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CONVERGENCE STUDY OF AN ITERATIVE JOINT DETECTOR FOR WAVELET PACKET MULTIPLE-ACCESS COMMUNICATION

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ABSTRACT

The joint detection of all users in a multiple access communication system is known to greatly enhance the system performance. The optimal joint detector has a complexity which is exponential in the number of users and is, therefore, impractical. Suboptimal algorithms attempt to achieve near optimal performance with reduced computational complexity. The suboptimal joint detection literature deals only with receiver design, allowing for the use of any set of signature waveforms. Using the structure of the wavelet packet transform [1] we are able to choose signal sets *and* design a detection algorithm which results in significantly lower computational complexity than other proposed suboptimal joint detectors. The wedding of detector development and wavelet packet signal set design, therefore, becomes an interesting problem. Convergence, performance and receiver complexity of suboptimal MA detectors crucially depend upon the signal set structure; it is, thus, of paramount importance to study the convergence of our iterative algorithm for our signal structure. In this paper, sufficient conditions for the convergence of an iterative joint detection scheme are developed.

1. INTRODUCTION

A multiple access communication system will typically support a large number of users over a given channel.

By allowing many users to transmit simultaneously in the same frequency band, we are inducing interference among users. This inter-user or multiple access interference (MAI) will degrade the performance of conventional receivers, and is, in fact, the major limiting factor when the number of users exceeds a threshold. In the current literature, much attention is being paid to overcoming the MAI effects, but, the signal design aspect and the receiver design aspect of the MA system are always considered separately.

Assuming that a single bit is transmitted by each user within a single time window, the received signal, $\mathbf{r}$, which is the aggregate of each user's transmission (plus noise), can be written as

$$\mathbf{r} = \sum_{k=1}^{K} b_k \mathbf{w}_k + \mathbf{n} = \mathbf{W}\mathbf{b} + \mathbf{n}, \qquad (1)$$

where K is the total number of users, b_k is either a 1 or -1 corresponding to a 1 or 0 information symbol of the kth user, $\mathbf{w}_k$ is the kth user's discrete waveform represented as a vector, $\mathbf{n}$ is an i.i.d., zero mean, Gaussian noise vector, and the following definitions are used: $\mathbf{b} = [b_1 \ b_2 \cdots b_K]^T$, $\mathbf{W} = [\mathbf{w}_1 \ \mathbf{w}_2 \cdots \mathbf{w}_K]$. We assume that all relative time shifts are known and incorporated in $\mathbf{w}_k$.

The conventional approach to receiver design is to use a matched filter,

$$\hat{b}_k = sgn[\mathbf{w}_k^T \mathbf{r}], \qquad (2)$$

where sgn represents the signum functional and $\hat{b}_k$ denotes the estimated/detected symbol of the kth user. The conventional matched filter (2) represents the optimal receiver in the case where the MAI is assumed normal and white. Under this assumption, as users are added to the system the MAI raises the noise level; this, in turn, limits the matched filter performance and, ultimately, the reliable throughput. For this reason, the

signal design aspect of MA is typically restricted to a general attempt at minimizing correlation among user waveforms.

The MAI, however, is not an additive white Gaussian noise process; it possesses a great degree of structure which can be exploited in achieving a greater transmission capacity (e.g. more simultaneous users) than the system which uses the conventional detector. The optimal joint detector (which accounts for the MAI) maximizes the log-likelihood function [2] and results in

$$\hat{\mathbf{b}} = \arg\left[\max_{\mathbf{b}\in\{1,-1\}^K} 2\mathbf{r}^T\mathbf{W}\mathbf{b} - (\mathbf{W}\mathbf{b})^T\mathbf{W}\mathbf{b}\right]. \quad (3)$$

This maximization is over 2^K possible $\mathbf{b}$ vectors (a computational complexity that is exponential in K, the number of users). This optimal method offers substantially higher performance than the conventional demodulator (2) (e.g. it will afford a relatively larger number of users successful utilization of an MA system), but is of little practical use on account of its computational burden [3].

Recent communication literature addresses the general notion of *suboptimal* joint detection which offers, with minimal performance loss, computational improvement relative to the optimal method while achieving a significant improvement over the conventional detector [4, 5, 6]. These approaches only address the receiver design problem. The convergence, performance, and receiver complexity of such suboptimal MA detectors, however, crucially depend upon the structure of the signal set $\mathbf{W}$. In light of this, our work in MA systems couples detection and waveform design.

Our method for waveform design takes full advantage of the hierarchical nature of the wavelet packet transform to control MAI by providing structure that can be exploited in receiver design. Our waveforms possess a nested-type of correlation structure which is well suited for iterative bit estimation. As shown in [7] by using our wavelet-packet-based signature waveforms in lieu of the well known direct sequence spread spectrum waveforms, a great reduction in computational complexity of other proposed detectors results.

2. THE JOINT DETECTOR

In the MA scenario described in Equation (1), we observe the signal

$$\mathbf{r} = \mathbf{W}\mathbf{b} + \mathbf{n}. \quad (4)$$

We wish to derive an iterative algorithm to estimate $\mathbf{b}$. We know that $\mathbf{b} \in \{1,-1\}^K$ and we use as the first estimate the result from the matched filter detector (2). Denoting the first estimate of the bit vector as $\hat{\mathbf{b}}(1)$, we have

$$\hat{\mathbf{b}}(1) = sgn[\mathbf{W}^T\mathbf{r}]. \quad (5)$$

Without noise, i.e. $\mathbf{n} = \mathbf{0}$, we have $\mathbf{r} = \mathbf{W}\mathbf{b}$ and our first estimate would be

$$\hat{\mathbf{b}}(1) = sgn[\mathbf{W}^T\mathbf{W}\mathbf{b}]. \quad (6)$$

For obvious reasons we require the true bit vector, $\mathbf{b}$, to be a fixed point of our iteration, at least in the noiseless case. In general, $\mathbf{b} \neq \mathbf{W}^T\mathbf{W}\mathbf{b}$. For the special case in which $\mathbf{b} = \mathbf{W}^T\mathbf{W}\mathbf{b}$, the user waveforms would be orthogonal and the matched filter would suffice. In the general case (in which users are correlated) we must add $(\mathbf{b} - \mathbf{W}^T\mathbf{W}\mathbf{b})$ to the argument of Equation (6) so that, in the noiseless case, we have

$$\mathbf{b} = sgn[\mathbf{W}^T\mathbf{W}\mathbf{b} + \mathbf{b} - \mathbf{W}^T\mathbf{W}\mathbf{b}]. \quad (7)$$

This leads to our iterative detector

$$\hat{\mathbf{b}}(m+1) = sgn[\mathbf{W}^T\mathbf{r} + \hat{\mathbf{b}}(m) - \mathbf{W}^T\mathbf{W}\hat{\mathbf{b}}(m)], \quad (8)$$

or

$$\hat{b}_k(m+1) = sgn[\mathbf{r}^T\mathbf{w}_k - \sum_{j\neq k}\mathbf{w}_k^T\mathbf{w}_j\hat{b}_j(m)], \quad (9)$$

where m denotes the iteration number and $\hat{\mathbf{b}}(0) = 0$ so that Equation (8) reduces to the matched filter estimate (5) for $\hat{\mathbf{b}}(1)$.

We offer a simple interpretation of our joint detector by looking at Equation (8) for noiseless reception, $\mathbf{r} = \mathbf{W}\mathbf{b}$,

$$\hat{\mathbf{b}}(m+1) = sgn[\hat{\mathbf{b}}(m) + \mathbf{W}^T\mathbf{W}(\mathbf{b} - \hat{\mathbf{b}}(m))]. \quad (10)$$

At each iteration, we update the previous estimate with a correction term which is based on the difference between the actual bit vector and the previous bit estimate.

The joint detection procedure defined by Equation (9) is similar to a more general algorithm proposed by Varanasi and Aazhang [4].

Commonly, MA systems employ direct sequence spread spectrum (DSSS) or pseudo-noise spread spectrum (PNSS) user waveforms (sequences) where any single user experiences interference from all other users [8]. Figure 1-a shows the cross correlation matrix, $\mathbf{W}^T\mathbf{W}$, for a set of 68 pseudo-noise (PN) waveforms in a 64 dimensional waveforms space (i.e. 64 samples per waveform duration). Each waveform, $\mathbf{w}_k$, was generated by a random number generator such that $\mathbf{w}_k \in \{-1,1\}^{64}$. We make two observations: 1) when $\mathbf{w}_k$ is a pseudo-noise spread spectrum (PNSS) waveform, $\mathbf{w}_k^T\mathbf{w}_j$ is, in general, nonzero for all values of j and k, therefore, Equation (9) must decode the bits of all K users to detect a single user's bit; 2) each user experiences the same

degree of MAI. We note that, typically, the number of users in a DSSS system does not exceed the number of dimensions in the waveform space. Additionally, Gold or Kasami sequences [8] which have been designed to have low cross correlations are typically used. We include this example of the PN waveform set in order to illustrate the general notion of DSSS and the cross correlation structure of PN sequences.

Figure 1-b shows the cross correlation matrix for one wavelet packet signal set in which 68 user waveforms exist in a 64 dimensional waveform space. (See [7] for the wavelet packet signal set development.) In contrast to the PNSS observations made above: 1) each of our wavelet packet waveforms, $\mathbf{w}_k$, is correlated with only a fraction of the signal set; 2) each user experiences a different degree of MAI. We immediately see that our waveforms offer two advantages in a joint detection MA system: 1) due to the reduction in the number of correlated users, a large degree of computational simplification of the joint detector is realized; 2) the hierarchical structure of MAI experienced by each user lends itself to the iterative nature of our algorithm.

In the next section we study the convergence of an iterative joint detection algorithm. We set a framework within which wavelet packet multiple access (WPMA) signal optimization as well as joint detection can be considered coincidentally.

3. STUDY OF CONVERGENCE

We are interested in the convergence of Equation (10), the recursive joint detection algorithm for the noiseless case. We rearrange Equation (10) and incorporate the scalar, μ, to allow for speedy convergence.

$$\hat{\mathbf{b}}(m+1) = \text{sgn}[\mathbf{b} + (\mu \mathbf{W}^T \mathbf{W} - I)(\mathbf{b} - \hat{\mathbf{b}}(m))]. \quad (11)$$

Proposition: The optimal convergence speed of the recursion in Equation (11) is achieved for $\mu = \frac{2}{A+B}$, where A and B are, respectively, the minimum and maximum eigenvalues of the matrix $\mathbf{W}^T \mathbf{W}$. Furthermore, the convergence is guaranteed if
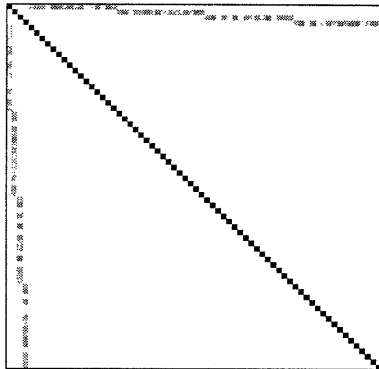
$$B < 3A.$$

Proof: Let $\mathbf{e}(m)$ denote the error vector with its jth element defined by

$$e_j(m) \triangleq \begin{cases} 0 & \text{, if } \hat{b}_j(m) = b_j \\ 1 & \text{, otherwise.} \end{cases}$$

The error vector indicates which bits are incorrect in the estimate $\hat{\mathbf{b}}(m)$, for $m \geq 1$. We set $\mathbf{e}(0) = 0.5\eta$,
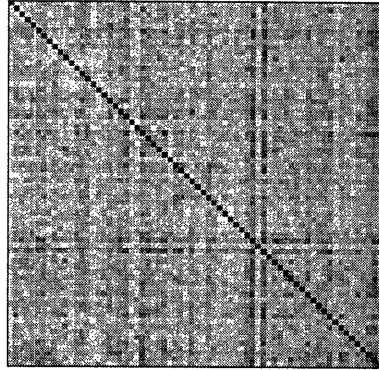
(a)

(b)

Figure 1: $\mathbf{W}^T \mathbf{W}$ displayed as a gray scale image where. White corresponds to zero and black corresponds to unit magnitude. a) PN sequences   b) WPMA waveforms

where $\eta \triangleq [1, \ldots, 1]^T$. Letting $\mathcal{B} \triangleq \text{diag}\{\mathbf{b}\}$ allows us to express the error in our estimate as

$$(\mathbf{b} - \hat{\mathbf{b}}(m)) = 2\mathcal{B}\mathbf{e}(m).$$

Let $\mathbf{Z} = (\mu \mathbf{W}^T \mathbf{W} - I)$ so that we may rewrite equation (11) as

$$\begin{aligned} \hat{\mathbf{b}}(m+1) &= \text{sgn}[\mathcal{B}\eta + 2\mathbf{Z}\mathcal{B}\mathbf{e}(m)] \\ &= \mathcal{B}\,\text{sgn}[\eta + 2\mathcal{B}\mathbf{Z}\mathcal{B}\mathbf{e}(m)], \quad (12) \end{aligned}$$

where we used the equality $\mathcal{B}\mathcal{B} = I$. From Equation (12), we see that an error is made, i.e. $\hat{b}_k(m+1)$ is incorrect and $e_k(m+1) = 1$, if

$$[2\mathcal{B}\mathbf{Z}\mathcal{B}\mathbf{e}(m)]_k < -1.$$

In more general terms, if

$$\| 2\mathcal{B}\mathbf{Z}\mathcal{B}e(m) \|^2 < p+1, \quad p \in \{0, 1, 2, \cdots\}$$

$$\Updownarrow$$

$$\| \ 2\mathcal{B}\mathbf{Z}\mathcal{B}e(m) \ \| < \sqrt{p+1},$$

then $\hat{b}(m+1)$ has at most $p+1$ incorrect bits and

$$\| \ e(m+1) \ \| \leq \sqrt{p+1}.$$

Now, if $\hat{b}(m+1)$ has *exactly* $p+1$ incorrect bits, then
$$\| \ e(m+1) \ \| = \sqrt{p+1}$$
and we have

$$\begin{aligned}\| \ 2\mathcal{B}\mathbf{Z}\mathcal{B}e(m+1) \ \| \ &\leq 2 \ \| \ \mathcal{B}\mathbf{Z}\mathcal{B} \ \| \| \ e(m+1) \ \| \\ &= 2 \ \| \ \mathcal{B}\mathbf{Z}\mathcal{B} \ \| \ \sqrt{p+1}.\end{aligned} \quad (13)$$

In the next iteration of the estimate, $\hat{b}(m+2)$, we tolerate, at most, p incorrect bits, i.e.

$$\| \ e(m+2) \ \| < \sqrt{p+1},$$

so that the number of bit errors decreases at each step of the recursion

$$\ldots < \| \ e(m+2) \ \| < \| \ e(m+1) \ \| < \ldots$$

and convergence is insured. Hence, for convergence, we need

$$\| \ \mathcal{B}\mathbf{Z}\mathcal{B} \ \| < \frac{1}{2}.$$

In particular, in order to have optimal convergence speed, we must minimize the norm of $(\mathcal{B}\mathbf{Z}\mathcal{B})$. We refer here to the matrix norm

$$\| \ C \ \| = \max_x \frac{\| \ Cx \ \|}{\| \ x \ \|} = |\lambda(C)|_{max},$$

where $\lambda(C)$ denotes the eigenvalue of the matrix C. It is easy to show that $\lambda(\mathcal{B}\mathbf{Z}\mathcal{B}) = \lambda(\mathbf{Z})$ and

$$\| \ \mathbf{Z} \ \| = |\lambda(\mathbf{Z})|_{max} = \max(|\mu A - 1|, |\mu B - 1|),$$

where $A = \lambda_{min}(\mathbf{W}^T\mathbf{W})$ and $B = \lambda_{max}(\mathbf{W}^T\mathbf{W})$, $0 \leq A \leq B < \infty$. Additionally, the norm of $\mathbf{Z}$ is minimal if we choose $\mu = \frac{2}{A+B}$ so that

$$\| \ \mathbf{Z} \ \| = \frac{B-A}{B+A}.$$

It follows that the recursion in (12) is guaranteed to converge if

$$\| \ \mathbf{Z} \ \| = \frac{B-A}{B+A} < 1/2 \quad \Leftrightarrow \quad B < 3A. \ \square$$

The convergence conditions derived above are sufficient, only. In [7], we arbitrarily chose WPMA signal sets for which $A = 0$, i.e. the user signature waveforms comprise a linearly dependent set, but convergence is observed in Monte Carlo simulations using the iterative detector of Equation (10). The goal of future work is to find both necessary and sufficient conditions on $\mathbf{W}$ to guarantee convergence of a joint detection algorithm. These conditions will elucidate modifications of the joint detector so that optimal performance is achieved for any given number of users communicating within a given time-bandwidth space.

4. CONCLUSION

The importance of joint detection of all correlated users in a multiple access communication system motivates our research in the area of the coincidental design of user waveforms and joint detection algorithms. The hierarchical correlation structure of wavelet-packet-derived signals appears to be well suited to the development of iterative joint detectors.

In this paper, we set a framework within which wavelet packet multiple access signal optimization as well as joint detection can be considered together. We derived one such iterative joint detector and developed sufficient conditions for its convergence.

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