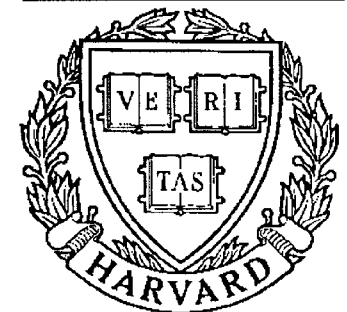


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Multireception Probabilities for FH/SSMA Communications

by T. Ketseoglou and E. Geraniotis

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MULTIRECEPTION PROBABILITIES FOR FH/SSMA COMMUNICATIONS

Thomas Ketseoglou and Evaggelos Geraniotis

ABSTRACT

Exact expressions for the probabilities $P(l, m-l | k)$ of l correct packet receptions and $m-l$ erroneous ones, out of total k packets contending in a slot, are presented for the case of frequency-hopped spread-spectrum random-access slotted networks employing random frequency hopping patterns. These expressions are difficult to evaluate numerically for values of $m > 3$. However, our numerical analysis indicates that under light traffic conditions these probability values are very close to the ones provided by the independent receiver operation assumption, under which, the distribution of multireception obeys the binomial law.

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I. Introduction

Spread-spectrum (SS) random access networks have been extensively studied during the last few years (see [2]-[6]). From the network performance point of view, their analysis has focused on the evaluation the throughput/delay trade-off. In [5], an analytical framework for the performance evaluation of slotted SS networks was presented, which made clear the impact of $P(l, m - l|k)$ on performance. However, the independent receiver operation assumption (IROA) was used in deriving the performance expressions in [5], an assumption, which is realistic in certain cases. However, there are situations in which a more thorough investigation is necessary for drawing definite conclusions.

In this paper, an exact analysis for the multireception probabilities $P(l - m, l|k)$ for frequency-hopped spread-spectrum (FH/SS) random access slotted networks with random frequency patterns is presented. The effects of Additive White Gaussian Noise (AWGN) are omitted, because the presense of AWGN channel will randomize each receiver operation more. Thus the primary source of interference is multiple-access (MA) inteference. Note that the effects of AWGN could be easily incorporated in our analysis (see [4], for example).

Slotted network operation is assumed throughout this paper but, at the symbol level, the users need not be synchronous (see [6])¹. Forward error conrol (FEC) coding using RS codes is employed, as is commonly done in FH/SS systems, and two modes of decoder operation are incorporated in our model: error correction and erasure correction.

The paper is organized as follows: in Section II, we present the expressions for the multi-reception probabilities; in Section III, we present our numerical results and a comparison with the IROA ones; in Section IV, we present our conclusions.

¹Although certain, more general situations could be incorporated in our model, we are concerned primarily with fully connected networks with paired-off user topology [5].

II. Derivation of Multireception Probabilities

We are interested in finding the probability $P(l, m-l|k)$ of l receivers receiving correctly and $m-l$ ones receiving erroneously, for a specific set of receivers. Due to the symmetry in the system, we can equivalently find the probability of the first l receivers correctly decoding, while the remaining $m-l$ receivers decode in error². We note that for MA applications, one is interested in finding the “total” probability of l receivers correctly receiving, $m-l$ ones erroneously receiving, for any configuration of l and $m-l$. The probability $P_T(l, m-l|k)$ can be found from $P(l, m-l|k)$ to be

$$P_T(l, m-l|k) = \binom{m}{l} \cdot P(l, m-l|k).$$

Moreover, under IROA this total probability obeys the binomial law. If we denote by $P_I(l, m-l|k)$ the probability of a particular set and $P_{T,I}(l, m-l|k)$ denotes the total probability under IROA, we have

$$P_{T,I}(l, m-l|k) = \binom{m}{l} \cdot P_I(l, m-l|k).$$

Obviously, $P_I(l, m-l|k)$ obeys a geometric law with respect to $P_c(k)$, the probability of correct decoding of each particular receiver.

For FH/SS MA communications, the probability of a coded symbol error is upper-bounded by the probability of a hit, which is a function of the available frequency slots q and the number of contending users k . We denote this probability by $P_{h,s}(k, q)$ and $P_{h,a}(k, q)$ for the synchronous and asynchronous cases, respectively. From [2] we have

$$P_{h,s}(q, k) = 1 - (1 - 1/q)^{k-1}$$

²In the sequel, we implicitly assume that $m \geq 2$. For $m = 1$, the model reduces to the single receiver model and corresponds to the analysis of [2].

and

$$P_{h,a}(q, k) = 1 - (1 - 2/q)^{k-1}$$

respectively. Subsequently, we denote by $P_h(k, q)$ the probability of a hit and, henceforth, treat both cases simultaneously.

The i th receiver receives correctly the transmitted packet, if the number of hits $h(i)$, for $1 \leq i \leq m$, satisfies

$$0 \leq h(i) \leq t \tag{1}$$

In (1), t denotes the correction capability of the code. For pure error-correction,

$$t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = \frac{N - K}{2} \tag{2}$$

while, for erasure correction,

$$t = d_{min} - 1 = N - K. \tag{3}$$

In the synchronous case, each receiver output depends only on other receivers outputs during the same dwell time. This applies also to the asynchronous case, if proper interleaving takes place (see [6]). Then the total system operation becomes memoryless.

In this section, we first present exact expressions for $P(l, m - l|k)$; then we show that IROA gives a good approximation to the probabilities $P(l, m - l|k)$ under conditions which are commonly met in practical applications.

IIa. Exact Analysis of $P(l, m - l|k)$

As amplified in Appendix I,

$$\begin{aligned} P(l, m - l|k) &= \\ &= P(0 \leq h(1) \leq t, \dots, 0 \leq h(l) \leq t, t + 1 \leq h(l + 1) \leq N, \dots, t + 1 \leq h(m) \leq N) = \end{aligned}$$

$$\begin{aligned}
&= \sum_{\ell_1} \sum_{\ell_2} \dots \sum_{\ell_{2^m-2}} \sum_{\ell_{2^m-1}} \binom{N}{\ell_1} \binom{N-\ell_1}{\ell_2} \dots \binom{N-\sum_{i=1}^{2^m-2} \ell_i}{\ell_{2^m-2}} \times \\
&\quad P_1^{\ell_1} \cdot P_2^{\ell_2} \dots P_{2^m}^{N-\sum_{i=1}^{2^m-1} \ell_i}
\end{aligned} \tag{4}$$

where $P_i = P(E_i)$ denotes the probability of the "simultaneous" event E_i , under which the demodulator outputs correspond to the binary representation of $i-1$ during the same numbered symbol. Of course, any other correspondence of "simultaneous" events and the natural numbers would work as well. ³

The range of ℓ_i for the sums in (4) can be found from a Diofantine analysis of the inequalities:

$$0 \leq \sum_{j=1}^{2^m-1} a_j^{(i)} \cdot \ell_j \leq t, \quad i = 1, 2, \dots, l \tag{4a}$$

and

$$t+1 \leq \sum_{j=1}^{2^m-1} a_j^{(i)} \cdot \ell_j \leq N, \quad i = l+1, l+2, \dots, m \tag{4b}$$

where $a_j^{(i)} = 1$ or 0, according to whether the probabilities of the form P_{E_j} that correspond to ℓ_j take part in the i th receiver error count or not. We have to add another constraint to the m constraints posed by (4a,4b), namely that

$$0 \leq \sum_{j=1}^{2^m-1} \ell_j \leq N. \tag{4c}$$

The purpose of that constraint is to ensure that we do not surpass the word length by permitting higher values of the ℓ_j s.

It remains to find expressions for all P_{E_i} , for $i = 1, 2, \dots, 2^m$. This is equivalent to finding the probability of having ρ demodulator outputs correct and $m - \rho$ ones in error

³Note that in (4), all events having same weight have equal probabilities, although this does not simplify the expression.

during the same numbered transmitted symbol, for $\rho = 1, 2, \dots, m$. These probabilities should be a function of ρ, m, k , and q . We denote them by $P_s(\rho, m, q, k)$. First we find $P_{E_1} = P_{c\dots c} = P_s(m, 0, k, q)$, that is, the probability of finding all the simultaneous symbols in all receivers correct. Because of the symmetry we get

$$\begin{aligned}
P_{\underbrace{c\dots c}_m} &= P(c|\underbrace{c\dots c}_{m-1}) \cdot P_{\underbrace{c\dots c}_{m-1}} = \\
&= P(c|\underbrace{c\dots c}_{m-1}) \cdot P(c|\underbrace{c\dots c}_{m-2}) \dots P(c|cc) \cdot P(c|c) \cdot P(c) = \\
&= \prod_{j=1}^m (1 - P_h(q - j + 1, k - j + 1)) \quad .
\end{aligned} \tag{5}$$

Let us now find $P_{E_2m} = P_{\underbrace{e\dots e}_m} = P_s(0, m, q, k)$. We get $P_{\underbrace{e\dots e}_m} =$

$$\begin{aligned}
&= P(e|\underbrace{e\dots e}_{m-1}) P_{\underbrace{e\dots e}_{m-1}} = \\
&= \left[1 - P(c|\underbrace{e\dots e}_{m-1}) \right] \cdot P_{\underbrace{e\dots e}_{m-1}} = \\
&= \left[1 - \frac{P(e\dots e|c) \cdot P(c)}{P_{\underbrace{e\dots e}_{m-1}}} \right] \cdot P_{\underbrace{e\dots e}_{m-1}} = \\
&= P_{\underbrace{e\dots e}_{m-1}} - P(c) \cdot P(\underbrace{e\dots e}_{m-1}|c) = \\
&= P_{\underbrace{e\dots e}_{m-1}} - (1 - P_h(q, k)) P_s(0, m-1, q-1, k-1) \Rightarrow \\
&\Rightarrow P_s(0, m, q, k) = P_s(0, m-1, q, k) - (1 - P_h(q, k)) \cdot P_s(0, m-1, q-1, k-1) \quad (6)
\end{aligned}$$

Equation (6) is a recursive formula for finding $P_s(0, m, q, k)$. The solution of this equation, as shown in Appendix II, is

$$P_s(0, m, q, k) = 1 + \sum_{i=1}^m (-1)^i \cdot \binom{m}{i} \cdot \prod_{j=1}^i (1 - P_h(q - j + 1, k - j + 1)) \quad (7)$$

Proceeding one step further we get the more general expressions

$$\begin{aligned}
P_{E_i} &= P_s(\rho, m - \rho, q, k) = \\
&= P_{\underbrace{e \dots e}_{m-\rho} \underbrace{c \dots c}_{\rho}} = \\
&= P(\underbrace{e \dots e}_{m-\rho} | \underbrace{c \dots c}_{\rho}) P_{\underbrace{c \dots c}_{\rho}} = \\
&= P_s(\rho, m - \rho, q - \rho, k - \rho) \cdot P_{\underbrace{c \dots c}_{\rho}} = \\
&= \prod_{j=1}^{\rho} (1 - P_h(q - j + 1, k - j + 1)) \times \\
&\quad \left[1 + \sum_{i=1}^{m-\rho} (-1)^i \cdot \binom{m-\rho}{i} \cdot \prod_{j=1}^i (1 - P_h(q - \rho - j + 1, k - \rho - j + 1)) \right]. \quad (8)
\end{aligned}$$

Equations (4) and (8), together with the constraints posed by (4a), (4b), (4c), give the solution to our problem. As it becomes clear from (4), the exact evaluation of $P(l, m - l | k)$ requires the computation of $2^m - 1$ dependent sums, in which the limits should be found through a Diofantine analysis of (4a), (4b), and (4c). In addition, the summands are powers of $P(E_i)$, which can be computed through (8). Due to those computational requirements, exact expressions are nearly impossible to evaluate $m \geq 4$. However, as the next subsection indicates, for a q that is large enough in comparison to k , IROA can be used as a good approximation.

IIb. Approximate Analysis

We shall show that, if the number of frequency slots is large enough in comparison to k ($q \gg k$), then $P(l, m - l | k)$ can be approximated closely by assuming that the receivers operate independently.

To prove this, we first notice that equation (4) is valid for any probability law on the events E_i . This means that (4) should be true, even if the demodulator outputs are

independent during the same symbol transmission. In this case, however, (4) simplifies to the binomial law. Therefore, IROA results for multireception probabilities become easy to compute as they obey a geometric law ⁴.

If we assume that $q \gg k$, then it suffices to show that $P_s(l, m-l, k, q)$ can be approximated by the corresponding expression, when all receivers operate independently. That is,

$$P_s(l, m-l, q, k) \simeq (1 - P_h(q, k))^l \cdot P_h(q, k)^{m-l}$$

We see that (9) is equivalent to the following two equations

$$P_s(m, 0, q, k) \simeq (1 - P_h(q, k))^m$$

and

$$P_s(0, m, q, k) \simeq P_h(q, k)^m.$$

The first approximation can be shown as follows

$$\begin{aligned} P_s(m, 0, q, k) &= \prod_{j=1}^m (1 - P_h(q - j + 1, k - j + 1)) \simeq \\ &\simeq (1 - P_h(q, k))^m \end{aligned}$$

where the approximation is valid through our assumption on q and the form of the expressions giving $P_h(q, k)$. Now, let us consider $P_s(0, m, q, k)$. We get

$$\begin{aligned} P_s(0, m, q, k) &= 1 + \sum_{i=1}^m (-1)^i \binom{m}{i} \cdot \prod_{j=1}^i (1 - P_h(q, k)) \simeq \\ &\simeq 1 + \sum_{i=1}^m (-1)^i \cdot \binom{m}{i} (1 - P_h(q, k))^i = \\ &= (1 - (1 - P_h(q, k)))^m = P_h(q, k)^m \end{aligned}$$

⁴The total probability of l correct receptions and $m-l$ erroneous ones obeys the binomial law.

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where the approximation holds for the same reasons as above and the last equation follows from the binomial theorem.

This establishes the validity of our approximation. Note that in many practical applications, the generated traffic is light. In such cases, the condition $q \gg k$ is easily satisfied.

III. Numerical Results

In this section, we present our numerical results and comparisons. As explained in previous section, the evaluation of $P(l, m - l|k)$ becomes prohibitive for $m \geq 4$, so that only results for $m = 2$ and $m = 3$ are presented. We consider asynchronous FH/SS with RS (32,16) coding and error-correction decoding ⁵.

In Table 1, we present our results for $m = 2$, for different values of q and k . Both exact and approximate results (IROA) are included for comparison. Table 1a contains the corresponding results for $P(2, 0|k)$. An examination of these results shows that IROA gives high accuracy, specially for $q \gg k$, in accordance to our approximation in Section II. This is also true for $P(1, 1|k)$. On the other hand, as the number of contending users k increases, the approximation becomes less accurate. However, for large values of k , both $P(l, m - l|k)$ and $P_I(l, m - l|k)$ become very small.

In Table 2, we make the same comparisons for $m = 3$. For this case, three subtables are included. Table 2a presents results for $P(3, 0|k)$, Table 2b for $P(2, 1|k)$, and Table 2c for $P(1, 2|k)$. We see that IROA gives results close to the exact ones with improving accuracy as q increases (???) with respect to k .

An interesting fact that, as we have discovered, holds true in all numerical analysis

⁵Larger blocklengths will randomize more each receiver operation.

we have performed, is that $P(m, 0|k)$ is higher than $P_I(l, m - l|k)$. In other words, IROA seems to give “pessimistic” results in comparison to the exact analysis. The interested reader can consult [4] for more numerical results.

Based on the results presented above, we can draw some conclusions, at least for the cases $m = 2$ and $m = 3$. IROA gives good approximation to the probability $P(l, m - l|k)$; the accuracy of the approximation depends on the specific values of q and k , for $q \gg k$, the corresponding results are almost the same with the exact ones.

The kind of behavior observed so far is expected to be true for higher values of m , as well. In addition, more complicated scenarios with AWGN and erasure decoding randomize each receiver operation even more.

IV. Conclusions

For FH/SSMA communications, we present exact expressions for the multireception probabilities $P(l, m - l|k)$. These expressions, however, are very difficult to evaluate for $m \geq 4$, as they require computation of $2^m - 1$ sums. At the same time, we establish the validity of the IROA, at least for the case $q \gg k$. Additionally, our numerical analysis indicates that IROA is a good approximation for the multireception probabilities, for $m = 2$ and $m = 3$. Therefore, it appears that IROA gives realistic results, while requiring minimal numerical effort.

Appendix I

In this appendix, we derive the expression for $P(l, m - l | k)$ as a function of the probabilities $P_{e \dots c}$. Let q, k denote the number of frequency slots and contending users in the slot, respectively.

For receiver i , $1 \leq i \leq m$, let $\underline{e}_i$ be a vector having 0 in the positions of correct symbol reception and 1 in the positions of error reception, that is,

$$\underline{e}_i = (e_{i1}, e_{i2}, \dots, e_{iN}).$$

For hard decision decoding, the i th receiver decodes correctly the received packet iff

$$0 \leq \sum_{j=1}^N e_{ij} \leq t$$

while it decodes erroneously iff

$$t + 1 \leq \sum_{j=1}^N e_{ij} \leq N.$$

Let us now turn our attention to the interreceiver operation. For the j th transmitted symbols, for $1 \leq j \leq m$, we define the vector of simultaneous event E_j as

$$E_j = (e_{1j}, \dots, e_{mj}).$$

As each e_{ij} takes on two possible values, there is a total of 2^m possible E_j , for each j . As slotted operation is assumed throughout, statistics are the same from symbol to symbol, so that the description of the system is independent of the particular symbol j . Then we can arbitrarily assign events E to symbol events e_i . However, we choose for clarity the correspondence

$$E_i = (e_1, \dots, e_m)$$

so that $i - 1$ is equal to the binary representation of $(e_1, \dots, e_m)$. If we define by l_i the number of times a particular event E_i occurs, we get, due to the memoryless operation assumption, the result given by (4).

Appendix II

In this appendix, we prove that $P_e(m, q, k)$, given in (7), is the solution to the recursive equation described by (6). For compactness, we denote the binomial coefficients $\binom{m}{i}$ by $C_{m,i}$.

First we observe that, for the binomial coefficients $C_{m,i}$, the following recursion is true

$$C_{m+1,i} = C_{m,i-1} + C_{m,i}. \quad (A2.1)$$

Then, by direct substitution of (8) to the right hand side of (7), we get

$$\begin{aligned} & 1 + \sum_{i=1}^{m-1} (-1)^i C_{m-1,i} \prod_{j=1}^i (1 - P_h(q - j + 1, k - j + 1)) - \\ & - (1 - P_h(q, k)) \left(1 + \sum_{i=1}^{m-1} (-1)^i C_{m-1,i} \prod_{j=1}^i (1 - P_h(q - j, k - j)) \right) = \\ & = 1 - m(1 - P_h(q, k)) + \sum_{i=2}^{m-1} (-1)^i (C_{m-1,i} + C_{m-1,i-1}) \prod_{j=1}^i (1 - P_h(q - j + 1, k - j + 1)) + \\ & + (-1)^m C_{m-1,m-1} \prod_{j=1}^m (1 - P_h(q - j + 1, k - j + 1)) = \\ & = 1 + \sum_{i=1}^m (-1)^i C_{m,i} \prod_{j=1}^m (1 - P_h(q - j + 1, k - j + 1)) \end{aligned} \quad (A2.2)$$

From (A2.2) we see that (6) has as solution the expression given in (7).

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Table 1aProbabilities $P(2,0|k)$ (exact) and $P_I(2,0|k)$ (under IROA)

for asynchronous FH/SSMA with RS (32,16) coding and error-correction decoding

q	$k = 5$		$k = 10$		$k = 20$		$k = 50$	
	Exact	IROA	Exact	IROA	Exact	IROA	Exact	IROA
10	4.4×10^{-8}	3.9×10^{-8}	1.6×10^{-29}	1.7×10^{-29}	0.0	0.0	0.0	0.0
50	0.9200	0.9152	0.1065	0.0967	9.1×10^{-7}	7.1×10^{-7}	2.4×10^{-29}	2.4×10^{-29}
100	0.9989	0.9988	0.8630	0.8612	7.4×10^{-2}	7.0×10^{-2}	2.6×10^{-10}	2.3×10^{-10}

Table 1bProbabilities $P(1,1|k)$ (exact) and $P_I(1,1|k)$ (under IROA)

for asynchronous FH/SSMA with RS (32,16) coding and error-correction decoding

q	$k = 5$		$k = 10$		$k = 20$		$k = 50$	
	Exact	IROA	Exact	IROA	Exact	IROA	Exact	IROA
10	9.8×10^{-5}	9.8×10^{-5}	4.1×10^{-15}	4.1×10^{-15}	0.0	0.0	0.0	0.0
50	3.8×10^{-2}	4.1×10^{-2}	0.2045	0.2142	8.4×10^{-4}	8.4×10^{-4}	4.9×10^{-15}	4.9×10^{-15}
100	5.4×10^{-4}	5.5×10^{-4}	6.4×10^{-2}	6.6×10^{-2}	0.1906	0.1906	1.5×10^{-5}	1.5×10^{-5}

Table 2aProbabilities $P(3,0|k)$ (exact) and $P_I(3,0|k)$ (under IROA)

for asynchronous FH/SSMA with RS (32,16) coding and error-correction decoding

q	$k = 5$		$k = 10$		$k = 20$		$k = 50$	
	Exact	IROA	Exact	IROA	Exact	IROA	Exact	IROA
10	9.3×10^{-11}	9.4×10^{-11}	0.0	0.0	0.0	0.0	0.0	0.0
50	0.8863	0.8857	3.97×10^{-2}	3.0×10^{-2}	1.2×10^{-9}	0.9×10^{-9}	0.0	0.0
100	0.9983	0.9983	0.8053	0.7993	2.1×10^{-2}	1.8×10^{-2}	4.9×10^{-15}	3.4×10^{-15}

Table 2bProbabilities $P(2,1|k)$ (exact) and $P_I(2,1|k)$ (under IROA)

for asynchronous FH/SSMA with RS (32,16) coding and error-correction decoding

q	$k = 5$		$k = 10$		$k = 20$		$k = 50$	
	Exact	IROA	Exact	IROA	Exact	IROA	Exact	IROA
10	9.6×10^{-9}	9.4×10^{-9}	1.6×10^{-29}	1.6×10^{-29}	0.0	0.0	0.0	0.0
50	3.9×10^{-2}	4.3×10^{-2}	6.6×10^{-2}	7.2×10^{-2}	7.0×10^{-7}	8.4×10^{-7}	2.4×10^{-29}	3.3×10^{-29}
100	5.5×10^{-4}	5.6×10^{-4}	6.2×10^{-2}	6.5×10^{-2}	5.1×10^{-2}	5.3×10^{-2}	2.2×10^{-10}	2.2×10^{-10}

Table 2cProbabilities $P(1,2|k)$ (exact) and $P_I(1,2|k)$ (under IROA)

for asynchronous FH/SSMA with RS (32,16) coding and error-correction decoding

q	$k = 5$		$k = 10$		$k = 20$		$k = 50$	
	Exact	IROA	Exact	IROA	Exact	IROA	Exact	IROA
10	9.8×10^{-5}	9.8×10^{-5}	4.1×10^{-15}	4.2×10^{-15}	0.0	0.0	0.0	0.0
50	1.7×10^{-3}	1.7×10^{-3}	0.1476	0.1476	8.3×10^{-4}	8.4×10^{-4}	4.9×10^{-15}	5.3×10^{-15}
100	3.0×10^{-7}	2.6×10^{-7}	4.8×10^{-3}	4.8×10^{-3}	0.1434	0.1430	1.1×10^{-5}	1.1×10^{-5}