic behavior look for cause of unnatural variability.

6. If values of the important variable appear to hug the center 1ine look for cause of unnatural variability.

It should be emphasized that these rules are generally acceptable in the use" of Statistical Control Charts. When a specific situation is being studied a careful analysis should be made of the cost of making incorrect decisions. In order to accomplish this task you must determine the cost of saying that the process is in control when it is not. Similarly you must determine the cost of saying the process is not in control when it is. Once these costs have been established the rules given above and the corresponding risks can be evaluated. In many cases the rules might have to be changed to reflect the costs.

4.4 Problems

I. Using the data presented for problem I of Chapter 2, construct a control chart for the average number of defects and plot the individual values. Determine if the process is in a state of statistical control. =

II. Using the data presented for problem III of Chapter 1, construct a control chart for percent defective and plot the 'individual values. Determine if the process is in a state of statistical control.

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