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OPTIMAL STATIONING OF RADAR PICKETS AND ANTI-BALLISTIC MISSILE DEFENDERS FOR LONG RANGE SURVEILLANCE AND TRACKING (LRS&T) AND BALLISTIC MISSILE DEFENSE (BMD) OPERATIONS

by

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September 2006

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We describe major enhancements to the missile defense planning aid "JOINT DEFENDER" (JDEF). JDEF is the first system that shows how to evaluate and exploit new and anticipated improvements in interceptors, long-range surveillance and tracking capabilities, networked communications, and the ability of detecting platforms to cue intercepting ones downrange. We want to improve system-wide effectiveness, gauged here by the reduction of expected damage inflicted. We defend an asset list (DAL) of targets, characterized by their locations and values to us. Our defenders include pure "LOOKERs," radars and sensors of enemy missile launches, and "SHOOTERs," platforms with means to both detect and intercept enemy launches. JDEF optimally positions platforms that can be moved, and prescribes what each platform should do. JDEF can estimate the value to either opponent of secrecy, deception, or intelligence. JDEF is the only missile defense planning system using formal optimization. Among many advantages this conveys, JDEF is able to unambiguously quantify the difference among disparate plans. Although the JDEF planner can manually control any detail, the planner is well advised to let optimization suggest where to start.

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TABLE OF CONTENTS

LIST OF FIGURES

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LIST OF TABLES

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LIST OF ACRONYMS AND ABBREVIATIONS

AADC	Area Air Defense Commander's model
ABM	Anti-Ballistic Missile
ABL	Airborne Laser
APL	Applied Physics Laboratory
AEGIS	Not an acronym, rather it is a reference to "The Shield of Zeus" or "Shield of the Gods" in Greek Mythology, now a U.S. Navy shipboard weapon system.
ASCII	American Standard Code for Interchange of Information
CG	Guided Missile Cruiser
CPLEX	A large-scale mathematical solver, registered trademark of ILOG CPLEX corporation.
CR/DR	Cross-range, down-range table
DAL	Defended Asset List
DDG	Guided Missile Destroyer
GAMS	General Algebraic Modeling System, a registered trademark of GAMS Development Corporation
GBI	Ground Based Interceptor
GUI	Graphic User Interface
JDEF	JOINT DEFENDER
JTF	Joint Task Force
LOOKER	Refers to a Theater Ballistic Missile Defense asset with the capability to detect an attack launch.
LRS&T	Long Range Search and Track
NSPD	National Security Presidential Directive
NWDC	Naval Warfare Development Center
PAC-3	Patriot Advanced Capability
RF	Radio Frequency
SHOOTER	Refers to a Theater Ballistic Missile Defense asset with the capability to organically intercept an attack launch
SM-2	Standard Missile 2
SM-3	Standard Missile 3
THAAD	Theater High Altitude Area Defense

TBM	Theater Ballistic Missile
TBMD	Theater Ballistic Missile Defense
VBASIC	Visual Basic ©, a registered trademark of Microsoft Corporation

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EXECUTIVE SUMMARY

As of this writing, at least 12 countries either possess Theater Ballistic Missiles (TBM's) or are assessed to be actively pursuing technology to acquire such weapons. These weapons may be armed with high explosive, chemical, biological, nuclear, or special-purpose warheads. National Security Presidential Directive 23 (NSPD-23) orders the Secretary of Defense and the Chairman of the Joint Chiefs of Staff to develop and maintain a ballistic missile defensive capability, and we are developing a global, integrated weapon and sensor network to defeat every variant of ballistic missile in flight.

The United States has not pursued an active Anti-Ballistic Missile (ABM) capability since signing the Strategic Arms Limitations Treaty I (SALT I) and Anti-Ballistic Missile (ABM) treaties in the early 1970's, so it must develop Theater Ballistic Missile Defense (TBMD) systems anew. TBMD weapons systems in procurement include the Airborne Laser (ABL), Ground-Based Interceptor (GBI), AEGIS Standard Missile 3 (SM-3), Patriot Advanced Capability 3 (PAC-3), and Theater High Altitude Area Defense (THAAD). GBI is a static system intended for initial deployment to Air Force Bases in Alaska, California and in central Europe. The rest of these weapons are mobile, intended for rapid deployment to areas of potential conflict. Detection systems include static land-based phased-array search radars, deployable X-band radar afloat, and Navy SPY-1 radars associated with the AEGIS weapons system.

The development of a defensive system of this complexity and scale requires completely new tactics, techniques, and procedures. The individual weapons and sensors each have unique capabilities and limitations, and as an integrated defensive system, there are myriad tactical memoranda available for their employment. There are tactical aids to optimally station a single defensive unit for best success against single or multiple threats. What about optimal, coordinated stationing of multiple defensive assets? Our platforms are limited in number and we need to take advantage of mutual support, maximized probabilities of attack nullification, and best use of tactical data links for exchange of targeting data.

This thesis addresses the optimal, coordinated stationing of multiple defensive assets of a variety of types in order to maximize their joint effectiveness. We extend an existing planning system "JOINT DEFENDER" (hereafter JDEF). JDEF was first proposed in the March, 2004, Naval Postgraduate School thesis by LT Douglas Diehl "How to Optimize Joint Theater Ballistic Missile Defense" and further documented in the September, 2005, Operations Research journal article "A Two-Sided Optimization for Theater Ballistic Missile Defense," by Gerald Brown and others. The first chapter here summarizes this previous work.

JDEF is not the only decision aid available to mission planners for stationing defensive assets; others in use include the Area-Air Defense Commander system (AADC) and the Theater Battle Management Core System (TBMCS). AADC employs a greedy, myopic heuristic that protects the defended assets in strict hierarchical order from most to least valuable, locating the best first defensive platform and then the second, and so forth, until the most important defended asset is protected to an acceptable level, then turning to the next defended asset, and so on. There are only a few AADC super-computer clusters available for planning, and funding for this program has been curtailed. TBMCS displays a planner-provided defensive positioning overlaid by a set of preprogrammed launch fans with an output suggesting the relative validity of the plan. TBMCS computers are more generally available, for instance in each air operations center worldwide. Neither of these tools uses formal optimization, and thus neither provides any qualitative assessment of the overall quality of the plan suggested; hence the planner will never know if some other, significantly improved plan remains to be discovered.

We have added cross-range, down-range intercept tables to JDEF identical to those used by other planning systems to estimate interceptor performance. For any spherical coordinate engagement triangle defined by launch, target, and interceptor location, JDEF interpolates these tables to determine the single-shot kill probability for an interceptor fired at an attacking missile.

We have added radar equations and a library of missile radar cross sections to JDEF, so we can emulate, for instance, the AEGIS mission planner.

Now that JDEF can determine what can be detected by each radar from each position, we can ask JDEF to optimally position LOOKER platforms (i.e., pickets) to detect, track, and cue launches for downrange SHOOTER platforms to intercept.

We present and motivate a mathematical justification for the influence a cue has on the kill chain leading to a successful intercept.

We then show how JDEF optimally positions LOOKERs to surveil potential launch locations, how these LOOKERs cue SHOOTERs downrange to increase the probability of attack nullification, and how all this can be optimized theater-wide in a unified fashion.

We introduce a graphical user interface for JDEF with which the planner can control every detail, every nuance of a theater scenario. Or, the planner can let JDEF optimally advise positioning of defenders, and how to engage attacks, in order to minimize expected damage to our defended asset list.

Two scenarios illustrate how JDEF works. One, a trivial example, shows how all the LOOKER and SHOOTER features interact to prescribe an optimized, unified defense plan. The other example is a large-scale, theater-wide attack that demonstrates how JDEF not only suggests where we should position our assets and how we should employ them, but also lends insight about our relative strengths and weaknesses, and what we can do about them.

JDEF offers three alternate flavors of planning. In a “surprise” scenario, the enemy launches a set of attacks optimized to achieve maximum expected target damage to our defended asset list. We then mount an optimized defense against this optimal attack, seeking to minimize this expected damage. A “greedy” scenario shows how every enemy launch site can attack every target on our defended asset list. This mathematically derives “launch fans” representing every enemy course of action. Such launch fans are used by some contemporary planning methods to plan defenses, so JDEF emulates this by placing defending LOOKERs and SHOOTERs in most-advantaged positions to defend from as much expected target damage as possible. Finally, a “two-sided” scenario evaluates a conservative case, and a worrisome one, where the enemy can see all our defensive preparations *before* launching his attacks. JDEF is the only planning tool today that can advise optimal defensive actions in such a disadvantaged situation.

We show how a LOOKER cue can be used to represent new technologies for fast detection, tracking, and automated engagement of missile attacks.

Finally, we show how JDEF can demonstrate the value of secrecy and deception to the defenders.

JDEF is implemented using Microsoft Excel © and runs on a standard WINTEL laptop. The planner is presented with a geographic display of the area under consideration and can manually control any feature via the Excel interface. However, the planner is well-advised to follow the defaults JDEF offers, and to let JDEF optimally advise LOOKER and SHOOTER platform positioning and intercept engagements. Although there are internal optimization modules, in particular, General Algebraic Modeling System (GAMS) and one or more licensed solvers --- there is absolutely no exposed mathematical detail at all: The planner needs to know and understand missile defense, not mathematical modeling of missile defense.

I. INTRODUCTION

As of this writing, at least 12 countries either possess Theater Ballistic Missiles (TBM's) or are assessed to be actively pursuing technology to acquire such weapons [Gorwitz 2005]. These weapons may be armed with high explosive, chemical, biological, nuclear, or special-purpose warheads. National Security Presidential Directive 23 (NSPD-23) orders the Secretary of Defense and the Chairman of the Joint Chiefs of Staff to develop and maintain a ballistic missile defensive capability [Bush 2002], and we are developing a global, integrated weapon and sensor network to defeat every variant of ballistic missile in flight.

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This thesis addresses the optimal, coordinated stationing of multiple defensive assets of a variety of types in order to maximize their joint effectiveness. We extend an existing planning system “JOINT DEFENDER” (hereafter JDEF) introduced by Diehl [2004] and described by Brown, et. al [2005]. The first chapter here summarizes this previous work.

JDEF is not the only decision aid available to mission planners for stationing defensive assets; others in use include the Area-Air Defense Commander system (AADC) and the Theater Battle Management Core System (TBMCS). AADC employs a greedy, myopic heuristic that protects the defended assets in strict hierarchical order from most to least valuable, locating the best first defensive platform, then the second, and so forth, until the most important defended asset is protected to an acceptable level, then turning to the next defended asset, and so on. There are only a few AADC super-computer clusters available for planning, and funding for this program has been curtailed. TBMCS displays a planner-provided defensive positioning overlaid by a set of preprogrammed launch fans with an output suggesting the relative validity of the plan. TBMCS computers are more generally available, for instance in each air operations center worldwide. Neither of these tools uses formal optimization, and thus neither provides any qualitative assessment of the overall quality of the plan suggested; hence the planner will never know if some other, significantly improved plan remains to be discovered.

In the sections that follow, we show the mathematical modeling underlying JDEF, a graphical user interface that makes JDEF easy to use by a planner, not just an analyst, new features added to represent radar, and embellishments to the optimization that position LOOKER platforms (i.e., pickets) to detect, track, and cue launches for downrange SHOOTER platforms to intercept. We demonstrate with two case studies, and show how these can be used to gain insight about the planning problem, and new technology proposed to enhance our capabilities in theater ballistic missile defense.

II. THE JDEF MODEL

A. JDEF MATHEMATICAL DEVELOPMENT

JDEF is based on a mathematical, two-sided, attacker-defender optimization model that seeks to minimize the maximum damage an intelligent enemy can achieve (Diehl 2004, Brown et al. 2005). JDEF assumes prior knowledge of potential enemy launch positions, a specific defended asset list (DAL hereafter), and knowledge of discrete locations where defending platforms may be positioned. Defending platforms currently include the Navy's AEGIS-capable ships, Army Patriot Advanced Capability (PAC-3) batteries, and ground-based interceptors. Addition or deletion of additional sensors and weapons systems is trivial.

The attacker controls a set of launch sites, and possesses $\overline{fixed}_{m,s}$ missiles of type $m \in M$ pre-positioned at site *s*, as well as a pool of $\overline{mobile}_m$ missiles that can be transported to any capable receiving launch site. Transport of the mobile missiles may be limited by $\underline{move}_{m,s}, \overline{move}_{m,s}$. Launch site *s* can launch no more than $\overline{fixed}_{m,s}$ + $\overline{mobile}_m$ missiles of type *m*, and during any planning epoch, can launch at most $\overline{launches}_{m,s}$. We assume an intelligent defender who knows which sites *s* can accept and launch missiles of type *m*. The defender guards a set of targets $t \in T$, with each target *t* having value $value_t$. An attack $a \in A$ consists of a launch form site $s_a \in S$ of a missile of type $m_a \in M$ at a target $t_a \in T$. This attack will hit the target with probability Pk_a, assuming the defender takes no action; Pk_a is the probability of kill for attack *a*. An upper bound $\overline{missiles}_t$ may be placed on the number of missiles the attacker will launch at target *t*. The attacker must decide which missiles to launch at which targets to maximize total expected target damage, weighted by target value $value_t$.

The defender controls a set of defending platforms $p \in P$, each of which is in a platform class $c_p \in C$. Each platform of class *c* can be pre-positioned at any one location $g \in G_c \subseteq G$. Each platform p carries loadout $loadout_{p,i}$ defensive interceptors of type $i \in I$. An attack *a* can be engaged with alternative defensive actions $d \in D$, where defense *d* launches $salvo_{a,c,d,i}$ interceptors of type(s) *i* and succeeds in thwarting the attack with probability of negation $Pn_{a,c,g,d}$. Each defensive engagement is conditional. If attack *a* is not launched, then any interceptors devoted to its engagement are not launched.

The defender wishes to optimize defensive pre-positioning for attack interception *while assuming the attacker will observe these preparations and optimize attacks to exploit any weaknesses observed in these defenses.* The defender's objective is to minimize the maximum total damage to targets. We note that this model is a conservative one for the defender because the defense is planned against the worst possible set of attacks. JDEF is also conservative for the attacker, because it assumes the best possible defense. However, variants of the model we describe later enable analysis of a range of situations for either opponent, from conservative to optimistic.

B. FORMULATION

The mathematical formulation of JDEF follows:

1. Indices and Index Sets

Attacker:

$m \in M$	attacking missile types
$s \in S$	attacker launch sites
$t \in T$	targets, ("defended assets")
$a \in A$	attack launching a missile at a target
$a \in A_{m,s} \subseteq A$	attacks launching a missile of type *m* from site *s*
$a \in A_t \subseteq A$	attacks *a* with target *t*
s_a	launch site of attack a, $s_a \in S$

m_a	missile type launched in attack a, $m_a \in M$
t_a	target of attack a, $t_a \in T$

Defender:

$p \in P$	defending platforms
$c \in C$	defending platform classes
c_p	class of platform p, $c_p \in P$
$g \in G$	candidate stationing positions for a defending platform
$g \in G_c \subseteq G$	candidate stationing location for a defending platform of class c
$i \in I$	defensive interceptor types
$d \in D$	defense options

2. Data [units]

Attacker:

$\overline{fixed}_{m,s}$	attacker’s total supply of stationary type m missiles at launch site s
$\overline{mobile}_m$	attacker’s total supply of mobile missile type m
$\underline{move}_{m,s}, \overline{move}_{m,s}$	minimum and maximum number of mobile missile type m that attacker can transport to launch site s
$\overline{launches}_{m,s}$	maximum launches of missile type m from launch site s
$\overline{missiles}_t$	maximum number of missiles that can attack target t
$value_t$	value of target t
Pk_a	probability that attack a hits target t

Defender:

$loadout_{p,i}$	type i interceptors carried by platform p
$salvo_{a,c,d,i}$	number of type i interceptors used against attack a by class c platform exercising defense option d

$\overline{engage}_p$	maximum number of engagements platform *p* can manage in a particular time epoch
$\overline{shoot}_p$	maximum number of interceptors platform p can shoot in a short period of time
$Pn_{a,c,g,d}$	probability that attack *a* would is negated if platform p, class c_p ,in position $g \in G_{c_p}$ exercises defense option *d*, e.g., probability of negation

3. Variables [units]

Attacker:

$W_{m,s}$	type *m* missiles transported to launch site *s*
$V_{m,s}$	total of stationary and mobile type *m* missiles available at launch site *s*
Y_a	1 if attack *a* is conducted, 0 otherwise (Y the vector of attacks is an “Attack Plan”)

Defender:

$X_{p,g}$	1 if platform *p* located at *g*, 0 otherwise
$R_{a,p,g,d}$	1 if attack *a* is engaged by platform *p* from position $g \in G_{c_p}$, exercising defense option *d*, 0 otherwise

C. FORMULATION OF JDEF MIN-MAX (JD-MINMAX)

$$Z^* = \min_{(X,R)\in XR} \left\{ \begin{array}{lllllr} \max\limits_{Y} & \sum\limits_t value_t \sum\limits_{a\in A_t} Pk_a \left(1-\sum\limits_{p,g,d} Pn_{a,p,g,d}\right) Y_a & & & & \text{(A0)} \\ s.t. & \sum\limits_s W_{m,s} & \le \overline{mobile}_m & \forall m & [\alpha_m] & \text{(A1)} \\ & -W_{m,s}+V_{m,s} & \le \overline{\text{fixed}}_{m,s} & \forall m,s & [\beta_{m,s}] & \text{(A2)} \\ & -V_{m,s}+\sum\limits_{a\in A_{m,}} Y_a & \le 0 & \forall m,s & [\gamma_{m,s}] & \text{(A3)} \\ & \sum\limits_{a\in A_t} Y_a & \le \overline{missiles}_t & \forall t,s & [\delta_{\text{t}}] & \text{(A4)} \\ & W_{m,s} & \in \{\underline{move}_{m,s},\ldots,\overline{move}_{m,s}\} & \forall m,s & [-\underline{\pi}_{m,s},\overline{\pi}_{m,s}] & \text{(A5)} \\ & \text{V}_{\text{m,s}} & \in \{0,\ldots,\overline{launches}_{m,s}\} & \forall m,s & [\rho_{\text{m,s}}] & \text{(A6)} \\ & Y_a & \in \{0,1\} & \forall a & [\theta_a] & \text{(A7)} \end{array} \right\}$$

$(X,R)\in XR$, which we describe in detail below, denotes all feasible pre-positioning and intercept preparations for the defender.

The attacker's objective (A0) expresses total expected target damage, assuming a cumulative effect for multiple missiles. Constraints (A1) limit the number of mobile missiles of each type that can be transported to launch sites. Constraints (A3) limit the number of missiles that can be launched from each launch site. Constraints (A4) limit the number of missiles that can attack each target. Bounds (A5) limit the number of mobile missiles of each type that can be transported to each launch site, (A6) the maximum number of missiles that can be launched in some limiting planning epoch, and (A7) stipulates binary launch decisions.

The objective (A0) expresses expected incremental target value damage inflicted as a consequence of each attacking missile. For an area target, such as a city or airfield, such a cumulative damage model is standard [e.g. Eckler and Burr 1972]. But a point target might be destroyed by any single attacking missile, and the lack of a joint probability expression for surviving more than one shot means that the attacker can "over-credited" with damage value. (This problem disappears if the attacker can launch no more that one missile at any target, which can be enforced through constraints (A4).) We believe that when it comes to weapons of mass destruction carried by TBMs, the damage to an economy and a society will continue to increase as the number of successful missile strikes increases. Thus, the cumulative damage model is appropriate, although there might be some diminishing returns to an attacker as the number of successful strikes on a target (in a target area) increases.

The defender's actions are limited by $(X,R) \in XR$, where XR is defined by the following set of constraints:

$$\sum_{g} X_{p,g} \le 1 \qquad \forall p \tag{D1}$$

$$\sum_{p} X_{p,g} \le 1 \qquad \forall g \tag{D2}$$

$$\sum_{\text{p,g,d}} R_{a,p,g,d} \le 1 \qquad \forall a \tag{D3}$$

$$\sum_{a,d} salvo_{a,c_p,d,i} R_{a,p,g,d} \le loadout_{p,i} X_{p,g} \qquad \forall i,g,p \in G_{c,p} \tag{D4}$$

$$\sum_{a,g,d} R_{a,p,g,d} \le \overline{engage}_{p} \qquad \forall p \tag{D5}$$

$$\sum_{a,g,d} salvo_{a,c_p,d,i} R_{a,p,g,d} \le \overline{shoot}_{p} \qquad \forall p \tag{D6}$$

$$\sum_{d} R_{a,p,g,d} \le X_{p,g} \qquad \forall a,p,g \tag{D7}$$

$$\text{all } X_{p,g} R_{a,p,g,d} \in \{0,1\} \tag{D8}$$

Each (D1) limits a platform to occupy at most one grid position, each constraint (D2) (*optionally*) limits a grid position to accommodate at most one platform, each constraint (D3) allows at most one interception of each attack, each constraint (D4) limits the number of interceptor engagements from each positioned platform and grid-point combination, each constraint (D5) (*optionally*) limits the total number of engagements that a platform can conduct in one planning epoch (a discrete time period), each constraint (D6) (*optionally*) limits the total number of interceptors that a platform can shoot in a short period of time, each constraint (D7) permits an engagement only from an occupied platform and grid-point combination, and constraints (D8) required binary decisions. Note that constraints (D3) do not require a response for every attack. Indeed, if defenses are overwhelmed, it may be impossible to intercept every attack, and we must allow for this eventuality.

The attacker plans to maximize expected damage, the defender plans to minimize the attackers maximum expected damage.

D. SOLVING JD-MINMAX WITH JD INTEGER LINEAR PROGRAM (JD-ILP)

Direct solution of a minimize-maximize integer linear program like JD-MINMAX is impossible with standard optimization software. We are fortunate in this case however, because; although the attacker's decision vectors **W** and **V** are integer, and **Y** is binary, the constraint matrix involving W, V, and Y is totally unimodular [Ahuja et al 1993]. All right-hand side data are integer, thus all solutions the linear-programming relaxation of the attacker's maximizing problem are intrinsically integer. Therefore, we replace constraints $W_{m,s} \in \{\underline{move}_{m,s},...,\overline{move}_{m,s}\}\ \forall_{\mathrm{m,s}}$ with $\underline{move}_{m,s} \le W_{m,s} \le \overline{move}_{m,s}, \forall_{m,s}$, replace $0 \le \mathrm{V}_{\mathrm{m,s}} \le \overline{launches}_{m,s}\ \forall_{\mathrm{m,s}}$, and replace binary constraints $Y_a \in \{0,1\}, \forall_a$ with $0 \le Y_a \le 1\}, \forall_a$, to create an inner maximization that is a linear program. We then define dual variables for that linear program, take the dual of that inner linear maximization to create a "minimize-minimize" problem, which is just a minimizing Integer-Linear Program (ILP) and solve that ILP using standard optimization software. This ILP is:

JD-ILP

$$\min_{\alpha,\beta,\gamma,\delta,\pi,\rho,\theta,X,R} \sum_{m} \overline{mobile}_m \alpha_m + \sum_{m,s,} \overline{fixed}_{m,s} \beta_{m,s} + \sum_{t} \overline{missiles}_t \delta_t$$

$$- \sum_{m,s} \underline{move}_{m,s} \underline{\pi}_{m,s} + \sum_{m,s} \overline{move}_{m,s} \overline{\pi}_{m,s} + \sum_{m,s} \overline{launches}_{m,s} \rho_{m,s} + \sum_{a} \theta_a \qquad \text{(T0)}$$

s.t.

$$\alpha_m - \beta_{m,s} - \underline{\pi}_{m,s} + \overline{\pi}_{m,s} \geq 0 \qquad \forall m,s \qquad \text{(T1)}$$

$$\beta_{\mathrm{m,s}} - \gamma_{m,s} + \rho_{m,s} \geq 0 \qquad \forall m,s \qquad \text{(T2)}$$

$$\gamma_{m_a,s_a} + \delta_{t_a} + \theta_a + \sum_{p,g,d} Pk_a value_{t_a} Pn_{a,c_p,g,d} R_{a,p,g,d} \geq Pk_a value_{t_a} \qquad \forall a \qquad \text{(T3)}$$

$$\sum_{g} X_{p,g} \leq 1 \qquad \forall p \qquad \text{(T4)}$$

$$\sum_{p} X_{p,g} \leq 1 \qquad \forall g \qquad \text{(T5)}$$

$$\sum_{p,g,d} R_{a,p,g,d} \leq 1 \qquad \forall a \qquad \text{(T6)}$$

$$\sum_{a,d} salvo_{a,c_p,d,i} R_{a,p,g,d} - loadout_{p,i} X_{p,g} \leq 0 \qquad \forall i,g,p \in G_{c_p} \qquad \text{(T7)}$$

$$\sum_{a,g,d} R_{a,p,g,d} \leq \overline{engage}_p \qquad \forall p \qquad \text{(T8)}$$

$$\sum_{a,g,d} salvo_{a,c_p,d,i} R_{a,p,g,d} \leq \overline{shoot}_p \qquad \forall p \qquad \text{(T9)}$$

$$R_{a,p,g,d} \leq X_{p,g} \qquad \forall a,g,p \qquad \text{(T10)}$$

$$\text{all } \alpha_m, \beta_{m,s}, \gamma_{m,s}, \delta_t, \underline{\pi}_{m,s}, \overline{\pi}_{m,s}, \rho_{m,s}, \theta_a \geq 0,$$

$$\text{all } X_{p,g},\ R_{a,p,g,d} \in \{0,1\} \qquad \text{(T11)}$$

The solution of **JD-ILP** yields an optimal defense pre-positioning plan $\mathbf{X}^*$ and interceptor-commitment plan $\mathbf{R}^*$. We recover the associated, optimal mobile missile transport $\mathbf{W}^*$ and attack plan $\mathbf{Y}^*$, by fixing $\mathbf{X}=\mathbf{X}^*$ and $\mathbf{R}=\mathbf{R}^*$ in JD-MINMAX, and solving the linear program that results.

JD-ILP can be embellished with additional features as long as the modifications can be expressed linearly in $(X,R) \in XR$ and the embellishments respect unimodularity. E.g., if the embellishments violate the intrinsically integer nature of the original model, a special decomposition would likely be required to achieve a solution for the enhanced model.

E. MODEL IMPLEMENTATION AND GRAPHICAL USER INTERFACE

The user interface is programmed in Microsoft Visual Basic (VBASIC) © using Microsoft Excel © [Microsoft 2006]. The user start-up screen is displayed in Figure 1.

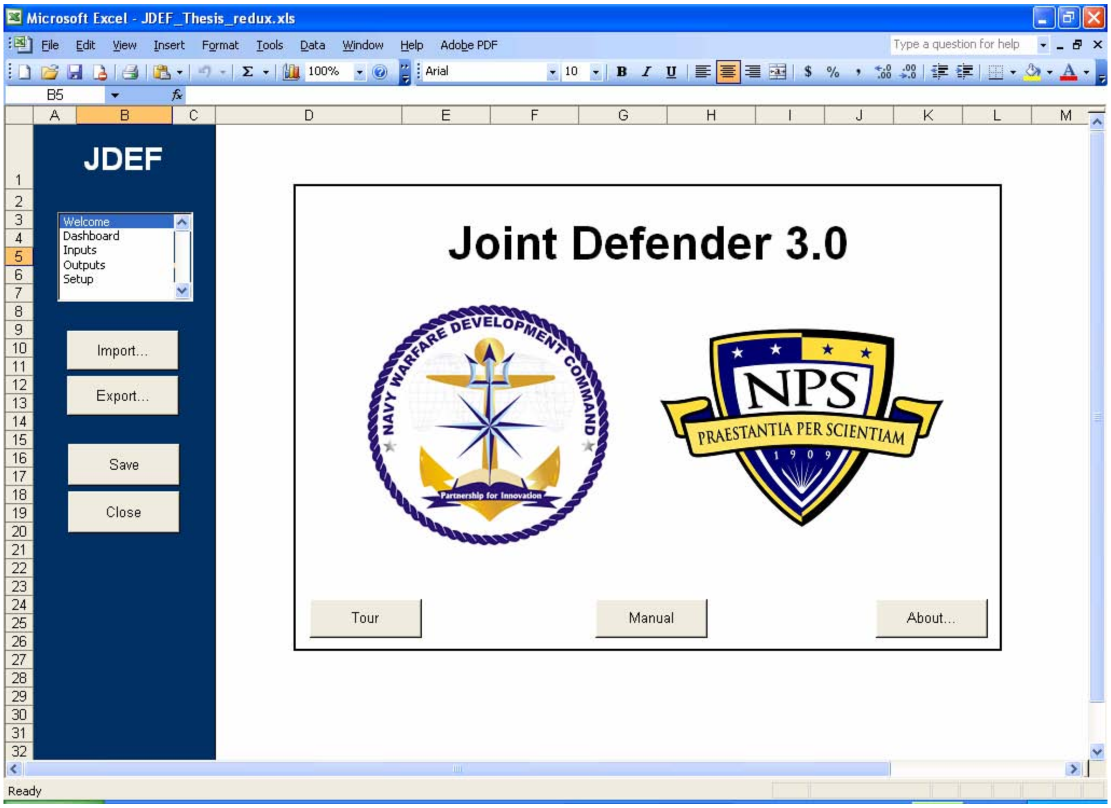


Figure 1. The JDEF User Interface. Planner controls are located at the left. The "Dashboard" option places the planner into a geographic (map) view of the theater of operations. The inputs option allows a planner to change any defender, defended asset, or attacker parameter to suit the situation.

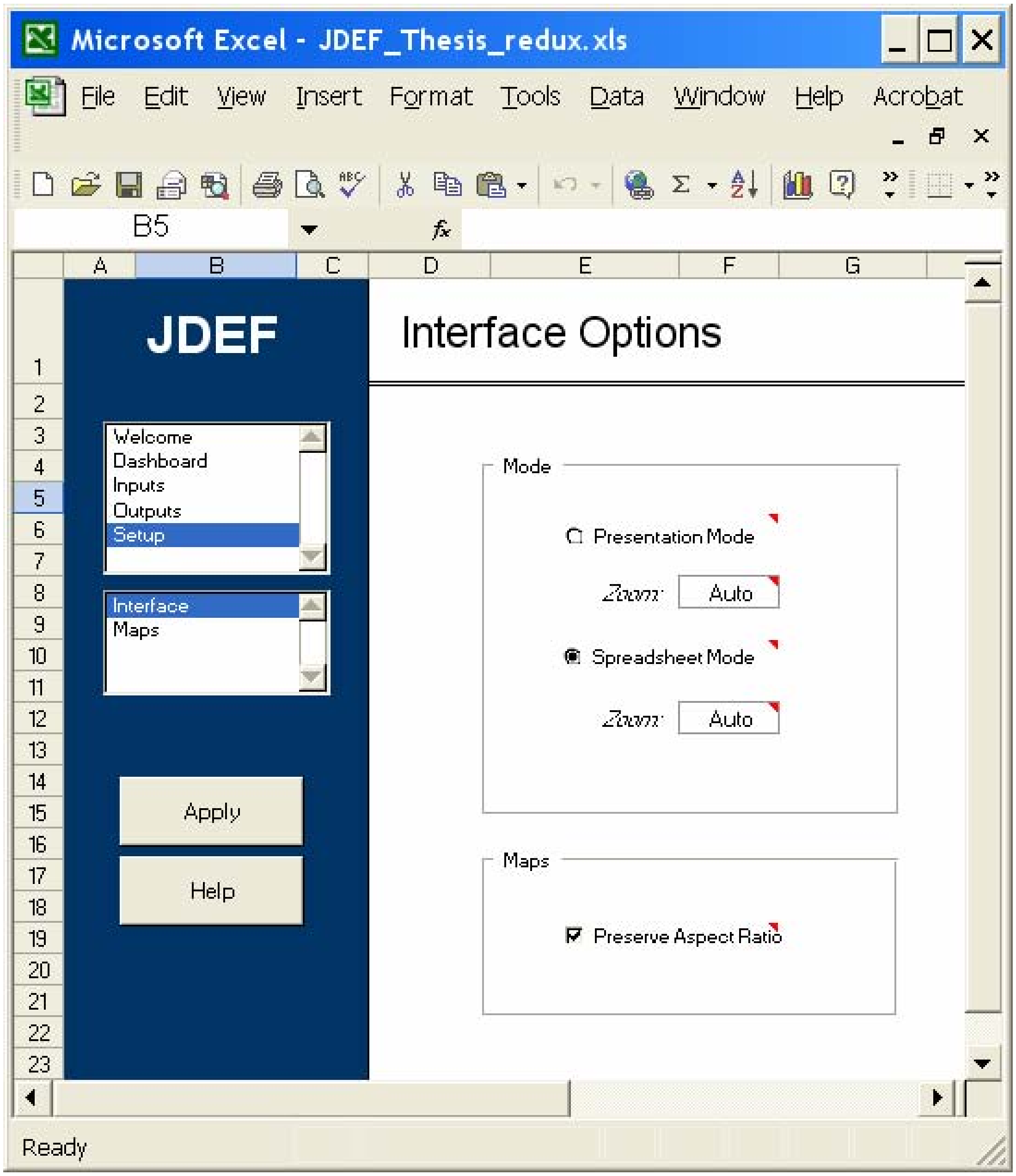
When "Solve" is invoked, planner-supplied data describing friendly force disposition, enemy launch positions, and a defended asset list is converted to an ASCII text file, and the interface invokes the JDEF GAMS (General Algebraic Modeling System © [2006]) script to import this. JDEF solves the requested planning model(s), providing solution(s) and diagnoses as more text files, and the appropriate parts of those outputs are cataloged back into the spreadsheet. From there, the planner is shown the recommended solution and asked whether or not to accept it. Upon selecting “Accept Solution”, the map slews to include only those actions the planner has approved. The model typically takes one to four minutes to arrive at a solution, and once that solution is achieved, geographic display of that recommended solution is instantaneous.

F. USER INPUT AND OUTPUT

The obvious advantage to using Excel as the input shell is that most U.S. Officers will be immediately familiar with general operating procedures. All required user input is placed in menu-driven spreadsheet cells and final output is available in the same format.

The planner first selects “New” and is asked to name the scenario and then proceed to one of four areas “Dashboard”, “Inputs”, “Outputs”, and “Setup”. The function of each is summarized below.

“Setup” invites the planner to select a map for the area of interest. JDEF invites the planner to toggle either a “Spreadsheet” mode or “Presentation” mode which governs the display of information to the screen. JDEF output may be displayed directly from the user interface via a digital projector or very simply pasted into any number of commercial presentation software packages.



Interface Options

Mode

○ Presentation Mode

Zoom: Auto

● Spreadsheet Mode

Zoom: Auto

Maps

☑ Preserve Aspect Ratio

Figure 2. The Setup Screen. This screen allows a planner to set the desired display parameters and select which map will be geographically displayed for use with the current scenario.

“Inputs” invites the planner to enter and place defensive platforms in starting positions. These positions may be fixed so JDEF cannot change them, or simply evaluated by JDEF in the course of its optimization. This is key: *the planner has complete control of this.*

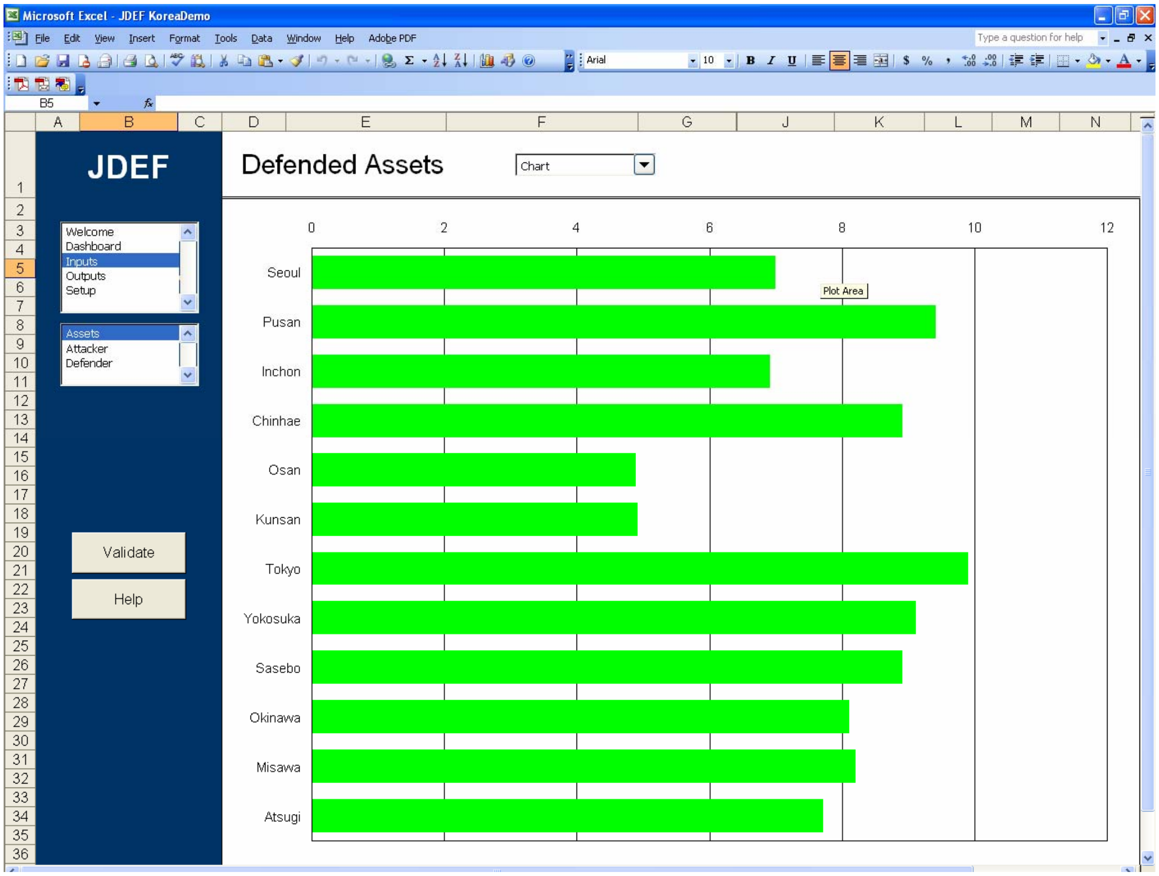


Figure 3. The Inputs Screen Displaying the Values of a Defended Asset List (DAL). This is a slide-bar display of the assigned defended asset values. By convention, we use continuous values in the interval [0,10], but a planner can use any values that appeal.

"Dashboard" is a geographic display of the planning area that either shows the initial positions of platforms, or those recommended by JDEF.

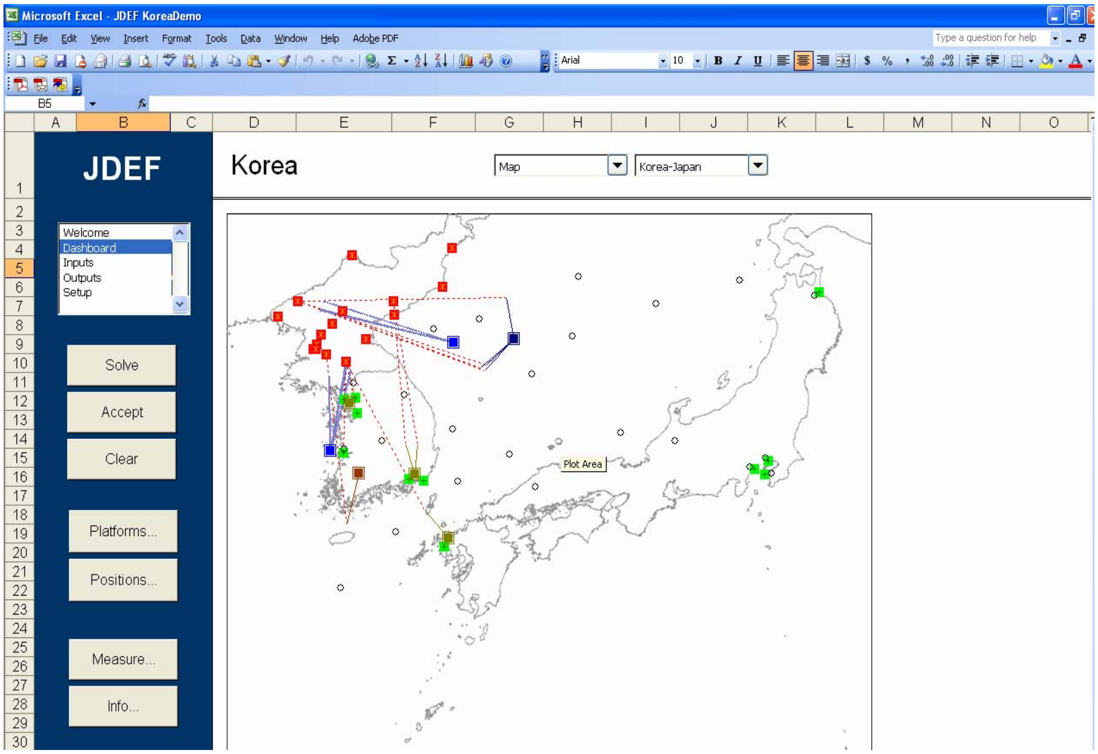


Figure 4. The JDEF Dashboard. This is the main geographic display for JDEF. A planner can see enemy launch sites and defended asset positions, as well as the current and recommended locations for every defender, and planned interception engagements. The small, empty circles are candidate defender locations, squares north-west are North Korean launch sites, circles in South Korea and Japan are defended asset locations. South-east squares denote defender platforms. Dashed and solid lines respectively represent attack launches, and defender engagements. The "Info" button at the lower left opens a window that permits the planner to fly over this map and identify any symbol with a mouse click.

"Outputs" displays the best achievable worst-case assessment of what can take place based on the underlying mathematical optimization. Outputs include a solution summary, the recommended positioning of defensive assets, and a summary of which threats each defensive asset is designated to engage.

The anticipated JDEF planner is an officer experienced in ballistic missile defense, not a mathematical modeler. The planner is insulated from the internal intricacies. The planner requires no mathematical sophistication or internal knowledge of the JDEF model.

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III. LONG-RANGE SEARCH AND TRACK EXTENSIONS OF THE JDEF MODEL

A. ENHANCEMENTS TO JDEF

Radar performance in legacy JDEF has been modeled using an engineering approximation that states a threat becomes detectable at distance “X” from the sensor location. We introduce a radar range equation, but one that can be adjusted or overridden by the planner. There has been no provision in legacy JDEF to differentiate between a defender platform possessing organic weapons and one whose mission is to primarily be a sensor. We distinguish between a sensor platform, referred to hereafter as a “LOOKER” and one that is primarily a “SHOOTER.” For practical purposes, and to isolate the effects of our new modeling paradigm, we position the LOOKERs, fix their locations, and then position SHOOTERs that have not already been located as LOOKERs. Our primary focus is the U.S. Navy’s AEGIS system, but the planning includes other fixed and mobile detection assets such as airborne laser, X-band radar, or deployable ground-based systems.

B. RADAR EQUATIONS

Several versions of the well-known radar range equation have been considered for use in approximating radar performance in JDEF. Two alternate equations are presented here along with the rationale for the one chosen. We first consider the classic equation suggested by Skolnik [2001]:

$$R_{\max}^4 = \frac{P_t G_t A_e \sigma_{m^2} I_i(n) G_p}{(4\pi)^2 k T_s B_m L_f L_s (S/N)_{\min}}$$

Where:

$\mathrm{R}_{\max}$ = Range of target from radar,
P_t = Power Out,
G_t = System Gain,
A_e = Antenna Aperture,
σ_{m^2} = Radar cross section of target in meters square,
$I_i(n)$ = Integration Constant,
G_p = Processing Gain,
kT_sB_m = Boltzman's Constant × System Temp (Degrees K) × Bandwidth,
L_f = Target Fluctuation Loss,
L_s = System Loss, and
$(S/N)_{\min}$ = Signal to Noise Ratio as if the detection were based on a single pulse.

We rearrange the basic equation to isolate radar cross section over signal-to-noise ratio and arrive at the following:

$$R_{\max}^4 = \frac{P_t G_t A_e I_i(n) G_p}{(4\pi)^2 kT_s B_m L_f L_s} \times \frac{\sigma_{m^2}}{(S/N)_{\min}}.$$

Then, using generic parameters derived from open sources [Forecast International 1998], [Skolnik 2001] to populate the variables we arrive at the following:

$$R_{\max}^4 = 4.5 \times 10^{13} \frac{\sigma_m^2}{(S/N)_{\min}}.$$

Our equation now establishes a functional relationship between three variables: radar cross section of the target, range of the target from the radar, and signal-to-noise ratio. Next, we estimate a reasonably achievable signal-to-noise ratio as a function of desired probability of detection and probable false alarm rate (P_{fa}). We adopt a probability of false alarm rate of 10^{-6} [Knorr 2006] which according to Skolnik equates to an average of one false alarm per 15 minutes assuming a clear operating environment (e.g., one free of counter-measures).

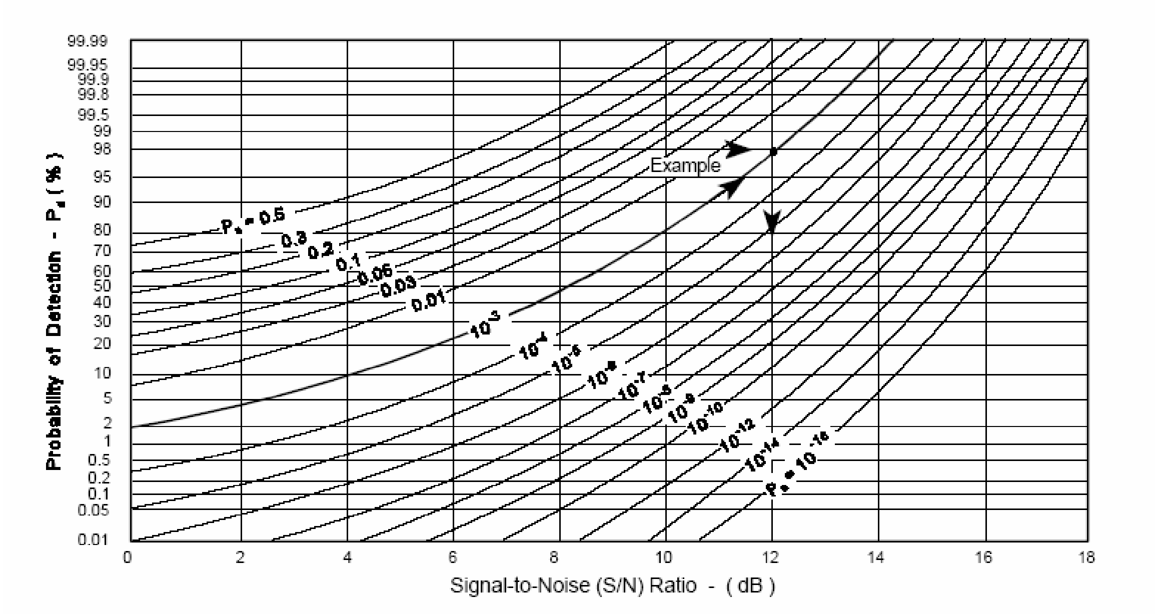


Figure 5. Nomograph of Signal-to-Noise (S/N) Ratio as a Function of Probability of Detection (P_d) and Probability of False Alarm Rate (P_f) [Naval Air Warfare Center 1992] The vertical axis represents a desired probability of detection (*Pd*) and the horizontal a signal-to-noise (S/N) ratio. The curved lines represent a known probability of false alarm (P_{fa}). In the example above, the desired P_d = .98, the P_{fa} = 10^{-6} which implies that the (S/N) ratio is 12 dB.

To achieve a desired 95 percent probability of detection and given P_{fa} of 10^{-6}, we make a pessimistic estimate of 14dB minimum required signal-to-noise-ratio from the nomogram presented in Figure 5. This leads to a radar equation of the following form:

$$R_m^4 = 4.5\times10^{13}\frac{\sigma_{m^2}}{14dB}.$$

Actual radar cross sections of potential threat missiles are closely-guarded military secrets, so we have made suitable estimates as surrogates for actual data. The estimates in Table 1 are from Spick [2001].

Aircraft	**RCS [dBsm]**	**RCS [m^2]**	**RCS [ft^2]**
F-16 Fighting Falcon	+7	5	54
B-1B Lancer	0	1	11
F-18E/F Super Hornet	0	1	11
Rafale	0	1	11
Typhoon	-3	0.5	5.5
AGM-86 ALCM	-6	0.25	2.5
BGM-109 Tomahawk	-13	0.05	0.5
SR-71 Blackbird	-18	0.015	0.15
F-22 Raptor	-22	0.0065	0.07
F-117 Nighthawk	-25	0.003	0.03
B-2 Spirit	-28	0.0015	0.02
AGM-129 ACM	-30	0.001	0.01
Boeing Bird of Prey	-70	0.0000001	0.000008

Table 1. Estimated Radar Cross Sections. From Spick [2001].

We choose the table estimate for an AGM- 86 Air-Launched Cruise Missile as an example with corresponding radar cross section of 0.25m^2 (-6 dBsm). Our radar equation now appears:

$$R_{km}^4 = 4.5 \times 10^{13} \frac{0.25_{m^2}}{14dB} \approx 946\,km\,.$$

This equation represents an optimistic estimate of the maximum likely detection range for an object of this size [Skolnik 2001].

A second radar equation has been proposed by Applied Physics Laboratory (APL) [Loy 2005]:

$$Range = ref\_range \times 10^{[(rcs\_tgt + snr\_ref - snr\_thr - rcs\_ref)/unit\_conversion\_factor]}$$

Where:

ref_range	Sets range units, kilometers here (1 km),
rcs_tgt	Radar Cross Section of the target (decibels/square meter),
snr_ref	Reference Signal to Noise Ratio (decibels),
snr_thr	Threshold Signal to Noise Ratio (decibels),
rcs_ref	Reference Radar Cross Section (decibels), and
unit_conversion_factor	converts output to appropriate units.

Using values derived from the example given for Skolnik's generic radar equation, we get:

$$10^{((-6dBsm+125dB-3dB-3dB)/40)} = 501 \text{ km.}$$

We have evaluated other radar cross sections as well, all with proportionate results. Depending on the radar cross section used, the APL-recommended equation produces results that are about 30 percent more pessimistic than the basic radar equation in Skolnik [2001]. After consultations with Professor Jeff Knorr (Naval Postgraduate School) and Mr. Todd Loy (APL), we conclude the following. The basic radar equation is a theoretical construct based on physics in a perfect world, albeit with provisions made to model noise, etc. The APL equation is a perturbation of the basic radar equation with some provision made for actual ranges experienced in the use of various tactical radars. The reader is invited to vary any or all of the parameters used to gain these results. We chose the APL equation to model radar performance in JDEF, because it more closely resembles radar performance in a less-than-perfect world.

C. JDEF LOOKERS AND SHOOTERS

1. Implementation of Cuing

The military platforms and weapons being developed for TBMD are intended to operate in a geographically dispersed manner, some intended to detect launches, some to intercept them, and some to do both. We now develop a scenario where one ship or aircraft (LOOKER) would autonomously detect a TBM, transmit that detection via digital data link to another ship or aircraft (SHOOTER), which would then assume engagement responsibility for that particular threat. This is referred to hereafter as a “cued engagement.”

In AEGIS lexicon, the implication of a cued engagement is that a ship possessing both a radar and TBMD weapons (SHOOTER) would be able to focus available radar resources over a set of radar observable areas. Additional units equipped with radar only (LOOKERs) could then surveil a separate set of radar observable areas; cuing the SHOOTER when a threat TBM launch is detected.

For practical reasons, this extension to JDEF is implemented as a sequential heuristic. First, we position the LOOKERs, fix their positions and then re-run the optimization to position the SHOOTERs. This does not guarantee optimality in the solution recommended, but we present an analysis of veracity in Chapter 3. If a scenario is presented to JDEF with no LOOKERs available, this is equivalent to using legacy JDEF, albeit with much-improved radar detection and engagement fidelity.

2. The Value of a LOOKER Cue

Complicated systems like missile interceptors rely on a long sequence of things going right in order to ultimately be successful. If Pn is the probability of nullification (i.e., engagement success) for such an interceptor system, one might imagine that $Pn=P_1P_2\ldots P_n$, where P_i is the probability that the i^{th} of n independent tasks is accomplished successfully in a *kill chain* of events. How do we account for an earlier or more accurate designation (*cue*) of an enemy missile launch that improves one link in this kill chain? Such an improvement should have the effect of increasing *Pn*, but by exactly how much?

In reality, the kill chain formula for *Pn* holds in principle, but neither the number of sequential factors nor their specific values are known. A stochastic model of the situation may be our only recourse. Let *X* be the number of things that go wrong in the kill chain, a random variable that must be 0 if the interceptor system is to succeed. As long as *n* is known to be large, even if its exact value is unknown, it is reasonable on theoretical grounds to think of *X* as a Poisson random variable. If the mean of *X* is *m*, then *Pn* is the probability that *X* is 0, which is e^{-m}. If *Pn* is 0.7, for example, then m=-ln(0.7)=0.357 . Now let *f* be the fraction of things that might go wrong that are eliminated by a cue. *X* is still Poisson, but its mean is now $m(1-f)$, and *Pn* becomes $e^{-m(1-f)}$. If f=0.2 and m=0.357, Then *Pn* becomes 0.752 (see Table 2). Quantitatively accounting for the contribution of a cue is thus reduced to making a judgment about *f*. One could argue, of course, that one might as well make a judgment about *Pn* in the first place, but *f* is not simply a “tuning parameter.” Parameter *f* has a well-defined meaning, and judgments about *f* may therefore be more accurate than direct judgments about *Pn* (Washburn, 2006).

		Prior P(N)								
Marginal Value of a Cue	0	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
	0.05	0.616	0.664	0.713	0.761	0.809	0.857	0.905	0.952	1
	0.1	0.631	0.679	0.725	0.772	0.818	0.864	0.91	0.955	1
	0.15	0.648	0.693	0.739	0.783	0.827	0.871	0.914	0.957	1
	0.2	0.665	0.709	0.752	0.794	0.837	0.878	0.919	0.96	1
	0.25	0.682	0.724	0.765	0.806	0.846	0.885	0.924	0.962	1
	0.3	0.699	0.74	0.779	0.818	0.855	0.893	0.929	0.965	1
	0.35	0.718	0.756	0.793	0.829	0.865	0.9	0.934	0.967	1
	0.4	0.736	0.772	0.807	0.842	0.875	0.907	0.939	0.97	1
	0.45	0.755	0.789	0.822	0.854	0.885	0.915	0.944	0.972	1
	0.5	0.775	0.806	0.837	0.866	0.894	0.922	0.949	0.975	1
	0.55	0.795	0.824	0.852	0.879	0.905	0.93	0.954	0.977	1
	0.6	0.815	0.842	0.867	0.891	0.915	0.937	0.959	0.98	1
	0.65	0.836	0.86	0.883	0.904	0.925	0.945	0.964	0.982	1
	0.7	0.858	0.879	0.899	0.917	0.935	0.952	0.969	0.985	1
	0.75	0.88	0.898	0.915	0.931	0.946	0.96	0.974	0.987	1
	0.8	0.903	0.918	0.931	0.944	0.956	0.968	0.979	0.99	1
	0.85	0.926	0.937	0.948	0.958	0.967	0.976	0.984	0.992	1
	0.9	0.95	0.958	0.965	0.972	0.978	0.984	0.99	0.995	1
	0.95	0.975	0.979	0.982	0.986	0.989	0.992	0.995	0.997	1
	1	1	1	1	1	1	1	1	1	1

Table 2. Probability of Negation (Pn) Conditioned by the Value of a Long-Range Search and Track (LRS&T) Poisson Cue. For instance, an LRS&T cue of 0.20 conditions (raises) a Pn of 0.700 to 0.752. This displays the influence of an LRS&T cue on reducing the Poisson uncertainty in a kill chain leading to a successful intercept. Such a cue relieves the intercepting platform from the distractions of initial detection and track of a missile launch, enabling this platform to concentrate attention and resources on just the intercept. If we call the entries in the first row Pn(cue=0), then every other row is computed Pn(cue|Pn(0))=exp(log(Pn(0))*(1-cue)).

JDEF applies a cue factor *f* that varies by LOOKER class, radar type, and radar cross section of an attacking missile.

3. Interceptor Performance in JDEF

The performance of defensive interceptors in JDEF is expressed by a cross-range, down-range table (CR/DR Table). One example of a CR/DR table entry is displayed at Table 3. The parameters used in this document are theoretical constructs derived from open sources but may be changed per the planner's requirements.

Missile	*Interceptor*	*Distance*	*Min Dr*	*Max Dr*	*Min Cr*	*Max Cr*	*Prob*
NoDong1	SM-3	50	375	9999	30	375	0.700
NoDong1	SM-3	1500	375	9999	30	375	0.700

Table 3. Sample Cross-Range, Down-Range Table Entries. The "Missile" column denotes an attacking missile type, "Interceptor" the defending missile type, and "Distance" the length of a downrange attack track. Given these, "Min Dr," "Max Dr," Min Cr," and "Max Cr" give limits on the relative position of an interceptor launch that can achieve the single-shot kill "Prob" (*Pssk*) shown. The notional rows shown here are for a NoDong1 attack track length of 50 and 1,500 km. If an SM-3 interceptor is fired from no further than 375 km and no closer than 30 km to the launch track, and from at least 375 km downrange from the NoDong1 launch point, it has a *Pssk* of 0.700. Not shown are additional limitations of the SM-3 that govern exactly where in the attack track we can achieve an exo-atmospheric kinetic kill. In general, for any missile-interceptor pair, there will be hundreds of entries like this from minimum to maximum attack track distance. JDEF interpolates the proximate entries for each single-shot probability of kill.

Each pairing of attack missile and defending interceptor has its own set of cross-range, down-range table entries. For an attack launch from some launch site to a target location, and from each defender position, JDEF evaluates the spherical engagement triangle and interpolates from these table entries to determine whether the interceptor can be used, and how well it will work. All these table entries are open to manual editing by the planner.

D. MATHEMATICAL DEVELOPMENT OF LOOKERS AND SHOOTERS

To position LOOKERs, we embellish the optimization presented in Chapter I with the following new notation.

1. Indices and Index Sets

Defender:

$s \in S_{p,g}$	observable launch sites by platform *p* at location *g*
$m \in M_{p,g}$	observable launch missiles by platform *p* at location *g*
$a \in A_{s,m,t}$	attacks from launch site *s* by missile *m* at target *t*

2. Data [units]

$\overline{max\_surveils}_p$ maximum launch sites platform *p* can surveil

3. Variables [units]

Defender:

$Q_{p,g,s}$	1 if platform *p* at location *g* surveils launch site *s*, 0 otherwise [binary]

4. Formulation

$$Q_{p,g,s} \le X_{p,g} \qquad \forall p, g, s \qquad \text{(L1)}$$

$$\sum_d R_{a,p,g,d} \le Q_{p,g,s} \qquad \forall p, g, \; s \in S_{p,g}, m \in M_{p,g}, \; a \in A_{s,m,t} \qquad \text{(L2)}$$

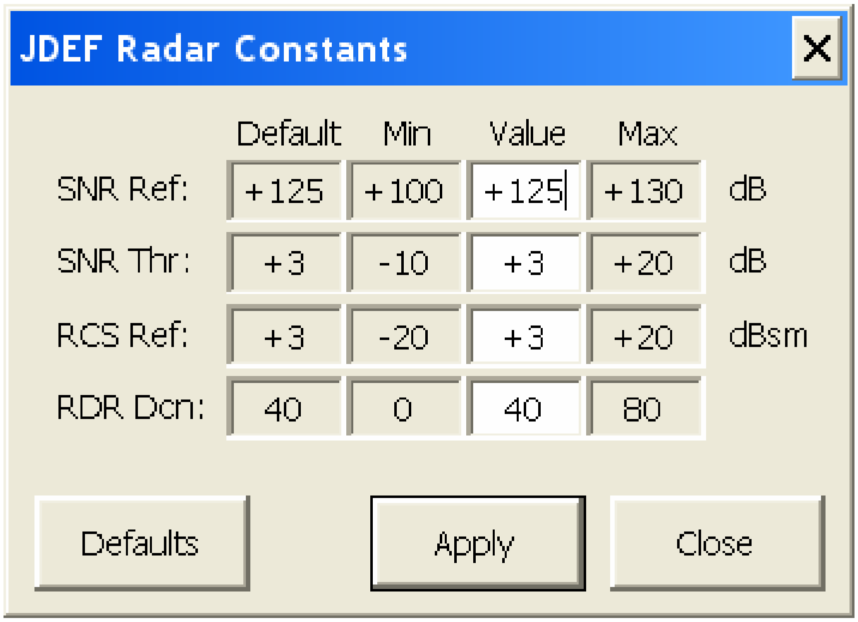
$$\sum_s Q_{p,g,s} \le \overline{\max\_ \, surveils}_p \qquad \forall p, g \qquad \text{(L3)}$$

$$Q_{p,g,s} \in \{0,1\} \qquad \forall s, p, g \qquad \text{(L4)}$$

Each constraint (L1) requires that platform *p* be located in position *g* before it can surveil launch site *s* from there. Each constraint (L2) limits radar detection by platform *p* at position *g* of a candidate launch of missile *m* from launch site *s* unless that launch can be detected and the launch site is surveiled. Each constraint (L3) limits the number of launch sites a platform *p* can simultaneously observe from location *g*: This represents radar hardware and software limitations. Domain restriction (L4) requires observation decisions to be binary. Any particular launch site may be surveiled by more than one platform.

This modification positions LOOKER platforms, selects a set of candidate launch sites to surveil, and devotes radar resources to detect any launches from the surveiled sites to cue shooters.

E. MODIFICATIONS IN SUPPORT OF LOOKERS AND SHOOTERS



JDEF Radar Constants

	Default	Min	Value	Max	
SNR Ref:	+125	+100	+125	+130	dB
SNR Thr:	+3	-10	+3	+20	dB
RCS Ref:	+3	-20	+3	+20	dBsm
RDR Dcn:	40	0	40	80	

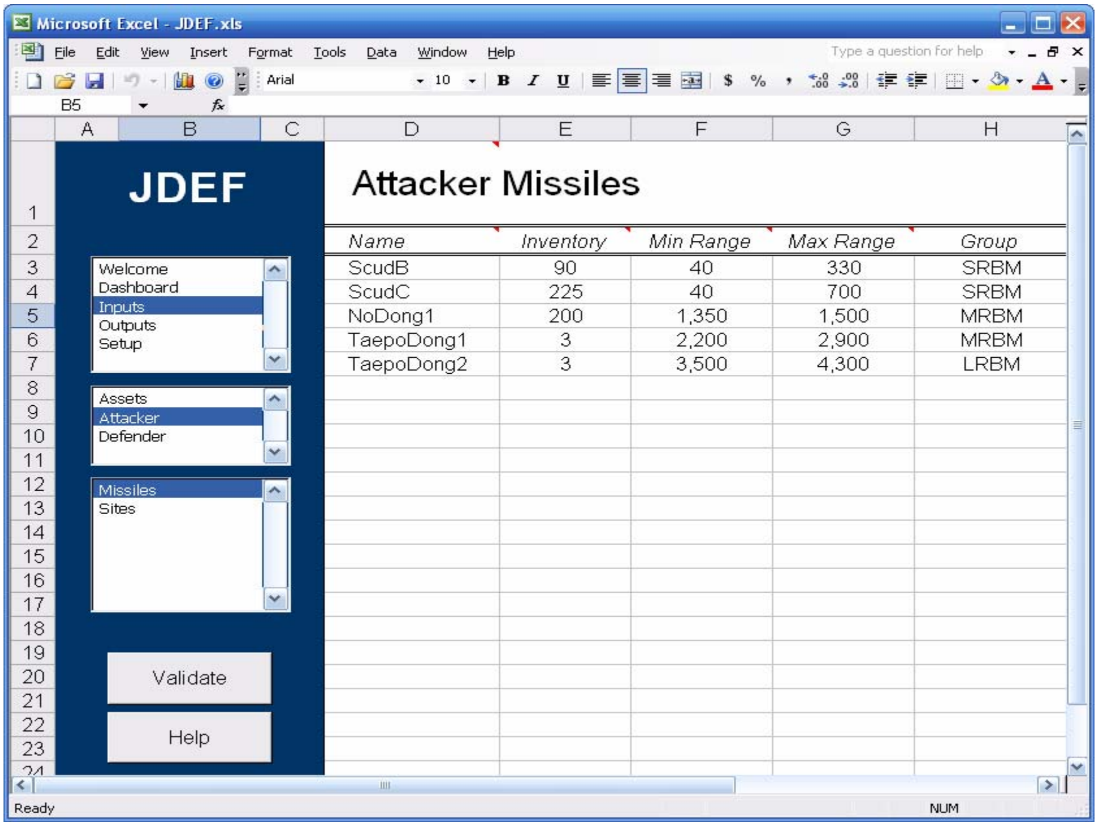
Figure 6. Radar Parameter Settings. This window allows the planner to alter default radar engineering parameters. The values required are Signal-to-Noise (S/N) ratio reference, (S/N) ratio threshold, Radar Cross Section (RCS) reference, and units constant (RDR Dcn). These values may be left at, or restored to default values with the "Defaults" button. The "Apply" button invokes the radar equation to generate detection ranges for a library of standard reference radar cross sections of attacking missiles.

A "defaults" option restores radar constants to a set of default values shown in Table 4.

Reference Range	1 km
Target RCS	0 dB
Reference SNR	115 dB
Threshold SNR	5 dB
Reference RCS	0 dB
Unit Conversion Factor	40

Table 4. JDEF Default Radar Parameters. These values are the current defaults in JDEF and may be restored by the planner with the "defaults" button in the JDEF radar constants window.

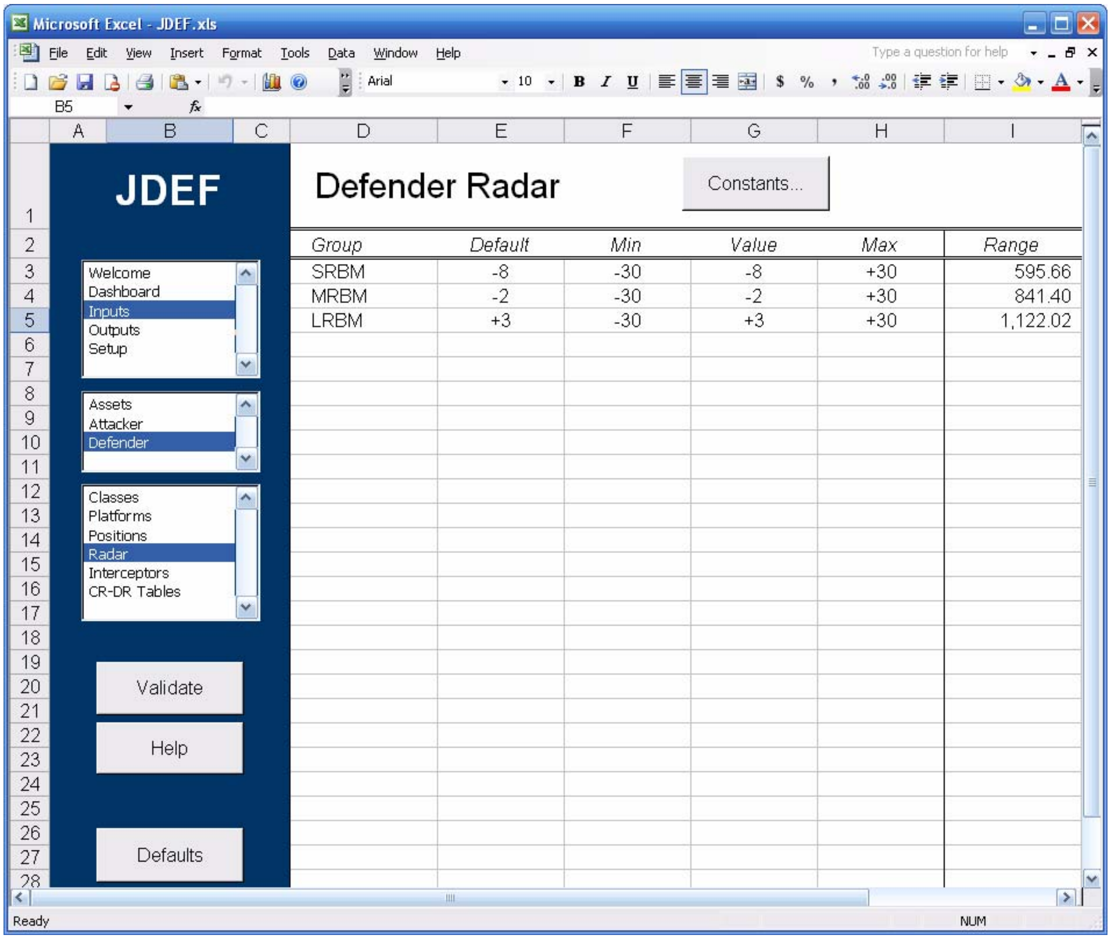
For convenience, threat missiles in JDEF have been aggregated into three categories, or *missile_groups*, that each share a common radar cross section. These are: Short Range Ballistic Missiles (SRBM's), Medium Range Ballistic Missiles (MRBM's), and Long Range Ballistic Missiles (LRBM's). These missile_groups can be expressed with higher fidelity, or even stated as each individual missile type. Suggested sources of data for unclassified scenario building include FAS [2006], UCS [2006], and Jane's [2006]. Figure 7 shows each North Korean missile type, its minimum and maximum range, and its missile group classification for radar cross section. Figure 8 shows the numerical radar cross section for each missile group, and the maximum detection range for a SPY-1 radar with current radar equation constants. The constants, or the detection ranges, can be changed by the planner.



Attacker Missiles

Name	Inventory	Min Range	Max Range	Group
ScudB	90	40	330	SRBM
ScudC	225	40	700	SRBM
NoDong1	200	1,350	1,500	MRBM
TaepoDong1	3	2,200	2,900	MRBM
TaepoDong2	3	3,500	4,300	LRBM

Figure 7. Attacker Missile Inventory, Ranges, and Radar Cross Section. For example, JDEF credits North Korea with 200 NoDong1 missiles, each with minimum range 1,350 km, maximum range 1,500 km, and the radar cross section of a medium range ballistic missile (MRBM).



Defender Radar

Group	Default	Min	Value	Max	Range
SRBM	-8	-30	-8	+30	595.66
MRBM	-2	-30	-2	+30	841.40
LRBM	+3	-30	+3	+30	1,122.02

Figure 8. Radar Detection Ranges for Missile Group Radar Cross Sections. For example, Using JDEF default radar constants, a medium range ballistic missile (MRBM) NoDong1 has a radar cross section of -2dBsm and can be detected by a SPY-1 radar at a range of 841 km. The planner can override these ranges.

IV. ANALYSIS OF OUTPUTS

A. SCENARIO BACKGROUND

JDEF applies one of three alternate planning paradigms, which are summarized as follows:

(1) Surprise: This mode approximates what military planners call “enemy’s most dangerous course of action”. JDEF answers the question, “What is the optimal friendly force disposition required to counter the worst possible attack on our Defended Asset List (DAL)?”

(2) Two-Sided: This mode assumes an intelligent enemy who observes defensive preparations and circumvents those preparations.

(3) Greedy: This model computes “all possible attacks” and defends as much expected target damage as possible from all these.

B. A SMALL ILLUSTRATIVE NORTH KOREAN SCENARIO

The following tables list the friendly forces, defended assets, and attackers involved in this small, illustrative scenario.

1. Attacker Launch Sites and Missiles

We have chosen three launch sites from unclassified sources (see Table 5). These attacker locations and missiles represent a possible short- or no-notice attack threat against U.S. interests in Japan.

Name	*Position*	NoDong1
Chunggangup	41° 46' N 126° 53' E	4
Mayang	40° 00' N 128° 11' E	4
Sangwon	38° 50' N 126° 05' E	4

Table 5. Attacker Locations for a Minimal Attack Scenario. Each launch site here has four NoDong1 missiles.

2. Defender List

Table 6 shows defenders. The DDG LOOKER has no interceptor missiles, but can surveil as many as three launch sites. SHOOTERs include a DDG and a CG, each endowed with six SM-3 interceptors, and a Patriot battery with 16 PAC-3 interceptors.

Class	*Looker*	*Shooter*	*Surveil*	*Lat*	*Lon*	SM-3	PAC-3
DDG	x		3	39.16667	130.0333	0	
DDG		x		40.56667	133.95	6	
CG		x		36.61667	130	6	
Patriot		x		33.36667	129.8667		16

Table 6. Defenders and Initial Position Assignments for a Minimal, Illustrative Attack Scenario. The DDG LOOKER has no interceptor missiles, but can surveil as many as three launch sites. SHOOTERs include a DDG and a CG, each endowed with six SM-3 interceptors, and a Patriot battery with 16 PAC-3 interceptors.

3. Defended Asset List

Table 7 shows the values we assign to each defended asset.

Name	*Position*	*Value*
Yokosuka	35° 17' N 139° 40' E	9
Sasebo	33° 09' N 129° 43' E	8
Misawa	40° 41' N 141° 20' E	7

Table 7. Defended Asset List (DAL). This shows the name, location and value of each defended asset.

4. Candidate Defender Positions

JDEF allows the planner to nominate any number of feasible defender locations as potential stations as shown in Figure 4. In this scenario, 17 potential locations are nominated for seaborne assets and 15 potential locations for the Patriot battery ashore. Here, we place these arbitrarily across the Sea of Japan, the Japanese Islands, and the South Korean peninsula.

5. A Baseline JDEF Solution with no LOOKER

Our first iteration has no LOOKER. and three SHOOTERs. This exercises JDEF in a manner identical to its legacy form, giving us a baseline for comparison of results. Figure 9 shows the JDEF dashboard for this case.

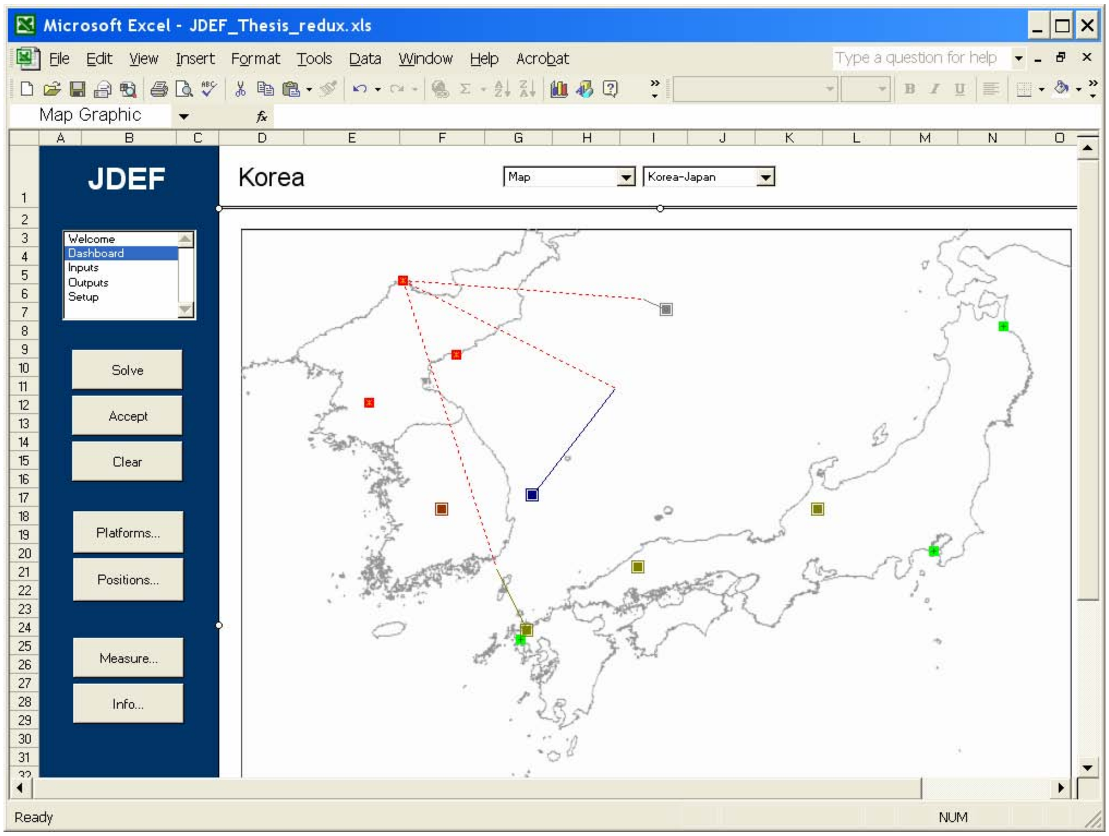
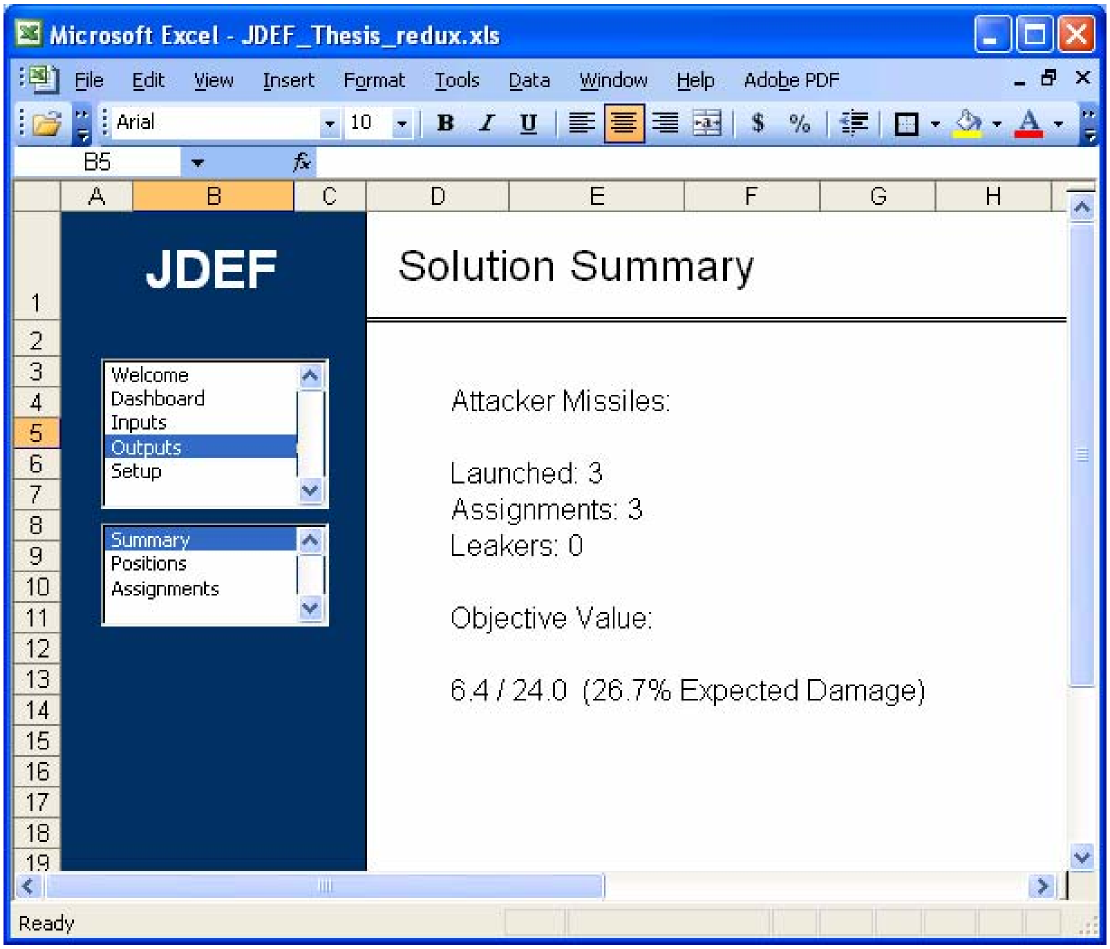


Figure 9. JDEF Minimal North Korean Scenario With No LOOKER. Square icons in North Korea represent potential launch sites. Square icons south-east represent a cruiser, a destroyer, and a Patriot Battery placed in positions recommended by JDEF. Each dashed line represents an attack launch, and each solid line denotes a defender interception. The “Info” button at the lower left opens a window that permits the planner to fly over this map and identify any symbol with a mouse click.

The formal objective value is displayed under “Outputs” >> “Summary” (see Figure 10):



Solution Summary

Attacker Missiles:

Launched: 3
Assignments: 3
Leakers: 0

Objective Value:

6.4 / 24.0 (26.7% Expected Damage)

Figure 10. The Minimal North Korean Scenario Solution Summary with No LOOKER Assigned. This is the summary page that displays the number of attacks launched, the number of attacks that JDEF is able to successfully intercept, and the expected damage to targets on our Defended Asset List (DAL).

The expected damage to each attacked target on our Defended Asset List (DAL) is computed as the product of that target value and the probability that the attack succeeds, even if we try to intercept it. We conservatively assume that the attacking missile probability of kill (*Pk*) is 1. See Table 8.

Asset	Value	Intercepting Missile	Assigned Pssk	Salvo Size	E[Defended Value]
Yokosuka	9	SM-3	0.7	1	2.7
Sasebo	8	PAC-3	0.8	1	1.6
Misawa	7	SM-3	0.7	1	2.1
Totals	24				6.4

Table 8. Expected Damage of the JDEF Objective Function. For example, the expected damage to Yokosuka above is 2.7 units based on an original asset value of 9.0, an assumed attacking missile probability of kill (*Pk*) of 1, and a probability of negation by our interception (*Pn*) of 0.7.

6. The Contribution of a LOOKER

The precise definition of a "cue" varies. A cue might include anything from a voice report to an automated threat detection triggering automated, electronic firing orders and resulting in a fully-automatic, remote interceptor launch. JDEF can now represent any such cue, expressed mathematically as a fraction that exponentially increases the probability of negation achieved by the cued SHOOTER. We use unclassified, notional numerical values of these cues. These values can be changed by the planner to represent fleet engineering data, expert judgment, or alternate estimates of performance of new technology.

We now activate the DDG LOOKER (the ship in Table 6 with no interceptors), and rerun the same scenario (see Figure 11).

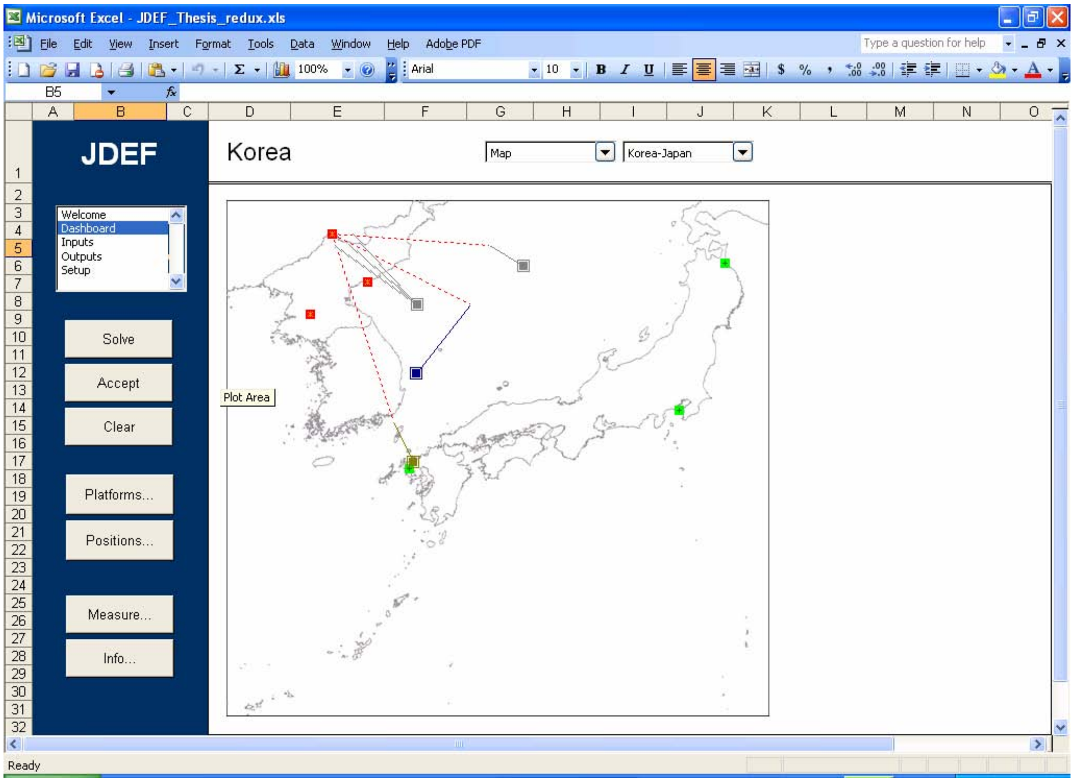


Figure 11. JDEF Minimal North Korean Scenario Results with one DDG LOOKER. The DDG LOOKER is the square icon positioned southeast of Mayang (upper-left most of the defenders) and features three lines representing detection and cuing of all three enemy launches.

We now assume a marginal cue value of 0.20 and amplify *Pn*'s based on the values displayed in Table 2. The LOOKER cues reduce expected target damage from 6.40 to 5.64. (See Table 9.)

Asset	Value	Intercepting Missile	Assigned Pssk	Pn with Cue	E[Value]
Yokosuka	9	SM-3	0.7	0.739	2.35
Sasebo	8	PAC-3	0.8	0.818	1.46
Misawa	7	SM-3	0.7	0.739	1.83
Totals	24				5.64

Table 9. Expected Damage with Cuing Added. The LOOKER cues an attack on each of these targets, and the interception of each attack has its probability of nullification (*Pn*) amplified by that cue as shown. Expected damage is reduced from 6.40 to 5.64.

C. THE INCREMENTAL VALUE OF CUING IN A THEATER-WIDE SURPRISE ATTACK

Our next scenario examines the value of incrementally increasing the number of LOOKERs in a robust, theater-wide, North Korean surprise-attack scenario. We generate a scenario with 12 potential attackers, two CGs, four DDGs, three Patriot Batteries and one THAAD Battery. The DDGs have no interceptor missiles, and are added incrementally in successive iterations as LOOKERs.

1. Attacker Launch Sites and Missiles

In Table 10 below, we expand the number of launch sites, and the variety of attacker missiles. In this scenario, we limit the number of attacks coming from each launch site to one per attacking missile type per planning time epoch. E.g., only one Scud B can be prepared and launched from Chihari, only one NoDong1, etc., but these can all be prepared and launched simultaneously.

Name	*Location*	ScudB	ScudC	NoDong 1	TaepoDong 1	TaepoDong 2
Chihari	38° 37' N 126° 41' E	15	20	10		
Chunggangup	41° 46' N 126° 53' E		10	10		
Kanggamchan	40° 24' N 125° 12' E		15	10		
Kanggye	40° 07' N 126° 35' E		15	10		
Mangyongdaeri	38° 59' N 125° 40' E	10	20	10		
Mayang	40° 00' N 128° 11' E		15	20	1	1
Namgungni	39° 08' N 125° 46' E	5	15	15		
NoDong	40° 50' N 129° 40' E		5	15	1	1
Okpyong	39° 17' N 127° 18' E	15	15	10		
Paegun	39° 58' N 124° 35' E		15	10		
Pyongyang	39° 00' N 125° 45' E	15	15	10		
Sangwon	38° 50' N 126° 05' E	15	20	10		
Sunchon	39° 25' N 125° 55' E	5	15	10		
Tokchon	39° 45' N 126° 15' E	5	15	15		
Toksong	40° 25' N 128° 10' E	5	15	15		
Yongodong	41° 59' N 129° 58' E			20	1	1

Table 10. Launch Sites for a Theater-Wide North Korean Missile Attack. This list of launch sites includes the location, type, and quantity of each potential attacking missile. Attacks are limited to a single launch per attacking missile type per time epoch in this scenario.

2. Defender List

Table 11 shows our defenders. The LOOKERs are introduced one at a time to see what contribution they make to the overall effectiveness of missile defense.

Class	*Looker*	*Shooter*	*Surveil*	*Engage*	*Launch*	SM-3	PAC-2	PAC-3	HTK
CG		X		5	10	30			
CG		X		5	10	30			
Patriot		X		4	8		6	32	
Patriot		X		4	8		6	32	
Patriot		X		4	8		6	32	
Thaad		X		4	8				10
DDG	X		3	2	4	0			
DDG	X		3	2	4	0			
DDG	X		3	2	4	0			
DDG	X		3	2	4	0			

Table 11. Defending Platforms. DDG LOOKERS have no interceptors. SHOOTER platforms are endowed with the interceptor loadouts shown.

3. Defended Asset List with Asset Values

Table 12 shows our defended asset list with locations and associated asset values.

Name	*Location*		*Value*
Seoul	37° 33' N	126° 59' E	6.9
Pusan	35° 05' N	129° 06' E	9.4
Inchon	37° 30' N	126° 38' E	6.9
Chinhae	35° 08' N	128° 38' E	8.9
Osan	37° 05' N	127° 02' E	4.8
Kunsan	35° 55' N	126° 37' E	4.9
Tokyo	35° 40' N	139° 46' E	9.9
Yokosuka	35° 17' N	139° 40' E	9.1
Sasebo	33° 09' N	129° 43' E	8.9
Okinawa	26° 12' N	127° 41' E	8.1
Misawa	40° 41' N	141° 20' E	8.2
Atsugi	35° 27' N	139° 21' E	7.7

Table 12. Defended Asset List. This shows the name, location and value of every target on our defended asset list for a theater-wide attack scenario.

4. Scenario Conduct and Results

We begin with no LOOKERS.

Iteration 0, No LOOKERs: North Korea can only reach ten of twelve targets on our DAL, Misawa and Atsugi are spared. A Patriot battery intercepts SCUDB's aimed at Seoul, Inchon, and Osan, another Patriot battery intercepts SCUDC's aimed at Pusan and Chinhae, the third Patriot intercepts a SCUDC aimed at Sasebo. Thaad intercepts a NoDong1 aimed at Yokosuka, and the CG's each intercept NoDong1s respectively aimed at Tokyo and Okinawa. Nine of 10 attacks are intercepted, and Kunsan is struck by a leaker. The expected damage is 16.7 of 93.7 (17.8%).

Iteration 1, One LOOKER: The LOOKER surveils Chihari, Kanggamchan, and Pyongyang, five of 10 attacks are cued. There is still one leaker striking Kunsan, but the expected damage decreases to 15.79 of 93.7 (16.9%), due to the marginal contribution of the cues.

Iteration 2, Two LOOKERs: One LOOKER surveils Chihari, Paegun, and Sangwon, the other surveils Kanggamchan, NoDong, and Pyongyang. Eight of ten attacks are cued. Kunsan is still struck by a leaker, and expected damage decreases to 15.39 of 93.7 (16.4%).

Iteration 3, Three LOOKERs: All ten attacks are cued, Kunsan is still struck by a leaker, and expected damage decreases to 15.32 of 93.7 (16.4%).

There is no value added by introducing additional LOOKERs beyond these three because each attack is already being surveiled and cued. Regardless of what marginal values are chosen for cued engagements, the objective function will remain constant once all engagements are cued.

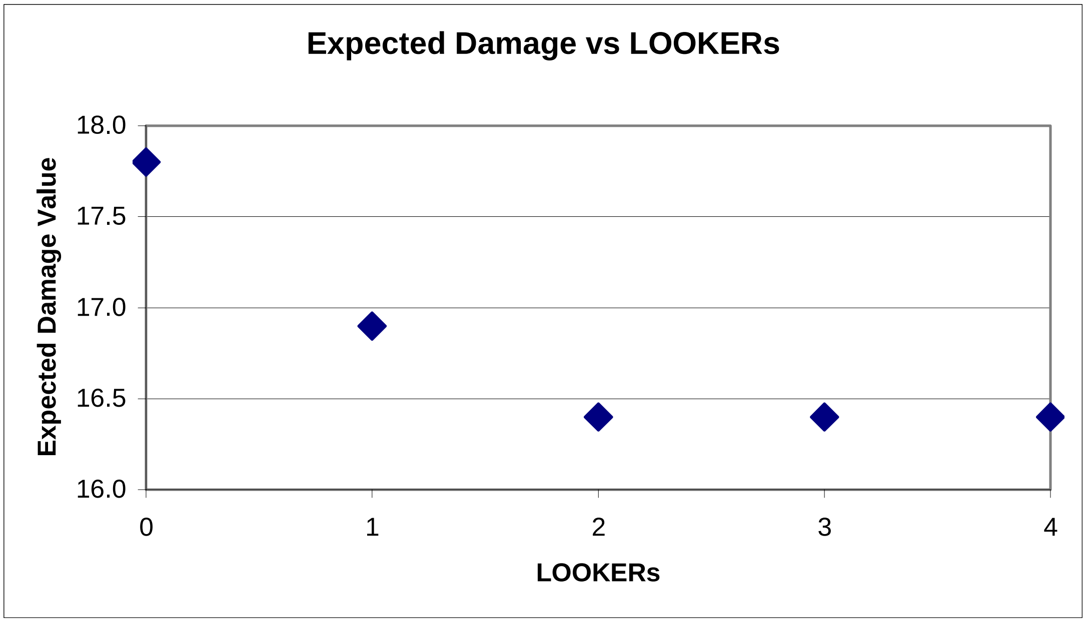


Figure 12. Expected Damage as a Percentage of Total Target Value on the Defended Asset List (DAL), Versus Number of LOOKERs Assigned. LOOKER cues reduce expected damage, but additional LOOKERs contribute diminishing returns, with no return at all for four or more

5. The JDEF Greedy Case Shows How Launch Fans Can Be Generated and Used for Defense Positioning

Figure 13 shows what happens when we ask JDEF for a “greedy” plan for our scenario. A greedy plan permits each enemy launch site to launch every available missile attacking its best target. The resulting “launch fans” illustrate the reach of such an enemy course of action. Defending LOOKER and SHOOTER platforms are positioned to protect against the maximal expected target damage. Here, five DDG LOOKERs cue SHOOTERs including three CGs, three Patriot batteries, and a Thaad battery. This “greedy” planning mode mathematically emulates manual placement of defenders to “cover enemy launch fans” used by, for instance, Theater Battle Management Core Systems (TBMCS).

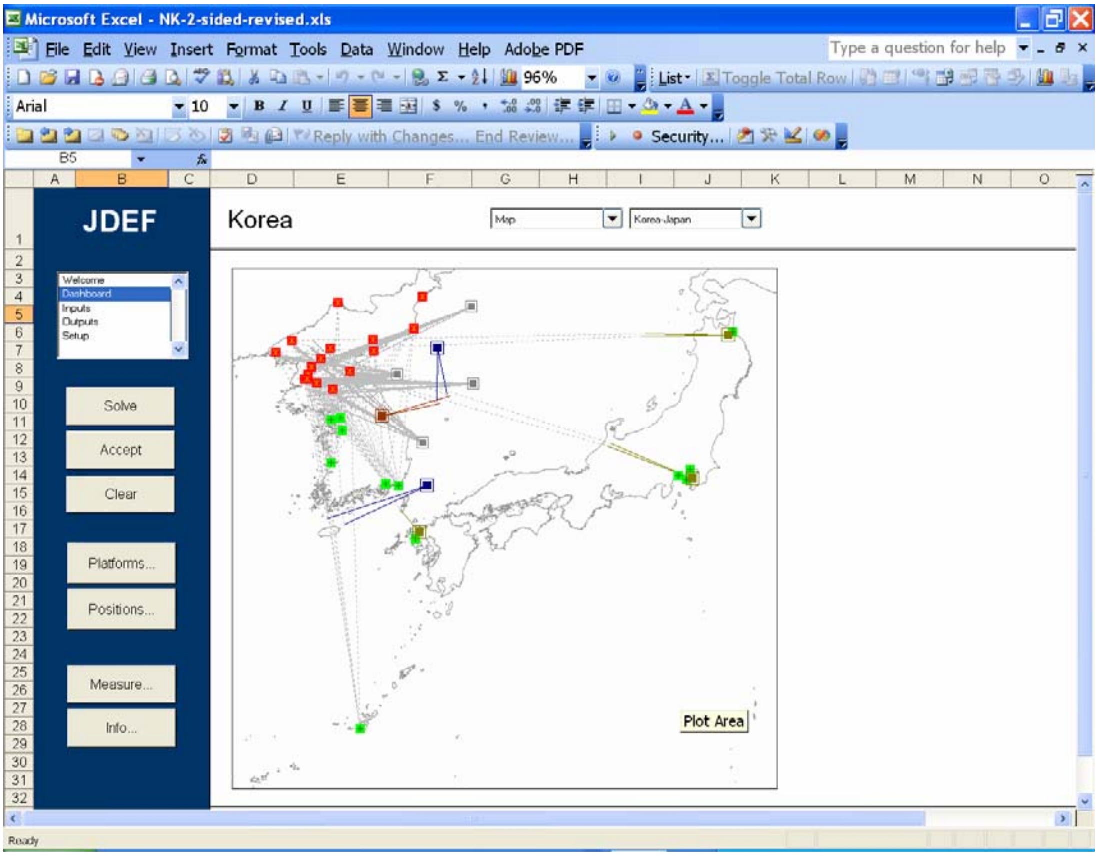


Figure 13. A JDEF "Greedy" Plan Permits Each Enemy Launch Site to Launch Every Available Missile Attacking its Best Target. The resulting "launch fans" illustrate the reach of such an enemy course of action. Defending LOOKER and SHOOTER platforms are positioned to protect against the maximal expected target damage. Here, five DDG LOOKERs cue SHOOTERs including three CGs, three Patriot batteries, and a Thaad battery. This "greedy" planning mode mathematically emulates manual placement of defenders to "cover enemy launch fans" used by, for instance, Theater Battle Management Core Systems (TBMCS).

6. The JDEF Two-Sided Case Shows How Secrecy and Deception Can Help

The JDEF two-sided case introduced by Diehl [2004] and amplified by Brown, et. al [2005], assumes the enemy will observe our defensive preparations before launching his attacks. This is a conservative case, and a worrisome one. Ground-based missile batteries and their supporting radars and equipment are hard to hide, especially given that Patriots are most advantageously located very near our defended assets. A ship using a SPY-1 radar can make no secret of this, and thus reveals its presence if not a telltale of its position.

To illustrate, we use a “surprise” case as a baseline. We deploy three AEGIS DDG LOOKERs, and SHOOTERs including three AEGIS CGs, three Patriot batteries and a Thaad battery. All these defenders are hidden from the enemy (“secret” in JDEF parlance).

The enemy launches three NoDong1, three SCUDB, and four SCUDC missiles. Two defended assets escape attack because they are not in range of any attacking missile from any potential launch site. Assuming (as we do) each attacking missile hits its target if not intercepted, expected damage from this attack is 83.0% of our defended asset value.

Our LOOKERs detect, track, and cue all ten attacks. A CG intercepts a NoDong1 aimed at Tokyo, another CG intercepts one aimed at Okinawa, the Thaad intercepts a NoDong1 aimed at Yokosuka, a single Patriot battery intercepts three SCUDBs aimed at Seoul, Inchon, and Osan, another Patriot intercepts two SCUDCs aimed at Pusan and Chinhae, and the third Patriot intercepts a SCUDC aimed at Sasebo. A SCUDC leaker hits Kunsan. See Figure 14.

In this baseline case, expected defended asset damage is about 16.4%.

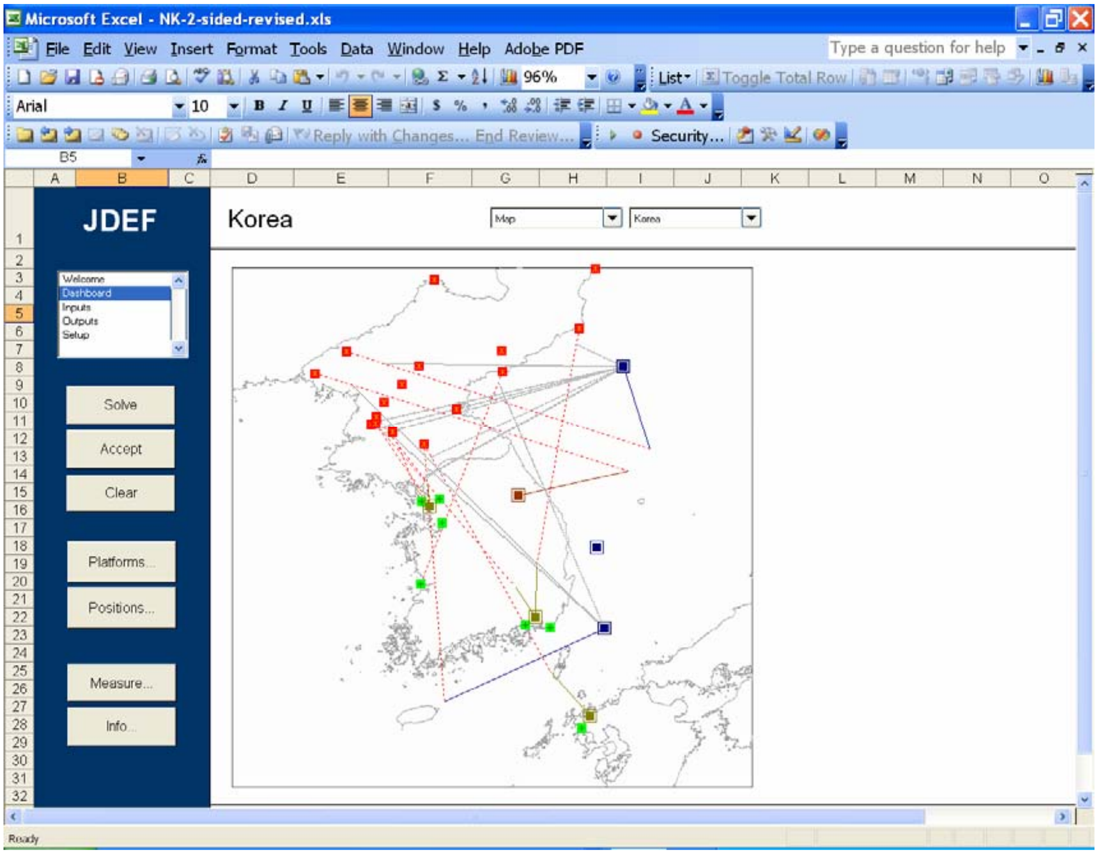


Figure 14. JDEF "Surprise" Baseline Case. The enemy launches three NoDong1, three SCUDB, and four SCUDC missiles. Two defended assets escape attack because they are not in range of any attacking missile from any potential launch site. Assuming (as we do) each attacking missile hits its target if not intercepted, expected damage from this attack is 83.0% of our defended asset value if we do not mount a defense. Our LOOKERs detect, track, and cue all 10 attacks. A CG intercepts a NoDong1 aimed at Tokyo, another CG intercepts one aimed at Okinawa, the Thaad intercepts a NoDong1 aimed at Yokosuka, a single Patriot battery intercepts three SCUDBs aimed at Seoul, Inchon, and Osan, another Patriot intercepts two SCUDCs aimed at Pusan and Chinhae, and the third Patriot intercepts a SCUDC aimed at Sasebo. A SCUDC leaker hits Kunsan. Defenders reduce expected damage to about 16.4%.

We now make all defenders visible ("seen" in JDEF parlance). When we do this, *expected damage increases from 16.4% to 76.2%*.

We now make all defenders visible (“seen” in JDEF parlance). When we do this, *expected damage increases from 21% to 76%*.

How did this happen? The enemy, observing our preparations, decides not to launch five of its NoDong1 missiles that we would certainly intercept (we call such instances “blocked” shots). He does launch one NoDong1 that we intercept and nine SCUDs, every one of which leaks through our seen defenses to its target.

The only defensive weapon we have that is effective against SCUD attacks is Patriot. With only three Patriot batteries, all seen by the enemy, and 12 targets to defend, he just shoots around us.

Making our ships “secret,” and leaving ground units “seen” is of scant help. Expected damage moderates from 76% to about 62%. The enemy launches three NoDong1 missiles, and our ships intercept these. Seven SCUDs are launched and leak.

You can see from these excursions that secrecy is of great value to the defender.

We can also use JDEF to evaluate deception by creating “seen” defenders that have no real ability to detect, track, cue, or intercept at all. JDEF can show how to position these “dummy defenders” to frustrate and weaken enemy attack plans.

7. Evaluating high Cue Values

As we have seen, intercepting SCUDs still presents a challenge. Patriot terminal defenders are the only purpose-built interceptors effective against SCUD. While we can possibly use a sea-based interceptor such as an upgraded SM-2 missile against a SCUD, this would require placing the SHOOTER ship very close to the attacking missile track. The downrange flyout time for a SCUD is only a few minutes, compressing our window of opportunity to identify, track, cue, and intercept.

Any new system we build to deal with SCUD, or any similar short-range, low altitude attacking missile threat, will have to accommodate extremely short decision cycles. Detect and track will need to be quick, and accurate.

We use JDEF to evaluate the influence of better detect-and-track cues. We endow our AEGIS DDG LOOKERs with respective cues of 0.5 for the SRBM missile_group radar cross section, 0.6 for MRBM, and 0.7 for LRBM.

With no LOOKER, and no cue, we intercept nine of 10 attacks with expected damage 83% of defended asset value. With one LOOKER, we cue four launches from two surveiled sites. We intercept only eight of 10 attacks, but expected damage drops to 24%. The cues are so good, we abandon un-cued attacks, the better to defend targets we can really protect well.

V. CONCLUSIONS, FUTURE ENHANCEMENTS AND RESEARCH OPPORTUNITIES

A. CONCLUSIONS

JDEF now accepts interceptor performance data exactly as it is expressed by standard engineering sources, reckons radar performance in a transparent, standard fashion, and incorporates LOOKERs to cue SHOOTERs and increase their effectiveness. Every detail, every nuance is completely documented. All this is presented in a graphical user interface, and every parameter is open to planner view and control. JDEF invites the planner to fix any condition, override any default, and completely manually control every detail. JDEF also offers an optimization-based mechanism to take partial guidance --- expert judgment and/or exogenous constraints --- and follow this while completing a theater-wide, joint missile defense plan in a minute or two. We know of no other planning tool that is documented as well, is as open as this to manual editing and control, and is as flexible to follow planner guidance, or merely use planner advice as a starting point to complete an optimal defense plan.

JDEF works on a WINTEL laptop. Cost per seat is about fifteen thousand dollars. All its features can be transported to any other reasonable, contemporary computing platform.

JDEF can be well-used by any planner with missile defense expertise. The only mathematical detail exposed in the graphical user interface is optional, and expressed in standard missile defense lexicon. All the rest of the optimization is automated, and no modeling experience is required.

JDEF is the only missile defense planner extant that provides a qualitative guarantee with its solution: it gives an expected damage, and an upper bound on this, and there can be no as-yet undiscovered plan that follows the rules the planner has dictated and betters this.

B. EARTH CURVATURE VERSUS RADAR DETECTION

When radar systems operate on or near the earth's surface and focus a radio frequency (RF) energy beam along or near the earth's surface, that energy is refracted along a well-understood path that extends the radar's range. This is normally accounted for by an approximation called the "four-thirds Earth approximation" [Wagner, et. al 1999] that assumes earth radius to be one-third larger than it actually is, and renders a more accurate radar range estimate. JDEF radar targets are ballistic missiles with relatively high trajectories that extend into thin to nearly no earth atmosphere. As RF energy propagates into areas of lesser atmospheric concentration, refraction wanes to nothing in a vacuum. One future enhancement would be to incorporate this atmospheric radar range gradient into the radar equations for a refined estimate of threat detectability

C. MISSILE KINEMATICS

The US Navy's SM-3 missile, the US Army's Theater High Altitude Area Defense missile system, and the US Air Force's Ground Based Interceptor are kinetic TBMD weapons that have either been fielded or are in production as of this writing. JDEF currently represents the effectiveness of any engagement by one of these interceptor missiles by interpolating single-shot kill probability entries in a cross-range, down-range table. Future research could model the actual kinematics of kinetic TBMD weapons in production.

D. RADAR RESOURCES VERSUS RADAR OBSERVABLE AREAS

The maximum number of launch sites that any one defensive platform p can monitor in JDEF is constrained by the planner-specified limit $\overline{max\_surveils}_p$. The maximum number of sites that one radar may surveil is, in reality; a function of radar resources available, the state of radar maintenance, and the ranges at which surveillance is attempted. Future research could insulate the planner from the number of radar-observable launch sites, requiring only the state of radar maintenance as a planner input and then calculating the maximum number of radar observable areas “on the fly.”

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