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A METHOD FOR INCLUDING CONTROL EFFECTOR INTERACTIONS IN THE CONTROL ALLOCATION PROBLEM (PREPRINT)

Michael W. Oppenheimer and David B. Doman

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14. ABSTRACT
Much emphasis has been placed on over-actuated systems for air vehicles. Over-actuating an air vehicle provides a certain amount of redundancy for the flight control system, thus potentially allowing for recovery from off-nominal conditions. Due to this redundancy, control allocation algorithms are typically utilized to compute a unique solution to the over-actuated problem. As the number of control effectors placed on a vehicle increases, the likelihood of the occurrence of control effector interactions increases. For example, deflection of an aerodynamic surface that is upstream of another aerodynamic surface may cause the forces and moments produced by the downstream effector to differ from those produced when the upstream effector is not deployed. Another example can be found on launch vehicles that can use a combination of differential throttles and gimballed nozzles for attitude control. The effectiveness of gimballed nozzles are clearly influenced by the engine thrust. The above are examples of the control effector interaction problem. In this work, a method is devised, which utilizes linear programming methods in an iterative framework, to take into account control effector interactions. While nonlinear programming techniques could be directly applied to such problems, the lack of convergence guarantees precludes their use in flight critical systems. The use of linear programming methods is appealing because an optimal solution to each linear programming sub-problem can be determined in a finite amount of time.

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A Method for Including Control Effector Interactions in the Control Allocation Problem

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I. Abstract

Much emphasis has been placed on over-actuated systems for air vehicles. Over-actuating an air vehicle provides a certain amount of redundancy for the flight control system, thus potentially allowing for recovery from off-nominal conditions. Due to this redundancy, control allocation algorithms are typically utilized to compute a unique solution to the over-actuated problem. As the number of control effectors placed on a vehicle increases, the likelihood of the occurrence of control effector interactions increases. For example, deflection of an aerodynamic surface that is upstream of another aerodynamic surface may cause the forces and moments produced by the downstream effector to differ from those produced when the upstream effector is not deployed. Another example can be found on launch vehicles that can use a combination of differential throttles and gimballed nozzles for attitude control. The effectiveness of gimballed nozzles are clearly influenced by the engine thrust. The above are examples of the control effector interaction problem. In this work, a method is devised, which utilizes linear programming methods in an iterative framework, to take into account control effector interactions. While nonlinear programming techniques could be directly applied to such problems, the lack of convergence guarantees precludes their use in flight critical systems. The use of linear programming methods is appealing because an optimal solution to each linear programming sub-problem can be determined in a finite amount of time.

II. Introduction

Conventional aircraft utilize an elevator for pitch control, ailerons for roll control, and a rudder for yaw control. As aircraft design has advanced, more control effectors (some unconventional) have been placed on vehicles. In some cases, certain control effectors may be able to exert significant influence upon multiple axes, e.g., ruddervators. When a system is equipped with more effectors than axes to control, the system may be over-actuated which means that control redundancy exist. The allocation, blending, or mixing of these control effectors to achieve some desired objectives defines the control allocation problem.

Due to over-actuation and coupling of control surface effects, it may not be straightforward to determine how to translate a moment or acceleration command from the flight control system into a set of commands to the control effectors. In addition, rate and position limits of the control surfaces must be considered in order to achieve a physically realizable solution. It is also desirable for a control allocator to enable the aircraft to recover from off-nominal conditions, such as a failed control surface, when physically possible. A great deal of work in reconfigurable control has been performed over the last decade .[1–6] Many reconfigurable control systems make use of control allocation algorithms to perform automatic distribution of the control power requests among a large number of control effectors, while ensuring that the rate and position limits of the actuators are not violated, and to potentially allow recovery from off-nominal conditions.

Some of the simplest control allocation techniques are explicit ganging, pseudo control, pseudo inverse, and daisy chaining. Unfortunately, each suffers from difficulty in guaranteeing that rate and position limits

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will not be violated and some can be difficult to apply due to the need to derive a control mixing law a priori. Another control allocation method, called direct allocation,[7] finds the control vector that results in the best approximation of the command vector in a given direction. Unconstrained least squares control allocation methods, that account for rate and position limits, through the use of penalty functions, have also been developed.[8] One of the first instances of linear programming based control allocators was from Paradiso.[9,10] In this work, Paradiso developed a selection procedure for determining actuator positions that was based on linear programming and limited actuator authority. More recently, the control allocation paradigm has been posed as a constrained optimization problem.[11] In this work, the control allocation problem was split into two sub-problems. The first was the error minimization part, which attempts to find the control vector, such that the control effector induced moments or accelerations match the desired moments or accelerations. If multiple solutions exist to the error minimization problem, the second problem attempts to find a unique solution by driving the control vector to a preference vector and optimizing a secondary objective. The linear control allocation problem has been extended to an affine problem[12] to account for nonlinearities in the moment-deflection curves. Quadratic programming has also been used in the past.[13] An excellent paper discussing control allocation, by Bodson,[14] provides a glimpse into numerous control allocation techniques.

Coupling of the effects of control effectors is a situation that has not been addressed in the past. In many instances, the coupling effects of aerodynamic surfaces are insignificant and may not require their inclusion in the control allocation paradigm. However, the coupling effects of engine thrust and thrust vectoring for an ascent vehicle typically cannot be ignored. In this work, a method is developed to include the coupling between two control effectors. The method utilizes the simplex algorithm so that guaranteed converge is achieved. An example using fourthrust vectored rocket engines is provided.

III. Linear Control Allocation

The linear control allocation problem can be posed as follows: find the control vector, $\boldsymbol{\delta} \in \mathbb{R}^n$, such that

$$\mathbf{B}\boldsymbol{\delta} = \mathbf{d}_{des} \tag{1}$$

subject to

$$\begin{aligned} \boldsymbol{\delta}_{min} \leq \boldsymbol{\delta} \leq \boldsymbol{\delta}_{max} \\ \dot{\boldsymbol{\delta}} \leq \dot{\boldsymbol{\delta}}_{max} \end{aligned} \tag{2}$$

where $\mathbf{B} \in \mathbb{R}^{m \times n}$ is a control effectiveness matrix, the lower and upper position limits are defined by $\boldsymbol{\delta}_{min} \in \mathbb{R}^n$ and $\boldsymbol{\delta}_{max} \in \mathbb{R}^n$, respectively, $\dot{\boldsymbol{\delta}} \in \mathbb{R}^n$ are the control rates, $\dot{\boldsymbol{\delta}}_{max} \in \mathbb{R}^n$ are the maximum control rates, $\mathbf{d}_{des} \in \mathbb{R}^m$ are the desired moments or accelerations (typically for inner-loop control laws, $\mathbf{d}_{des} \in \mathbb{R}^3$), n is the number of control effectors, and m is the number of axes to control. Equation 2 provides the position and rate limits for the control effectors. In a digital computer implementation, the rate limits are converted to effective position limits. The combined limits become the most restrictive of the rate or position limits and are specified as

$$\underline{\boldsymbol{\delta}} \leq \boldsymbol{\delta} \leq \overline{\boldsymbol{\delta}} \tag{3}$$

where

$$\begin{aligned} \overline{\boldsymbol{\delta}} &= \min\left(\boldsymbol{\delta}_{max}, \boldsymbol{\delta} + \Delta t \dot{\boldsymbol{\delta}}_{max}\right) \\ \underline{\boldsymbol{\delta}} &= \max\left(\boldsymbol{\delta}_{min}, \boldsymbol{\delta} - \Delta t \dot{\boldsymbol{\delta}}_{max}\right) \end{aligned} \tag{4}$$

Here, $\overline{\boldsymbol{\delta}} \in \mathbb{R}^n$, $\underline{\boldsymbol{\delta}} \in \mathbb{R}^n$, and $\overline{\boldsymbol{\delta}}$, $\underline{\boldsymbol{\delta}}$ are the most restrictive upper and lower control effector limits, respectively.

A necessary condition for a system to be over-actuated is the number of columns of $\mathbf{B}$, n, must be greater than the number of rows of $\mathbf{B}$, m. The true test of over-actuation is that the number of linearly independent columns of $\mathbf{B}$ be greater than the number of rows of $\mathbf{B}$. For inner-loop control laws, the $\mathbf{B}$, or control effectiveness, matrix typically becomes

$$\mathbf{B} = \begin{bmatrix} \frac{\partial L}{\partial \delta_1} \frac{\partial L}{\partial \delta_2} \cdots \frac{\partial L}{\partial \delta_n} \\ \frac{\partial M}{\partial \delta_1} \frac{\partial M}{\partial \delta_2} \cdots \frac{\partial M}{\partial \delta_n} \\ \frac{\partial N}{\partial \delta_1} \frac{\partial N}{\partial \delta_2} \cdots \frac{\partial N}{\partial \delta_n} \end{bmatrix} \tag{5}$$

where L, M, and N are the rolling, pitching, and yawing moments, respectively and $\boldsymbol{\delta} = [\delta_1, \delta_2, \ldots, \delta_n]$.

In some practical cases, it may be impossible to satisfy the equality of Eq. 1 subject to the inequality constraints of Eq. 2. Furthermore, on systems where control redundancy exists, there is seldom a unique solution to the problem. In order to address these practical issues, a mixed optimization formulation of the control allocation problem, developed by Bodson,[14] can be used. The mixed optimization control allocation problem is formulated as a linear programming problem. The primary objective is minimization of the difference between $\mathbf{B}\boldsymbol{\delta}$ and $\mathbf{d}_{des}$ and the secondary objective is to drive the control effectors to some preferred position $\boldsymbol{\delta}_p$, subject to the constraints of rate and position limits. Mathematically, the mixed optimization problem is posed as:

$$\min_{\boldsymbol{\delta}} \|\mathbf{B}\boldsymbol{\delta} - \mathbf{d_{des}}\|_\mathbf{1} + \|\mathbf{W}_{\boldsymbol{\delta}}(\boldsymbol{\delta} - \boldsymbol{\delta}_\mathbf{p})\|_\mathbf{1} \tag{6}$$

subject to Eq. 3. This can be converted to a standard linear programming (LP) problem:

$$\min_{\boldsymbol{\delta}_{s1}, \boldsymbol{\delta}_{s2}} \begin{bmatrix} 0 & 0 & \ldots & 0 & w_1 & w_2 & \ldots & w_n & 1 & 1 & \ldots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}_{s1} \\ \boldsymbol{\delta}_{s2} \end{bmatrix} \tag{7}$$

subject to

$$\begin{bmatrix} -\boldsymbol{\delta}_{s1} \\ -\boldsymbol{\delta}_{s2} \\ -\boldsymbol{\delta} \\ \boldsymbol{\delta} \\ \mathbf{B}\boldsymbol{\delta} - \boldsymbol{\delta}_{s2} \\ -\mathbf{B}\boldsymbol{\delta} - \boldsymbol{\delta}_{s2} \\ \boldsymbol{\delta} - \boldsymbol{\delta}_{s1} \\ -\boldsymbol{\delta} - \boldsymbol{\delta}_{s1} \end{bmatrix} \leq \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -\underline{\boldsymbol{\delta}} \\ \overline{\boldsymbol{\delta}} \\ \mathbf{d}_{des} \\ -\mathbf{d}_{des} \\ \boldsymbol{\delta}_p \\ -\boldsymbol{\delta}_p \end{bmatrix} \tag{8}$$

where $w_1, w_2, \ldots, w_n$ are the diagonal elements of $\mathbf{W}_{\boldsymbol{\delta}}$.

IV. Linear Programming and Control Effector Interactions

Typically, the effect of one control effector on another is ignored in the control allocation problem. In many applications, these interaction effects are small as compared to the forces and moments generated by each control effector acting individually. However, cases exist where the interactions should not be ignored. In aircraft applications where the aerodynamic surfaces are used to modulate forces and moments, instances where control surfaces lie in close proximity to other surfaces or cases where one control surface lies downstream of another, are examples of situations where the effectiveness of some surfaces are influenced by the deflection of other surfaces. In automotive applications, there exists significant coupling between the yaw and axial acceleration effectiveness of the steering angle and front wheel braking forces.[15] Another example can be found on launch vehicles that can use a combination of differential throttles and gimballed nozzles for attitude control. The effectiveness of gimballed nozzles are clearly influenced by the engine thrust. Hence, a control allocation method, which can include the coupling effects of multiple control effectors, is desired. For an example, consider the vehicle shown in Fig. 1. This launch vehicle has control surfaces which interact with each other. In particular, the flaperons and ruddervators. Hence, in addition to the conventional forces and moments from the individual deflection of these surfaces, there are also forces and moments from the combined deflection. For example, the pitching moment coefficient is the sum of the base or wing-body pitching moment, plus the sum of the incremental pitching moments produced by each control effector taken one at a time, plus the incremental pitching moments caused by any interactions between control surfaces:

$$\begin{aligned} C_m = C_{m_{Base}}(\alpha, \beta, M) + \Delta C_{m_{RF}}(\alpha, \beta, M, \delta_{RF}) + \Delta C_{m_{LF}}(\alpha, \beta, M, \delta_{LF}) + \Delta C_{m_{RR}}(\alpha, \beta, M, \delta_{RR}) + \\ \Delta C_{m_{LR}}(\alpha, \beta, M, \delta_{LR}) + \Delta C_{m_{RF,RR}}(\alpha, \beta, M, \delta_{RF}, \delta_{RR}) + \Delta C_{m_{LF,LR}}(\alpha, \beta, M, \delta_{LF}, \delta_{LR}) \end{aligned} \tag{9}$$

where the subscripts RF, LF, RR, LR imply right flap, left flap, right rudder, and left rudder. The last two terms in Eq. 9 are the interactions, that is, a combined flaperon and rudder deflection cause forces and

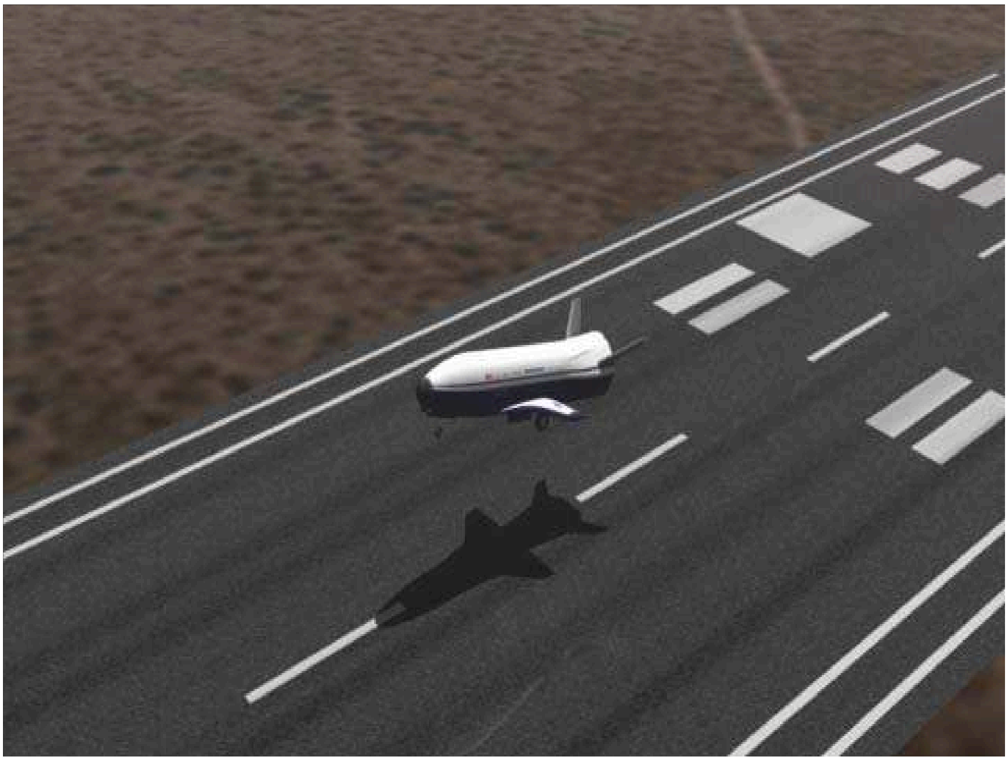

Figure 1. Reusable Launch Vehicle.

moments. Interactions do not fit into a linear control allocation scheme since by definition they are non-separable nonlinear functions of two control deflections. In a number of important cases, the interactions can be described by a bilinearity, i.e., of the form $\Delta C_M(\delta_i, \delta_j) = \frac{\partial^2 C_M}{\partial \delta_i \partial \delta_j} \delta_i \delta_j$. In such cases, one can pose a control allocation problem similar to that posed in Eq. 6 except that the control effectiveness matrix $\mathbf{B}$ is replaced by a control dependent matrix $\mathbf{A}(\boldsymbol{\delta})$.

$$\min_{\boldsymbol{\delta}} \|\mathbf{A}(\boldsymbol{\delta})\boldsymbol{\delta} - \mathbf{d_{des}}\|_\mathbf{1} + \|\mathbf{W}_{\boldsymbol{\delta}}(\boldsymbol{\delta} - \boldsymbol{\delta}_\mathbf{p})\|_\mathbf{1} \tag{10}$$

subject to Eq. 3, where

$$\mathbf{A}(\boldsymbol{\delta}) \triangleq \left\{ \mathbf{\frac{1}{2}} \begin{bmatrix} \boldsymbol{\delta}^T \mathbf{Q}_L \\ \boldsymbol{\delta}^T \mathbf{Q}_M \\ \boldsymbol{\delta}^T \mathbf{Q}_N \end{bmatrix} + \mathbf{B} \right\} \tag{11}$$

and $\mathbf{Q}_L, \mathbf{Q}_M, \mathbf{Q}_N$ are the contributions to the rolling, pitching, and yawing moments from the combined actuation of 2 control surfaces. More specifically, $\mathbf{Q}_L$ can be expressed as

$$\mathbf{Q}_L = \frac{1}{2} \begin{bmatrix} \frac{\partial^2 L}{\partial \delta_1^2} \, \frac{\partial^2 L}{\partial \delta_1 \delta_2} \cdots \frac{\partial^2 L}{\partial \delta_1 \delta_n} \\ \frac{\partial^2 L}{\partial \delta_2 \delta_1} \, \frac{\partial^2 L}{\partial \delta_2^2} \cdots \frac{\partial^2 L}{\partial \delta_2 \delta_n} \\ \vdots \\ \frac{\partial^2 L}{\partial \delta_n \delta_1} \, \frac{\partial^2 L}{\partial \delta_n \delta_2} \cdots \frac{\partial^2 L}{\partial \delta_n^2} \end{bmatrix} \tag{12}$$

The matrices $\mathbf{Q}_M$ and $\mathbf{Q}_N$ are similar to Eq. 12. For the case where bilinear interaction terms are the only nonlinearities of interest, the main diagonal terms will be zero. However, this form can also accommodate cases where forces or moments are separable quadratic functions of individual control deflections, e.g., individual aileron or flap contributions to yawing moment at low angles-of-attack.[16]

Because the matrix $\mathbf{A}$ itself is a function of $\boldsymbol{\delta}$, the control allocation problem is, strictly speaking, nonlinear. Rather than directly applying nonlinear programming techniques to this problem, we propose to iteratively solve a series of linear programming (LP) subproblems (each with guaranteed convergence properties) in order to progressively improve approximations to the solution of the original nonlinear programming problem. Define the control deflection vector, $\boldsymbol{\delta}_k$, as the most recent solution to an LP subproblem that is computed as part of the iterative procedure. We now pose the following LP subproblem whose solution yields an updated estimate, $\boldsymbol{\delta}_{k+1}$, of the control deflection vector that solves the original nonlinear programming problem:

$$\min_{\boldsymbol{\delta}_{s1},\boldsymbol{\delta}_{s2}} \begin{bmatrix} 0 & 0 & \dots & 0 & w_1 & w_2 & \dots & w_n & 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta}_{k+1} \\ \boldsymbol{\delta}_{s1} \\ \boldsymbol{\delta}_{s2} \end{bmatrix} \tag{13}$$

Subject to:

$$\begin{bmatrix} -\boldsymbol{\delta}_{s1} \\ -\boldsymbol{\delta}_{s2} \\ -\boldsymbol{\delta}_{k+1} \\ \boldsymbol{\delta}_{k+1} \\ \mathbf{A}(\boldsymbol{\delta}_\mathbf{k})\boldsymbol{\delta}_\mathbf{k+1} - \boldsymbol{\delta}_\mathbf{s2} \\ -\mathbf{A}(\boldsymbol{\delta}_\mathbf{k})\boldsymbol{\delta}_\mathbf{k+1} - \boldsymbol{\delta}_\mathbf{s2} \\ \boldsymbol{\delta}_{k+1} - \boldsymbol{\delta}_{s1} \\ -\boldsymbol{\delta}_{k+1} - \boldsymbol{\delta}_{s1} \end{bmatrix} \leq \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -\underline{\boldsymbol{\delta}} \\ \overline{\boldsymbol{\delta}} \\ \mathbf{d}_{des} \\ -\mathbf{d}_{des} \\ \boldsymbol{\delta}_p \\ -\boldsymbol{\delta}_p \end{bmatrix} \tag{14}$$

The LP subproblems are solved until either the flight control system requires a solution at the end of a control update frame or until the following convergence criteria is satisfied:

$$\frac{|\boldsymbol{\delta}_{k+1} - \boldsymbol{\delta}_k|}{|\boldsymbol{\delta}_k|} \leq \text{tol} \tag{15}$$

which indicates that further iterations will not result in significant improvements in the solution estimate.

V. Example

In this section, the method described above is applied to a hypothetical vehicle with 4 rocket engines that have independent thrust settings along with the capability to gimbal each engine in the pitch axis. Figure 2 shows the engine layout. The control vector is

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_{g_1} & \delta_{g_2} & \delta_{g_3} & \delta_{g_4} & T_1 & T_2 & T_3 & T_4 \end{bmatrix} \tag{16}$$

The pitch gimbal angles, denoted by δ_{g_i}, are limited to $\pm 8°$, while the throttle for each engine, T_i, is limited to values between 0 and 1 which represents a fraction of the maximum thrust T_{max} that can be achieved in any engine (for these initial results, T_{max} is set to 1). It is assumed that the distance from each engine to the center of gravity is 1 in all directions, thus, the moment arms for each engine are

$$\begin{aligned} \mathbf{r}_1 &= r_{1_x}\hat{i} + r_{1_y}\hat{j} + r_{1_z}\hat{k} = 1\hat{i} + 1\hat{j} + 1\hat{k} \\ \mathbf{r}_2 &= r_{2_x}\hat{i} + r_{2_y}\hat{j} + r_{2_z}\hat{k} = 1\hat{i} + 1\hat{j} - 1\hat{k} \\ \mathbf{r}_3 &= r_{3_x}\hat{i} + r_{3_y}\hat{j} + r_{3_z}\hat{k} = 1\hat{i} - 1\hat{j} - 1\hat{k} \\ \mathbf{r}_4 &= r_{4_x}\hat{i} + r_{4_y}\hat{j} + r_{4_z}\hat{k} = 1\hat{i} - 1\hat{j} + 1\hat{k} \end{aligned} \tag{17}$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the X, Y, and Z directions, respectively. The moment produced by the engines is given by:

$$\begin{aligned} M = \sum_{i=1}^{4} \mathbf{F} \times \mathbf{r} = \\ \sum_{i=1}^{4} \left[\left(-T_i T_{max} \sin\delta_{g_i} r_{i_y}\right)\hat{i} - \left(T_i T_{max} \cos\delta_{g_i} r_{i_z} - T_i T_{max} \sin\delta_{g_i} r_{i_x}\right)\hat{j} + \left(T_i T_{max} \cos\delta_{g_i} r_{i_y}\right)\hat{k} \right] \end{aligned} \tag{18}$$

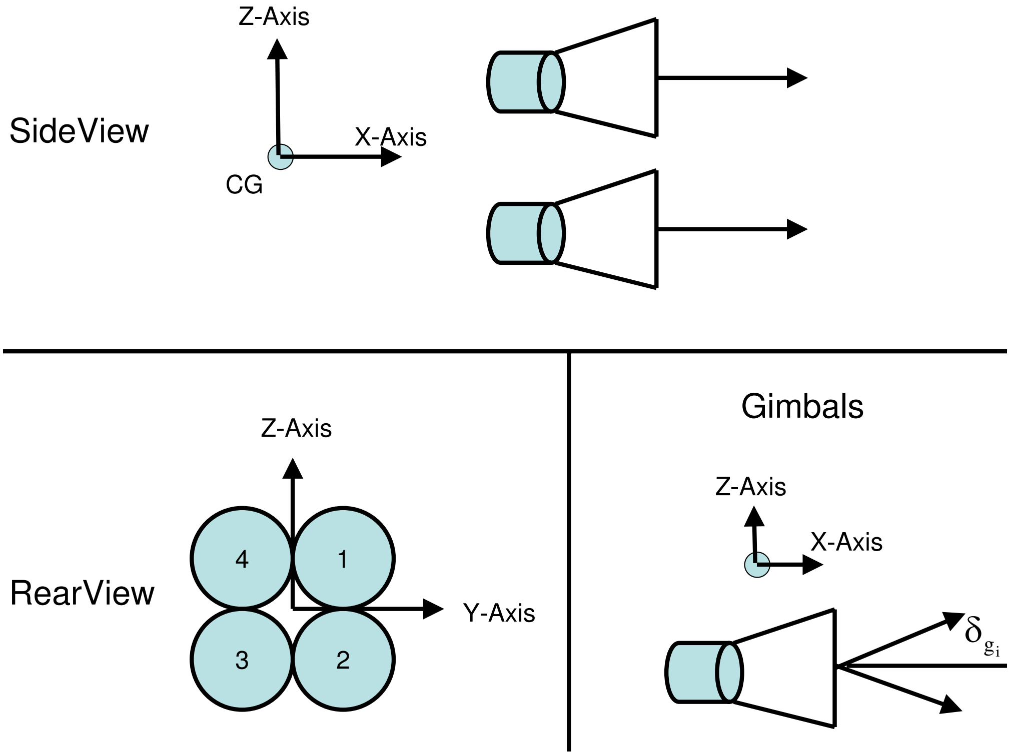

Figure 2. Reusable Launch Vehicle Engine Layout.

The components of Eq. 18 are the rolling, pitching, and yawing moments produced by the engine throttles and pitch gimbal angles. Note that there are 8 control effectors for the purposes of the control allocation problem. The interaction effects are evident here as the products of throttle and gimbal angle are included in each moment. Entries of the **B** matrix are

$$\begin{array}{lll} \frac{\partial L}{\partial \delta_{g_i}} = T_i T_{max} \cos \delta_{g_i} r_{i_y} & \frac{\partial M}{\partial \delta_{g_i}} = -\left(T_i T_{max} \sin \delta_{g_i} r_{i_z} + T_i T_{max} \cos \delta_{g_i} r_{i_x}\right) & \frac{\partial N}{\partial \delta_{g_i}} = T_i T_{max} T_{max} \sin \delta_{g_i} r_{i_y} \\ \frac{\partial L}{\partial \delta_{T_i}} = -T_{max} \sin \delta_{g_i} r_{i_y} & \frac{\partial M}{\partial \delta_{T_i}} = -T_{max} \cos \delta_{g_i} r_{i_z} + T_{max} \sin \delta_{g_i} r_{i_x} & \frac{\partial N}{\partial \delta_{T_i}} = T_{max} \cos \delta_{g_i} r_{i_y} \end{array} \tag{19}$$

Then, using Eq. 5, the **B** matrix can be constructed. For this example problem, the form of the $\mathbf{Q}_L$ matrix is

$$\mathbf{Q}_L = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\partial^2 L}{\partial \delta_{g_1} \partial T_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 L}{\partial \delta_{g_2} \partial T_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 L}{\partial \delta_{g_3} \partial T_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 L}{\partial \delta_{g_4} \partial T_4} \\ \frac{\partial^2 L}{\partial \delta_{g_1} \partial T_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 L}{\partial \delta_{g_2} \partial T_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial^2 L}{\partial \delta_{g_3} \partial T_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial^2 L}{\partial \delta_{g_4} \partial T_4} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

where

$$\frac{\partial^2 L}{\partial \delta_{g_i} \partial T_i} = T_{max} \cos \delta_{g_i} r_{i_y} \tag{21}$$

In other words, there is no coupling between the i^{th} gimbal angle and the j^{th} throttle when $i \neq j$. The $\mathbf{Q}_M$ and $\mathbf{Q}_N$ matrices have the same structure as $\mathbf{Q}_L$ in Eq. 20 with their entries being

$$\begin{aligned} \frac{\partial^2 M}{\partial \delta_{g_i} \partial T_i} &= -\left(T_{max} \sin \delta_{g_i} r_{i_z} + T_{max} \cos \delta_{g_i} r_{i_x}\right) \\ \frac{\partial^2 N}{\partial \delta_{g_i} \partial T_i} &= T_{max} \sin \delta_{g_i} r_{i_y} \end{aligned} \tag{22}$$

The desired command vector, $\mathbf{d}_{des}$ is set to

$$\mathbf{d}_{des} = \mathbf{d}_{des} + 0.05 * rand \tag{23}$$

where $rand$ is a uniformly distributed random vector on the unit interval and the initial value for $\mathbf{d}_{des}$ is

$$\mathbf{d}_{des} = \begin{bmatrix} .1 \\ .2 \\ .3 \end{bmatrix} \tag{24}$$

Two problems are solved in order to make a comparison. The first is the linear programming problem, where the objective is to find $\boldsymbol{\delta}$ such that Eq. 1 holds subject to Eq. 3. In this case, the control allocator does not have any information about the control effector interactions. If the interactions are large, then the linear control allocator should produce a control vector which yields large moment errors. The linear programming problem produces $\boldsymbol{\delta}_{linear}$ and the results of this are a set of moments given by

$$\mathbf{B}\boldsymbol{\delta}_{linear} \tag{25}$$

The second problem solved is Eq. 10 subject to Eq. 3. Here, a sequential LP is solved so that the control effector couplings are taken into account. This computation produces $\boldsymbol{\delta}_{quad}$ and the results of this are a set of moments given by

$$\left\{ \frac{1}{2} \begin{bmatrix} \boldsymbol{\delta}^T_{quad}\mathbf{Q}_L \\ \boldsymbol{\delta}^T_{quad}\mathbf{Q}_M \\ \boldsymbol{\delta}^T_{quad}\mathbf{Q}_N \end{bmatrix} + \mathbf{B} \right\} \boldsymbol{\delta}_{quad} \tag{26}$$

Figures 3, 4, and 5 show the results for each channel of the problem. For instance, Fig. 3 shows the first entry in $\mathbf{d}_{des}$ along with the the first entries from Eqs. 25 and 26. Clearly, the strictly linear problem is not able to compute the correct control setting to achieve the desired values in $\mathbf{d}_{des}$. On the other hand, incorporating the interaction terms into a sequential linear programming problem yields results which are essentially equivalent to the commands (the traces from Eq. 26 and those of $\mathbf{d}_{des}$ lie on top of each other. In an effort to more closely show how the sequential linear programming problem performs, Fig. 6 shows the results from all channels, but does not include the linear programming results. Here, it can be seen that slight differences exist between the sequential linear programming results and the commands, but the performance is clearly superior to the linear programming results.

VI. Conclusions

The work presented here details a method to take into account the interactions between two control effectors. The problem is solved using a standard linear programming solver, however, the quadratic terms are included and the solution is obtained after sequentially calling the LP solver. Results indicate that the solutions are superior to using a standard linear programming problem.

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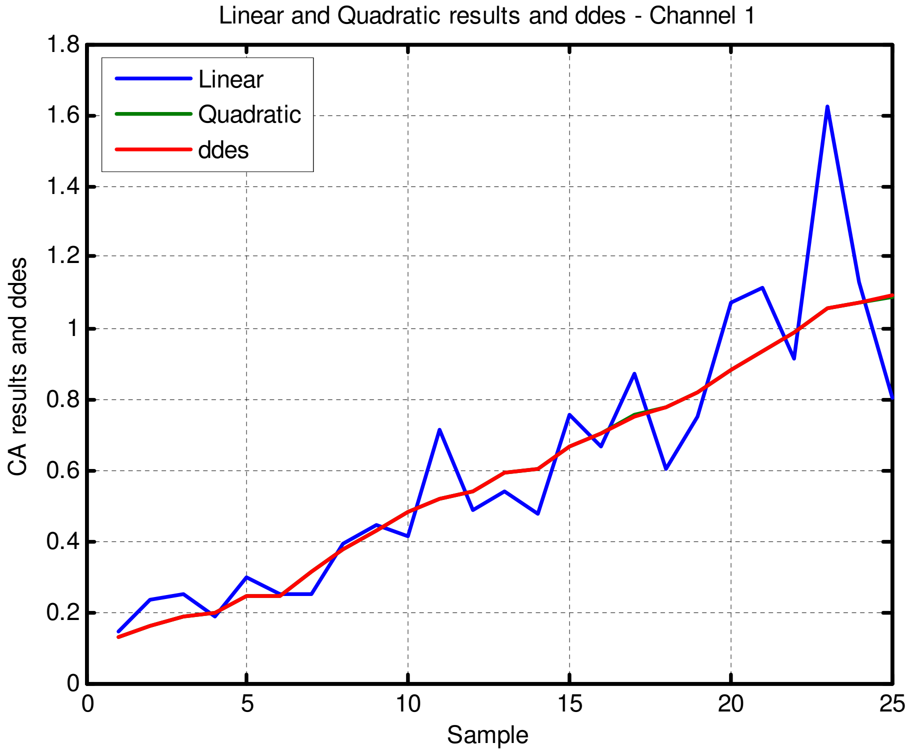

Figure 3. Linear and Quadratic Control Allocation Results and d_{des} - Channel 1.

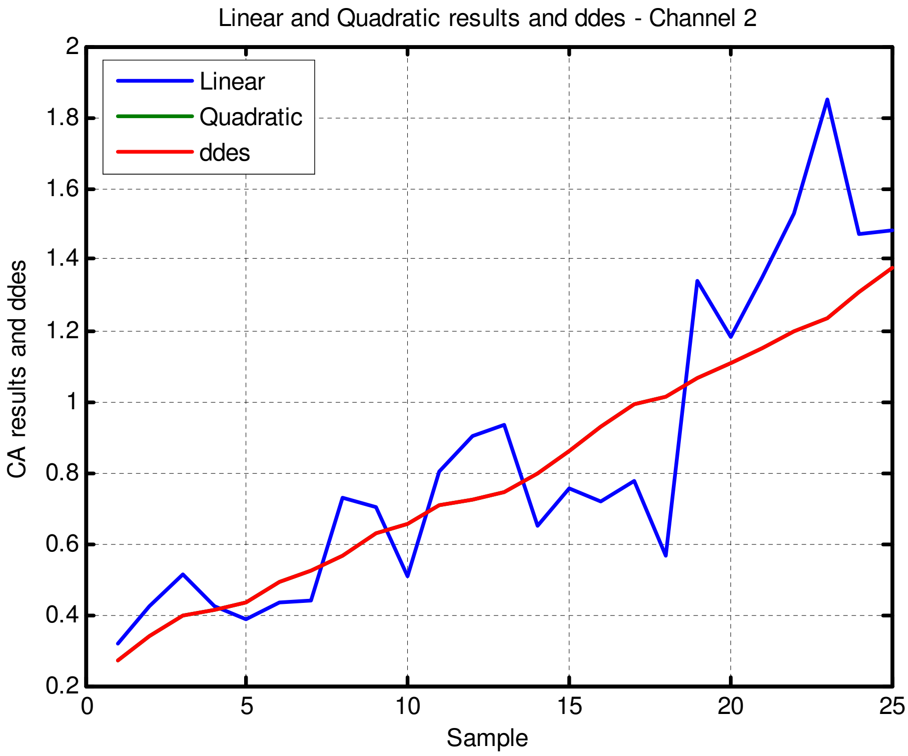

Figure 4. Linear and Quadratic Control Allocation Results and d_{des} - Channel 2.

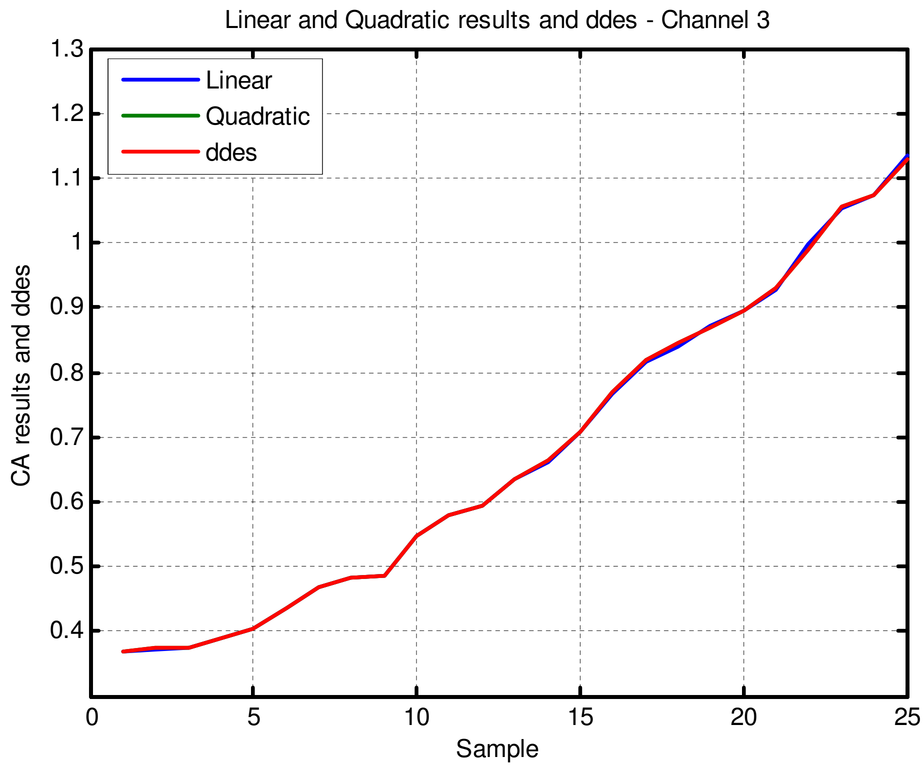

Figure 5. Linear and Quadratic Control Allocation Results and d_{des} - Channel 3.

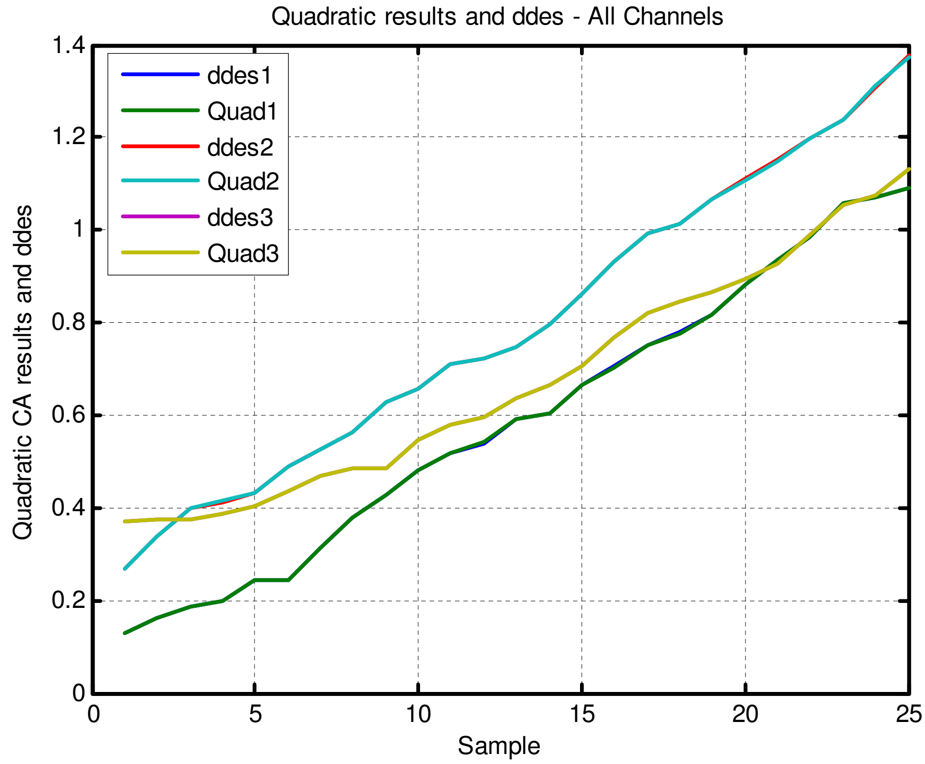

Figure 6. Quadratic Control Allocation Results and d_{des} - All Channels.

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