



NRL/MR/6750--06-8986

Antenna Impedance Measurements in a Magnetized Plasma. Part I: Spherical Antenna

DAVID D. BLACKWELL

DAVID N. WALKER

SARAH J. MESSER

WILLIAM E. AMATUCCI

Charged Particle Physics Branch

Plasma Physics Division

October 16, 2006

Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.					
1. REPORT DATE (DD-MM-YYYY) 16-10-2006		2. REPORT TYPE Memorandum Report		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE Antenna Impedance Measurements in a Magnetized Plasma. Part I: Spherical Antenna				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) David D. Blackwell, David N. Walker, Sarah J. Messer, and William E. Amatucci				5d. PROJECT NUMBER 67-3419-16	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Research Laboratory 4555 Overlook Avenue, SW Washington, DC 20375-5320				8. PERFORMING ORGANIZATION REPORT NUMBER NRL/MR/6750--06-8986	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Naval Research 875 North Randolph Street Arlington, VA 22203-1995				10. SPONSOR / MONITOR'S ACRONYM(S)	
				11. SPONSOR / MONITOR'S REPORT NUMBER(S) ONR	
12. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The input impedance of a metal sphere immersed in a magnetized plasma is measured with a network analyzer at frequencies up to 1 GHz. The experiments were done in the Space Physics Simulation Chamber at the Naval Research Laboratory. The hot-filament argon plasma was varied between weakly ($\omega_{ce} < \omega_{pe}$) and strongly ($\omega_{ce} > \omega_{pe}$) magnetized plasma with electron densities in the range 10^7 - 10^{10} cm ⁻³ . It is observed that the lower-frequency resonance of the impedance characteristic previously associated with series sheath resonance ω_{sh} in the unmagnetized plasma occurs at a hybrid sheath frequency of $\omega_r^2 = \omega_{sh}^2 + \kappa \omega_{ce}^2$, with κ a constant $0.5 < \kappa < 1$. As seen in previous experiments, the higher frequency resonance associated with the electron plasma frequency ω_{pe} in the unmagnetized plasma is relocated to the upper hybrid frequency $\omega_{uh}^2 = \omega_{pe}^2 + \omega_{ce}^2$. As with the unmagnetized plasma, the maximum power deposition occurs at the lower frequency resonance ω_r .					
15. SUBJECT TERMS Plasma diagnostics Plasmas Antennas					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UL	18. NUMBER OF PAGES 16	19a. NAME OF RESPONSIBLE PERSON David Blackwell
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER (include area code) (202) 767-6781

I. INTRODUCTION

Antenna impedance in a plasma has been an active area of research for many years, with the largest amount of work, both theoretical and experimental, conducted before 1980 [1]-[8]. Most of this research was for spacecraft and rocket mounted plasma diagnostics for electron density measurements in the ionosphere. Recently, there has been renewed interest in this subject as advances in measurement and computer technology have allowed scientists to collect more accurate readings and model antennas more realistically than was previously possible [9]-[18]. The use of impedance measurements for space physics research is still a fairly active field, but there have also been new applications; for example, plasma electron density measurements under experimental conditions that make it difficult to get Langmuir probe readings, such as plasma thrusters [11] and processing plasmas [14]. There is also a desire to understand antenna-plasma coupling better with the application of developing improved methods for plasma wave launching and detection in ionospheric and space environments [15]. Finally, there are basic plasma physics issues that can be uncovered with antenna impedance measurements [17].

It is such applications that motivate our current work with antenna impedance measurements. As most previous experimental results have been from sounding rocket applications, the data is limited to plasma conditions found in the ionosphere. There are far fewer controlled laboratory experiments where changes in the impedance curve can be tracked over a large range of frequencies. There are also only limited experimental results in strongly magnetized plasmas where the cyclotron frequency is greater than the plasma frequency ($\omega_{ce} > \omega_{pe}$). And we could find no experimental or theoretical results where the antenna size transitions from short, meaning significantly smaller than wavelengths expected in the plasma, to long, meaning the antenna is on the order of the plasma wavelength.

In the first part of this two part paper, we will examine antenna impedance under such variations in experimental parameters for an antenna with spherical geometry, which is a natural extension of previous experiments done in unmagnetized plasmas [13] [16]. In part II, we will present results from a simple dipole antenna.

II. THEORY

A sphere of radius R suspended in a cold, uniform, collisionless, unmagnetized plasma has an impedance given by [1]

$$Z(\omega) = \frac{1}{j\omega C_{sh}} + \frac{1}{j\omega 4\pi\epsilon_0\epsilon_p R} \quad (1)$$

with

$$\epsilon_p = 1 - \frac{\omega_{pe}^2}{\omega^2} \quad (2)$$

where ω_{pe} is the electron plasma frequency, and C_{sh} the sheath capacitance which for an ion sheath of thickness s can be approximated by

$$C_{sh} = 4\pi\epsilon_0 R \left(\frac{R+s}{s} \right) = C_0 \left(\frac{R+s}{s} \right), \quad (3)$$

defining $C_0 = 4\pi\epsilon_0 R$ as the vacuum capacitance of the sphere. Expression 1 has two resonant frequencies, known as the parallel resonance at $\omega = \omega_{pe}$, and the series resonance at

$$\omega = \omega_{sh} = \omega_{pe} \sqrt{\frac{s}{R+2s}} \quad (4)$$

With no magnetic field present, a good approximation for the ion sheath thickness s is $s \approx 5\lambda_d$, with λ_d the Debye length. When a magnetic field is introduced, two things happen: the sheath around the sphere may become deformed such that the expression for C_{sh} changes, and the expression for ϵ_p will be expression 2 in the direction parallel to the magnetic field, while perpendicular to the field we have

$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}, \quad (5)$$

with ω_{ce} the electron cyclotron frequency. In an early paper by Crawford [2], the argument was made that for smaller magnetic fields, such that the electron gyroradius ρ_e is large compared to the sheath thickness, no correction is necessary for the expression for the sheath capacitance C_{sh} . This condition is met for $\frac{\rho_e}{\lambda_d} = \frac{\omega_{pe}}{\omega_{ce}} \gg 1$. In our experiment we sometimes operate with $\omega_{ce} \geq \omega_{pe}$, but as a starting point will use the unmagnetized sheath resonance for comparison as well.

Substituting 5 into 1 changes the parallel resonance to the upper hybrid frequency $\omega_{uh}^2 = \omega_{pe}^2 + \omega_{ce}^2$, while the series resonance becomes

$$\omega_r^2 = \omega_{sh}^2 + \omega_{ce}^2 \quad (6)$$

where ω_{sh} is the unmagnetized series resonance. This result is only valid for a probe with only perpendicular electric fields with respect to the magnetic field, i.e., a planar probe. A spherical probe involves scaling the coordinate system such that the point charge potential is written as

$$V = \frac{q}{4\pi\epsilon_0\sqrt{\epsilon_p\epsilon_\perp x^2 + \epsilon_p\epsilon_\perp y^2 + \epsilon_\perp z^2}} \quad (7)$$

Equation 7 can then be integrated over the surface of a sphere to get a value for the impedance. This was done by Balmain[5], and the expression for impedance, including the sheath, is

$$Z = \frac{1}{j\omega C_{sh}} + \frac{1}{j\omega C_0\epsilon_\perp m} \ln\left(\frac{1+m}{1-m}\right) \quad (8)$$

with

$$m = \sqrt{1 - \frac{\epsilon_p}{\epsilon_\perp}}$$

III. EXPERIMENTAL ARRANGEMENT

The impedance probe used is a 2-cm diameter aluminum sphere mounted on a length of 0.25 in. 50- Ω semi rigid coax. Measurement of the sphere impedance has been described in detail in previous papers [13][16], and in this experiment little has been changed. The complex reflection coefficient Γ is measured with a network analyzer and the impedance calculated according to

$$Z = Z_0 \frac{1 + \Gamma}{1 - \Gamma} \quad (9)$$

where Z_0 is the analyzer internal impedance of 50 Ω . The difficulty in this measurement lies in eliminating the effects of the cabling and body of the probe from equation 9. This is done by building a matched load, an electrical short, and open termination and calibrating out the unwanted stray capacitance and cabling such that we get Γ values of 0, -1, and 1, respectively.

The probe is positioned near the center of the Space Physics Simulation Chamber plasma. The chamber is a 2-m diameter, 5-m long vacuum vessel surrounded by five 3-m diameter magnet coils. Improvements have been made to the plasma source in that the 1-m square filament array now consists of 72 shorter filaments connected in parallel to a power supply capable of supplying up to 300 A. Extra filaments were subsequently added around the

perimeter of this array to create as uniform a density profile as possible. Additional diagnostics are a heated Langmuir probe, an emissive probe, and magnetic probes, although the latter were not used in these measurements. The chamber is filled to a pressure of $p \approx 10^{-4}$ Torr Argon. A weakly ionized ($n \approx 10^6 - 10^{10}$) plasma is created by the heated filaments biased at -45 V with respect to a grounded grid. The magnetic field can be raised to a maximum value of 44 Gauss. Typical electron and ion temperatures are typically $T_e \approx 0.5$ eV and $T_i \approx 0.05$ eV. A drawing of the experimental setup is shown in figure 1.

IV. RESULTS

Figure 2 shows the magnitude and phase of the sphere impedance versus frequency as the ratio of ω_{pe}/ω_{ce} decreases from a factor of three to approximately 1/4. For comparison purposes, we have normalized the magnitudes to unity and normalized the frequency axes versus the respective values of ω_{pe} . Also shown are theoretical plots corresponding to equation 8. The conditions chosen here are for ω_{ce} comparable to ω_{pe} , with plasma density $1.2 \times 10^7 \text{ cm}^{-3}$. The effect of the magnetic field can be observed as a flattening of the magnitude and increase in phase at frequencies below the lower resonance. As the magnetic field is further increased, both the upper and lower resonances move to the right on the frequency axis due to the integration of ω_{ce} into both resonance frequencies. Figure 3 shows experimental and theoretical impedance characteristics at higher plasma density ($n \approx 5 \times 10^9 \text{ cm}^{-3}$). The smaller ratio of ω_{ce} to ω_{pe} results in a wider distance between the two resonances; however, the effects of the magnetic field are still apparent below the lower resonance. Again, there is very good agreement between measurements and calculations as the magnetic field is changed.

The sheath resonance frequency ω_{sh} can be calculated from the plasma density according to equation 4. The lower resonance found experimentally, normalized by this frequency, is plotted against ω_{ce} in figure 4. The excellent linear fit shows that the resonance frequency can be expressed as $\omega_r^2 = \omega_{sh}^2 + \kappa \omega_{ce}^2$, with $\kappa \approx 0.82$. This agrees with previous results [refs] in which the lower resonance was bounded by $\omega_{sh} < \omega_r < \omega_{sh}^2 + \omega_{ce}^2$. Thus, a simple substitution of $\epsilon_{\perp}$ for ϵ_p in equation 8 which gives the result $\kappa = 1$ is actually a fairly good approximation of the probe behavior.

The reflection coefficient Γ is plotted vs frequency for the various plasma conditions in

the experiment in figure 5. A sharp dip can be seen at the lower series resonance under all conditions including $\omega_r > \omega_{pe}$.

V. CONCLUSIONS

Experimental measurements of impedance for a spherical antenna in a magnetized plasma have been made over a large range of plasma conditions. Previous experiments have been limited to plasmas with $\omega_{sh} < \omega_{ce} < \omega_{pe}$; this experiment extends that work by considering larger magnetic fields ($\omega_{ce} > \omega_{pe}$). By doing this we were able to make observations regarding the nature of the probe impedance which heretofore have not been published.

It is observed that the low-frequency series resonance, referred to as the sheath resonance for the unmagnetized plasma, becomes a hybrid sheath resonance with frequency $\omega_r^2 = \omega_{sh}^2 + \kappa\omega_{ce}^2$. The parallel resonance which for the unmagnetized plasma occurred at ω_{pe} is shifted to the upper hybrid frequency ω_{uh} . With a sufficiently strong magnetic field, both resonances can actually be higher than the plasma frequency. The power deposition into the plasma is still maximum at this lower resonance regardless of where it occurs. Thus, while the probe in the unmagnetized plasma always had maximum power into the plasma below the plasma frequency, the magnetized plasma allows for this to occur at any frequency between the unmagnetized series sheath resonance and the upper hybrid frequency, including $\omega > \omega_{pe}$. Also, we can see that using the unmagnetized sheath capacitance in the expression for impedance is a good approximation even when $\omega_{ce} \geq \omega_{pe}$.

These results are directly applicable to plasma conditions found at higher altitudes in the ionosphere, and also in high density, high magnetic field processing discharges such as helicons.

-
- [1] Harp RS, Crawford FW, *Journal Of Applied Physics* **35** (12): 3436 (1964)
 - [2] Crawford FW, *J. Appl. Phys* **36** (10) 3142, (1965)
 - [3] Dote T, Ichimiya T, *Journal of Applied Physics* **36** (6): 1866 (1965)
 - [4] Oya H, Obayashi T, *Report of Ionosphere and Space Research in Japan* **20** (2): 199 (1966)
 - [5] Balmain, K. *IEEE Transactions on Antennas and Propagation* vol.AP-14, no.3 : 402 (1966)
 - [6] Uramoto J *Physics Of Fluids* **13** (3): 657 (1970)

- [7] T. H. Y. Yeung and J. Sayers, *Proc. Phys. Soc. London*, Sect. B 70, 663 (1957)
- [8] K. Takayama, H. Ikegami, and S. Miyazaki. *Phys. Rev. Lett.*, **5**, 238 (1960)
- [9] Jensen MD, Baker KD, *Journal of Spacecraft and Rockets* **29** (1): 91 (1992)
- [10] Steigies CT, Block D, Hirt M, Hipp B, Piel A, Grygoczuk J, *Journal of Physics D* **33** (4): 405 (2000)
- [11] Bilen, S.G., Haas, J.M., Gulczinski, F.S. Gallimore, A.D., Letoutchaia, J.N., *AIAA-99-2714*, 35th Joint Propulsion Conference, Los Angeles, CA, June 1999.
- [12] Kim J, Seong D, Lim J, Chung K, *Applied Physics Letters* **83** (23): 4725 (2003)
- [13] Blackwell,D.D., Walker, D.N., Amatucci, W.E., *Review of Scientific Instruments* vol.76, no.2 : 23503-1-6 , (2005)
- [14] Kokura H, Nakamura K, Ghanashev IP, Sugai H, *Japanese Journal of Applied Physics* **38** (9A): 5262 (1999)
- [15] Ward J., Swenson C., Furse C., *IEEE Transactions on Antennas and Propagation* vol.53, no.8, pt.2 : 2711-18, Aug. 2005
- [16] Blackwell, D.D., Walker, D.N., Messer, S.J., Amatucci, W.E. *Physics of Plasmas* vol.12, no.9 : 93510-1-7, (2005)
- [17] Walker, D.N.; Fernsler, R.F.; Blackwell, D.D.; et al. *Physics of Plasmas* vol.13, no.3 : 32108-1-9, March 2006
- [18] Ku VPT, Annaratone BM, Allen JE, *Journal of Applied Physics* **84** (12): 6536 (1998)
- [19] Amatucci et. al., *Journal of Geophysical Research* **103** (A6): 11711 (1998)

LIST OF FIGURES

Figure 1: Sketch of the Space Plasma Simulation Chamber used for the antenna impedance measurements.

Figure 2: Experimental (solid lines) and theoretical (dashed lines) plots of (a) magnitude and (b) phase of the impedance of a 2 cm sphere in a plasma vs frequency with electron density $n \approx 1.5 \times 10^7 \text{cm}^{-3}$ as the magnetic field is increased.

Figure 3: Experimental (solid lines) and theoretical (dashed lines) plots of (a) magnitude and (b) phase in a plasma vs frequency with electron density $n \approx 5 \times 10^8 \text{cm}^{-3}$ as the magnetic field is increased.

Figure 4: The lower resonance ω_r taken from the data in figures 2 and 3. The quantity $\frac{\omega_r^2}{\omega_{sh}^2}$ is plotted vs $\frac{\omega_{ce}^2}{\omega_{sh}^2}$ to demonstrate the linear relationship according to $\omega_r^2 = \omega_{sh}^2 + \kappa\omega_{ce}^2$. The quantity ω_{sh} is the sheath resonance frequency calculated according to equation 4 which does not include magnetic field effects.

Figure 5: Reflection coefficient of the spherical antenna vs frequency under the lower density ($n \approx 5 \times 10^8 \text{cm}^{-3}$) condition with increasing magnetic field showing a minimum at the lower resonance frequency ω_r . This minimum corresponds to a maximum energy deposition into the plasma.

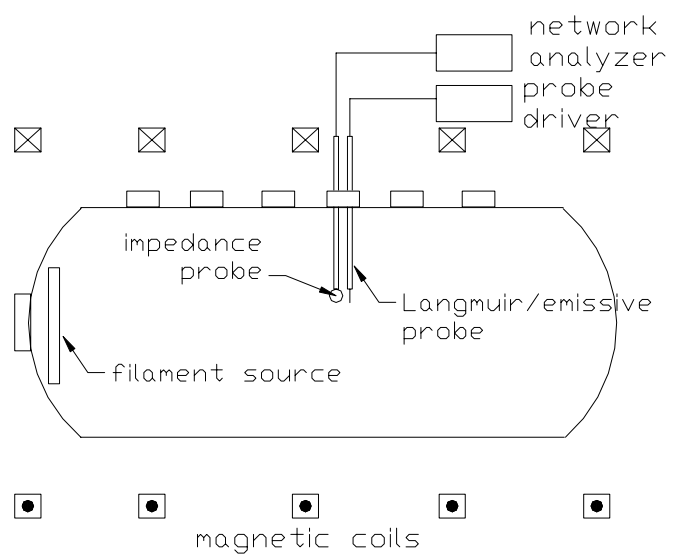


figure 1

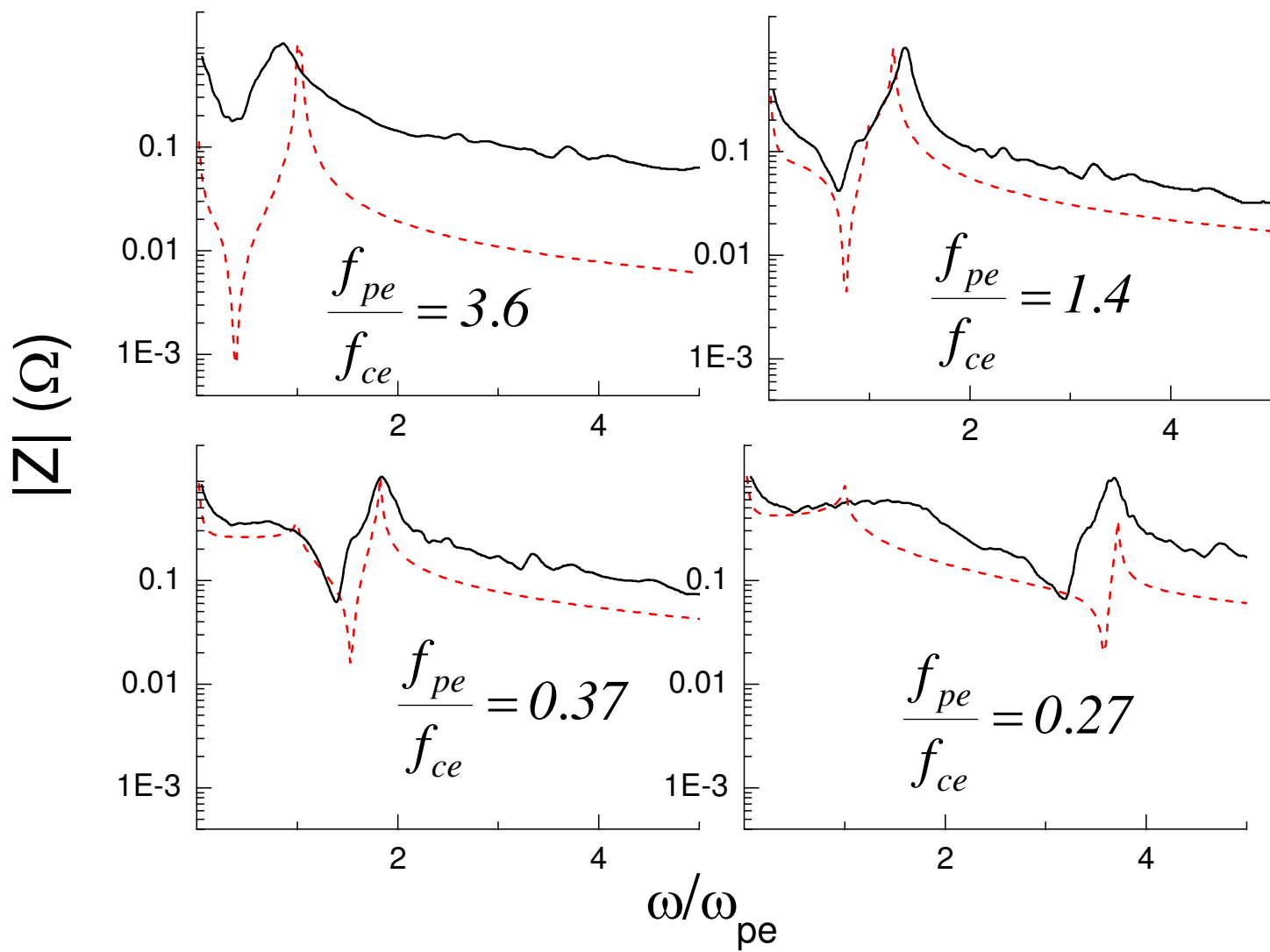


figure 2 (a)

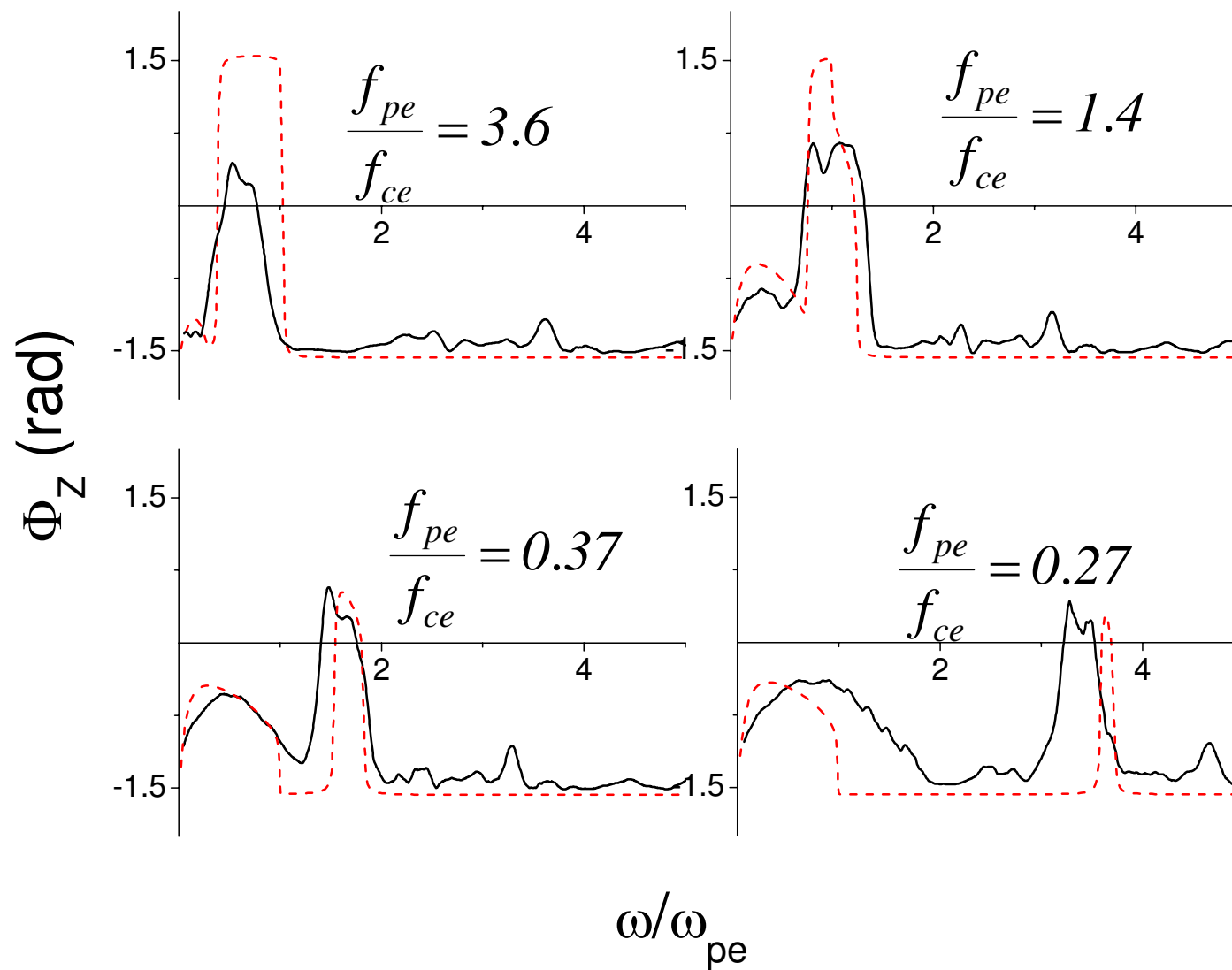


figure 2 (b)

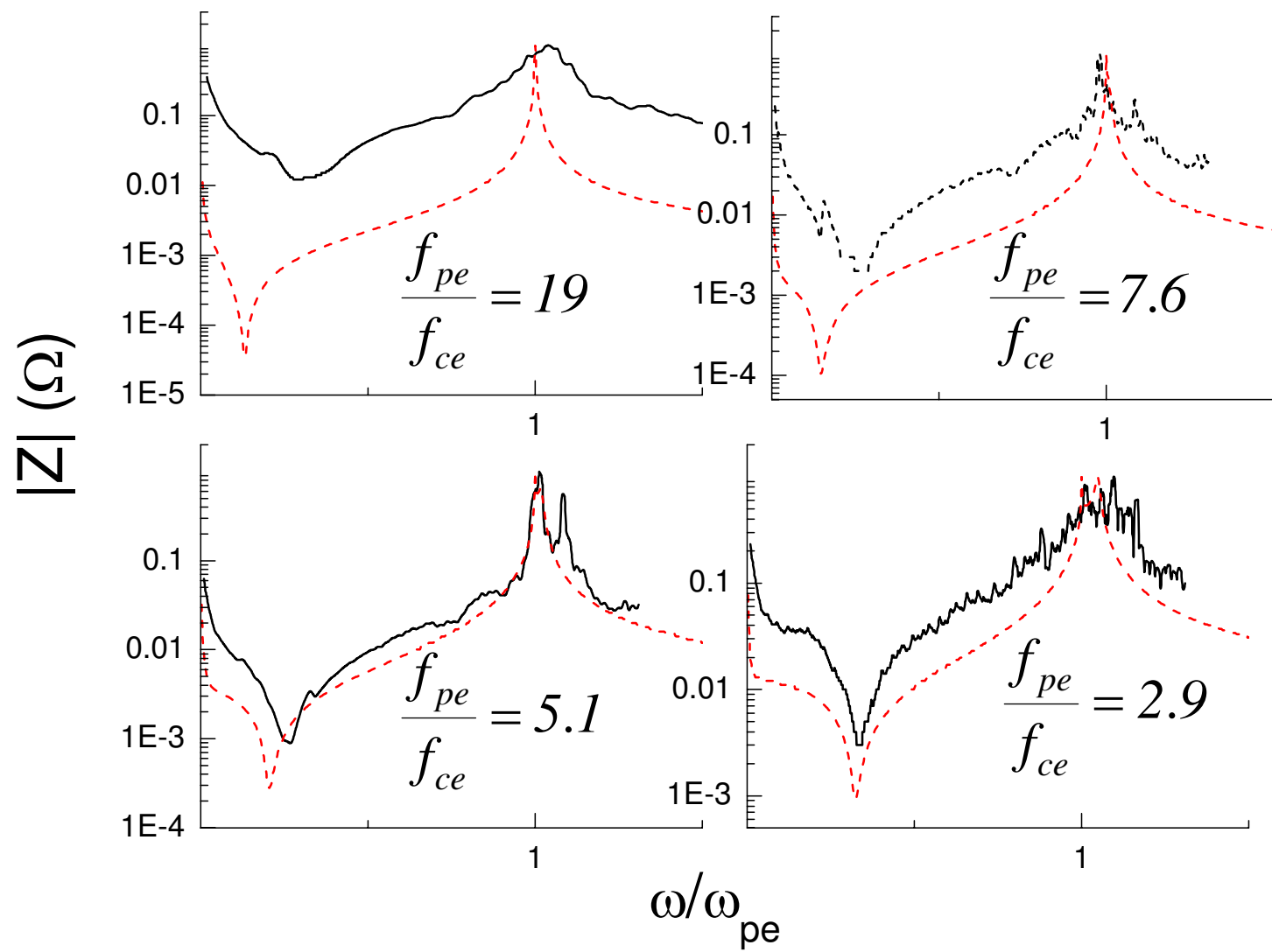


figure 3 (a)

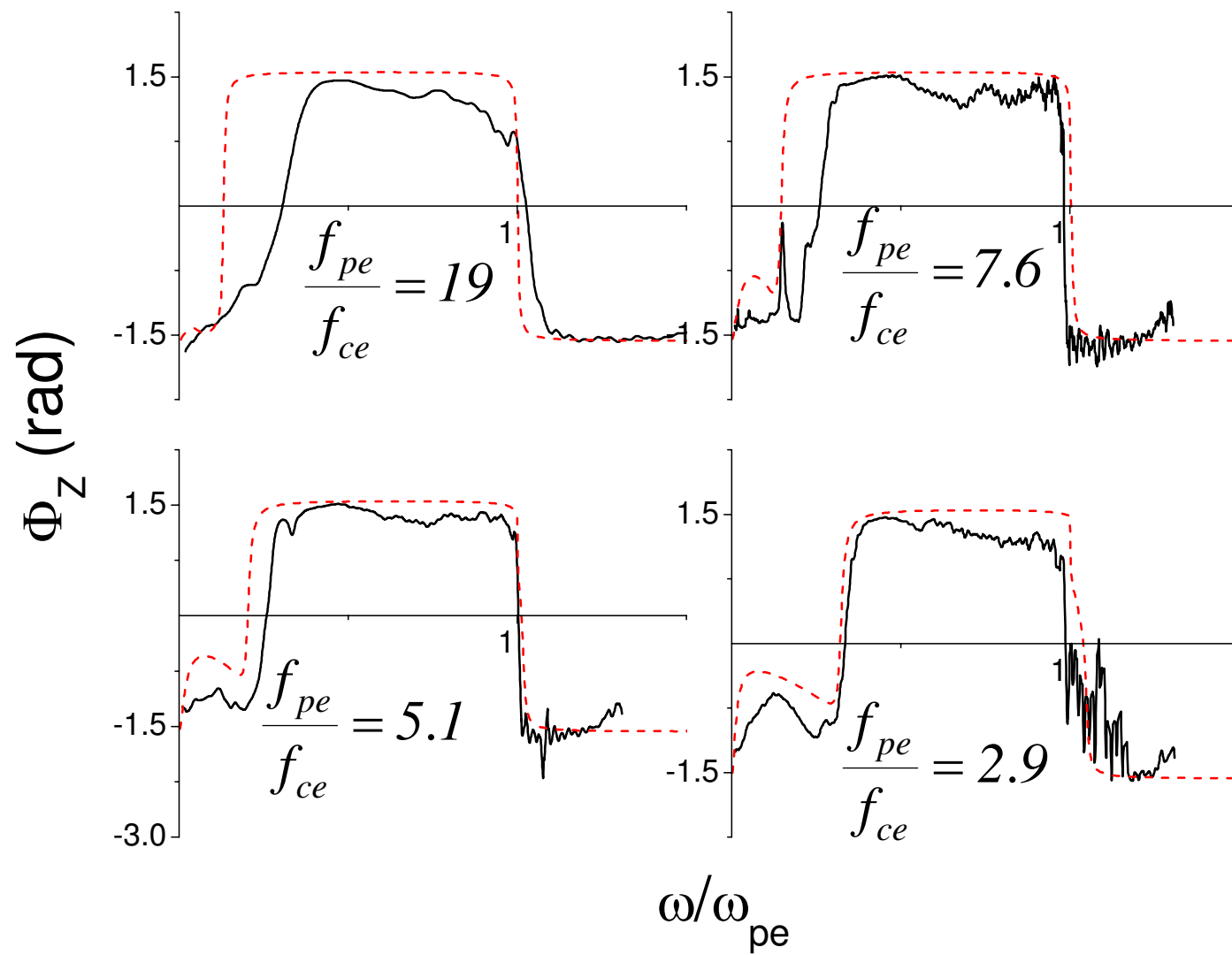


figure 3 (b)

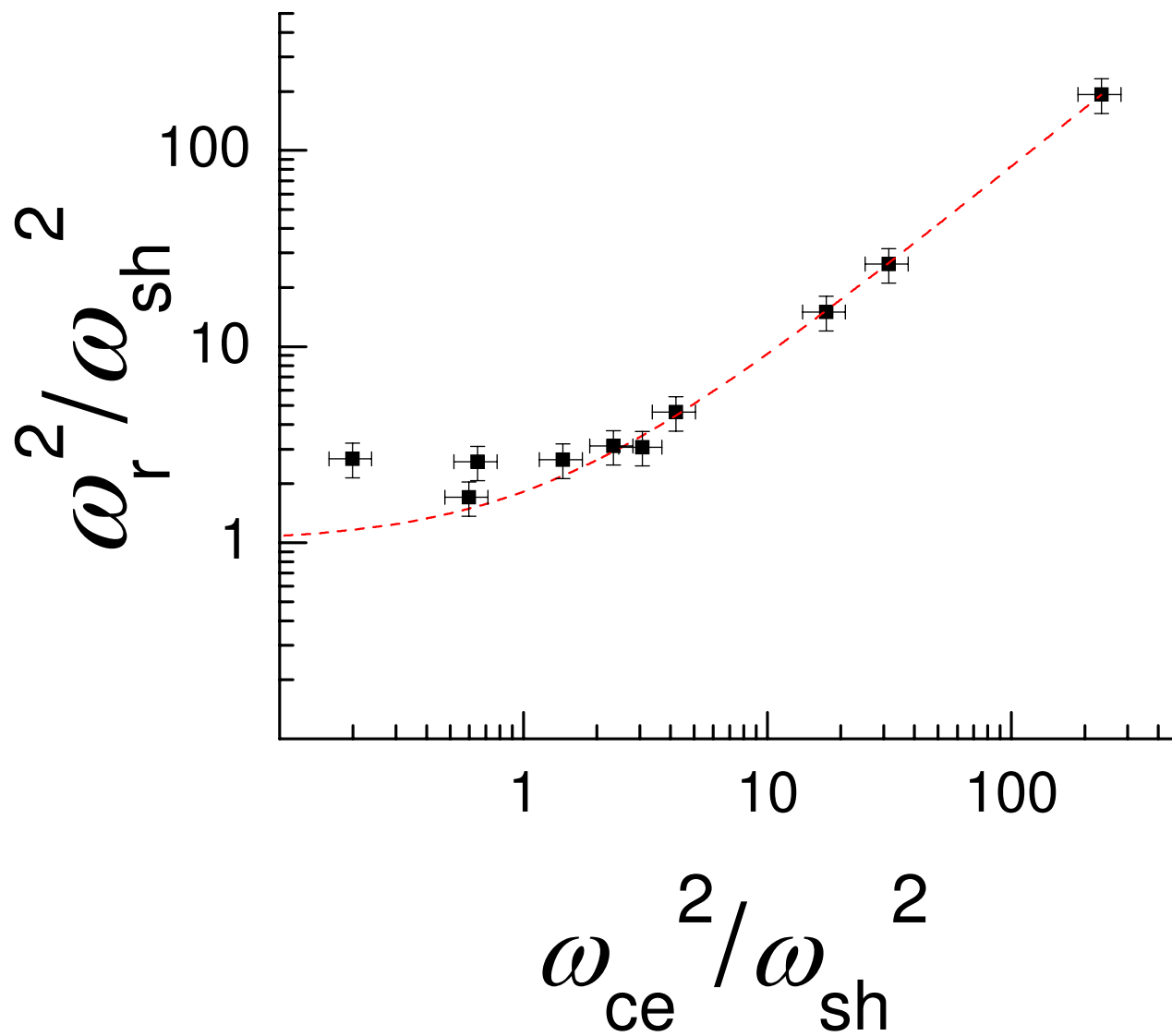


figure 4

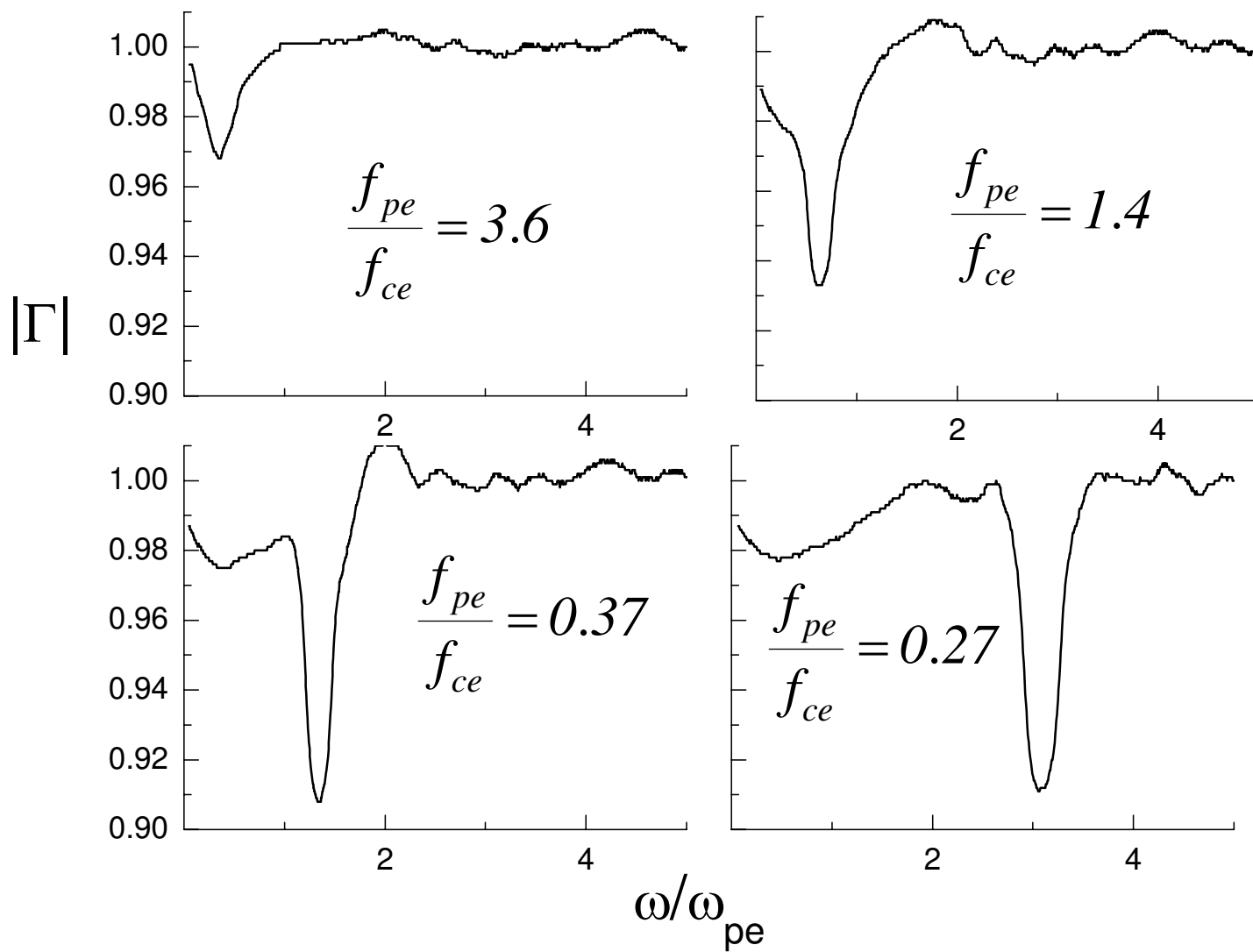


figure 5