

Exploitation of ISAR Imagery in Euler Parameter Space

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ABSTRACT

Efforts are being made to exploit the full-polarimetric radar scattering nature of ground targets to extract maximum information, enabling target identification and classification. These efforts have taken varied approaches to decomposing the polarimetric scattering matrix into more meaningful, phenomenological parameter spaces. The Euler parameters have potential value in target classification but have historically met with limited success due to ambiguities that arise in the decomposition as well as the parameters sensitivity to noise and target movement. Using polarimetric ISAR signatures obtained from stationary targets in compact radar ranges at the University of Massachusetts Lowell Submillimeter Technology Laboratory (STL)^{1,2,3,4} and the U.S. Army National Ground Intelligence Center (NGIC), correlation studies were performed in the Euler parameter space to assess its impact on improving target classification. Methods for deriving explicit transform equations that minimize ambiguities will be presented, as well as the results of the correlation studies.

Keywords: HRR, Ka-Band, polarimetric, signature, ISAR, Euler

1.0 INTRODUCTION

The U.S. Army National Ground Intelligence Center (NGIC) sponsored and directed a radar imaging project exploring the reproducibility of high-resolution target signatures, specifically of main battle tanks (MBT). The project entailed acquiring full-polarimetric Ka-band radar signature data at Edlin AFB, in addition to its sub millimeter-wave compact radar range equivalent. Using exact 1/16th scale replicas designed and fabricated through the ERADS program, the equivalent signatures were collected using the NGIC Ka-band compact radar range in conjunction with the University of Massachusetts Lowell Submillimeter Technology Laboratory (STL). Under NGIC's sponsorship, the full-polarimetric signatures were processed and analyzed to evaluate methods for improving target identification. The foremost method comprises correlating ISAR images of the targets in magnitude space. The effort to transform the Ka-band MBT ISAR images into Euler parameter space is an extension of the original project.

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2.0 ISAR IMAGERY IN MAGNITUDE SPACE

A typical Inverse Synthetic Aperture Radar (ISAR) configuration contains a coherent radar source and receiver co-aligned for the purpose of measuring the back-scattered radiation. When the radar sweeps through a band of frequencies, the Doppler effect enables a Fourier transform of the resulting data to reveal a down-range profile of the target's spatial scattering distribution. Similarly, when the scattering target is rotated through a small angle, the Doppler effect enables a Fourier transform of the resulting data to reveal a cross-range profile of the target's spatial scattering distribution. When both approaches are combined, a two-dimensional spatial image of the target's scattering properties may be created. The use of a full-polarimetric ISAR system enables the measurement of the magnitude and phase of the reflected signal for an orthogonal set of polarization states. A two-dimensional ISAR image can be formed, wherein each resolution cell contains all of the scattering information available for the target in that cell. This information is represented by the scattering matrix S ,

$$S = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \quad (1)$$

where, S_{vh} for example, corresponds to the amplitude and phase of back-scattered radiation measured when horizontally-polarized waves are transmitted and vertically-polarized waves are received. Traditionally, the phases are neglected and the magnitudes are imaged separately (Figure 1). To completely characterize a target, a set of ISAR images at incremental azimuths are obtained, spanning the full 360° rotation. Correlation studies of ISAR images are being pursued under the support of NGIC, for the purpose of improving target identification.⁴ When two separate targets are imaged at the same center frequency, resolution, azimuth and elevation, the similarity of the ISAR images is an indication of the similitude of the targets. For a given corresponding resolution cell on both targets, the difference of the RCS values in dBsm divided by the sum of the values gives the relative difference in magnitude scattering between the two targets. The average of all of the cell comparisons in the image gives the average percent difference for that azimuth look angle. When such an image to image correlation is made for all of the ISAR images over the 360° azimuth sweep, the resulting average percent difference can be taken to represent the total physical difference between the two targets. Target recognition can be achieved when the average percent difference between a known and unknown target becomes minimal when compared to a host of targets.

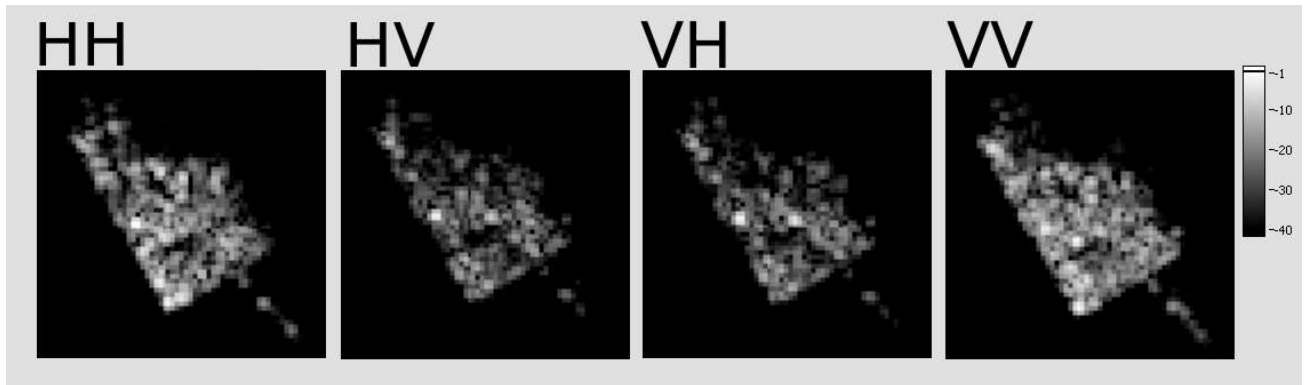


Figure 1. ISAR RCS Magnitude Images (dBsm) of the T-72BK Tank Fingerprint

3.0 ISAR IMAGERY IN EULER PARAMETER SPACE

In magnitude space, the scattering information has limited physical meaning and technological usefulness. The problem is to transform the scattering matrix so as to establish more physically relevant parameters, and to reduce the limitations seemingly inherent in the transform process. Autonne⁵ showed that a complex symmetric matrix S , can be diagonalized by applying a consimilarity transform in the following way,

$$S_D = U^T S U \quad (2)$$

The unitary transform matrix U is constructed out of the conjugate eigenvectors, $\mathbf{x}$, that satisfy the conjugate eigenvalue equation,

$$S \mathbf{x}_i = \lambda_i \mathbf{x}_i^* \quad (3)$$

Kennaugh⁶ related this mathematical concept to radar polarimetry by designating S as the scattering matrix, and by describing the conjugate eigenvectors as optimal polarization states. Once the scattering matrix is decomposed into a transform matrix U and a diagonal eigenvalue matrix S_D , physically meaningful parameters can be defined using

$$S_D = \begin{bmatrix} m e^{i2v} & 0 \\ 0 & m \tan^2(\gamma) e^{-i2v} \end{bmatrix} \quad (4)$$

$$U = \begin{bmatrix} \cos(\psi) \cos(\tau) - i \sin(\psi) \sin(\tau) & -\sin(\psi) \cos(\tau) + i \cos(\psi) \sin(\tau) \\ \sin(\psi) \cos(\tau) + i \cos(\psi) \sin(\tau) & \cos(\psi) \cos(\tau) + i \sin(\psi) \sin(\tau) \end{bmatrix} \quad (5)$$

These are the Euler parameters and have the following physical meanings as established by Huynen⁷: m is the maximum magnitude of target reflectivity. The parameter ψ denotes the orientation angle at which the maximum reflectivity occurs. Furthermore, τ is the symmetry angle, v is the bounce angle, and γ represents the polarizability angle. The goal is to find the explicit transform equations that define the Euler parameters in terms of the original scattering matrix. Once found and applied, the ISAR images in magnitude space can be transformed into images in Euler parameter space.

The scattering matrix S (1) can be simplified and put into more useful notation. Reciprocity for mono static measurements ensures $S_{hv} = S_{vh}$. This reduces the meaningful data in the scattering matrix to three complex numbers, or six real components. There is an overall phase factor which represents the distance to the object. Because it is dependent on the radar and not the object itself, it is meaningless in our present analysis and is factored out. The scattering matrix becomes

$$S = e^{ig} \begin{bmatrix} a e^{ib} & c \\ c & d e^{if} \end{bmatrix} \quad \text{where} \quad a = |S_{hh}|, \quad b = \text{Arg}(S_{hh}) - \text{Arg}(S_{hv}), \quad c = |S_{hv}|, \quad d = |S_{vv}|, \quad (6)$$

$$f = \text{Arg}(S_{vv}) - \text{Arg}(S_{hv}), \quad g = \text{Arg}(S_{hv})$$

Thus the scattering matrix has been reduced to five meaningful parameters, a, b, c, d, f . For future convenience, the following parameters can be defined:

$$M = \sqrt{a^4 + d^4 + 4a^2c^2 + 4c^2d^2 - 2a^2d^2 + 8ad c^2 \cos(b+f)} \quad (7)$$

$$N = a^2 + 2c^2 + d^2 \quad (8)$$

$$L = \sqrt{(a^2 - d^2)^2 + 4c^2(a \cos(b) + d \cos(f))^2} \quad (9)$$

The coneigenvalue equation (3) can be cast into a modified form and treated as a regular eigenvalue equation.

$$S S^* \mathbf{x}_i = |\lambda_i|^2 \mathbf{x}_i \quad (10)$$

The eigenvalues of the above equation can easily be found, which then yields the square of the magnitude of the eigenvalues of the original coneigenvalue equation. From these, the first two Euler parameters, m and γ are found,

$$m = \sqrt{\frac{N+M}{2}} \quad (11)$$

$$\gamma = \tan^{-1} \left[\frac{N-M}{N+M} \right]^{1/4} \quad (12)$$

In order to reduce ambiguities and simplify derivation, an intermediate form for the transform matrix U is used,

$$U = \frac{1}{\sqrt{1+|p|^2}} \begin{bmatrix} e^{i\alpha} & -p^* e^{-i\alpha} \\ p e^{i\alpha} & e^{-i\alpha} \end{bmatrix} \quad (13)$$

The approach consists of deriving the final variables in terms of the intermediate variables and the intermediate variables in terms of the original known variables. Once found, the intermediate variables may be removed and the Euler parameters can be known in terms of the scattering matrix data. The final variables can be related to the intermediate variables by equating the transform matrix U in the final form (5) to the transform matrix in the intermediate form (13). After equating components, the following relations are derived.

$$\tau = \frac{1}{2} \sin^{-1} \left(\frac{2 \Im(p)}{1+|p|^2} \right) \quad (14)$$

$$\psi = \tan^{-1} \left(\frac{2 \Re(p)}{\sqrt{(1+|p|^2)^2 - 4 \Im(p)^2} + 1 - |p|^2} \right) \quad (15)$$

$$\tan(\alpha) = -\tan(\psi) \tan(\tau) \quad (16)$$

To derive the intermediate variables p and α in terms of the known parameters the consimilarity transform equation (2) is

used. Substituting the intermediate form of the transform matrix U (13), the known scattering matrix S (6), and the diagonalized scattering matrix S_D (4) into the transform equation, the following form results,

$$\begin{bmatrix} \sqrt{N+M} e^{i2\nu} & 0 \\ 0 & \sqrt{N-M} e^{-i2\nu} \end{bmatrix} = \frac{\sqrt{2} e^{ig}}{1+|p|^2} \begin{bmatrix} e^{i\alpha} & p e^{i\alpha} \\ -p^* e^{-i\alpha} & e^{-i\alpha} \end{bmatrix} \begin{bmatrix} a e^{ib} & c \\ c & d e^{if} \end{bmatrix} \begin{bmatrix} e^{i\alpha} & -p^* e^{-i\alpha} \\ p e^{i\alpha} & e^{-i\alpha} \end{bmatrix} \quad (17)$$

It should be noted that the magnitude of the eigenvalues in the diagonalized scattering matrix on the left side have already been found and substituted in. After multiplying out the matrices and equating the bottom left components, the set of equations is solved for p

$$p = \frac{2c(a e^{ib} + d e^{-if})}{M + (a^2 - d^2)} \quad (18)$$

With relationships between intermediate variables and final variables as well as between original variables and intermediate variables, the two systems can be linked to eliminate the intermediate variables. Substituting p (18) into the equations for τ (14) and ψ (15), the final forms of the next two Euler parameters are found,

$$\tau = \frac{1}{2} \sin^{-1} \left(\frac{2c(a \sin(b) - d \sin(f))}{M} \right) \quad (19)$$

$$\psi = \tan^{-1} \left(\frac{2c(a \cos(b) + d \cos(f))}{a^2 - d^2 + L} \right) \quad (20)$$

Applying the results found above and the relations in (16), the intermediate variable α is established,

$$\alpha = \tan^{-1} \left[\left(\frac{a \sin(b) - d \sin(f)}{a \cos(b) + d \cos(f)} \right) \frac{a^2 - d^2 - L}{M + L} \right] \quad (21)$$

Returning to the coneigenvalue equation (3) and using the intermediate transform matrix, two matrix equations result. After substituting the values for p (18) and α (21), one can solve for the last Euler parameter,

$$\nu = l + n + \tan^{-1} \left[\left(\frac{a \sin(b) - d \sin(f)}{a \cos(b) + d \cos(f)} \right) \frac{a^2 - d^2 - L}{M + L} \right] + \frac{1}{4} \tan^{-1}(y/x) \quad \text{where} \quad (22)$$

$$y = (2d^2 - N - M)a^2 \sin(2b) - (2a^2 - N + M)d^2 \sin(2f) + ((a^2 - d^2 + M)^2 - 8c^4) \frac{ad}{2c^2} \sin(b-f)$$

$$x = (2d^2 - N - M)a^2 \cos(2b) + (2a^2 - N + M)d^2 \cos(2f) + ((a^2 - d^2 + M)^2 - 8c^4) \frac{ad}{2c^2} \cos(b-f)$$

$$l = 0 \text{ if } x > 0, \quad \pi \text{ if } x < 0 \text{ and } y > 0, \quad -\pi \text{ if } x < 0 \text{ and } y < 0$$

$$n = -\pi/2, 0, \pi/2 \text{ chosen so that } \frac{-\pi}{4} \leq \nu \leq \frac{\pi}{4}$$

The final equations relating the Euler parameters to the scattering matrix have been found (11), (12), (20), (19),

(22). Using these transform equations, an ISAR image in magnitude space can be converted to an image in Euler parameter space. A simple test object known as Slicy (Figure 2) contains flat plates, sharp edges, corners, cylinders, and trihedral. The ISAR images of Slicy in Euler parameter space at 40° elevation and 90° azimuth (Figure 3) allow for an intuitive visualization of ISAR images in Euler parameter space.

In the imaging process, the maximum magnitude image is used to threshold away meaningless data cells in the remaining images. Physically, this implies that data cells with a reflectivity that is below the noise threshold will not have meaningful values for the other parameters. While the use of a simple object allows for an intuitive understanding of the meaning of each Euler parameter, a more realistic object, the T-72BK Main Battle Tank conveys the more typical appearance of ISAR images in Euler parameter space (Figure 4).

Figure 2. Slicy

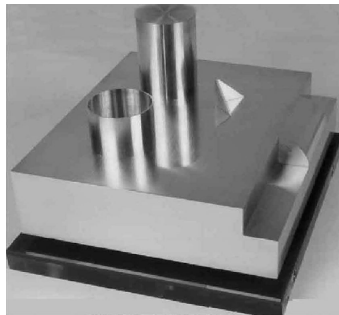
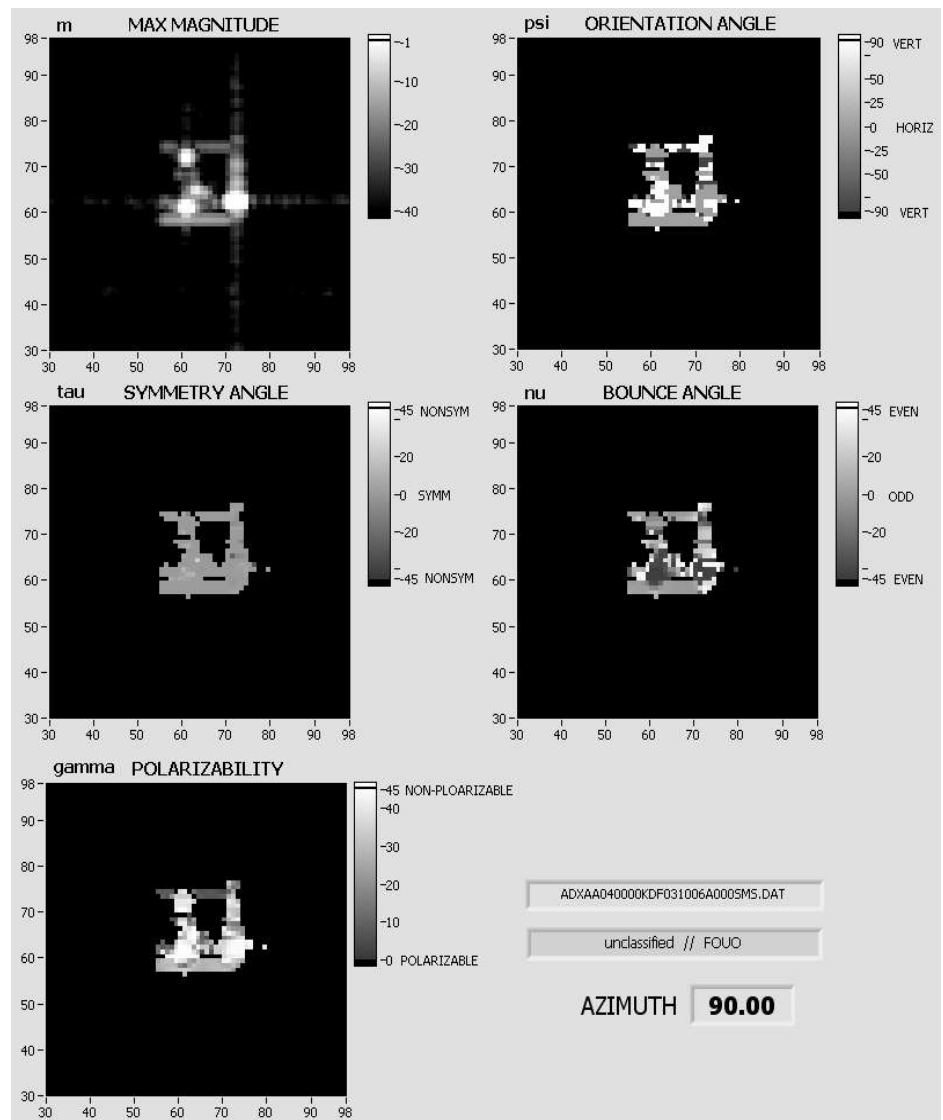


Figure 3. ISAR Images of Slicy in Euler Parameter Space



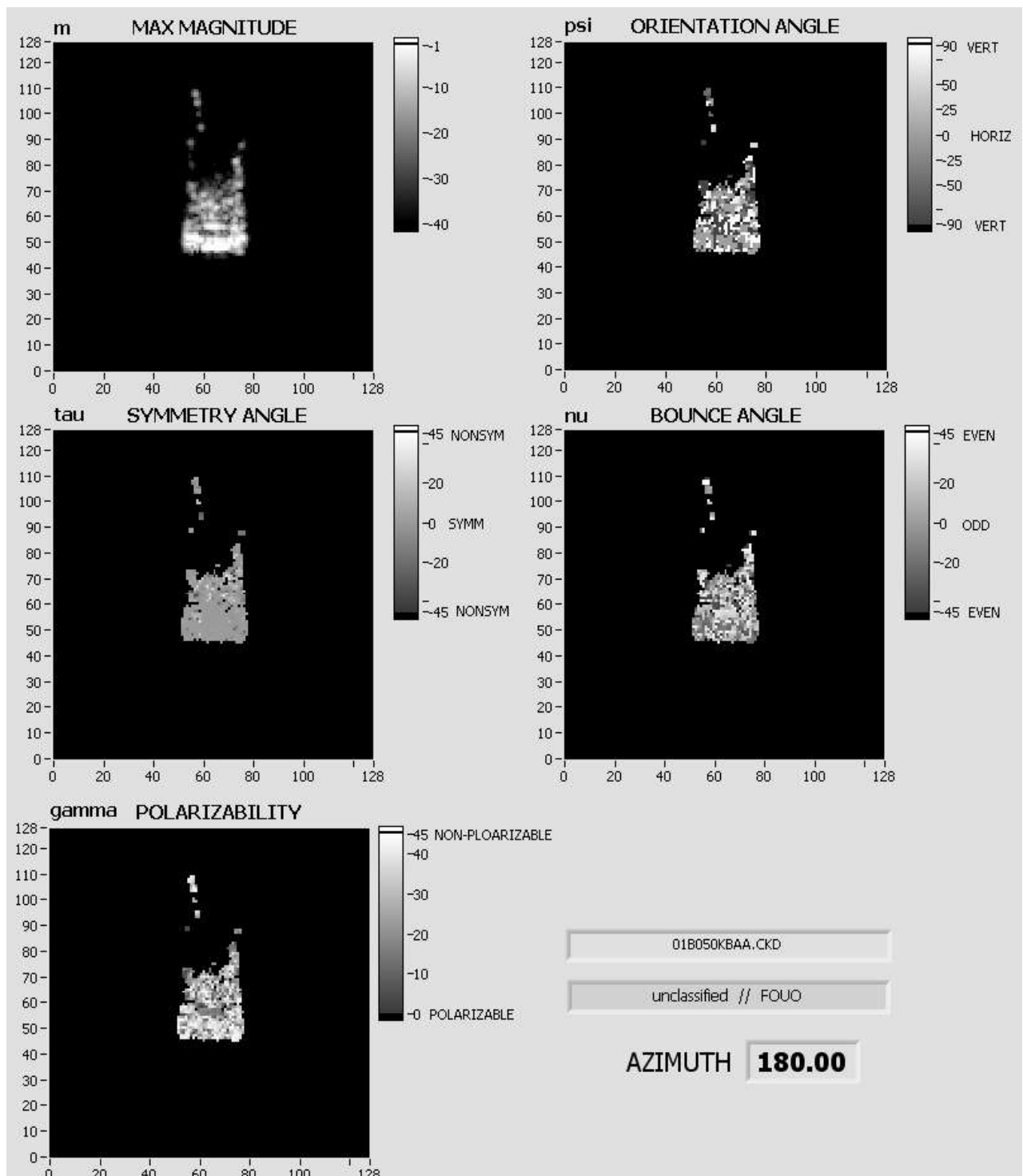


Figure 4. ISAR Images in Euler Parameter Space of the T-72BK Tank Fingerprint

4.0 TRANSFORM EQUATION AMBIGUITIES

In order to test the accuracy of the derivations and assess the extent of ambiguities leading to false results, a numerical analysis can be performed. For any known set of Euler parameters, the corresponding scattering matrix can be computed using $S = U^* S_D U^\dagger$. Using the transform equations derived above, the corresponding Euler parameters can then be rederived and compared to the original in order to assess the accuracy of the transform equations.

When tested with 1,000,000 random sets of Euler parameters, each transform equation was shown to be 99.9999% accurate or better. The absence of 100% accuracy indicates the presence of a limited set of singular points that give rise to ambiguities. To investigate the ambiguities, 1,000,000 random sets of Euler parameters were used again, rounded to the nearest integer degree and the nearest 5 degrees (Table 1).

	Random	Random, rounded to integer degrees	Random, rounded to nearest 5 degrees
m	100.00	100.00	100.00
ψ	100.00	97.26	87.19
τ	100.00	98.91	94.72
ν	99.9999	95.71	80.09
γ	100.00	100.00	100.00

Table 1. % Accuracy of Transform Equations for 1 Million Tests

The results indicate that the ambiguous parameter sets lie at extremes, such as the maximum values, minimum values, or multiples of 5 degrees. While the accuracy appears high enough for real-world application purposes, it must be remembered that man made targets contain discrete shapes and will tend to have parameter values near the extremes. Future work will include characterizing and removing these ambiguities.

5.0 CORRELATION STUDIES

Correlation studies between similar and dissimilar targets have been performed to assess the usefulness of using ISAR images in Euler parameter space to enhance target classification. The correlation equation used depends on the physical meaning of the parameter used (Table 2), where the magnitude parameters use the difference divided by the sum in dbSM, and the angular parameters use the difference divided by the maximum sum in degrees.

$ S_{hh} $	$\frac{ x_2 - x_1 }{ x_2 + x_1 }$
m	$\frac{ x_2 - x_1 }{ x_2 + x_1 }$
ψ	$\frac{ x_2 - x_1 }{180}$
τ	$\frac{ x_2 - x_1 }{90}$
ν	$\frac{ x_2 - x_1 }{90}$
γ	$\frac{ x_2 - x_1 }{45}$

Table 2. Percent Difference Equations used in Correlation Studies

The appropriate comparison equation is used to find the percent difference between corresponding two resolution cells on the two targets to be compared in a certain parameter space. The procedure is carried out for all cells in the ISAR image and an average percent difference (APD) results. The process is repeated for a full 360° azimuth sweep and a histogram is created representing the overall APD as a function of azimuth between two targets in a certain parameter space. The end purpose is a minimal APD for similar targets and high APD for dissimilar targets in order to improve target recognition. All comparisons were made between T-72 class tanks and their 16th-scale fingerprint models (Figures 5-10).

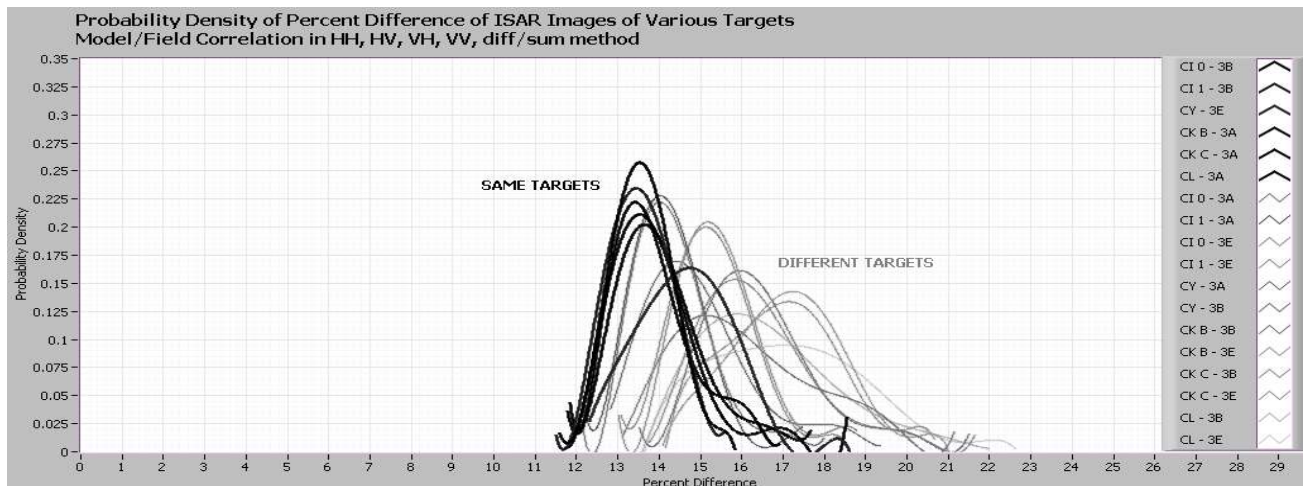


Figure 5. Magnitude Space Correlations of Similar and Dissimilar Targets

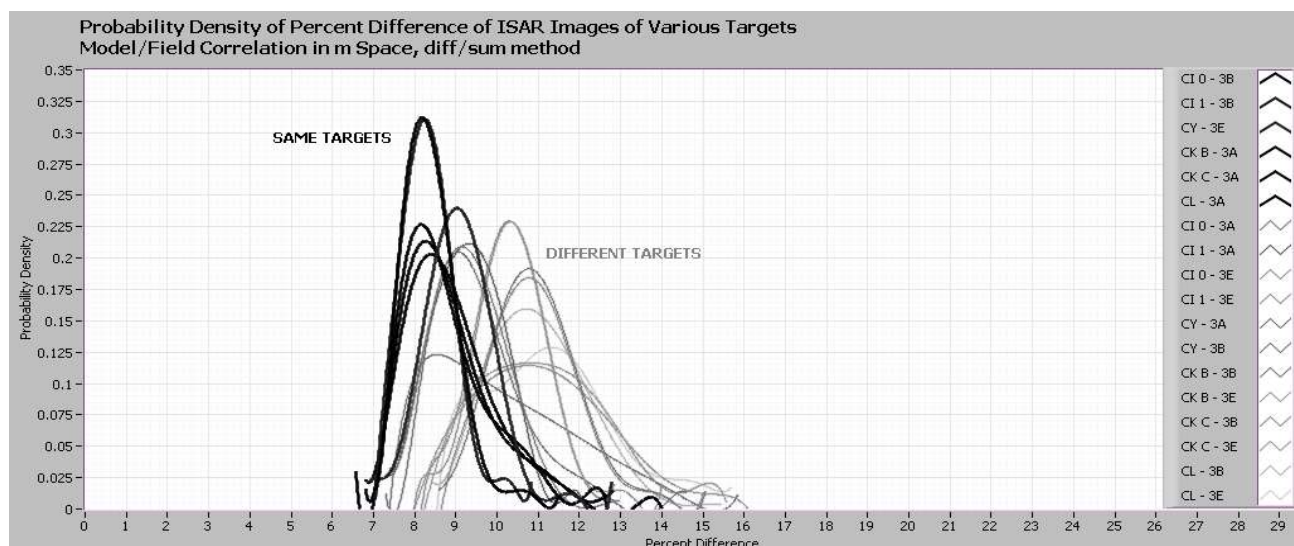


Figure 6. m Space Correlations of Similar and Dissimilar Targets

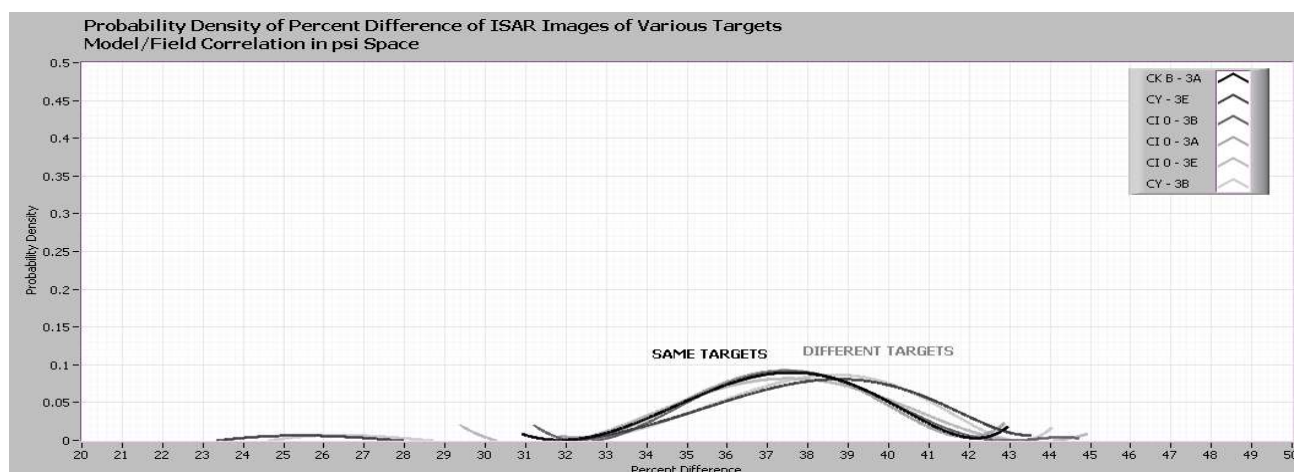


Figure 7. psi Space Correlations of Similar and Dissimilar Targets

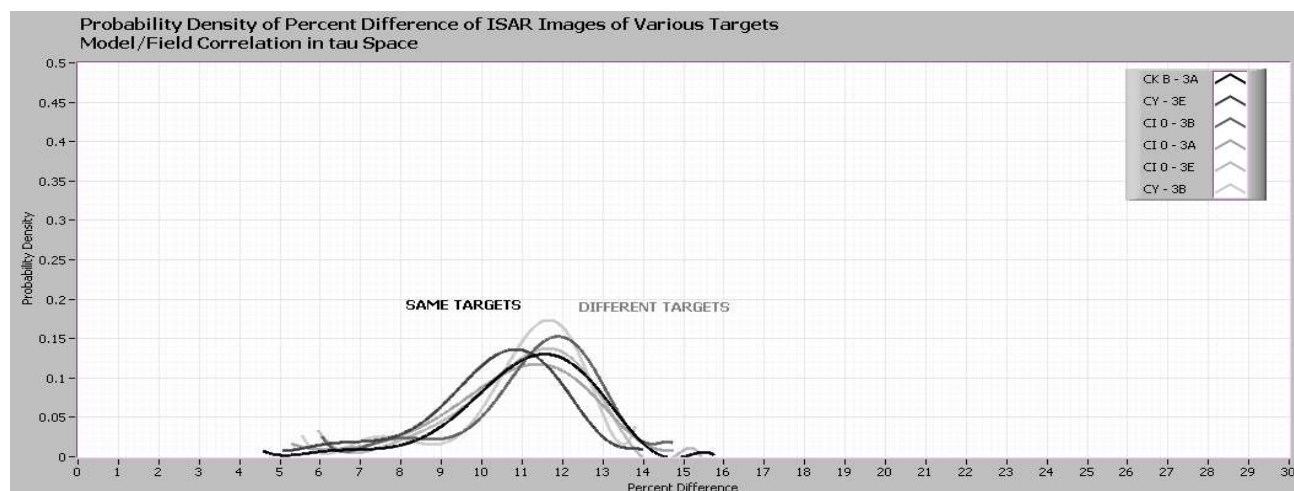


Figure 8. tau Space Correlations of Similar and Dissimilar Targets

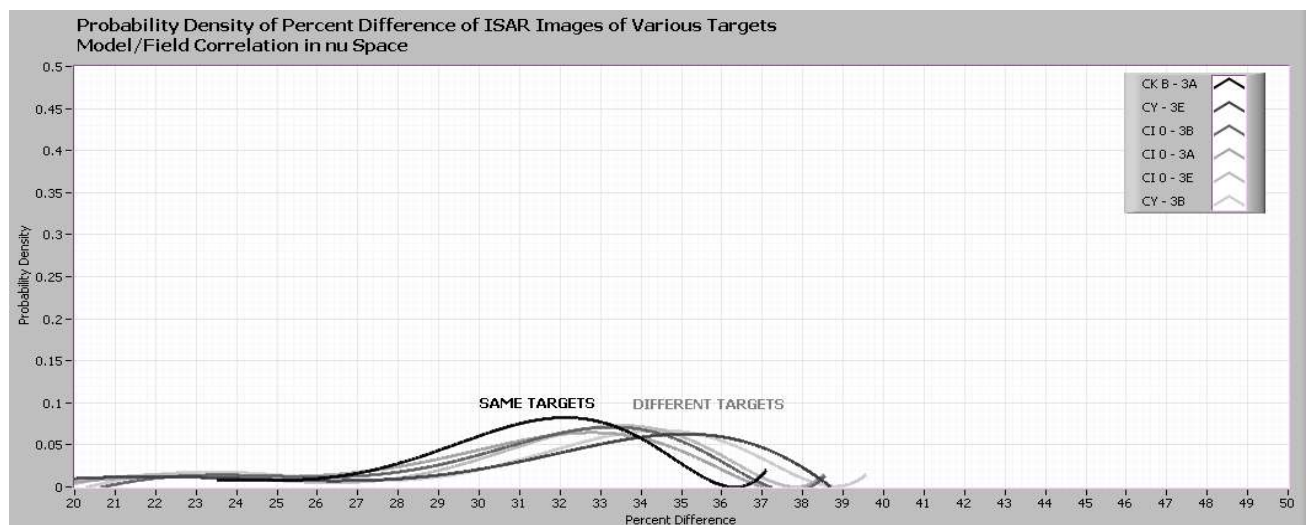


Figure 9. nu Space Correlations of Similar and Dissimilar Targets

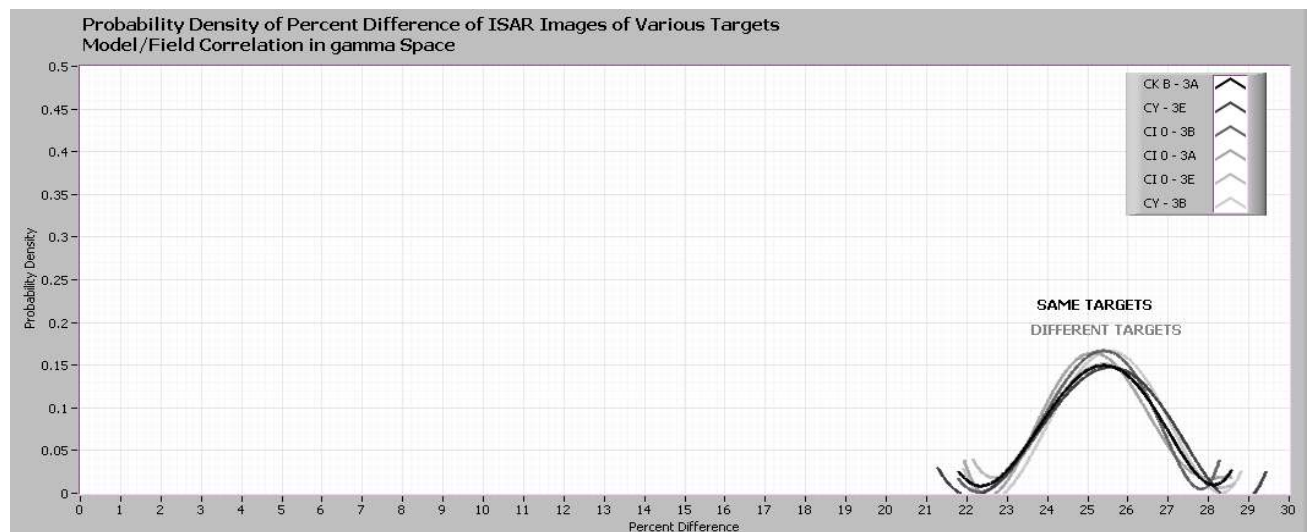


Figure 10. gamma Space Correlations of Similar and Dissimilar Targets

6.0 CONCLUSIONS

The transform equations have been explicitly derived relating the Euler parameters to the original scattering matrix. Upon numerical analysis, the derived transform equations show high accuracy for non-integer Euler parameters, but degraded accuracy for integer parameters due to ambiguities. The results of the correlation studies indicate that in the current form, use of ISAR images in Euler parameter Space for target identification is inconclusive. Further analysis is required to characterize the ambiguities and assess the possibility of removing them.

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