AFRL-VA-WP-TP-2006-346

COMBINING COLLISION AVOIDANCE AND OPERATOR WORKLOAD REDUCTION WITH COOPERATIVE TASK ASSIGNMENT AND PATH PLANNING (PREPRINT)

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OCTOBER 2006

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
October 2006	Conference Paper Preprint	06/01/2006 – 09/20/2006

4. TITLE AND SUBTITLE

COMBINING COLLISION AVOIDANCE AND OPERATOR WORKLOAD REDUCTION WITH COOPERATIVE TASK ASSIGNMENT AND PATH PLANNING (PREPRINT)

5a. CONTRACT NUMBER
In-house

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
62201F

6. AUTHOR(S)
Jeffery B. Saunders, Steven J. Rasmussen, and Corey J. Schumacher

5d. PROJECT NUMBER
A03D

5e. TASK NUMBER

5f. WORK UNIT NUMBER
0B

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Control Design and Analysis Branch (AFRL/VACA)
Control Sciences Division
Air Vehicles Directorate
Air Force Materiel Command, Air Force Research Laboratory
Wright-Patterson Air Force Base, OH 45433-7542

8. PERFORMING ORGANIZATION REPORT NUMBER
AFRL-VA-WP-TP-2006-346

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Vehicles Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson Air Force Base, OH 45433-7542

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL-VA-WP

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-VA-WP-TP-2006-346

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
Conference paper submitted to the Proceedings of the 2007 American Control Conference (ACC 2007).

 PAO Case Number: AFRL/WS 06-2506 (cleared October 18, 2006). Paper contains color.

14. ABSTRACT

This paper develops a method of assignment and path allocation that incorporates a priori collision avoidance and operator workload reduction in assigning multiple tasks to cooperative unmanned aerial vehicles (UAV). The problem is posed as a combinatorial optimization problem. A branch and bound tree search algorithm is implemented for a satisfying solution using a cost function that integrates distance traveled, proximity to other UAVs, and target visitation times. The results demonstrate that the assigned path is near optimal with respect to distance traveled, significantly increases the expected proximity distance to other UAVs, and significantly increases the difference between visitation times of targets. The algorithm runs in less than a tenth of a second allowing on the fly replanning.

15. SUBJECT TERMS
UAVs, collision avoidance, task planning

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	b. ABSTRACT	c. THIS PAGE			Corey Schumacher
Unclassified	Unclassified	Unclassified	SAR	12	19b. TELEPHONE NUMBER *(Include Area Code)* N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

Combining Collision Avoidance and Operator Workload Reduction with Cooperative Task Assignment and Path Planning

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***Abstract*— This paper develops a method of assignment and path allocation that incorporates a priori collision avoidance and operator workload reduction in assigning multiple tasks to cooperative unmanned aerial vehicles (UAV). The problem is posed as a combinatorial optimization problem. A branch and bound tree search algorithm is implemented for a satisficing solution using a cost function that integrates distance travelled, proximity to other UAVs, and target visitation times. The results demonstrate that the assigned path is near optimal with respect to distance travelled, significantly increases the expected proximity distance to other UAVs, and significantly increases the difference between visitation times of targets. The algorithm runs in less than a tenth of a second allowing on the fly replanning.**

INTRODUCTION

Micro aerial vehicles (MAVs) have received increasing amounts of attention in the last few years. Low cost and simple operation give them application for military and civilian purposes alike. Small onboard autopilots allow for accurate autonomous navigation removing the need to place pilots in danger. The next logical step is deployment of teams of MAVs to cooperatively and synergistically accomplish a series of tasks.

A cooperative team of MAVs has the potential to produce greater and more efficient results than a team of independent MAVs. They can adapt to changing situations, replan task assignments when a MAV is lost, communicate over large areas, and quickly complete scenarios. We would like to use as much of that potential as possible. The goal of the work documented in this paper is to produce a method of task assignment such that we can produce a flight plan a priori and potentially replan mid-flight to adapt to changing situations.

Many military intelligence and reconnaissance mission scenarios involving teams of MAVs have been introduced in the past few years [1]. Each scenario involves varying degrees of coupling between the goals of team coordination, task priority, and planning feasible paths to accomplish tasks. Finding a general optimal solution is difficult. Many methods result in combinatorial problems that are difficult to solve. Algorithms in [2] and [3] use mixed integer linear programming to solve the combinatorial problem. Both methods involve a high amount of computation and cannot integrate other coupled factors without adding to computation time. The capacitated transhipment network solver in [4] is unable to generate multiple task assignments. The iterated capacitated transhipment network solver in [5] is heuristic in nature, terminates after finding an initial solution, and can not guarantee an optimal solution.

The desired solution must have the ability couple all constraining aspects of the problem, and be expandable to include other desired constraints. The branch and bound tree search described in [6] is expandable to include additional constraints. We propose to improve the algorithm to include additional costs to compliment the distance cost already introduced in [6]. We will show that the combined additional costs produce near optimal results while successfully adding needed constraints.

Ref [7] describes a research program called Cooperative Operations in Urban Terrain (COUNTER). For the COUNTER concept, a small unmanned aerial vehicle (SAV) transports MAVs to an area of interest, usually an urban area. The SAV flies over the urban terrain, or some designated area, and releases battery powered MAVs to observe objects of interest at closer proximity. The MAVs are released at points along the path of the SAV after which the MAVs fly to various objects and transmit video footage of the objects back to an operator. The MAVs cooperate together to observe all the objects in a timely manner after which they either return to a recover point or continue to another assigned point. The objective of the scenario is to view all objects as quickly as possible. One concern is that all of the MAVs may visit their assigned objects near the same time which will over-load the operator. Another concern is that the MAVs have no method of avoiding collisions with each other. This paper addresses both of these concerns through the introduction of new costs in the task allocation and path optimization algorithm.

The organization of this paper is as follows. The notation used in the paper is described in section I. A full description of the proposed scenario is in section II followed by a review of the branch and bound tree search in section III. Sections IV and V describe the collision prediction algorithm and simulation environment with a review of the results in section VI. We conclude with some points of interest in

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section VII.

I. Notation

Here we present the notation that will be used throughout the rest of the paper. This is intended to be a reference for the rest of the text.

$\mathbf{r}_i$ – Position of the i^{th} MAV.
$\mathbf{R}_i$ – Total distance travelled by the i^{th} MAV.
$\mathbf{R}$ – MAV turn radius.
$\mathbf{U}$ – The set of all MAVs.
V_i – Velocity (groundspeed) of the i^{th} MAV.
V_{ie} – Velocity (groundspeed) max error of the i^{th} MAV.
$\mathbf{w}_{i,j}$ – Waypoint j of the i^{th} MAV.
r_d – Two dimensional safety radius of a MAV. No MAV should be less than r_d from other MAVs.
$d_{i,j}$ – The minimum distance between MAVs i and j.

II. Problem Description

The motivation for this work is to develop algorithms to enhance the COUNTER solution for intelligence and reconnaissance missions [7]. The COUNTER research program proposes to have an operator control one SAV and four MAVs as they perform the mission. The mission is to find and investigate operator selected objects, suspected targets, in the city in order to identify targets. The task for the SAV is to orbit above the city providing video to the operator and acting as a relay for the MAVs. The MAVs are tasked to visit the selected objects in the city in order to convict or acquit those objects as being targets. Since there are many UAVs involved and only one operator, the algorithms developed for COUNTER were designed to utilize the MAVs to maximize the number of objects acquitted or convicted.

COUNTER is a cooperative control environment in which MAVs must cooperate together to quickly observe objects. The cooperative environment involves many coupled problems that may not be so obvious at first glance including minimizing distance travelled, collision avoidance, staggered target arrival (to optimize operator efficiency), and revisiting poorly observed targets. To perform this mission COUNTER's cooperative control algorithms are divided into two parts, object allocation/path planning and object visitation control. This paper attempts to solve the coupling of minimizing distance travelled, collision avoidance, and staggered target arrival. Other coupled parts of the problem may also be solvable by the same methods, but are not explored here.

III. Branch and Bound Algorithm

The Branch and Bound tree search is described fully in [6] and is presented here for completeness.

Combinatorial problems can be completely represented by a decision tree. A decision tree is defined by a set of nodes $T = (N, E)$ where N is the set of all nodes and E is the set of all edges. Each edge is directed, meaning it can only be traversed in one direction. Every node can have parent nodes from which an edge is traversed, or children node to which an edge is traversed. The root node is the single node without a parent node from which all nodes derive. Leaf nodes are nodes without children nodes. The root node begins the tree and the leaf nodes terminate the tree. The leaf nodes represent possible solutions.

Each node in the tree is given a cost based on the scenario it represents. The cost function determines which solutions are optimal. The leaf node with the lowest cost is considered the optimal solution.

Branch and bound assumes the cost of nodes monotonically increases, i.e. all children of a node have a higher cost than the node itself. Using a best first search, a feasible solution can quickly be found. Any nodes with a larger cost than the initial solution can be pruned from the tree significantly reducing the number of nodes to expand. Large parts of the tree can be pruned. If a new leaf node with a lower cost is found, it becomes the current optimal solution with which to compare other nodes. The search can continue till either the tree is completely searched and the optimal solution is found, or some time limit is reached at which the current best solution is used.

In the COUNTER scenario, the root node represents all the vehicles and targets without any assignments. Each edge traversal adds one target assignment to a vehicle. The leaf nodes represent complete sets of target assignments to vehicles. In [6], the cost of a node is the maximum distance flown by any one MAV. The optimal solution minimizes the maximum distance flown by any one MAV.

IV. Cost Function Algorithm

Our goal is to find a set of target assignments that minimize the distance travelled by any of the vehicles while maintaining a large separation distance between MAVs and staggering the MAVs arrival at objects. The J_2 (min-max) cost function of [6]

$$J_2 = \underset{i \in U}{max} R_i > 0 \tag{1}$$

is used to minimize the distance travelled by any of the MAVs. In order to meet the other objectives of the goal, we will add one cost representing the proximity distance to other MAVs (collision cost) and another representing the minimum time between target arrivals (arrival time cost).

A. Collision Cost

We assume all paths flown by MAVs between targets are Dubbin's paths [8]. Each path consists of turns of radius R and straight lines. Lines and turns will be considered different waypoints to simplify calculations even though they represent a single Dubbin's path. The following descriptions estimate the minimum distance between MAVs flying combinations of straight line paths and turning paths.

1) Two Straight Line Paths: Straight line paths can be described parametrically by:

$$\bar{\mathbf{m}}_i = \mathbf{w}_{i,j} - \mathbf{w}_{i,j-1},$$

$$\mathbf{m}_i = \frac{\bar{\mathbf{m}}_i}{\| \bar{\mathbf{m}}_i \|} = \frac{\mathbf{w}_{i,j} - \mathbf{w}_{i,j-1}}{\| \mathbf{w}_{i,j} - \mathbf{w}_{i,j-1} \|},$$

and

$$\mathbf{r}_i = V_i t \mathbf{m}_i + \mathbf{w}_{i,j-1}, \tag{2}$$

where j is the current waypoint and $t = 0$ when $r_i = w_{i,j-1}$. Equation 2 represents a MAV perfectly tracking a waypoint path starting at $\mathbf{w}_{i,j-1}$ when $t = 0$ and finishing at $\mathbf{w}_{i,j}$ when $t = \frac{\|\mathbf{w}_{i,j} - \mathbf{w}_{i,j-1}\|}{V_1}$.

We want to know the distance between two MAVs at any point along the path,

$$\| \mathbf{r}_1 - \mathbf{r}_2 \|^2 = d^2,$$

$$\| (V_1 t \mathbf{m}_1 + \mathbf{w}_{1,j-1} - V_2 t \mathbf{m}_2 - \mathbf{w}_{2,j-1}) \|^2 = d^2. \tag{3}$$

and take the minimum. The minimum can be found by taking the derivative and setting equal to zero. The result is:

$$t = \frac{V_1 \mathbf{m}_1^T (\mathbf{w}_{2,j-1} - \mathbf{w}_{1,j-1}) + V_2 \mathbf{m}_2^T (\mathbf{w}_{1,j-1} - \mathbf{w}_{2,j-1})}{V_1^2 \mathbf{m}_1^T \mathbf{m}_1 + V_2^2 \mathbf{m}_2^T \mathbf{m}_2 - 2 V_1 V_2 \mathbf{m}_1^T \mathbf{m}_2}. \tag{4}$$

The minimum distance can be found by substituting t into

$$d = \| \mathbf{r}_1 - \mathbf{r}_2 \|. \tag{5}$$

2) Turn and Straight Line Path: The minimum distance between a turn and a straight path is somewhat more difficult algebraically. A gradient descent could be useful for accuracy, but introduces a large amount of computation. We can estimate the proximity of a turn path and a straight path to within an error of $2R$ by finding the geometric intersections of the paths and checking the times of those intersections.

Consider the equations of a circle and line:

$$(r_e - w_{1,j-1,e})^2 + (r_n - w_{1,j-1,n})^2 = (R + r_d)^2, \tag{6}$$

and

$$r_n = m(r_e - w_{2,j-1,e}) + w_{2,j-1,n}. \tag{7}$$

These can be simultaneously solved and the minimum can be found using the quadratic equation. Substitute

$$a = 1 + m^2, \tag{8}$$

$$b = 2(w_{2,j-1,n} - w_{1,j-1,n} - m w_{2,j-1,e}) - 2 w_{1,j-1,e}, \tag{9}$$

and

$$c = (w_{2,j-1,n} - w_{1,j-1,n} - m w_{2,j-1,e})^2 + w_{1,j-1,e}^2 - (R + r_d)^2 \tag{10}$$

into

$$r_e = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{11}$$

r_n can be found by substituting r_e back into equation 7. If the result is imaginary, then the circle and line don't intersect. If the intersection isn't in the time frame of the waypoints, then they don't intersect. In these cases, the distance from the center of the circle to the MAV on the waypoint path in the correct time frame can estimate the distance.

3) Two Turns: For the distance between two turns, the distance between the circles is sufficient if they are in the same time frame. This introduces a maximum error of $4R$.

4) Cost Function: The cost function needs an adjustable parameter such that a desired minimum distance is imposed on the MAVs. An inverse square distance relationship allows the cost to greatly increase as distance decreases. Multiplication by β adds a tunable parameter to influence the distance at which the cost becomes significant. The collision cost function is,

$$J_{CC} = \frac{\beta}{\min_{i,j} (d_{i,j})^2} > 0, \{i,j\} \in U, i \neq j, \tag{12}$$

B. Arrival Time Cost

The arrival time at each target is calculated using the length of the path travelled to the target and a constant velocity assumption. We find the minimum difference between any two arrival times and substitute that into an inverse square formula for the cost. Once again we multiply the result by a constant γ to tune the cost. The arrival time cost is,

$$J_{AT} = \frac{\gamma}{\min_{i,j} (t_{i,j})^2}, \{i,j\} \in U, i \neq j \tag{13}$$

where $t_{i,j}$ is the difference in arrival time between targets i and j.

C. Combined Arrival Time and Collision Cost

The combined cost function combines the min-max cost with the two new costs into one function. The constants β and γ in Eq 12 and Eq 13 allow for tuning. The function is,

$$J = J_2 + J_{CC} + J_{AT} \tag{14}$$

V. Simulation

We ran 100 Monte Carlo simulations to demonstrate the effectiveness of the collision cost, arrival time cost, and combined cost. Each run consists of three vehicles and seven targets. The number of targets and vehicles is small to allow time to search the tree for the optimal solution in each run. The optimal solution could then be compared with the time limited branch and bound solution.

Each run used a uniform distribution to place targets and vehicle starting positions on square map. The map size is 3,000 feet by 3,000 feet for all calculations. However, simulations were also run on maps of up to 7,000 feet by

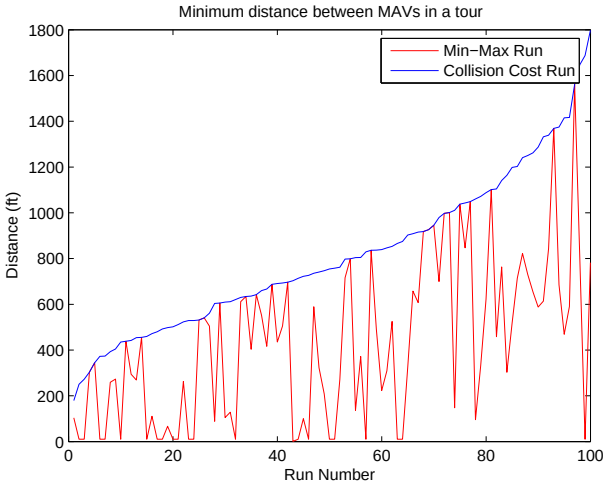

Fig. 1. The minimum distance between MAVs over a 100 tours is shown.

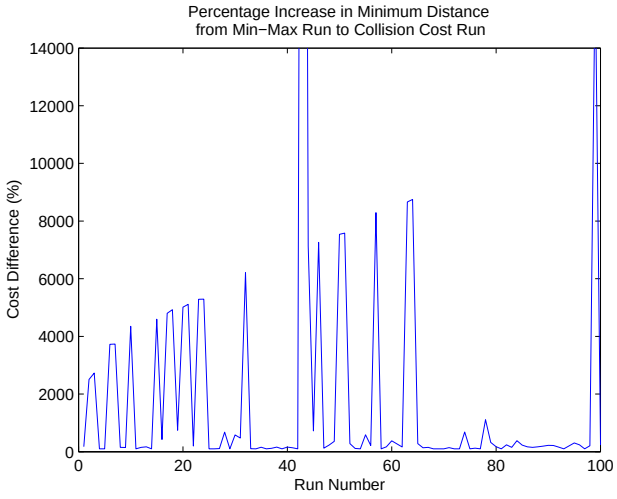

Fig. 2. The percentage increase in the minimum distance from the min-max cost run to the collision cost run.

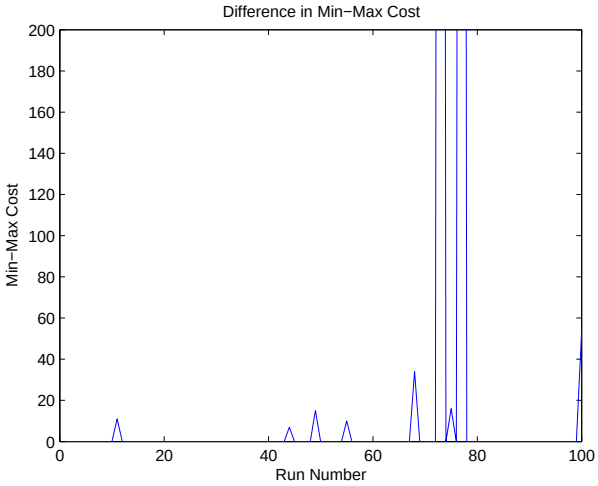

Fig. 3. The difference in the min-max costs between the min-max run and the collision cost run.

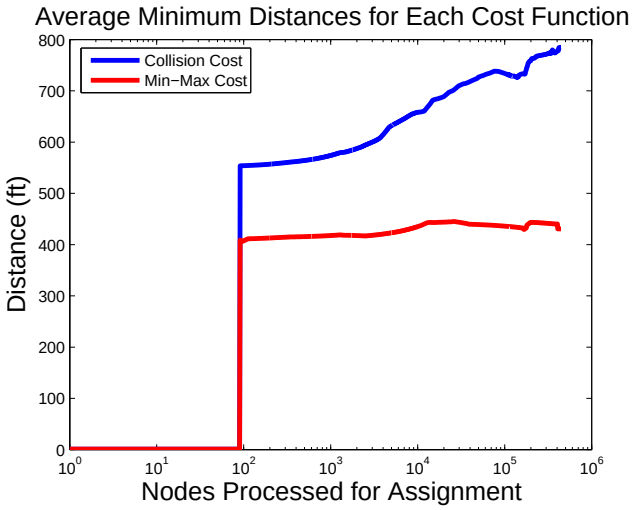

Fig. 4. The average minimum distance between MAVs as nodes are processed is shown. The collision cost results in increased distance between MAVs.

7,000 feet to show the path planner effectiveness at larger map sizes. Each MAV is assumed to have the same constant velocity.

The path planner was executed to the optimal solution four times for each run, once for min-max cost, collision cost, arrival time cost, and combined cost. With each cost function computed on the same data, they could be compared with more accuracy. We chose for parameters $\beta = 1000000$ and $\gamma = 15000$.

VI. Results

A. Collision Cost

Our goal with the collision cost was to increase the proximity distance of the MAVs with minimal change in the min-max cost. The distance between MAVs was increased with an average of 90% compared to the min-max cost runs. The average increase in min-max cost was 0.8%.

Figure 1 shows the distance increase for each of the 100 runs with the percentage increase in figure 2. The runs are ordered with increasing distance. Most of the runs result in a significant increase in distance. The runs without change in distance already have a large distance between MAVs, with the smallest at 300 feet.

The difference in min-max cost between the two runs is displayed in figure 3. Little difference is made in min-max cost between cost functions. Notice the two spikes in the graph. This is the result of two MAVs with starting positions in close proximity. If the MAVs have similar starting positions, the planner will remove one of them from flight to prevent collision. This is expected, however undesirable.

All of the above calculations result from tree searches to find the optimal solution. The optimal solution can take large periods of time to find due to the combinatorial nature of the

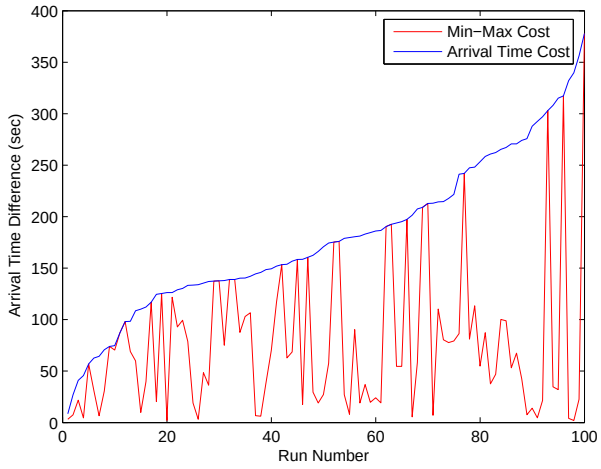

Fig. 5. The minimum difference in arrival times between targets is plotted for each run. The arrival time cost function produces arrival time differences better than or equal to the min-max cost function.

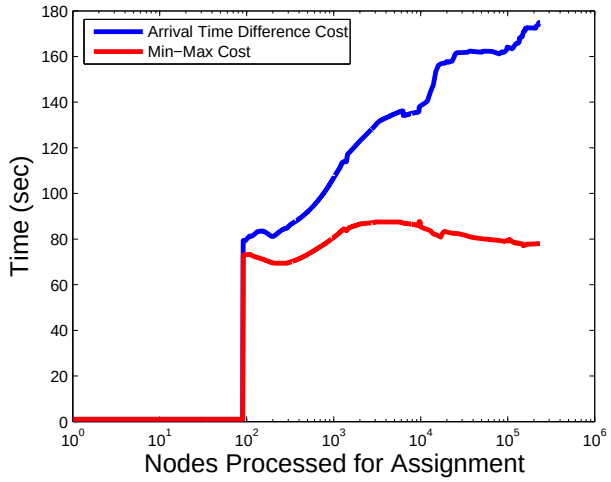

Fig. 6. The minimum difference in arrival times between targets is plotted against number of nodes processed.

problem. However, the branch and bound search can find a near optimal solution in tenths of a second. Figure 4 shows the increase in average minimum distance across the runs as nodes are processed. A larger amount of nodes implies a larger amount of time to process. The minimum distance from the collision cost runs tends to increase as more nodes are processed. Notice the minimum distance in the min-max costs runs show little change over number of processed nodes.

B. Arrival Time Cost

The arrival time cost has similar results to the collision cost. Figure 5 shows the arrival time difference between the arrival time cost function and the min-max cost function. The arrival time cost function results in arrival time differences greater than or equal to that of the mix-max cost function. Arrival time differences are increased by 133%. The min-max cost is increased by 0.06%.

The arrival time difference is plotted with the nodes processed in figure 6. Once again, they are similar to the results of the collision cost. The initial solution has an initial increase in arrival time difference and the solution improves over time.

C. Combined Cost

The combined cost has the potential to improve proximity of the aircraft and stagger target arrival times while keeping the distance flown by the MAVs close to optimal. The performance of these goals can be measured by comparing the results of the combined cost function to that of the other cost functions.

Figure 7 compares the expected proximity distance of the MAVs as nodes are processed. The combined cost function results are similar to the results of the collision cost function. Adding another cost had very little effect on proximity results.

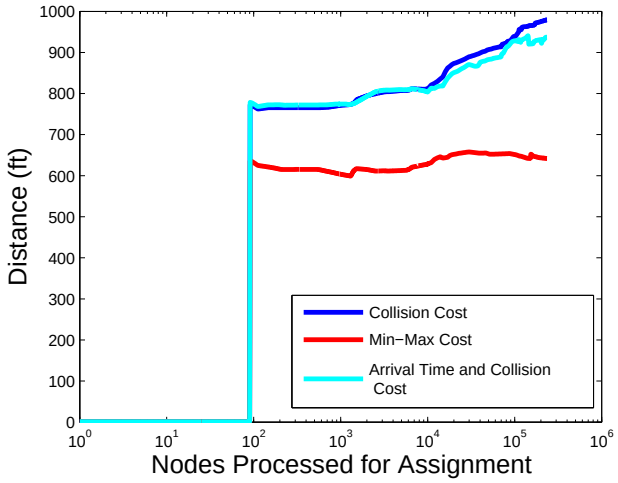

Fig. 7. The minimum expected distance between MAVs is plotted against number of nodes processed. The results for the combined cost function are very similar to the results of the collision cost function.

The differences in target arrival times are shown in figure 8. The combined cost function results are similar to the arrival time cost function. The combined cost has done little to negatively affect the target arrival time difference.

The min-max cost increased by 0.64% from the min-max cost run to the combined cost run. This is a small difference and worth the large increase in proximity distance and staggered target arrival.

VII. Conclusion

In this paper we have demonstrated a method of assigning targets to multiple vehicles while balancing objectives of distance travelled, collision avoidance, and staggered target arrival. The results show significantly improved expected distance between MAVs and a large increase in staggered arrival time. Both of these features may be added simultaneously

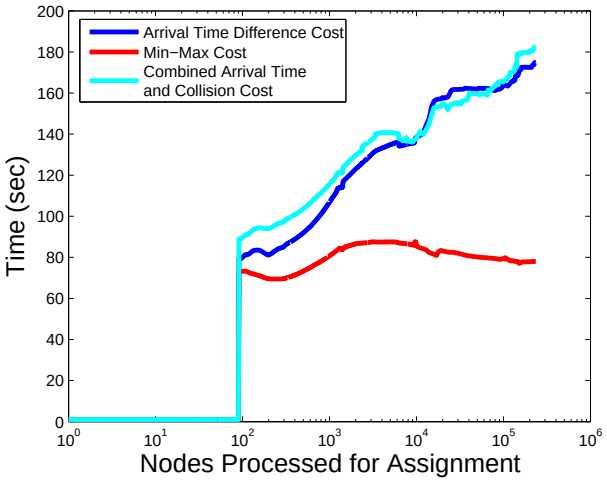

Fig. 8. The expected minimum difference in arrival times is plotted against number of nodes processed. The arrival time and combined cost functions have similar results.

with little difference in results. Suboptimal solutions may be found quickly using a time limited branch and bound search.

The success of the additional costs suggests additional research into other desired constraints that may vary between different scenarios and objectives. The combinatorial nature of the problem allows for a large amount of near optimal solutions that may satisfy a variety of constraints.

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