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Nonlinear Stabilization of High Angle-of-Attack Flight Dynamics Using Bifurcation Control

by E.H. Abed and H.C. Lee

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Nonlinear Stabilization of High Angle-of-Attack Flight Dynamics Using Bifurcation Control

Eyad H. Abed and Hsien-Chiarn Lee

Department of Electrical Engineering
and the Systems Research Center
University of Maryland, College Park, MD 20742 USA

Abstract

We consider the problem of designing stabilizing control laws for flight over a broad range of angles-of-attack which also serve to signal the pilot of impending stall. The paper employs bifurcation stabilization coupled with more traditional linear control system design. To focus the discussion, a detailed analysis is given for a model of the longitudinal dynamics of an F-8 Crusader.

I. Introduction

Several authors have studied the nonlinear phenomena that arise commonly in aircraft flight at high angle-of-attack (alpha). The literature on high alpha flight dynamics, control and aerodynamics has grown at a rapid pace. Of particular relevance here are references [4]-[6], [9] and [10]. The direct linkage of aircraft stall and divergence, as well as other nonlinear aircraft motions in high incidence flight, to bifurcations of the governing dynamic equations is a goal of many previous investigations. In particular, both stationary and Hopf bifurcations are reported and/or studied for several aircraft models in [4], [5], [9]; and a Hopf bifurcation occurring in the lateral dynamics of a slender-wing aircraft has been studied in [10], [5], [1].

In this note, we study the stabilization of the trim condition of an aircraft arbitrarily close to the stall angle, in a manner which also provides an *impending stall warning signal* to the pilot. This signal is a small-amplitude, stable limit cycle-type pitching motion of the aircraft which persists to within a prescribed margin from impending divergent stall. This is a Hopf-bifurcated periodic solution of the system dynamics, which is stabilized using the methods of bifurcation control [2], [3]. A brief summary of bifurcation control is given in the next section.

II. Bifurcation Control Laws

Local bifurcation control [2], [3] deals with the modification of stability characteristics of bifurcated periodic solutions by feedback control. Thus, we can contain to a local neighborhood of an unstable equilibrium the transients of a nonlinear system, even in cases wherein the transient would otherwise exhibit divergence. The feedback control designs of [1], [2] result in transforming a subcritical (unstable) bifurcation to a supercritical, and hence stable, bifurcation. (For background on bifurcations, see for instance [7].)

Specifically, local bifurcation control deals with the design of smooth control laws $u = u(x)$ which stabilize a bifurcation occurring in a one-parameter family of systems

$$\dot{x} = f_\mu(x, u). \tag{1}$$

These control laws exist generically, even if the critical eigenvalues of the linearized system at the equilibrium of interest are uncontrollable. (The critical eigenvalues are those lying on the imaginary axis.) This approach has been employed in the design of stabilizing control laws for a tethered satellite system in the station-keeping mode.

III. Bifurcation Control of Longitudinal Dynamics

From [6, Eqs. (10), (11)], we obtain the following model for pitching motions of a model F-8 Crusader aircraft in nearly level flight (i.e., for pitch angle remaining small). Here, α = angle-of-attack, θ = pitch angle, $\dot{\theta}$ = pitching moment, and δ = the instantaneous elevator control surface deflection.

$$\begin{aligned}\dot{\alpha} =& \dot{\theta} - \alpha^2\dot{\theta} - 0.088\alpha\dot{\theta} - 0.877\alpha + 0.47\alpha^2 + 3.846\alpha^3 \\ & - 0.215\delta_e + 0.28\delta_e\alpha^2 + 0.47\delta_e^2\alpha + 0.63\delta_e^3 \qquad (2a)\\ \ddot{\theta} =& - 0.396\dot{\theta} - 4.208\alpha - 0.47\alpha^2 - 3.564\alpha^3 \\ & - 20.967\delta_e + 6.265\delta_e\alpha^2 + 46\delta_e^2 + 61.4\delta_e^3 \qquad (2b)\end{aligned}$$

We have studied the stability of this model as a function of δ_e viewed as a *parameter*, as well as stabilization of the trim condition using elevator deflection as a feedback control signal which can either be linear or nonlinear. In either case, we seek control laws which have a negligible effect on the trim condition, which itself depends on δ_e. To achieve this, we require a certain form of dependence of the control signal on the state, namely

$$\delta_e(x) = \delta_{eC} + \{ \text{ a polynomial in } (x_1 - x_{10}(\delta_{eC})) \text{ and } (x_2 - x_{20}(\delta_{eC}))\}. \qquad (3)$$

Here, x_1 and x_2 are the state variables α and $\dot{\theta}$, respectively, δ_{eC} is the constant *commanded* value of δ_e, and subscripts 0 indicate equilibrium (trim) values of state variables, which depend on δ_{eC}. In our example, curve fitting gives the following approximations for the trim condition as a function of δ_{eC}:

$$\alpha_0 = -4.6092\delta_{eC}, \qquad (4a)$$

$$\dot{\theta}_0 = 630.8146\delta_{eC}^3 - 5.0498\delta_{eC}. \qquad (4b)$$

The design procedure aims to result in an increased range of stable angles-of-attack. First, a linear feedback complying with the general form (3) is designed to stabilize the trim condition for all values of δ_{eC} up to a value which verges on stall. Next, a nonlinear controller is designed to control the stability of the bifurcation which occurs at the point of instability just prior to stall. This bifurcation is a Hopf bifurcation to periodic solutions. By ensuring a small amplitude stable periodic solution in the neighborhood of the unstable trim condition, a signal of incipient stall is produced. This signal consists of the small amplitude sustained pitching oscillations induced. These oscillations do not lead to a divergence instability, but are a warning signal of an impending such instability. The figures best illustrate the conclusions.

In Figures 1a and 1b, the dependence of the trim condition and several other equilibria on δ_{eC} is shown. (In both Figs. 1 and 2, an S indicates a stable equilibrium, while a U indicates instability of an equilibrium.) Fig. 1 gives the equilibria of the open-loop system.

Note the presence of a Hopf bifurcation for the critical parameter value $\delta_{eC} = -0.064$, for which the angle-of-attack $\alpha = 0.305$ ($= 17.48°$). At this bifurcation, the eigenvalues are given by $\pm j2.212$. Moreover, the positivity of the "bifurcation stability coefficient" $\beta_2 = 3.123$ (see Fig. 1) implies instability of the bifurcated periodic solutions. Thus, for $|\delta_{eC}| > 0.064$, transients beginning near trim *diverge*. This divergence of the uncontrolled system is shown in the simulation of Fig. 3.

To remedy this, we can either use linear feedback to stabilize the trim condition for a useful range of angles-of-attack, or use bifurcation control laws to render the bifurcated periodic solutions stable and of small amplitude for such a range of angles-of-attack. With the latter design, the aircraft would continually experience an oscillatory pitching motion, which is not acceptable. With the former, the Hopf bifurcation is delayed to a greater value of trim angle-of-attack, and operating at higher than that new critical α might result in divergence as well. Thus, we employ a linear-plus-nonlinear feedback. The linear part of the feedback is chosen as above (to delay the Hopf bifurcation), and the nonlinear terms are chosen to stabilize (if necessary) the Hopf bifurcation at the new higher critical angle.

The system equilibria will be modified by feedback control laws of the type considered. However, by design, the trim condition will experience a limited deformation. Moreover, a "windowing" operation (in state space) can be used to result in control laws which have a negligible effect on the non-trim equilibria as well. Such an operation is not discussed in detail here.

Fig. 2 shows the post-linear feedback equilibria, where the stabilizing linear feedback is chosen to result in a (Hopf) critical α of 0.5 ($= 28.65°$). The linear feedback chosen here is given by

$$\delta_e = \delta_{eC} + k_1(\alpha - \alpha_0(\delta_{eC})) + k_2(\dot{\theta} - \dot{\theta}_0(\delta_{eC})), \tag{5}$$

where $k_1 = 0.3317$ and $k_2 = 0.0836$. The critical value of the bifurcation parameter at this bifurcation is $\delta_{eC} = -0.109$, and the eigenvalues of the linearization are given by $\pm j2.158$. However, the Hopf bifurcation occurring in the linearly controlled system remains subcritical, as indicated by a positive value of the bifurcation stability coefficient β_2 ($\beta_2 = 32.064$, as noted in Fig. 2). Moreover, Fig. 4 illustrates the result of a simulation starting near the trim condition for the post-critical parameter value of $\delta_e = -0.1095$. As can be seen from Fig. 4, the trajectory diverges.

To stabilize the Hopf bifurcation, and thus result in containment of post-critical trajectories to within a neighborhood of trim, nonlinear terms are added to the linear feedback above. Specifically, we have chosen to add certain quadratic and cubic terms to the linear feedback, as follows:

$$\begin{aligned}\delta_e =& \delta_{eC} + k_1(\alpha - \alpha_0(\delta_{eC})) + k_2(\dot{\theta} - \dot{\theta}_0(\delta_{eC})) \\ &+ q_1(\alpha - \alpha_0(\delta_{eC}))^2 + h_1(\alpha - \alpha_0(\delta_{eC}))^3 \\ &+ h_2(\dot{\theta} - \dot{\theta}_0(\delta_{eC}))^3\end{aligned} \tag{6}$$

Here, $q_1 = h_1 = h_2 = 0.8$, resulting in a bifurcation stability coefficient of value $\beta_2 = -320.639$. Thus, the Hopf bifurcation for the controlled system has been stabilized. Fig. 5 shows the convergence of the system trajectory to a stable limit cycle for the post-critical parameter value $\delta_e = -0.11318$, thus significantly extending the operating envelope over that achieved with the purely linear feedback noted above. The limit cycle then becomes a homoclinic orbit and disappears. The simulation of Fig. 6 shows a trajectory of the system started near trim for the parameter value $\delta_e = -0.11319$. The trajectory no longer converges to a stable limit cycle, but now diverges.

Acknowledgment

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FIGURE 1a. α AT OPEN-LOOP EQUILIBRIA

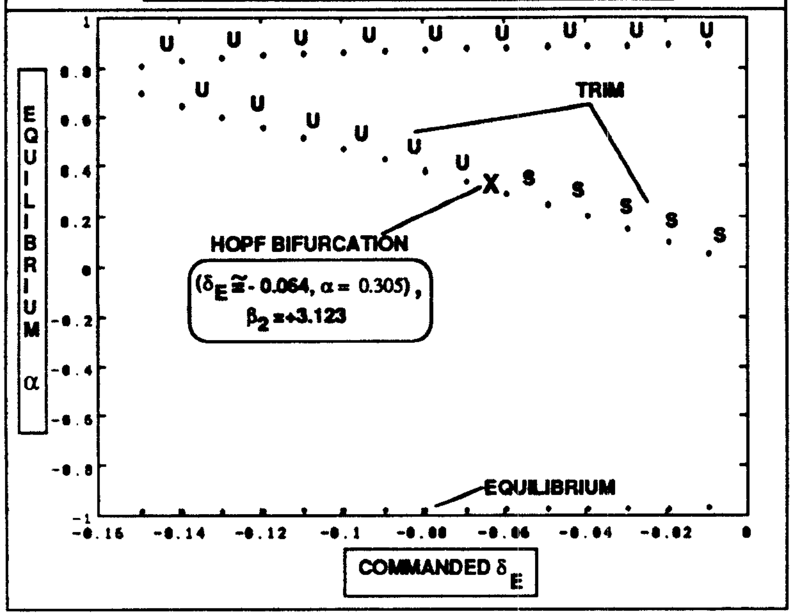

FIGURE 1b. $\dot{\theta}$ AT OPEN-LOOP EQUILIBRIA

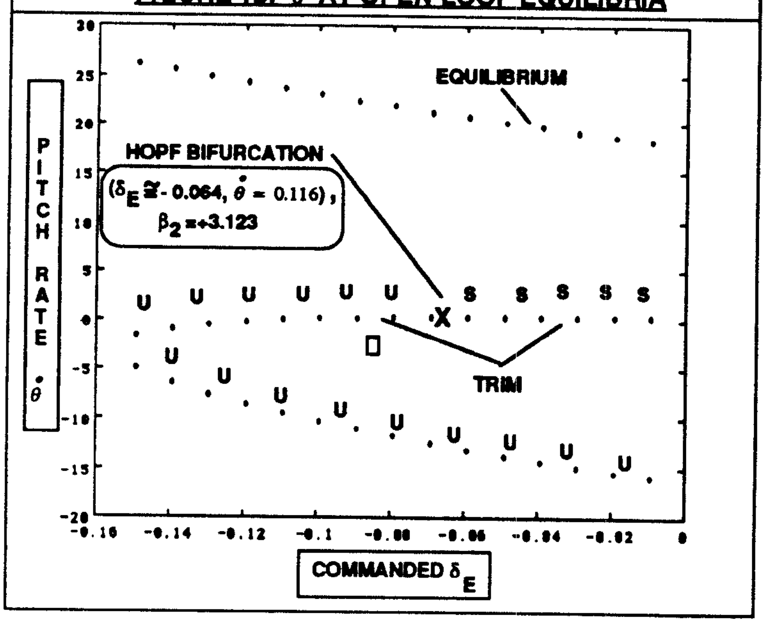

FIGURE 2a. α AT EQUILIBRIA UPON LINEAR FEEDBACK

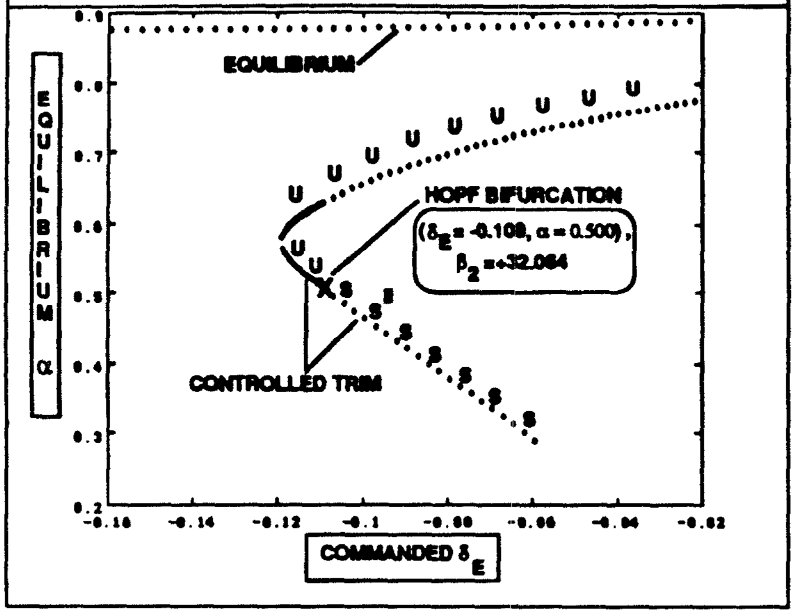

FIGURE 2b. $\dot{\theta}$ AT EQUILIBRIA UPON LINEAR FEEDBACK

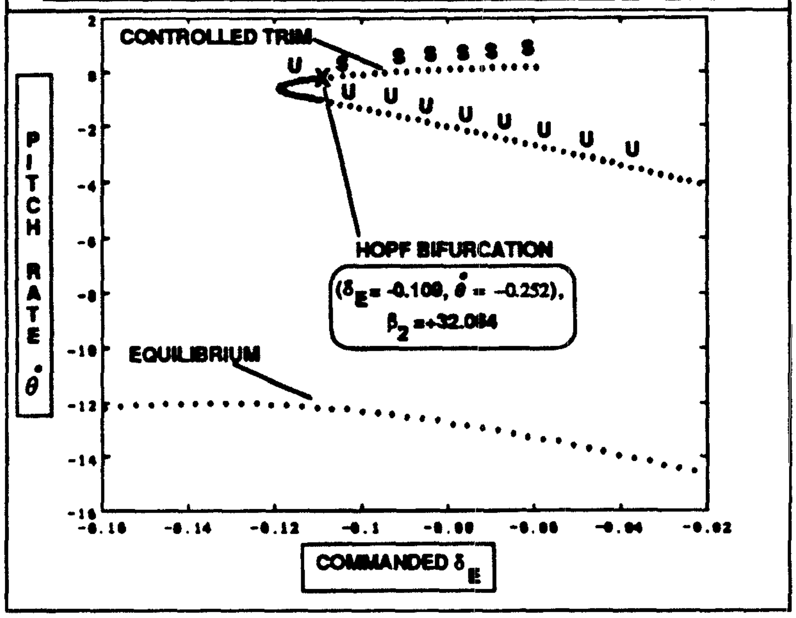

FIG. 3. SHOWING DIVERGENCE OF UNCONTROLLED SYSTEM

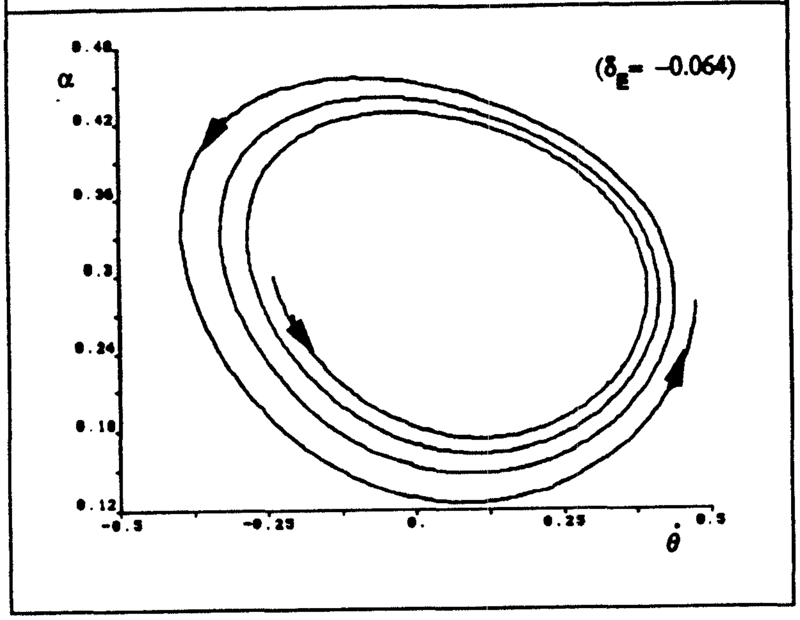

FIG. 4. SHOWING DIVERGENCE UNDER LINEAR FEEDBACK

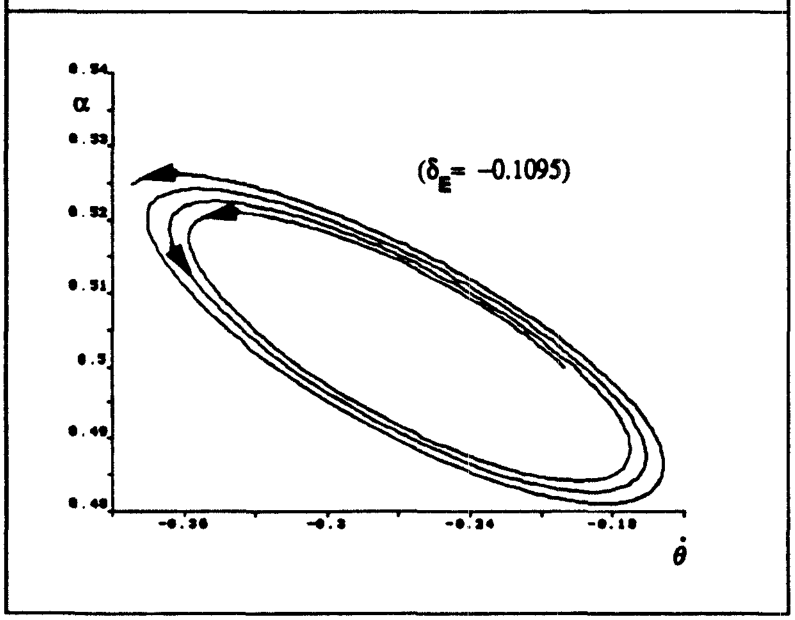

FIG. 5. SHOWING CONVERGENCE TO LIMIT CYCLE UNDER LINEAR-PLUS-NONLINEAR FEEDBACK

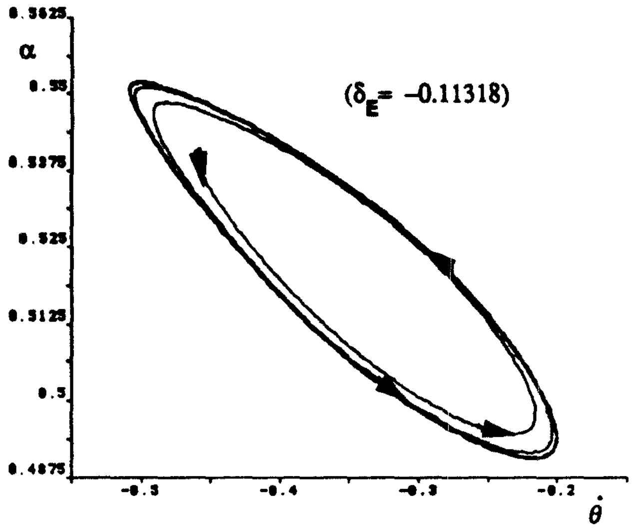

FIG. 6. SHOWING DIVERGENCE AFTER APPEARANCE OF HOMOCLINIC ORBIT