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**ANALYTICAL MODELING OF LAMB  
WAVES FOR STRUCTURAL HEALTH  
MONITORING (PREPRINT)**

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## ABSTRACT

Structural health monitoring techniques are being developed to reduce cost, increase availability, and maintain safety of current and future air vehicle systems. Various techniques have been investigated depending on the scale of the damage to be detected. For example, damage such as fastener failure may have a more global effect on the structural dynamics and therefore modal-based damage detection techniques may be suitable. This paper focuses on detecting smaller scale damage, such as cracking or corrosion, which typically has a highly localized effect on the system dynamics. The use of Lamb waves, guided elastic waves in a plate, has shown promise in detecting such highly localized damage due to the relatively short wavelengths of the propagating waves. However, the Lamb wave behavior is fairly complex as various waveforms may exist and the waves are dispersive, so the wave speed is a function of frequency. To examine the complex Lamb wave behavior, analytical models are being developed. This paper explores the use of explicit time integration finite element analysis. Key modeling issues are addressed including appropriate time increments and element lengths for accurate, yet efficient, solutions and the material properties used for the media through which the wave propagates. With these issues addressed, attention is focused on the effects of damage on the Lamb waves and the use various excitation waveforms. Lastly, potential improvements through advanced techniques, such as beamforming, are discussed.

## INTRODUCTION

The use of Lamb waves, guided elastic waves in a plate, has shown promise as a structural health monitoring (SHM) technique to detect highly localized damage, such as cracking or corrosion. However, the Lamb wave behavior is fairly complex as various waveforms may exist and the waves are dispersive, so the wave speed is a function of frequency. Significant literature exists on Lamb waves, so only a brief review of the relevant behavior is given below. To examine the complex Lamb

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wave behavior, explicit finite element analyses have been performed. Key modeling issues, such as temporal and spatial resolution, material properties, excitation waveforms, and the effects of damage, are addressed. Lastly, potential improvements through advanced techniques, such as beamforming and beamsteering, are discussed.

## **DAMAGE DETECTION USING LAMB WAVES**

Lamb waves are guided elastic waves that occur in a free plate. Because of their behavior, Lamb waves are particularly useful for investigating damage in plate and shell structures. Lamb waves can exhibit symmetric and antisymmetric waveforms, based on whether the out-of-plane displacements on either side of the plate are in or out of phase. As guided waves, Lamb waves also are dispersive and the wave speed, or phase velocity, is a function of frequency. Due to dispersion, Lamb waves become distorted as they propagate. This results in various waves with slightly different frequencies and wavelengths being excited in a structure. These waves interact and the resulting wave propagates at a group velocity which may differ from the individual phase velocities. For damage detection purposes, the group velocity can be thought of as the signal velocity, or the velocity at which energy is conveyed through a structure.

As frequency increases, the number of simultaneously existing waveforms also increases. To limit the number of coexisting waveforms, the excitation frequency is typically kept relatively low such that only the  $S_0$  and  $A_0$  waveforms will exist. These waveforms,  $S_0$  and  $A_0$ , are referred to as the fundamental modes and are typically used for Lamb wave damage detection. For damage detection purposes, it is also useful to analyze only a single waveform. As a general rule, elastic waves can be used to detect damage on the order of the wavelength. The  $A_0$  Lamb mode has a much lower wave speed than the  $S_0$  mode, and therefore has a smaller wavelength making it more sensitive to smaller levels of damage. However, the symmetric waveform has a substantially greater group velocity and would therefore arrive at a sensor well before the antisymmetric waveform. As a result, reflections of the  $S_0$  wave may arrive at a sensor simultaneously with the initial  $A_0$  wave, corrupting the measured signal. In addition to these issues, consideration must be given to the distribution of the particle displacements through the thickness of the material for the selected mode, as these displacements relate to the defect sensitivity (through the thickness). Giurgiutiu [1] has shown that, by adjusting the excitation frequency, it is possible to tune certain transducers to excite a single mode (either  $S_0$  or  $A_0$ ) dominantly.

## **ANALYTICAL MODELING**

One approach to modeling Lamb wave propagation is to numerically solve the governing wave equations with the appropriate boundary conditions. This approach can be taken for simple geometries, but becomes difficult for more complicated geometries or when damage is included. In such cases, different computational

techniques can be used to analyze wave propagation. Explicit finite element methods, which step through the solution in time, are one of the more popular techniques, since numerous finite element codes exist and it is not necessary to develop specialized code. Major advantages of the analytical modeling over experimental testing include the ability to investigate different wave propagation cases relatively quickly, as well as the capability to examine the wave behavior through the entire thickness of the material rather than solely based on surface measurements.

To model guided wave propagation using the finite element method, various three-dimensional (3D) and two-dimensional (2D) models can be utilized. Full 3D models typically can capture all aspects of the wave propagation behavior such as the waveform, reflections off boundaries or damage, or any other wave interactions. However, such models often are relatively large, require significant computational resources, and may become impractical. As a result, 2D plane strain and shell models have been utilized. For the 2D plane strain case, models are created in the length and thickness directions. These models can be used to investigate the motion of the upper and lower surfaces of the plate (e.g. to examine symmetric and antisymmetric waveforms), but do not incorporate any effects in the width direction. The elimination of any influences in the width direction can be useful, particularly to examine basic wave propagation down a plate. Shell models can be utilized when reflections and the interferences of various waveforms are important, but detailed modeling of the behavior of the upper and lower surfaces of a plate is not required.

The following paragraphs address some of the critical modeling issues, including temporal and spatial resolution, material properties, excitation waveforms, and the effects of damage. Specific details relating to each of these areas are given below. Lastly, to demonstrate analytical Lamb wave modeling, an example problem is presented and analytical predictions are compared with theoretical and experimental results.

### Temporal and Spatial Resolution

The stability and accuracy of the finite element solution are dependent on the temporal and spatial resolution used for the analysis. To avoid numerical instability, the CFL condition (named for Courant, Friedrichs, and Lewy) must be satisfied [2]. This condition prevents a longitudinal wave from traveling completely through an element during a single time step, and can be stated:

$$\Delta t < L_{\min} / C_L \quad (1)$$

where  $\Delta t$  is the time step used for the finite element solution,  $L_{\min}$  is the minimum element edge length used in the finite element model, and  $C_L$  is the longitudinal wave speed. In addition to the CFL condition, the time step should also obey the following guideline [3]:

$$\Delta t < 1 / (20 f_{\max}) \quad (2)$$

where  $\Delta t$  is again the time step used for the finite element solution and  $f_{\max}$  is the maximum wave frequency of interest. This relation ensures that there are at least 20 time steps taken during the cycle of a wave at the highest frequency. Lastly, a limit is placed on the size of the elements used in the finite element model [4]:

$$L_e < \lambda_{\min} / 10 \quad (3)$$

where  $L_e$  is the typical edge length of the finite elements used for the solution and  $\lambda_{\min}$  is the smallest wavelength to be analyzed. This relation ensures that there are at least 10 elements across the smallest wavelength of interest (although others recommend using at least 20 elements [3]).

### **Material Properties**

For accurate analytical solutions, particular attention must be given to the material properties used for the simulations. The general solutions to the elastic wave equations are based on homogeneous, isotropic properties. However, in many cases, some level of anisotropy exists. As a result, analytical simulations may predict the wave behavior more precisely than the theoretical dispersion curves calculated assuming isotropic properties. The elastic modulus typically affects both the symmetric and antisymmetric waves, whereas the shear modulus typically has a much larger effect on the antisymmetric waves due to their flexural behavior. For cases with significant anisotropy, such as in most composite materials, analytical modeling may offer significant benefits.

### **Excitation Waveform**

The excitation signals used to excite Lamb waves are typically Hanning-windowed sine bursts. Pulsed, rather than continuous signals are used to allow time of flight calculations and prevent unwanted interference between waves. By using a higher number of cycles, the signal strength can typically be increased. However, the cycles span a greater time period, increasing the likelihood that reflections may occur and interfere with the measured response. Often an additional  $\frac{1}{2}$  cycle is added to an integer number of cycles to provide a peak at the center of the signal. The Hanning-window is used to reduce the amount of energy at frequencies other than the excitation frequency.

### **Effects of Damage**

Although Lamb wave techniques are typically performed at lower frequencies than those used for highly localized ultrasonic NDE testing, similar damage detection methods can be utilized. Ultrasonic NDE techniques are based on the propagation and reflection of elastic waves, with the assumption that damage in the structure will alter the behavior of the waves. Typical ultrasonic methods include pitch-catch and pulse-echo techniques. For pitch-catch techniques, elastic waves are generated using an actuating transducer at one location on the structure, and the response is recorded using a sensing transducer at a different location. Damage is

detected by examining the attenuation and/or dispersion of the transmitted wave. The severity of the damage can be assessed based on the wave attenuation or dispersion. However, to locate the damage, various pairs of pitch-catch transducers may be required. For pulse-echo techniques, the same transducer used to create the elastic waves is used to measure the response at a later time. Damage is detected by investigating echoes in the measured response due to wave reflections off damaged regions. The time of flight can be used to locate the damage and the amplitude of the reflected signal can be used to assess the severity of the damage.

### Modeling Example

To demonstrate the application of finite element modeling, simulated analytical results have been compared with theoretical values as well as measured results from laboratory experiments. Figure 1 shows a schematic of the aluminum sheet specimen used for experimental Lamb wave studies. Finite element simulations have been performed using ABAQUS/Explicit [5], an explicit time integration finite element code based on the central difference method. Both 2D plane strain and shell models have been investigated. For the 2D plane strain case, models are created in the length and thickness directions using four-node plane strain elements (CPE4 elements in ABAQUS). The shell models contain four-node, bilinear shell elements (S4R elements in ABAQUS).

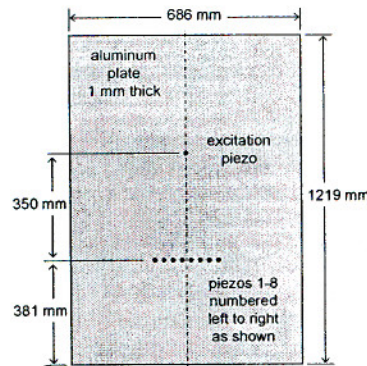


Figure 1. Aluminum sheet test article with piezoelectric transducers used for experimental and analytical studies

Lamb waves are excited using surface-mounted piezoelectric transducers which are bonded to the sheet at various locations. Excitation of the models is accomplished using point forces and moments at locations around the perimeter of the piezoelectric discs. For 2D plane strain models, point forces are applied at either edge of the disc to represent the expansion or contraction of the disc due to the applied voltage. For shell models, point forces and moments are applied at the nodes around the perimeter of the disc. Moments are required since the midplane of the aluminum sheet is modeled, but the discs are attached to the outer surface of the structure. For both models, the piezoelectric discs are not explicitly modeled and any contribution to the mass or stiffness of the structure due to the disc is assumed to be negligible. The piezoelectric discs also are used to sense the strains created due

to the Lamb waves. Based on the piezoelectric characteristics of the ceramic disc, the measured output voltage should be proportional to the radial strain experienced by the disc.

Initial simulations have been performed using a 2D plane strain model with a minimum (and typical) element edge length of 0.1 mm. Based on the longitudinal wave speed, Equation 1 requires that the time step used for the solution should be less than 16.5 nsec. A time step of 5.0 nsec has been used for the simulations. Simulations have been performed at frequencies up to 2 MHz. Therefore, per Equation 2, the time step should be less than 25.0 nsec to have at least 20 time steps per cycle. The 5.0 nsec time step used for the simulations meets this criterion. Lastly, based on the shear wave velocity and a maximum frequency of 2 MHz, the smallest wavelength can be estimated as 1.53 mm. As mentioned above, these studies use a model with a typical element edge length of 0.1 mm, which is less than the 0.15 mm edge length specified by Equation 3.

The test specimen shown in Figure 1 consists of a sheet of cold rolled aluminum. Cold rolling typically contributes to minor anisotropy which is usually ignored. Results from experimental testing and analytical modeling assuming isotropic properties show that the measured and predicted wave speeds, particularly for the  $A_0$  mode, do not correlate well. The analytical  $A_0$  wave arrives much sooner than the measured wave. To account for the anisotropic behavior of the aluminum sheet, which likely results due to the cold rolling process, it has been assumed that the aluminum is a non-Hookean cubic material. As such, the elastic modulus, shear modulus, and Poisson's ratio are needed to fully describe the elastic behavior of the sheet. Unlike the isotropic case, however, these three parameters are independent of each other. Subsequent analytical modeling for this work used an elastic modulus of 68.95 GPa, a shear modulus of 19.44 GPa, a Poisson's ratio of 0.33, and a density of 2.768 g/cm<sup>3</sup>.

Analytical simulations have been performed using the 2D plane strain model with cubic material properties at excitation frequencies ranging from 100 kHz to 1.5 MHz. A five cycle, Hanning-windowed, sine wave excitation signal has been utilized. Figure 2 shows a comparison between experimental and simulated results at excitation frequencies of 100, 200, 300, and 400 kHz. As shown in the figure, the analytical results capture the arrival of the initial  $S_0$  wave at the various excitation frequencies quite well. At later times, the analytical waveforms are typically much cleaner than the experimental results, since the 2D plane strain models cannot capture any interactions off the boundaries of the plate. These results illustrate the potential corruption of the  $A_0$  wave due to simultaneous arrival of reflections of the  $S_0$  wave. The group velocity of the  $S_0$  wave has been calculated based on the travel time between the excitation and response transducers. Due to dispersion, the waveform arriving at the response piezoelectric may differ from the five cycle excitation signal. Therefore, the center of the energy of the wave burst has been used to calculate the speed. Experimental testing also has been performed and a similar technique used to process the measured data. Figure 3 shows that the predicted group velocities from the analytical simulations compare well with corresponding results from experimental testing as well as the theoretical  $S_0$  dispersion curves.

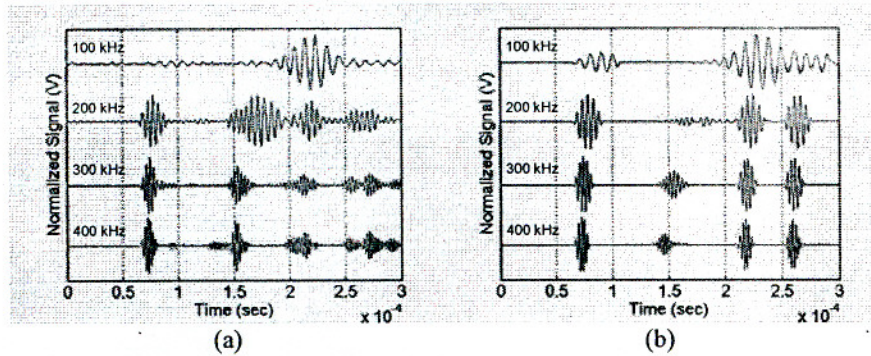


Figure 2. Experimental (a) and simulated 2D plane strain (b) results using five cycle, Hanning-windowed, sine wave at various excitation frequencies

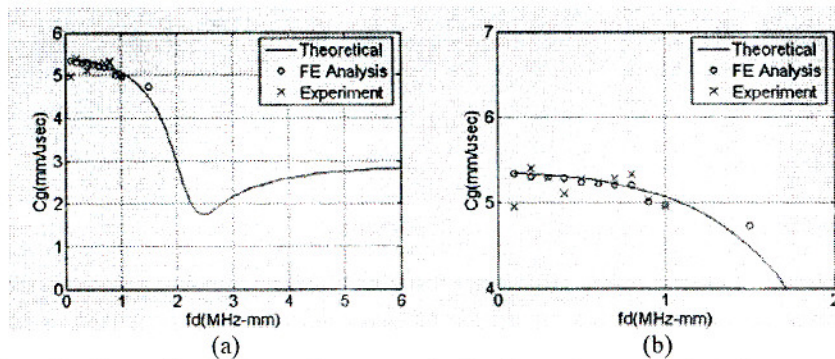


Figure 3. Comparison between  $S_0$  group velocity dispersion curves from theory, analysis, and experiment (a) and zoomed plot around region of interest (b)

To demonstrate the potential use of Lamb waves to detect damage, simulations have been performed using a shell model with a 9 mm, horizontal, through-crack centered between the excitation and response piezoelectric transducers and located on the plate centerline. A five cycle, Hanning-windowed, sine wave excitation at 300 kHz has been used. Figure 4 shows a comparison between the predicted results at piezoelectric 4 for the healthy (undamaged) plate and in the presence of a crack. As shown in the figure, the effects of damage can be seen using the pitch-catch technique when the damage is located between the excitation and response transducers.

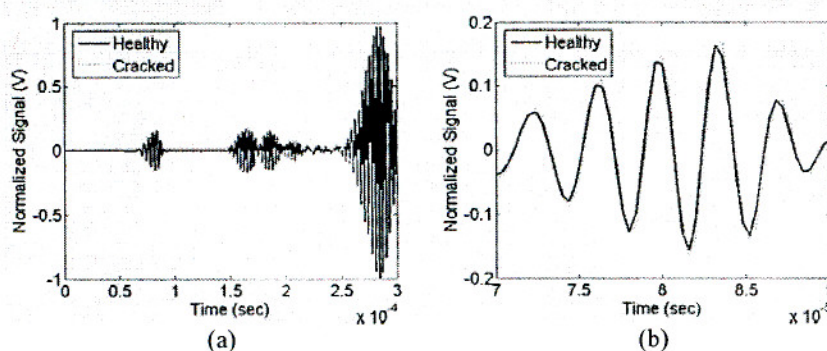


Figure 4. Comparison between healthy and cracked simulation results (a) and zoomed plot around region of interest (b)

## ADVANCED TECHNIQUES

Advanced techniques under consideration include using an array of piezoelectric transducers to generate or sense elastic waves in the structure under inspection. By adjusting the spacing and/or phasing between the piezoelectric transducers, the transmitted or received wave can be focused in a specific direction. The techniques to generate and sense elastic waves using transducer arrays are often referred to as beamsteering and beamforming, respectively. In beamsteering, time delays are applied to the excitation signal sent to each element in an array to focus the transmitted energy in a specific direction. Similarly, in beamforming, time delays are applied to the received signals from each element in an array to reinforce the desired signals arriving from a specific direction or reduce unwanted signals arriving from specific directions. For both beamsteering and beamforming, the arrays can be collinear or two-dimensional. Additional benefits can be achieved by using adaptive arrays, where adjustable weights are applied to the signals of each element of the array.

## CONCLUSIONS

Lamb waves can be utilized for SHM to detect highly localized damage, such as cracking or corrosion. However, Lamb wave behavior is fairly complex and must be well understood if such waves are to be used. The use of explicit finite element analyses for accurate, yet efficient, simulation of Lamb wave behavior has been demonstrated. Key modeling issues, such as temporal and spatial resolution, material properties, excitation waveforms, and the effects of damage, have been addressed and appropriate analytical techniques to account for these issues have been presented. With the basic Lamb wave behavior understood, advanced techniques such as beamforming and beamsteering can be utilized to improve the performance of SHM systems.

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