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Advanced ESM Angle Tracker: Volume I — Theoretical Foundations

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14. ABSTRACT

This report summarizes the NRL implementation of a filter used for ESM angle tracking. The filter is an Interactive Multiple Model (IMM) filter that uses three standard Kalman filters as a basis for filtering and predicting emitter bearing angle. Filter selection for the IMM is based on the expected variation of emitter trajectories and the imposed spherical coordinate system. The derivation of the IMM is well documented and is not addressed in this report; only implementation aspects of the IMM are addressed here.

The ESM angle tracker was developed to operate with the NRL ESM-ATD, which provides highly accurate but intermittent bearing and elevation reports. ESM parameters are tracked with first-order filters and provide the basis for associating intermittent bearing/elevation tracks in dense scenarios. The tracker is fully automatic, uses sequential Bayesian hypothesis testing for track initiation, and uses maximum likelihood association implemented across all filters to maximize single-event association probabilities.

15. SUBJECT TERMS

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ADVANCED ESM ANGLE TRACKER

VOLUME I – THEORETICAL FOUNDATIONS

1. INTRODUCTION

This report summarizes the Naval Research Laboratory (NRL) implementation of tracking filters used for tracking bearing, elevation, and electronic support measure (ESM) parameters. It describes the theory behind the NRL MATLAB and C code used for ESM angle tracking, developed to operate with the NRL ESM-Advanced Technology Demonstration (ESM-ATD) and later transitioned to Lockheed Martin Corporation for integration into the Advanced Integrated Electronic Warfare System (AIEWS) program. This code was intended to act as the ESM tracker for AIEWS and to provide feedback control for the front-end gating of the ESM receiver.

The Advanced ESM Angle Tracker was developed at NRL and embedded within the Joint Maritime Command Information System (JMCIS) as a prototype segment. This report presents the theory used for the tracker and represents Volume I of a three-volume set.

Volume II is a user's guide [1] for running the remote tracker via JMCIS. It describes how users set up and control the tracker from JMCIS and display real-time track information, and discusses JMCIS Tactical Database Manager (TDBM) display integration. Volume III is a developer's guide [2] that documents the software development of the Advanced ESM Angle Tracker. It describes the tracker's graphical user interface, the TCP/IP communications required to interface with the remote process, and the implementation of the tracker algorithms as a remote process.

The filters and tracking algorithm described in this report are unique in several ways. The bearing filter is an Interactive Multiple Model (IMM) filter [3, 4] that utilizes three Kalman filters of different orders to track emitter bearing angle. The requirements for combining first-, second-, and third-order tracks had not been defined and implemented prior to this work. Another unique feature of this tracker is that it includes all parameters (bearing, elevation, and ESM parameters) in the decision process for correlation and association of new reports with existing tracks. In addition, a new technique developed for determining plant noise in the ESM parameters filter estimates plant noise as a function of ESM parameter agility.

The IMM bearing filter uses three standard Kalman filters as a basis for filtering and predicting emitter bearing angle. Filter selection for the IMM is based on the expected variation of emitter trajectories and the imposed spherical coordinate system. The derivation of the IMM is well documented and is not addressed here; only the unique NRL implementation aspects of the IMM are addressed here.

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Each of the three bearing Kalman filters is tuned to a different trajectory model. The first of the three Kalman filter plant (hereafter referred to as trajectory) models is a simple first-order filter that is optimal for tracking stationary (land-based) emitters. This filter is also suitable for tracking non-maneuvering, radially inbound or outbound platforms. The second Kalman filter is based on filtering both bearing and bearing rate. The third Kalman filter tracks bearing, bearing rate, and bearing acceleration so as to minimize filter lag for crossing emitters and other highly maneuvering trajectories. Platforms tend to fly in straight line paths; crossing platforms will produce an infinite number of derivatives in a polar coordinate system, leading to significant filter lag. The defined second- and third-order Kalman filter trajectory models minimize these effects.

Since it is not known *a priori* which model is appropriate for any particular platform in track, the NRL IMM filter estimates the probability of each trajectory model's applicability for the particular platform under observation. These trajectory model probabilities are computed based on a hidden Markov model (HMM) as well as on outputs from each of the three Kalman filters used to filter and predict platform motion.

The elevation filter is a second-order Kalman filter that tracks elevation and elevation rate. The ESM parameters filters are first-order Kalman filters that can track either constant or agile parameters by automatically adjusting the filter plant noise. The ESM parameters filters provide the basis for the full likelihood association used in associating intermittent bearing/elevation tracks in dense scenarios.

2. IMM BEARING FILTER IMPLEMENTATION

IMM bearing filter update consists of the following steps.

1. IMM bearing filter initialization, which consists of IMM and Kalman filter initialization.

2. The IMM track update process, which consists of state estimator mixing (interaction), Kalman filter updates, model probabilities update, and combining or merging of the three state estimates.

2.1 IMM Bearing Filter Initialization

The IMM bearing filter assumes a hidden Markov model that controls trajectory model switching with a switching probability that is assumed known. This transition matrix is defined as

$$P_{ij} = \left[p_{ij} \right], \tag{1}$$

where the transition probabilities are

$$p_{ij} = \text{Prob}\left\{ M_j(k) \middle| M_i(k-1) \right\}, \quad i, j = 1,2,3, \tag{2}$$

and $M_j(k)$ stands for "model j in effect during the period ending at time k."

The above expression should be interpreted as the probability that the trajectory model for the track at time *k* equals model *j* given that at the previous update, the trajectory model was *i* where (*i, j*) range from 1 to 3. For example, in the NRL model, the three elements in the first row of the matrix indicate the transition probability from the bearing-only model to the bearing-only model (the 1,1 element), to the second-order model (the 1,2 element), and to the third-order model (the 1,3 element), respectively. The second and third rows should be interpreted in a similar manner.

The NRL IMM bearing filter initializes this Markov model transition matrix as

$$P_{ij} = \begin{bmatrix} p_{11} & (1-p_{11})/2 & (1-p_{11})/2 \\ (1-p_{22})/2 & p_{22} & (1-p_{22})/2 \\ (1-p_{33})/2 & (1-p_{33})/2 & p_{33} \end{bmatrix}. \tag{3}$$

This matrix is initialized by setting

$$\begin{bmatrix} p_{11} \\ p_{22} \\ p_{33} \end{bmatrix} = \begin{bmatrix} .90 \\ .95 \\ .90 \end{bmatrix}. \tag{4}$$

The transition probability p_{11} should be interpreted as the probability that the first-order model transitions to itself (or stays a first-order model) during the bearing track update process. Due to the form of the matrix specified above, there is also a small and equal probability that the current first-order platform model will transition to the second-order or third-order models during track update. The two remaining transition probabilities, p_{22} and p_{33}, should be interpreted in a similar manner. We note that the NRL model gives preference to the second-order bearing filter by setting the return probability to a slightly higher value. Thus, a track that is second-order (maintains bearing and bearing rate) is more likely to stay second-order during the track update process than a first- or third-order track.

The initial probability distribution associated with the three trajectory models (hereafter referred to only as models) is also assumed known. In the NRL model, this initial distribution is given by

$$p_j = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}, \tag{5}$$

where this vector is initialized to

$$p_j = \begin{bmatrix} .05 \\ .85 \\ .10 \end{bmatrix}. \tag{6}$$

Again, the NRL model gives preference to initiating second-order tracks. The **assoc_f** routine correlates current cluster descriptors (CDs) with prior CDs that have been previously stored and that have not correlated with existing tracks (defined as first points). A CD is equivalent to an ESM contact consisting of extracted information from a set of post-processed pulse descriptor words (PDWs). If a correlation exists in the appropriate parameter space, a new tentative bearing track is initiated. Although the transition matrix is a constant, the model probabilities are updated on each track update as discussed below.

2.2 IMM Track Update Process

Computations associated with the IMM track update process are discussed below.

2.2.1 State Estimator Mixing (Interaction) and State Prediction

For any initiated track, once an associated contact is identified, the first step in the IMM bearing update process is mixing the previous filter estimates stored in the extended track file (ETF). This process is known as mixing, or interaction, and allows the IMM to achieve the accuracy associated with a second-order generalized pseudo-Bayesian (GPB2) filter [4] while maintaining three rather than nine separate Kalman filters.

The 3 × 3 mixing probability matrix is given by

$$P_{i|j} = \left[p_{i|j} \right] = \begin{bmatrix} \frac{p_{11}}{\bar{c}_1} \cdot p_1 & \frac{p_{12}}{\bar{c}_2} \cdot p_1 & \frac{p_{13}}{\bar{c}_3} \cdot p_1 \\ \frac{p_{21}}{\bar{c}_1} \cdot p_2 & \frac{p_{22}}{\bar{c}_2} \cdot p_2 & \frac{p_{23}}{\bar{c}_3} \cdot p_2 \\ \frac{p_{31}}{\bar{c}_1} \cdot p_3 & \frac{p_{32}}{\bar{c}_2} \cdot p_3 & \frac{p_{33}}{\bar{c}_3} \cdot p_3 \end{bmatrix}, \tag{7}$$

where $\bar{c}$ is a 3 × 1 normalizing constant given by

$$\bar{c} = \begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \end{bmatrix} = P'_{ij} p_j \tag{8}$$

and P'_{ij} is the complex conjugate transpose of P_{ij}.

Using the above mixing probabilities, the mixed filter estimates are obtained as

$$\hat{x}_j^m(k) = \sum_i \hat{x}_i(k)\, p_{i|j}(k), \tag{9}$$

where $\hat{x}_j^m(k)$ is the mixed estimate for model *j*, $\hat{x}_i(k)$ is the i^{th} filtered estimate (*i*,*j* = 1,2,3) obtained from the ETF, and $p_{i|j}$ are the mixing probabilities defined above. The associated scalar covariances are given by

$$v_j^m(k) = \sum_i p_{i|j}(k)\left\{v_i(k) + \left[\hat{x}_i(k) - \hat{x}_j^m(k)\right]\left[\hat{x}_i(k) - \hat{x}_j^m(k)\right]'\right\}. \tag{10}$$

It should be noted that for $j = 1$, both $\hat{x}_1^m(k)$ and $v_1^m(k)$ are 1 × 1 or scalars. For $j = 2,3$, this is not the case. Since, in general, the filter estimates vary in size from 1 × 1 for the first-order filter to 3 × 1 for the third-order filter, mixing the ETF estimates is not straightforward. Before computing the above mixing equations, it is necessary to resize the filter estimates and their associated covariance to appropriate sizes for mixing. This can be called upsizing and downsizing. For example, assume that the form of the ETF estimate and covariance for the third-order filter may be written as

$$\hat{x}_3(k) = \begin{bmatrix} \hat{x}_3^{(1)}(k) \\ \hat{x}_3^{(2)}(k) \\ \hat{x}_3^{(3)}(k) \end{bmatrix} = \begin{bmatrix} \hat{b}_3(k) \\ \hat{\dot{b}}_3(k) \\ \hat{\ddot{b}}_3(k) \end{bmatrix} \tag{11}$$

and

$$v_3(k) = \begin{bmatrix} v_3^{(1,1)}(k) & v_3^{(1,2)}(k) & v_3^{(1,3)}(k) \\ v_3^{(2,1)}(k) & v_3^{(2,2)}(k) & v_3^{(2,3)}(k) \\ v_3^{(3,1)}(k) & v_3^{(3,2)}(k) & v_3^{(3,3)}(k) \end{bmatrix}, \tag{12}$$

where $\hat{b}_3(k)$ is the filtered bearing estimate. Downsizing is simply the elimination of the appropriate row and column before mixing.

Downsizing the third-order filter for mixing to obtain a mixed first-order estimate is accomplished by defining the intermediate downsized third-order filter estimate as

$$\hat{x}_{31}^d(k) = \left[\hat{x}_3^{(1)}(k)\right] \tag{13}$$

and associated covariance as

$$v_{31}^d(k) = \left[v_3^{(1,1)}(k)\right]. \tag{14}$$

Upsizing in the NRL implementation implies the addition of zero elements to expand the filter estimate and associated covariance for mixing. Consider the $j = 3$ case above. Both the first- and second-order filters require upsizing before the mixing process. To upsize the first-order filter, an intermediate vector is defined as

$$\hat{x}_{13}^{u}\left(k\right)=\begin{bmatrix}\hat{x}_1\left(k\right)\\0\\0\end{bmatrix} \tag{15}$$

and the associated 3 × 3 upsized covariance is defined as

$$v_{13}^{u}\left(k\right)=\begin{bmatrix}v_1\left(k\right) & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}. \tag{16}$$

Similarly, the second-order filter is upsized to

$$\hat{x}_{23}^{u}\left(k\right)=\begin{bmatrix}\hat{x}_2^{(1)}\left(k\right)\\\hat{x}_2^{(2)}\left(k\right)\\0\end{bmatrix} \tag{17}$$

and the associated 3 × 3 upsized covariance is defined as

$$v_{23}^{u}\left(k\right)=\begin{bmatrix}v_2^{(1,1)}\left(k\right) & v_2^{(1,2)}\left(k\right) & 0\\v_2^{(2,1)}\left(k\right) & v_2^{(2,2)}\left(k\right) & 0\\0 & 0 & 0\end{bmatrix}. \tag{18}$$

All other upsizing and downsizing are handled in a similar manner.

2.2.2 Kalman Filter Updates

The three Kalman filters should be treated as subroutines for the IMM bearing filter. Therefore, Kalman filter updates may be performed by passing in the bearing data and the mixed estimates $\hat{x}_j^m\left(k\right)$ and their covariance $v_j^m\left(k\right)$ in lieu of the normal ETF values. The corresponding output of the Kalman filters for time *k* are defined as $\hat{x}_j\left(k\right)$ and $v_j\left(k\right)$, respectively. These outputs are used for merging the estimates using the updated model probabilities. These values are also saved in the ETF along with the merged estimate and its covariance. In addition, each of the three Kalman filters sends back the innovation sequence and its covariance, which are used for the update of model probabilities.

2.2.3 Update of Model Probabilities

The model probabilities are updated in two steps. First, the likelihood associated with each of the filter updates is computed. This likelihood is defined as

$$\Lambda_j(k) = P\left\{ z(k) \middle| M_j(k), Z^{k-1} \right\}, \tag{19}$$

where $z(k)$ is the current bearing measurement from the associated contact and Z^{k-1} is a vector representing all the data through time $k-1$. This may be computed as

$$\Lambda_j(k) = \frac{\exp\left\{ -\frac{1}{2}\left(\gamma_j' S_j^{-1} \gamma_j \right) \right\}}{\left\{ \det\left[2\pi S_j \right] \right\}^{1/2}}, \tag{20}$$

where γ_j is the innovation obtained during the update of the j^{th} Kalman filter and S_j is its associated covariance. Since the dimensionality of S_j is always equal to the dimensionality of the measurement space, S_j is always a scalar and the inverse indicated above never needs to be computed. Therefore, the above may be simplified to

$$\Lambda_j(k) = \frac{\exp\left\{ -\frac{1}{2S_j}\left(\gamma_j' \gamma_j \right) \right\}}{\sqrt{(2\pi)(S_j)}}. \tag{21}$$

Based on the likelihoods defined above, the updated model probabilities may be computed as

$$p_j(k) = \frac{\Lambda_j(k)\,\bar{c}_j}{c}, \tag{22}$$

where a second normalizing constant is defined as

$$c = \sum_j \Lambda_j(k) \cdot \bar{c}_j \tag{23}$$

and $\bar{c}_j$ was previously defined.

2.2.4 Merging the State Estimates

The merged estimates and their covariances are obtained by computations similar to that discussed above for mixing (Section 2.2.1). The merged minimum mean-squared error (MMSE) filter estimate is given by

$$\hat{x}(k) = \sum_i \hat{x}_i(k) p_i(k), \tag{24}$$

and its associated covariance by

$$v(k) = \sum_i p_i(k-1)\left\{v_i(k) + \left[\hat{x}_i(k) - \hat{x}(k)\right]\left[\hat{x}_i(k) - \hat{x}(k)\right]'\right\}, \tag{25}$$

where the individual estimates and their covariance must be upsized and downsized appropriately as discussed in Section 2.2.1. The correlation and association process uses the estimates associated with the individual filters.

2.3 Extended Track File Entries

The track file database must save the three separate Kalman filter estimates, $\hat{x}_j(k)$, which vary in size from 1 × 1 to 3 × 1, and their associated covariances, $v_j(k)$, which vary in size from 1 × 1 to 3 × 3. In addition, the merged estimate $\hat{x}(k)$ and its covariance $v(k)$ should be saved.

2.4 The Single-State Bearing Filter

The following describes the mathematics for the recursive, single-state bearing angle filter. This report describes both the initialization and the update of the filter. The purpose of the filter is to estimate bearing for stationary or radially inbound emitters and to compute optimal time-varying bearing gates.

The bearing filter is a first-order Kalman filter that models only bearing. This is one of the three independent bearing filters used in the IMM architecture. The filter assumes that the measurement error variance is properly defined for the ESM bearing angle as a function of measured frequency and signal-to-noise ratio (SNR).

2.4.1 Overview

The bearing filter described is a first-order Kalman filter. The filter's general equation corresponds to an alpha filter and is given by

$$\hat{x}_1(k) = \Phi_1(k,k-1)\hat{x}_1(k-1) + K_1(k)\left[z(k) - H_1(k)\Phi_1(k,k-1)\hat{x}_1(k-1)\right], \tag{26}$$

where the subscript indicates the first-order filter. For the first-order bearing filter we define

$$\hat{x}_1(k) = \left[\hat{b}_1(k)\right], \tag{27}$$

$$\Phi_1(k, k-1) = [1], \tag{28}$$

$$z(k) = b_m(k), \tag{29}$$

and

$$H_1(k) = [1]. \tag{30}$$

In the above, $\hat{x}_1(k-1)$ is the 1 × 1 vector of filtered bearing estimates valid at time $(k-1)$ from the previous iteration and valid before the current bearing data is received. Time $(k-1)$ is the last time the track was updated (time of previous cluster update), and time k is the current cluster measurement time. $z(k) = b_m(k)$ is the bearing angle measurement from the current CD. $\Phi_1(k, k-1)$ is a 1 × 1 state or plant transition model and describes the emitter motion from time segment $(k-1)$ to k. $H_1(k)$ is a 1 × 1 measurement transformation vector, and due to the form of H_1, $K_1(k)$ is a 1 × 1 vector of Kalman gain coefficients that weight the measurement residuals.

The above general equation for the filter update (Eq. 26) may also be simplified by introducing the predicted estimates and the predicted measurement terms. In terms of the predicted estimates, the general form of the filter equation may be written as

$$\hat{x}_1(k) = \hat{x}_1(k, k-1) + K_1(k)\left[z(k) - H_1(k)\hat{x}_1(k, k-1)\right], \tag{31}$$

where

$$\hat{x}_1(k, k-1) = \Phi_1(k, k-1)\hat{x}_1(k-1) = \hat{b}_1(k, k-1), \tag{32}$$

and $\hat{b}_1(k,k\text{-}1)$ is defined as the predicted bearing. Defining the predicted measurement as

$$\hat{z}_1(k, k-1) = H_1(k)\hat{x}_1(k, k-1) = \hat{b}_1(k, k-1) \tag{33}$$

allows the final form of the general Kalman filter equation to be written in compact form as

$$\hat{x}_1(k) = \hat{x}_1(k, k-1) + K_1(k)\left[z(k) - \hat{z}_1(k, k-1)\right]. \tag{34}$$

Due to the form of H_1 above, the form of the Kalman gain equation is recognized as 1 × 1. Taking this into account and expanding the above yields one simple filter equation for bearing:

$$\hat{b}_1(k) = \hat{b}_1(k,k-1) + K_1(k)\cdot\left[b_m(k) - \hat{b}_1(k,k-1)\right]. \tag{35}$$

The sequence $\gamma_1(k) = z(k) - \hat{z}_1(k,k-1)$ is known as the innovation sequence and represents the new information to the filter. This term is also passed up to the IMM bearing filter for model probability updates. Finally, expanding the predicted estimate defined as $\Phi_1(k,k-1)\hat{x}_1(k-1)$ yields the expansion for the filter predictions as

$$\hat{b}_1(k,k-1) = \hat{b}_1(k-1). \tag{36}$$

2.4.2 Initialization

The single-state bearing angle Kalman filter may be initialized using one of two equivalent methods.

Method 1

In this method, the bearing filter is initialized with a large variance:

$$\hat{x}_1(0) = 0 \text{ and } v_1(0) = 10^{12}, \tag{37}$$

where $v_1(0)$ is the initial 1 × 1 filtered covariance value. A large covariance value simply indicates no *a priori* information and, therefore, large uncertainty.

Method 2

In this method, the bearing filter is initialized from the first cluster measurement of bearing angle. In this case we define

$$\hat{x}_1(1) = b_m(1) \text{ and } v_1(1) = v_{mc}(1), \tag{38}$$

where $b_m(1)$ is the bearing measurement provided with the first cluster and $v_{mc}(1)$ is the computed measurement variance associated with the same bearing measurement discussed in Section 2.4.5. The first measurement is used to initialize the filter and the filter iteration starts at time $k = 2$ with the second measurement, while in Method 1, the iteration starts at time $k = 1$ with the first measurement.

2.4.3 Predicted Estimate

When a new measurement is associated with the bearing track, the first step in the filter update process requires that the ETF estimate be predicted to the time of the current associated measurement (time alignment). For the bearing filter, the predicted estimate is obtained from the transition matrix defined above (Eq. 28), the filtered estimate $\hat{x}_1(k-1)$ from the previous ETF update, and the time difference between the associated contact and the ETF time (time of the last track update) as

$$\hat{x}_1(k,k-1) = \Phi_1(k,k-1)\,\hat{x}_1(k-1) \tag{39}$$

or, equivalently,

$$\hat{b}_1(k,k-1) = \hat{b}_1(k-1), \tag{40}$$

where the 1 × 1 vector $\hat{x}_1(k,k-1)$ is the prediction of $\hat{x}_1(k-1)$ to the current CD time and $\hat{x}_1(k-1)$ is the previous filtered estimate obtained from the ETF.

2.4.4 Predicted Variance and Plant Noise Estimates

Once the prediction is computed, computation of the Kalman filter gain is required to merge the bearing CD data with the bearing angle of a track. The Kalman gain computation requires an estimate of the predicted variance and the measurement variance. The 1 × 1 predicted variance is given by

$$v_1(k,k-1) = v_1(k-1) + \tilde{v}_1(k), \tag{41}$$

where $\tilde{v}_1(k)$ is the 1 × 1 plant noise estimate and $v_1(k-1)$ is obtained from the track file (ETF). For the bearing filter, the plant noise is set to

$$\tilde{v}_1(k) = b_1^{(a)}(k)\cdot w_1^{(a)}\cdot b_1^{(a)}(k)', \tag{42}$$

where

$$b_1^{(a)}(k) = 1. \tag{43}$$

$w_1^{(a)}$ is a constant and should be approximately set to

$$w_1^{(a)} = 0.1\pi/180/40 \cong 0.00005. \tag{44}$$

Therefore,

$$\tilde{v}_1(k) = 0.00005. \tag{45}$$

As a final note, $w_1^{(a)}$ is a tuning parameter that controls filter bandwidth and should be optimized during testing.

2.4.5 Filter Gain and Measurement Noise Estimate

The 1 × 1 Kalman gain K_1 is given by

$$K_1(k) = \frac{v_1(k,k-1)H_1'}{H_1 v_1(k,k-1)H_1' + v_{mc}(k)}. \tag{46}$$

We note here that a simple divide is used and that no matrix inverse operation is required since the term $H_1 v_1(k,k-1)H_1'$ is a scalar. However, to compute this gain, an estimate of CD bearing measurement noise is required. If the CD bearing measurement noise is not included in the cluster message it can be computed from the single pulse specification variance according to the following equation:

$$v_{mc}(k) = \frac{v_m(k)}{n}, \tag{47}$$

where the variable $v_{mc}(k)$ is the measurement noise variance in bearing associated with the cluster estimate. The variable $v_m(k)$ is the single pulse measurement accuracy (equivalent to the PDW accuracy) of the bearing angle, and *n* is the number of pulses reported in the CD message. For the bearing estimate, the measurement accuracy is a function of frequency, SNR, and measured emitter bearing angle. These features must be represented as such in a suitable tabular format. This representation should be obtained from system testing and should follow the form

$$v_m(k) = \left(\sigma_a \frac{f_0}{f_m(k)}\right)^2, \tag{48}$$

where σ_a should be characterized and is the base bearing accuracy, which is a function of measured emitter bearing angle, and f_0 is the constant frequency at which σ_a is measured. The term $f_m(k)$ is the frequency measurement associated with the bearing angle provided in the cluster descriptor. $v_m(k)$ should be inversely proportional to SNR and limited to a constant that represents best-case bearing accuracy.

The actual value of σ_a must be obtained from system measurements and testing and used here if reliable tracking is to be expected. This same comment applies to the elevation filters. Note also that σ_a should be an over-bounded approximate estimate of accuracy in bearing angle as a function of frequency and measured bearing angle.

The Kalman gain may also be written in terms of the covariance of the innovation sequence as

$$K_1(k) = \frac{v_1(k,k-1)H_1'}{S_1(k)}, \tag{49}$$

where $S_1(k)$ is the scalar covariance of the innovation sequence defined above and is given by the equation

$$S_1(k) = H_1 v_1(k,k-1)H_1' + v_{mc}(k). \tag{50}$$

This term is also passed up to the IMM bearing filter for model probability updates.

2.4.6 The Filtered Estimate

After the Kalman gains are computed, the new filtered or ETF track estimate is given by

$$\hat{x}_1(k) = \hat{x}_1(k,k-1) + K_1(k)\{z(k) - \hat{z}_1(k,k-1)\} \tag{51}$$

or

$$\hat{x}_1(k) = \hat{x}_1(k,k-1) + K_1(k)\gamma_1(k), \tag{52}$$

where $\hat{x}_1(k)$ is the filtered bearing estimate, $z(k)$ is the current bearing cluster measurement, and $\gamma_1(k)$ is the innovation sequence. As mentioned above (Eq. 35), this is equivalent to computing

$$\hat{b}_1(k) = \hat{b}_1(k,k-1) + K_1(k)\gamma_1(k). \tag{53}$$

2.4.7 Filtered Variance

The filtered variance of this estimate is simply

$$v_1(k) = [1 - K_1(k)H_1]v_1(k,k-1), \tag{54}$$

which may be expanded as

$$v_1(k) = [1 - K_1(k)]v_1(k,k-1). \tag{55}$$

2.4.8 Track Coasting

There will be times when an emitter bearing estimate is not available. For example, this may be due to ESM system failures, improper dwell scheduling, and emitter fading due to low SNR. Since, in these cases, bearing estimates will not be available, the bearing track should be coasted to the time associated with the bearing update. The coasting duration time is a function of track confidence as well as track status. This is discussed in a different document.

If the bearing ETF track is in coast mode, the corresponding elevation and ESM parameters must also be in coast mode. It is possible, and at times necessary, to update a bearing track while coasting the corresponding elevation track.

It should also be noted that for these reasons, the initiate and drop track criteria for bearing angle should be evaluated separately from the associated elevation track. That is, bearing tracks should be allowed to initiate and drop without reference to the elevation track. However, as mentioned earlier, if a bearing track does not continue to exist, then the corresponding elevation and ESM parameter track will also cease to exist.

The simplest way to derive the filter for coasting is to recognize that coasting is equivalent to receiving a measurement with zero accuracy (or infinite measurement variance). Using this analogy implies the Kalman gain (see the appropriate equation above) goes to zero and the filtered estimate simply becomes the predicted estimate. The sequence for filtering during a bearing coast is the same for each of the three bearing filters. Coasting a track is achieved by setting the filter gain to zero (equivalent to setting the measurement variance to infinity).

With the Kalman gain set to zero, the remaining equations for filtered estimate and variance are the same as those defined above (Eqs. 51-53). In this case, the filtered estimate is given by the prediction

$$\hat{x}_1(k) = \hat{x}_1(k, k-1) \tag{56}$$

or, equivalently,

$$\hat{b}_1(k) = \hat{b}_1(k, k-1), \tag{57}$$

and the filtered variance takes the form

$$v_1(k) = v_1(k, k-1). \tag{58}$$

2.5 The Two-State Bearing Filter

The following describes the theory used for the recursive, two-state bearing angle filter. This section describes both the initialization and the update of the filter. The purpose of the filter is to estimate bearing for nominal trajectory emitter platforms and to compute optimal time-varying bearing gates.

The bearing filter is a second-order Kalman filter that models both bearing and bearing rate. This is one of the three independent bearing filters used in the IMM architecture. The first- and third-order bearing angle Kalman filters are described in separate sections. The filter assumes that the measurement error variance is properly defined for the ESM bearing angle as a function of measured frequency and SNR.

2.5.1 Overview

The bearing filter described is a second-order Kalman filter. The filter's general equation corresponds to an alpha-beta filter and is given by

$$\hat{x}_2(k) = \Phi_2(k,k-1)\hat{x}_2(k-1) + K_2(k)\left[z(k) - H_2(k)\Phi_2(k,k-1)\hat{x}_2(k-1)\right], \tag{59}$$

where for the bearing filter we define

$$\hat{x}_2(k) = \begin{bmatrix} \hat{b}_2(k) \\ \hat{\dot{b}}_2(k) \end{bmatrix}, \tag{60}$$

$$\Phi_2(k,k-1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \tag{61}$$

$$z(k) = b_m(k), \tag{62}$$

and

$$H_2(k) = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{63}$$

In the above, $\hat{x}_2(k-1)$ is the 2 × 1 vector of filtered bearing estimates valid at time $k-1$ from the previous iteration and valid before the current bearing data is received. Δt is the difference between the current cluster measurement time and the last time the track was updated (time of previous cluster update). $z(k) = b_m(k)$ is the bearing angle measurement from the current CD. $\Phi_2(k,k-1)$ is a 2 × 2 state or plant transition model and describes the emitter motion from time segment $k-1$ to k. $H_2(k)$ is a 1 × 2 measurement transformation vector, and due to the form of H_2 , $K_2(k)$ is a 2 × 1 vector of Kalman gain coefficients that weight the measurement residuals.

The above general equation for the filter update may also be simplified by introducing the predicted estimates and the predicted measurement terms. In terms of the predicted estimates, the general form of the filter equation may be written as

$$\hat{x}_2(k) = \hat{x}_2(k,k-1) + K_2(k)\left[z(k) - H_2(k)\hat{x}_2(k,k-1)\right], \tag{64}$$

where

$$\hat{x}_2(k,k-1) = \Phi_2\left(k,k-1\right)\hat{x}_2\left(k-1\right) = \begin{bmatrix} \hat{b}_2(k,k-1) \\ \hat{\dot{b}}_2(k,k-1) \end{bmatrix} = \begin{bmatrix} \hat{b}_2\left(k-1\right) + \Delta t \cdot \hat{\dot{b}}_2\left(k-1\right) \\ \hat{\dot{b}}_2\left(k-1\right) \end{bmatrix}, \tag{65}$$

where $\hat{b}_2(k,k-1)$ is defined as the predicted bearing estimate and $\hat{\dot{b}}_2(k,k-1)$ is defined as the predicted bearing rate estimate. Defining the predicted measurement as

$$\hat{z}_2\left(k,k-1\right) = H_2\left(k\right)\hat{x}_2\left(k,k-1\right) = \hat{b}_2\left(k,k-1\right) \tag{66}$$

allows the final form of the general Kalman filter equation to be written in compact form as

$$\hat{x}_2\left(k\right) = \hat{x}_2\left(k,k-1\right) + K_2\left(k\right)\left\{z\left(k\right) - \hat{z}_2\left(k,k-1\right)\right\}. \tag{67}$$

Due to the form of H_2 above, the form of the Kalman gain is recognized as 2 × 1. Taking this into account and expanding the above yields two simple filter equations for the bearing and bearing rate filters:

$$\begin{cases} \hat{b}_2(k) = \hat{b}_2(k,k-1) + K_2^{(1)}\left(k\right)\left\{z\left(k\right) - \hat{z}_2\left(k,k-1\right)\right\} \\ \hat{\dot{b}}_2(k) = \hat{\dot{b}}_2(k,k-1) + K_2^{(2)}\left(k\right)\left\{z\left(k\right) - \hat{z}_2\left(k,k-1\right)\right\}. \end{cases} \tag{68}$$

The sequence $\gamma_2\left(k\right) = z\left(k\right) - \hat{z}_2\left(k,k-1\right)$ is known as the innovation sequence and represents the new information to the filter. This term is also passed up to the IMM bearing filter for model probability updates.

Finally, expanding the predicted estimate defined as $\Phi_2\left(k,k-1\right)\hat{x}_2\left(k-1\right)$ yields the expansion for the filter predictions as

$$\begin{bmatrix} \hat{b}_2(k,k-1) \\ \hat{\dot{b}}_2(k,k-1) \end{bmatrix} = \begin{bmatrix} \hat{b}_2\left(k-1\right) + \Delta t \cdot \hat{\dot{b}}_2\left(k-1\right) \\ \hat{\dot{b}}_2\left(k-1\right) \end{bmatrix}. \tag{69}$$

2.5.2 Initialization

The two-state bearing angle Kalman filter may be initialized using any one of three equivalent methods.

Method 1

In this method, the bearing filter covariance matrix is initialized with large values on the diagonal:

$$\hat{x}_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } v_2(0) = 10^{12}\,\mathrm{I}_2\,, \tag{70}$$

where

$$\mathrm{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$v_2(0)$ is the initial 2 × 2 filtered covariance value and I_2 is a 2 × 2 identity matrix. A large value simply indicates no *a priori* information and, therefore, large uncertainty.

Method 2

In this method, the bearing filter is initialized from the first cluster measurement of bearing angle. In this case we define

$$\hat{x}_2(1) = \begin{bmatrix} b_m(1) \\ 0 \end{bmatrix} \text{ and } v_2(1) = \begin{bmatrix} v_{mc}(1) & 0 \\ 0 & 10^{12} \end{bmatrix}, \tag{71}$$

where $b_m(1)$ is the bearing measurement provided with the first cluster and $v_{mc}(1)$ is the computed measurement variance associated with the same bearing measurement. The filter iteration starts at time $k = 2$ with the second measurement.

Method 3

In this method, the bearing filter is initialized from the first two cluster measurements of bearing angle. In this case, we define

$$\hat{x}_2(2) = \begin{bmatrix} b_m(2) \\ \dfrac{b_m(2) - b_m(1)}{\Delta t} \end{bmatrix} \tag{72}$$

and

$$v_2(2) = \begin{bmatrix} v_{mc}(2) & \dfrac{v_{mc}(2)}{\Delta t} \\ \dfrac{v_{mc}(2)}{\Delta t} & \dfrac{2v_{mc}(2)}{(\Delta t)^2} \end{bmatrix}. \tag{73}$$

The filter iteration starts at time $k = 3$ with the third measurement.

2.5.3 Predicted Estimate

When a new CD is associated with the bearing track, the first step in the filter update process requires that the ETF estimate be predicted to the time of the current associated CD (time alignment). For the bearing filter, the predicted estimate is obtained from the transition matrix defined above, the filtered estimate $\hat{x}_2(k-1)$ from the previous ETF update, and the time difference between the associated CD and the ETF time (time of the last track update) as

$$\hat{x}_2(k,k-1) = \Phi_2(k,k-1)\,\hat{x}_2(k-1) \tag{74}$$

or, equivalently,

$$\begin{bmatrix} \hat{b}_2(k,k-1) \\ \hat{\dot{b}}_2(k,k-1) \end{bmatrix} = \begin{bmatrix} \hat{b}_2(k-1) + \Delta t \cdot \hat{\dot{b}}_2(k-1) \\ \hat{\dot{b}}_2(k-1) \end{bmatrix}, \tag{75}$$

where the 2 × 1 vector $\hat{x}_2(k,k-1)$ is the prediction of $\hat{x}_2(k-1)$ to the current CD time and $\hat{x}_2(k-1)$ is the previous filtered estimate obtained from the ETF.

2.5.4 Predicted Variance and Plant Noise Estimates

Once the prediction is computed, computation of the the Kalman filter gain is required to merge the bearing CD data with the bearing angle of a track. The Kalman gain computation requires an estimate of the predicted variance and the measurement variance. The 2 × 2 predicted variance is given by

$$v_2(k,k-1) = v_2(k-1) + \tilde{v}_2(k), \tag{76}$$

where $\tilde{v}_2(k)$ is the 2 × 2 plant noise estimate and $v_2(k-1)$ is obtained from the track file (ETF). For the bearing filter, the plant noise is set to

$$\tilde{v}_2(k) = b_2^{(a)}(k) \cdot w_2^{(a)} \cdot b_2^{(a)}(k)', \tag{77}$$

where

$$b_2^{(a)}(k) = \begin{bmatrix} \Delta t \\ 1 \end{bmatrix}. \tag{78}$$

$w_2^{(a)}$ is a constant and should be approximately set to

$$w_2^{(a)} = 0.1\pi/180/40 \cong 0.00005. \tag{79}$$

Therefore,

$$\tilde{v}_2(k) = 0.00005 \cdot \begin{bmatrix} \Delta t^2 & \Delta t \\ \Delta t & 1 \end{bmatrix}. \tag{80}$$

As a final note, $w_2^{(a)}$ is a tuning parameter that controls filter bandwidth and should be optimized during testing.

2.5.5 Filter Gain and Measurement Noise Estimate

The 2 × 1 Kalman gain K_2 is given by

$$K_2(k) = \frac{v_2(k,k-1)H_2'}{H_2 v_2(k,k-1)H_2' + v_{mc}(k)}. \tag{81}$$

We note here that a simple divide is used and that no matrix inverse operation is required since the term $H_2 v_2(k,k-1)H_2'$ is a scalar. However, to compute this gain, an estimate of CD bearing measurement noise is required. If the CD bearing measurement noise is not included in the cluster message it can be computed from the single pulse specification variance according to the following equation:

$$v_{mc}(k) = \frac{v_m(k)}{n}, \tag{82}$$

where the variable $v_{mc}(k)$ is the measurement noise variance in bearing associated with the cluster estimate. The variable $v_m(k)$ is the single pulse measurement accuracy (equivalent to the PDW accuracy) of the bearing angle, and n is the number of pulses reported in the CD message. The computation of $v_m(k)$ is discussed in Section 2.4.5.

The Kalman gain may also be written in terms of the covariance of the innovation sequence as

$$K_2(k) = \frac{v_2(k,k-1)H_2'}{S_2(k)}, \tag{83}$$

where $S_2(k)$ is the scalar covariance of the innovation sequence defined above and is given by the equation

$$S_2(k) = H_2 v_2(k,k-1)H_2' + v_{mc}(k). \tag{84}$$

2.5.6 The Filtered Estimate

After the Kalman gains are computed, the new filtered or ETF track estimate is given by

$$\hat{x}_2(k) = \hat{x}_2(k,k-1) + K_2(k)\left\{z(k) - \hat{z}_2(k,k-1)\right\}, \tag{85}$$

where $x_2(k)$ is the filtered bearing estimate and $z(k)$ is the current bearing cluster measurement. As mentioned above, this is equivalent to computing

$$\begin{cases} \hat{b}_2(k) = \hat{b}_2(k,k-1) + K_2^{(1)}(k)\left\{z(k) - \hat{z}_2(k,k-1)\right\} \\ \hat{\dot{b}}_2(k) = \hat{\dot{b}}_2(k,k-1) + K_2^{(2)}(k)\left\{z(k) - \hat{z}_2(k,k-1)\right\} \end{cases} \tag{86}$$

or, equivalently,

$$\begin{cases} \hat{b}_2(k) = \hat{b}_2(k,k-1) + K_2^{(1)}(k)\gamma_2(k) \\ \hat{\dot{b}}_2(k) = \hat{\dot{b}}_2(k,k-1) + K_2^{(2)}(k)\gamma_2(k), \end{cases} \tag{87}$$

where $\gamma_2(k)$ is the innovation sequence.

2.5.7 Filtered Variance

The filtered variance of this estimate is simply

$$v_2(k) = \left[I_2 - K_2(k)H_2\right] \cdot v_2(k,k-1), \tag{88}$$

which may be expanded as

$$v_2(k) = \begin{bmatrix} 1 - K_2^{(1)}(k) & 0 \\ -K_2^{(2)}(k) & 1 \end{bmatrix} \cdot v_2(k,k-1), \tag{89}$$

where I_2 is the 2 × 2 identity matrix, and $v_2(k,k-1)$ was defined above.

2.5.8 Track Coasting

As discussed in Section 2.5.5, the filter gain is set to

$$K_2(k) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{90}$$

With the Kalman gain set to zero, the remaining equations for filtered estimate and variance are the same as those defined above. In this case, the filtered estimate is given by the prediction

$$\hat{x}_2(k) = \hat{x}_2(k, k-1) \tag{91}$$

or, equivalently,

$$\begin{cases} \hat{b}_2(k) = \hat{b}_2(k, k-1) \\ \hat{\dot{b}}_2(k) = \hat{\dot{b}}_2(k, k-1), \end{cases} \tag{92}$$

and the filtered variance takes the form

$$v_2(k) = v_2(k, k-1). \tag{93}$$

2.6 The Three-State Bearing Filter

The following describes the theory for the recursive, three-state bearing angle filter. This section describes both the initialization and the update of the filter. The purpose of the filter is to estimate bearing for highly maneuvering emitter platforms and to compute optimal time-varying bearing gates.

The bearing filter is a third-order Kalman filter that models bearing, bearing rate and bearing acceleration. This is one of the three independent bearing filters used in the IMM architecture. The first- and second-order bearing angle Kalman filters are described in separate sections. The filter assumes that the measurement error variance is properly defined for the ESM bearing angle as a function of measured frequency and SNR.

2.6.1 Overview

The bearing filter described is a third-order Kalman filter. The filter's general equation corresponds to an alpha-beta-gamma and is given by

$$\hat{x}_3(k) = \Phi_3(k, k-1)\hat{x}_3(k-1) + K_3(k)\left\{z(k) - H_3(k)\Phi_3(k, k-1)\hat{x}_3(k-1)\right\}, \tag{94}$$

where for the bearing filter we define

$$\hat{x}_3(k) = \begin{bmatrix} \hat{b}_3(k) \\ \hat{\dot{b}}_3(k) \\ \hat{\ddot{b}}_3(k) \end{bmatrix}, \tag{95}$$

$$\Phi_3(k, k-1) = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}, \tag{96}$$

$$z(k) = b_m(k), \tag{97}$$

and

$$H_3(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \tag{98}$$

In the above, $\hat{x}_3(k-1)$ is the 3 × 1 vector of filtered bearing estimates valid at time $k-1$ from the previous iteration and valid before the current bearing data is received. Δt is the difference between the current cluster measurement time and the last time the track was updated (time of previous cluster update). $z(k) = b_m(k)$ is the bearing angle measurement from the current cluster descriptor (CD). $\Phi_3(k, k-1)$ is a 3 × 3 state or plant transition model and describes the emitter motion from time segment $(k-1)$ to k. $H_3(k)$ is a 1 × 3 measurement transformation vector and due to the form of H_3, $K_3(k)$ is a 3 × 1 vector of Kalman gain coefficients that weight the measurement residuals.

The above general equation for the filter update may also be simplified by introducing the predicted estimates and the predicted measurement terms. In terms of the predicted estimates, the general form of the filter equation may be written as

$$\hat{x}_3(k) = \hat{x}_3(k, k-1) + K_3(k)\left\{z(k) - H_3(k)\hat{x}_3(k, k-1)\right\}, \tag{99}$$

where

$$\hat{x}_3(k, k-1) = \Phi_3(k, k-1)\hat{x}_3(k-1) = \begin{bmatrix} \hat{b}_3(k, k-1) \\ \hat{\dot{b}}_3(k, k-1) \\ \hat{\ddot{b}}_3(k, k-1) \end{bmatrix}$$
$$= \begin{bmatrix} \hat{b}_3(k-1) + \Delta t \cdot \hat{\dot{b}}_3(k-1) + 0.5 \cdot \Delta t^2 \cdot \hat{\ddot{b}}_3(k-1) \\ \hat{\dot{b}}_3(k-1) + \Delta t \cdot \hat{\ddot{b}}_3(k-1) \\ \hat{\ddot{b}}_3(k-1) \end{bmatrix}, \tag{100}$$

where $\hat{b}_3(k,k-1)$ is defined as the predicted bearing estimate, $\hat{\dot{b}}_3(k,k-1)$ is defined as the predicted bearing rate estimate, and $\hat{\ddot{b}}_3(k,k-1)$ is defined as the predicted bearing acceleration estimate. Defining the predicted measurement as

$$\hat{z}_3\left(k,k-1\right)=H_3\left(k\right)\hat{x}_3\left(k,k-1\right)=\hat{b}_3\left(k,k-1\right) \tag{101}$$

allows the final form of the general Kalman filter equation to be written in compact form as

$$\hat{x}_3\left(k\right)=\hat{x}_3\left(k,k-1\right)+K_3\left(k\right)\left\{z\left(k\right)-\hat{z}_3\left(k,k-1\right)\right\}. \tag{102}$$

Due to the form of the H_3 matrix, the Kalman gain is recognized as a 3 × 1 matrix. Taking this into account and expanding the above yields three simple filter equations for the bearing, bearing rate and bearing acceleration filters:

$$\begin{cases}\hat{b}_3(k)=\hat{b}_3(k,k-1)+K_3^{(1)}\left(k\right)\left\{z\left(k\right)-\hat{z}_3\left(k,k-1\right)\right\}\\ \hat{\dot{b}}_3(k)=\hat{\dot{b}}_3(k,k-1)+K_3^{(2)}\left(k\right)\left\{z\left(k\right)-\hat{z}_3\left(k,k-1\right)\right\}\\ \hat{\ddot{b}}_3(k)=\hat{\ddot{b}}_3(k,k-1)+K_3^{(3)}\left(k\right)\left\{z\left(k\right)-\hat{z}_3\left(k,k-1\right)\right\}.\end{cases} \tag{103}$$

The sequence $\gamma_3\left(k\right)=z\left(k\right)-\hat{z}_3\left(k,k-1\right)$ is known as the innovation sequence and represents the new information to the filter. This term is also passed up to the IMM bearing filter for model probability updates.

Finally, expanding the predicted estimate defined as $\Phi\left(k,k-1\right)\hat{x}\left(k-1\right)$ yields the expansion for the filter predictions as

$$\begin{bmatrix}\hat{b}_3(k,k-1)\\ \hat{\dot{b}}_3(k,k-1)\\ \hat{\ddot{b}}_3(k,k-1)\end{bmatrix}=\begin{bmatrix}\hat{b}_3\left(k-1\right)+\Delta t\cdot\hat{\dot{b}}_3\left(k-1\right)+0.5\cdot\Delta t^2\cdot\hat{\ddot{b}}_3\left(k-1\right)\\ \hat{\dot{b}}_3\left(k-1\right)+\Delta t\cdot\hat{\ddot{b}}_3\left(k-1\right)\\ \hat{\ddot{b}}_3\left(k-1\right)\end{bmatrix}. \tag{104}$$

2.6.2 Initialization

The three-state bearing angle Kalman filter may be initialized using one of three methods.

Method 1

In this method, the bearing filter covariance matrix is initialized with large values on the diagonal:

$$\hat{x}_3(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } v_3(0) = 10^{12} \cdot \mathrm{I}_3, \quad (105)$$

where

$$\mathrm{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$v_3(0)$ is the initial 3 × 3 filtered covariance value and I_3 is a 3 × 3 identity matrix. A large value simply indicates no *a priori* information and, therefore, large uncertainty.

Method 2

In this method, the bearing filter is initialized from the first cluster measurement of bearing angle. In this case, we define

$$\hat{x}_3(1) = \begin{bmatrix} b_m(1) \\ 0 \\ 0 \end{bmatrix} \text{ and } v_3(1) = \begin{bmatrix} v_{mc}(1) & 0 & 0 \\ 0 & 10^{12} & 0 \\ 0 & 0 & 10^{12} \end{bmatrix}, \quad (106)$$

where $b_m(1)$ is the bearing measurement provided with the first cluster and $v_{mc}(1)$ is the computed measurement variance associated with the same bearing measurement. The filter iteration starts at time $k = 2$.

Method 3

In this method, the bearing filter is initialized from the first two cluster measurements of bearing angle. In this case, we define

$$\hat{x}_3(2) = \begin{bmatrix} b_m(2) \\ \dfrac{b_m(2) - b_m(1)}{\Delta t} \\ 0 \end{bmatrix} \quad (107)$$

and

$$v_3(2) = \begin{bmatrix} v_{mc}(2) & \frac{v_{mc}(2)}{\Delta t} & 0 \\ \frac{v_{mc}(2)}{\Delta t} & \frac{2v_{mc}(2)}{(\Delta t)^2} & 0 \\ 0 & 0 & 10^{12} \end{bmatrix}. \tag{108}$$

The filter iteration starts at time $k = 3$ with the third measurement.

2.6.3 Predicted Estimate

When a new CD is associated with the bearing track, the first step in the filter update process requires that the ETF estimate be predicted to the time of the current associated CD (time alignment). For the bearing filter, the predicted estimate is obtained from the transition matrix defined above, the filtered estimate $\hat{x}_3(k-1)$ from the last ETF update, and the time difference between the associated CD and the ETF time (time of the last track update) as

$$\hat{x}_3(k,k-1) = \Phi_3(k,k-1)\,\hat{x}_3(k-1) \tag{109}$$

or, equivalently,

$$\begin{bmatrix} \hat{b}_3(k,k-1) \\ \hat{\dot{b}}_3(k,k-1) \\ \hat{\ddot{b}}_3(k,k-1) \end{bmatrix} = \begin{bmatrix} \hat{b}_3(k-1) + \Delta t \cdot \hat{\dot{b}}_3(k-1) + 0.5 \cdot \Delta t^2 \cdot \hat{\ddot{b}}_3(k-1) \\ \hat{\dot{b}}_3(k-1) + \Delta t \cdot \hat{\ddot{b}}_3(k-1) \\ \hat{\ddot{b}}_3(k-1) \end{bmatrix}, \tag{110}$$

where the 3 × 1 vector $\hat{x}_3(k,k-1)$ is the prediction of $\hat{x}_3(k-1)$ to the current CD time and $\hat{x}_3(k-1)$ is the previous filtered estimate obtained from the ETF.

2.6.4 Predicted Variance and Plant Noise Estimates

Once the prediction is computed, computation of the the Kalman filter gain is required to merge the bearing CD data with the bearing angle of a track. The Kalman gain computation requires an estimate of the predicted variance and the measurement variance. The 3 × 3 predicted variance is given by

$$v_3(k,k-1) = v_3(k-1) + \tilde{v}_3(k), \tag{111}$$

where $\tilde{v}_3(k)$ is the 3 × 3 plant noise estimate and $v_3(k-1)$ is obtained from the track file (ETF). For the bearing filter, the plant noise is set to

$$\tilde{v}_3(k) = b_3^{(a)}(k) \cdot w_3^{(a)} \cdot b_3^{(a)}(k)', \tag{112}$$

where

$$b_3^{(a)}(k) = \begin{bmatrix} \Delta t^2/2 \\ \Delta t \\ 1 \end{bmatrix}. \tag{113}$$

$w_3^{(a)}$ is a constant and should be approximately set to

$$w_3^{(a)} = 0.1\pi/180/40 \cong 0.00005. \tag{114}$$

Therefore,

$$\tilde{v}_3(k) = 0.00005 \cdot \begin{bmatrix} \Delta t^4/4 & \Delta t^3/2 & \Delta t^2/2 \\ \Delta t^3/2 & \Delta t^2 & \Delta t \\ \Delta t^2/2 & \Delta t & 1 \end{bmatrix}. \tag{115}$$

As a final note, $w_3^{(a)}$ is a tuning parameter that controls filter bandwidth and should be optimized during testing.

2.6.5 Filter Gain and Measurement Noise Estimate

The 3 × 1 Kalman gain K_3 is given by

$$K_3(k) = \frac{v_3(k,k-1)H_3'}{H_3 v_3(k,k-1)H_3' + v_{mc}(k)}. \tag{116}$$

Note here that a simple divide is used and that no matrix inverse operation is required since the term $H_3 v_3(k,k-1)H_3'$ is a scalar. However, to compute this gain, an estimate of CD bearing measurement noise is required. If the CD bearing measurement noise is not included in the cluster message it can be computed from the single pulse specification variance according to the equation

$$v_{mc}(k) = \frac{v_m(k)}{n}, \tag{117}$$

where the variable $v_{mc}(k)$ is the measurement noise variance in bearing associated with the cluster estimate. The variable $v_m(k)$ is the single pulse measurement accuracy (equivalent to the PDW accuracy) of the bearing angle, and *n* is the number of pulses reported in the cluster descriptor message. The computation of $v_m(k)$ is discussed in Section 2.4.5.

The 3 × 1 Kalman gain may also be written in terms of the covariance of the innovation sequence as

$$K_3(k) = \frac{v_3(k,k-1)H_3'}{S_3(k)}, \tag{118}$$

where $S_3(k)$ is the scalar covariance of the innovation sequence defined above and is given by the equation

$$S_3(k) = H_3 v_3(k,k-1)H_3' + v_{mc}(k). \tag{119}$$

2.6.6 The Filtered Estimate

After the Kalman gains are computed, the new filtered or ETF track estimate is given by

$$\hat{x}_3(k) = \hat{x}_3(k,k-1) + K_3(k)\left\{z(k) - \hat{z}_3(k,k-1)\right\}, \tag{120}$$

where $\hat{x}_3(k)$ is the filtered bearing estimate and $z(k)$ is the current bearing cluster measurement. As mentioned above, this is equivalent to computing

$$\begin{cases} \hat{b}_3(k) = \hat{b}_3(k,k-1) + K_3^{(1)}(k)\left\{z(k) - \hat{z}_3(k,k-1)\right\} \\ \hat{\dot{b}}_3(k) = \hat{\dot{b}}_3(k,k-1) + K_3^{(2)}(k)\left\{z(k) - \hat{z}_3(k,k-1)\right\} \\ \hat{\ddot{b}}_3(k) = \hat{\ddot{b}}_3(k,k-1) + K_3^{(3)}(k)\left\{z(k) - \hat{z}_3(k,k-1)\right\} \end{cases} \tag{121}$$

or, equivalently,

$$\begin{cases} \hat{b}_3(k) = \hat{b}_3(k,k-1) + K_3^{(1)}(k)\gamma_3(k) \\ \hat{\dot{b}}_3(k) = \hat{\dot{b}}_3(k,k-1) + K_3^{(2)}(k)\gamma_3(k) \\ \hat{\ddot{b}}_3(k) = \hat{\ddot{b}}_3(k,k-1) + K_3^{(3)}(k)\gamma_3(k), \end{cases} \tag{122}$$

where $\gamma_3(k)$ is the innovation sequence.

2.6.7 Filtered Variance

The filtered variance of this estimate is simply

$$v_3(k) = \left[I_3 - K_3(k)H_3\right]v_3(k,k-1), \tag{123}$$

which may be expanded as

$$v_3(k) = \begin{bmatrix} 1-K_3^{(1)}(k) & 0 & 0 \\ -K_3^{(2)}(k) & 1 & 0 \\ -K_3^{(3)}(k) & 0 & 1 \end{bmatrix} \cdot v_3(k,k-1), \tag{124}$$

where I_3 is the 3×3 identity matrix, and $v_3(k,k-1)$ was defined above.

2.6.8 Track Coasting

As discussed in Section 2.5.8, when a track is to be coasted, the filter gain is set to

$$K_3(k) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{125}$$

With the Kalman gain set to zero, the remaining equations for filtered estimate and variance are the same as those defined above. In this case, the filtered estimate is given by the prediction

$$\hat{x}_3(k) = \hat{x}_3(k,k-1) \tag{126}$$

or, equivalently,

$$\begin{cases} \hat{b}_3(k) = \hat{b}_3(k, k-1) \\ \hat{\dot{b}}_3(k) = \hat{\dot{b}}_3(k, k-1) \\ \hat{\ddot{b}}_3(k) = \hat{\ddot{b}}_3(k, k-1), \end{cases} \tag{127}$$

and the filtered variance takes the form

$$v_3(k) = v_3(k, k-1). \tag{128}$$

3. THE ELEVATION FILTER

The following describes the mathematics for the recursive elevation filter. Both the initialization and the update of the elevation filter are described. The purpose of the filter is to estimate elevation and to compute time-varying elevation gates.

The elevation filter is a second-order Kalman filter that models both elevation and elevation rate. This same filter structure is used for the second-order bearing filter. The filter assumes that the measurement error variance is properly defined for elevation angle as a function of measured frequency and SNR.

3.1 Overview

The elevation filter is a second-order Kalman filter. The filter equation corresponds to an alpha-beta filter and is given by

$$\hat{x}(k) = \Phi(k, k-1)\hat{x}(k-1) + K(k)\left[z(k) - H(k)\Phi(k, k-1)\hat{x}(k-1)\right], \tag{129}$$

where for the elevation filter, we define

$$\hat{x}(k) = \begin{bmatrix} \hat{e}(k) \\ \hat{\dot{e}}(k) \end{bmatrix}, \tag{130}$$

$$\Phi(k, k-1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \tag{131}$$

$$z(k) = e_m(k), \tag{132}$$

and

$$H(k) = [1 \quad 0]. \tag{133}$$

In the above, $\hat{x}(k)$ is a 2 × 1 vector of filtered estimates valid at time k-1 from the previous iteration and valid before the current elevation data is received. Δt is the difference between the current cluster measurement time and the last time the track was updated (time of previous cluster update). $z(k) = e_m(k)$ is the elevation angle measurement from the current CD. $\Phi(k, k-1)$ is a 2 × 2 state or plant transition model and describes the emitter motion from time segment $k-1$ to k . $K(k)$ is a 2 × 1 vector of Kalman gain coefficients and $H(k)$ is a 1 × 2 measurement vector.

The above equation for the filter update may also be written as

$$\hat{x}(k) = \hat{x}(k, k-1) + K(k)\left[z(k) - H(k)\hat{x}(k, k-1)\right], \tag{134}$$

where the above is written in terms of the predicted estimate

$$\hat{x}(k, k-1) = \hat{e}(k, k-1) . \tag{135}$$

Expanding the above yields the two equations for elevation and elevation rate as

$$\begin{cases} \hat{e}(k) = \hat{e}(k, k-1) + K^{(1)}(k)\left[z(k) - \hat{z}(k, k-1)\right] \\ \hat{\dot{e}}(k) = \hat{\dot{e}}(k, k-1) + K^{(2)}(k)\left[z(k) - \hat{z}(k, k-1)\right], \end{cases} \tag{136}$$

where $K^{(1)}(k)$ is equivalent to the gain, α, used by an alpha-beta filter and $K^{(2)}(k)$ is equivalent to $\beta/\Delta t$.

In addition, the predicted estimates presented above are defined as follows:

$$\hat{e}(k, k-1) = \hat{e}(k-1) + \Delta t \cdot \hat{\dot{e}}(k-1) \tag{137}$$

is the predicted elevation angle,

$$\hat{\dot{e}}(k, k-1) = \hat{\dot{e}}(k-1) \tag{138}$$

is the predicted elevation angle rate, and

$$\hat{z}(k, k-1) = H(k)\Phi(k, k-1)\hat{x}(k-1) = \hat{e}(k, k-1) \tag{139}$$

is the predicted elevation cluster measurement.

The sequence $z(k)-\hat{z}(k,k-1)$ is known as the innovation sequence and represents the new information to the filter.

3.2 Initialization

The elevation filter may be initialized using one of three equivalent methods.

Method 1

In this method, the bearing filter covariance matrix is initialized with large values on the diagonal:

$$\hat{x}(0)=\begin{bmatrix}0\\0\end{bmatrix} \text{ and } v_e(0)=10^{12}\cdot \mathrm{I}_2\,, \tag{140}$$

where

$$\mathrm{I}_2=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$

$v_e(0)$ is the initial 2 × 2 filtered covariance value and I_2 is a 2 × 2 identity matrix. A large value simply indicates no *a priori* information.

Method 2

In this method, the elevation filter is initialized from the first cluster measurement of elevation angle. In this case, we define

$$\hat{x}(1)=\begin{bmatrix}e_m(1)\\0\end{bmatrix} \text{ and } v_e(1)=\begin{bmatrix}v_{ce}(1) & 0\\0 & 10^{12}\end{bmatrix}, \tag{141}$$

where $e_m(1)$ is the elevation measurement provided with the first cluster and $v_{ce}(1)$ is the measurement accuracy associated with the elevation measurement. The filter iteration starts at time $k=2$ with the second measurement.

Method 3

In this method, the elevation filter is initialized from the first two cluster measurements of elevation angle. In this case, we define

$$\hat{x}(2) = \begin{bmatrix} e_m(2) \\ \dfrac{e_m(2) - e_m(1)}{\Delta t} \end{bmatrix} \text{ and } v_e(2) = \begin{bmatrix} v_{ce}(2) & \dfrac{v_{ce}(2)}{\Delta t} \\ \dfrac{v_{ce}(2)}{\Delta t} & \dfrac{2v_{ce}(2)}{(\Delta t)^2} \end{bmatrix}. \tag{142}$$

The filter iteration starts at time $k = 3$. This is the preferred method since it starts the filter when a CD elevation measurement has correlated with a first point. Therefore, the initial elevation CD is not the first ETF track entry.

3.3 Predicted Estimate

When a new CD is associated with the elevation track, the first step in the filter update process requires that the ETF estimate be predicted to the time of the current associated CD (time alignment). For the elevation filter, the predicted estimate is obtained from the transition matrix defined above, the filtered estimate $\hat{x}(k)$, and the time difference between the associated CD and the ETF time (time of the last track update) as

$$\hat{x}(k,k-1) = \Phi(k,k-1)\,\hat{x}(k-1) = \begin{bmatrix} \hat{e}(k,k-1) \\ \hat{\dot{e}}(k,k-1) \end{bmatrix} = \begin{bmatrix} \hat{e}(k-1) + \Delta t \cdot \hat{\dot{e}}(k-1) \\ \hat{\dot{e}}(k-1) \end{bmatrix}, \tag{143}$$

where $\hat{x}(k,k-1)$ is the prediction of $\hat{x}(k-1)$ to the current CD time and $\hat{x}(k-1)$ is the filtered estimate obtained from the ETF.

3.4 Predicted Variance and Plant Noise Estimates

Once the prediction is computed, computation of the Kalman filter gain is required to merge the elevation CD data with the elevation track. The Kalman gain computation requires an estimate of the predicted variance and the measurement variance. The predicted variance is given by

$$v_e(k,k-1) = v_e(k-1) + \tilde{v}_e(k), \tag{144}$$

where $\tilde{v}_e(k)$ is the plant noise estimate. For the elevation filter, the plant noise is set to

$$\tilde{v}_e(k) = b_e(k) \cdot w_e \cdot b_e(k)', \tag{145}$$

where

$$b_e(k) = \begin{bmatrix} \dfrac{\Delta t^2}{2} \\ \Delta t \end{bmatrix} \quad \text{and} \quad w_e = \sigma_e^2. \tag{146}$$

σ_e^2 is a constant and should be approximately set to

$$\sigma_e^2 = 0.1 \cdot \pi / 180 / 40 \cong 0.00005 . \tag{147}$$

Therefore,

$$\tilde{v}_e(k) = 0.00005 \cdot \begin{bmatrix} \left(\frac{\Delta t^2}{2}\right)^2 & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 \end{bmatrix} . \tag{148}$$

3.5 Filter Gain and Measurement Noise Estimate

The Kalman gain is given by

$$K(k) = \frac{v_e(k, k-1) H'}{H v_e(k, k-1) H' + v_{ce}(k)} . \tag{149}$$

We note here that a simple divide is used and that no matrix inverse operation is required since the term $H v_e(k, k-1) H'$ is a scalar. However, to compute this gain, an estimate of CD elevation measurement noise is required. If the CD elevation measurement noise is not included in the cluster message, it can be computed from the single pulse specification variance according to the equation

$$v_{ce}(k) = \frac{v_{me}(k)}{n} , \tag{150}$$

where $v_{ce}(k)$ is the measurement noise variance in elevation associated with the cluster estimate, $v_{me}(k)$ is the single pulse measurement accuracy (equivalent to the PDW accuracy) of the elevation angle, and n is the number of pulses reported in the cluster descriptor message. The computation of $v_{me}(k)$ was discussed previously.

The Kalman gain may also be written in terms of the innovation sequence as

$$K(k) = \frac{v_e(k,k-1) \cdot H'}{S(k)}, \tag{151}$$

where $S(k)$ is the scalar covariance of the innovation sequence defined above and is given by the scalar

$$S(k) = H \cdot v_e(k,k-1) \cdot H' + v_{ce}(k). \tag{152}$$

3.6 The Filtered Estimate

After the Kalman gains are computed, the new filtered or ETF track estimate is given by

$$\hat{x}(k) = \hat{x}(k,k-1) + K(k)\left[z(k) - \hat{z}(k,k-1)\right], \tag{153}$$

where $\hat{x}(k)$ is the filtered elevation estimate and $z(k)$ is the current elevation cluster measurement. As mentioned above, this is equivalent to computing

$$\begin{cases} \hat{e}(k) = \hat{e}(k,k-1) + K^{(1)}(k)\left[z(k) - \hat{z}(k,k-1)\right] \\ \hat{\dot{e}}(k) = \hat{\dot{e}}(k,k-1) + K^{(2)}(k)\left[z(k) - \hat{z}(k,k-1)\right]. \end{cases} \tag{154}$$

3.7 Filtered Variance

The filtered variance of this estimate is simply

$$v_e(k) = \left[I_2 - K(k)H\right]v_e(k,k-1), \tag{155}$$

where I_2 is the 2×2 identity matrix, and $v_e(k,k-1)$ was defined above.

3.8 Track Coasting

To coast a track, the filter gain is set to

$$K(k) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{156}$$

With the Kalman gain set to zero, the remaining equations for filtered estimate and variance are the same as those defined above. In this case, the filtered estimate is given by the prediction

$$\hat{x}(k) = \hat{x}(k, k-1) \tag{157}$$

and the filtered variance takes the form

$$v_e(k) = v_e(k, k-1)\,. \tag{158}$$

4. THE ESM PARAMETERS FILTER

The ESM parameters filter is a simple first-order Kalman filter that may be used to update both agile and non-agile parameters as well as automatically generate gates for ESM front-end sorting. The filter equation for all ESM parameters is identical to an alpha-only filter and is given by

$$x(k) = x(k-1) + \alpha\left[z(k) - x(k-1)\right], \tag{159}$$

where *x* is the parameter to be stored in the track file and may be represented by frequency, pulsewidth, pulse repetition interval (PRI), or amplitude, and *z* is the current measured and averaged parameters from the cluster descriptor.

4.1 Initialization

Each ESM parameters filter may be initialized using one of two equivalent methods.

Method 1

The filter is initialized with

$$x(0) = 0 \ \text{ and } \ v_f(0) = 10^{12}$$
$$k = 0$$
$$Gate \ = \ \text{ESM parameter default size,}$$

where $v_f(k)$ is the associated filtered covariance value. A large value simply indicates no *a priori* information. In this case, $v_f(k)$ is a scalar since the filters are one-dimensional. *Gate* is the default ESM gate size associated with the front-end default gates for the parameter of interest. The default gate results in initial clusters, which represent first points to the ETF.

Method 2

In this method, the ESM parameters filter is initialized from the first cluster measurement. In this case, we define

$$x(1) = z_m(1) \text{ and } v_f(1) = v_{cz}(1), \tag{160}$$

where $z_m(1)$ is the ESM parameter measurement of interest (frequency, PRI, pulsewidth, or amplitude) and $v_{cz}(1)$ is its associated scalar covariance. $v_{cz}(k)$ is the measurement accuracy associated with the ESM parameter measurement provided in the cluster descriptor. The filter iteration starts at time $k = 2$.

4.2 Predicted Estimate

For the ESM parameters filter, the predicted estimate is simply the filtered estimate since no higher-order derivatives are used for ESM parameter tracking:

$$x(k, k-1) = x(k-1), \tag{161}$$

where $x(k, k-1)$ is the one step prediction of $x(k-1)$.

4.3 Predicted Variance, Measurement Noise, and Plant Noise Estimates

The predicted variance is given by

$$v_p(k, k-1) = v_f(k-1) + \tilde{v}_p(k), \tag{162}$$

where $\tilde{v}_p(k)$ is the plant noise estimate. For the case with jitter, the cluster plant noise may be estimated as

$$v_{pc}(k) = \frac{(x_{\max} - x_{\min})^2}{12}, \tag{163}$$

where $v_{pc}(k)$ refers to the measured plant noise or spread (max minus min) obtained from the parameter measurements provided in the cluster descriptor, and $(x_{\max}, x_{\min})$ are the largest and the smallest parameter measurement in the cluster descriptor, respectively. The average plant noise may be obtained by using an appropriate alpha filter with gain limited to 0.1, or

$$\tilde{v}_p(k) = \tilde{v}_p(k-1) + 0.1 \cdot \left[\frac{(x_{\max} - x_{\min})^2}{12} - \tilde{v}_p(k-1) \right], \tag{164}$$

where this filter is initialized with the first measurement of spread provided in the cluster descriptor.

This will accurately estimate the plant noise if the measurement noise is negligible by comparison (always the case for a jittered measurement). If the estimate of parameter spread is small, then no agility is present and the plant noise should be set to one-tenth of the measurement noise for heavier smoothing. This is checked for by implementing the test described below.

For

$$v_{pc}(k) \le 4 v_{cz}(k) \;\text{ or }\; \text{Freq Type} = \text{'Fixed'} ,$$

set

$$\tilde{v}_p(k) = \frac{v_{cz}(k)}{10} \;.$$

4.4 Filter Gain

The filter gain, alpha, is a function of the predicted variance. This gain is computed after the cluster is received and begins the filter iteration for the current update. The Kalman gain is given by

$$\alpha(k) = \frac{v_p(k,k-1)}{v_p(k,k-1) + v_{cz}(k)} . \tag{165}$$

However, to compute this gain, an estimate of parameter measurement noise is required. This is computed as

$$v_{cz}(k) = \frac{v_{mz}(k)}{n} , \tag{166}$$

where $v_{cz}(k)$ is the measurement noise variance associated with the cluster estimate, $v_{mz}(k)$ is the single pulse (PWD) measurement accuracy of the parameter of interest, and *n* is the number of pulses reported in the cluster descriptor message.

If unknown, the cluster measurement variance may be approximated by

$$v_{cz}(1) = \frac{(1.5 \cdot res)^2}{n} , \tag{167}$$

where *res* is the resolution of the parameter of interest.

4.5 Filtered Estimate

The filtered estimate is given by

$$x(k) = x(k-1) + \alpha(k)\left[z(k) - x(k-1)\right], \tag{168}$$

where $x(k)$ is the filtered ESM parameters estimate and $z(k)$ is the current ESM cluster measurement. Due to the initialization, for the first ESM parameter update this should yield

$$\alpha \cong 1 \text{ and } x(1) \cong z(1). \tag{169}$$

4.6 Filtered Variance

The filtered variance of this estimate is simply given by

$$v_f(k) = \left[1 - \alpha(k)\right] v_p(k, k-1). \tag{170}$$

For the first cluster update, this will be approximately equal to the initial cluster measurement variance

$$v_f(1) \cong v_{cz}(1). \tag{171}$$

5. SUMMARY

An ESM angle tracker was developed to provide highly accurate angle and angle rate estimates to a combat system. The tracker was developed to operate with the NRL ESM-ATD that provides accurate but intermittent bearing and elevation reports. This report describes the theory behind the bearing, elevation, and ESM parameters filters contained within the tracker.

The bearing filter is an IMM filter that automatically adjusts to optimally track stationary, constant-velocity, or constant-acceleration targets. The IMM bearing filter uses three standard Kalman filters of different orders as a basis for filtering and predicting emitter bearing angle. Details on how to combine estimates from the first-, second-, and third-order Kalman filters within the IMM are developed and described in this report.

The inclusion of ESM parameters in the tracking algorithm makes it possible to associate and correlate intermittent bearing/elevation measurements with existing tracks in dense scenarios. The ESM parameters filters are first-order Kalman filters, but they have the ability to track both constant and jittered parameters by automatically adjusting the filter plant noise. The technique for estimating plant noise from the received measurements is described in this report.

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