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# Playing Abstract games with Hidden States (Spatial and Non-Spatial).

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## Report Documentation Page

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## Outline

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- Our domain of research.
- The mathematics of strategically complex game playing
  - Move evaluation,
  - Min/max depth search,
  - Temporal difference learning.
- Application to our network checkers game.
- The HARD PROBLEM: hidden spatial states.
  - Information theoretic advisors,
  - Combine with TD(0)
- Open research questions.

## Our domain of research

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- Broadly described as medium resolution war-gaming.
  - Maneuver of forces, hidden forces, ambiguously defined end-states.
- Use machine learning techniques to develop strategies.
- Also use machine learning to capture and *generalize* expertise of human players
- Monte Carlo simulation to develop risk analysis.
- Currently looking at chess/checkers variants
  - Networked agents, Hidden agents.

# The Mathematics of Strategically Complex Gaming

- Often classical game theory won't work, notably because of the "tyranny of dimension."
  - Too many states and strategic paths over time.
- Solving tabular stochastic dynamic games is impossible.
- Checkers (a game of complexity in the lower end)
  - Has maneuver, materiel diversity, complex strategy
- $\approx 50^8 \approx 10^{14}$  possible strategic paths over the course of a game.
- Move to the theatre chess game (operational war-game).
- Gaming can still be viewed as approximating Bellman's state-action equation ( for the optimal sequence of actions at each state and time  $S_t, S_{t+1}, \dots, S_T$  ) with various methods.

## Evaluation

- Bellman's equation for optimal policy  $\pi^*$  satisfies

$$Q(s_t, a_t) = E^{\pi^*} \left\{ R(s_t, a_t) + \max_{a'} Q(s_{t+1}, a_{t+1}) \right\}$$

- The function  $Q(s, a)$  is called the action-value function, specifying the *total future* reward of taking action  $a$  from state  $s$ . Games like chess/checkers no intermediate, only terminal reward.
- Gaming playing – approximation to the action-value function by a function of a **linear sum of weighted features**.

$$Q \approx \tilde{Q} = f \left( \sum w_i \phi_i(s) \right)$$

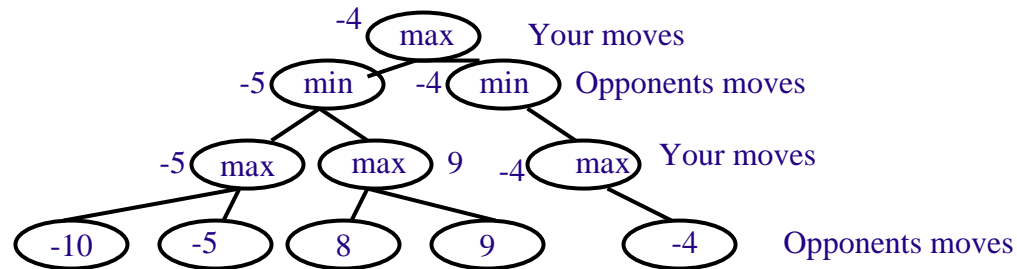
- The weights  $w_i$  are called the "**advisors**" to "**feature**"  $\phi_i(s)$  of state  $s$  (force balance).

## Min-Max depth search.

- Essentially you “project into the future” to a limited horizon. The optimal action is chosen to be

$$\arg \max_a \left\{ \min \left( \max \left( \dots \left( \min \tilde{Q}(s''''', a''''') \right) \right) \right) \right\}$$

- Best way to explain this is tree search, with the value at the leaves “backed up” to the root node through mini-max search.
- Limited by the branching factor of the move (checkers roughly 8, chess, 36, go 80).



## Temporal difference learning.

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- Remembering Bellman's equation, at the optimal policy with no intermediate rewards the difference,

$$E^{\pi^*} (Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t) | s_t, a_t) = 0$$

- In temporal difference learning you learn by “shifting” the past approximation  $\tilde{Q}(s_t, a_t)$  towards the future approximation  $\tilde{Q}(s_{t+1}, a_{t+1})$
- This incrementally minimises the “temporal difference” as is required by Bellman.
- Done by setting (if you have state numbers)

$$\tilde{Q}_{new}(s_t, a_t) = \alpha \tilde{Q}_{old}(s_{t+1}, a_{t+1}) + (1 - \alpha) \tilde{Q}_{old}(s_t, a_t).$$

- The learning rate  $\alpha$  must satisfy some simple stochastic convergence conditions.

## Temporal Difference in Game Playing.

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- Too many states to solve, so do TD learning on the advisors  $w_i$  so we can generalize to new novel states.

- Here we use gradient descent, incrementing the vector of advisors by

$$\Delta \vec{w} = \alpha \left( \tilde{Q}(s_{t+1}, a_{t+1}) - \tilde{Q}(s_t, a_t) \right) \nabla_{\vec{w}} \tilde{Q}(s_t, a_t).$$

- Changes in advisor weights can be done on-line (as the game is played) or off-line (at the end of each game).
- Here  $\tilde{Q}(s_t, a_t) = \Pr(\text{winning game} \mid s_t, a_t)$
- Equivalently get a terminal reward of 1 if win, 0 if loss.

## Function Approximation

- We approximate this probability by

$$\tilde{Q}(s, a) = 1 / \left( 1 + \exp \left( - \sum w_i \phi(s) \right) \right)$$

- Terminal state

$$\tilde{Q}(s_T, \bullet) = \begin{cases} 1 \text{ for a win,} \\ 1/2 \text{ for a draw,} \\ 0 \text{ for a loss.} \end{cases}$$

- The advisors usually reflect importance of balance in pieces, mobility, ..... Other features of the game.

- $w_1 (N_1 - N_2) + \text{other terms}$
- $N_i$  number of pieces of side  $i$

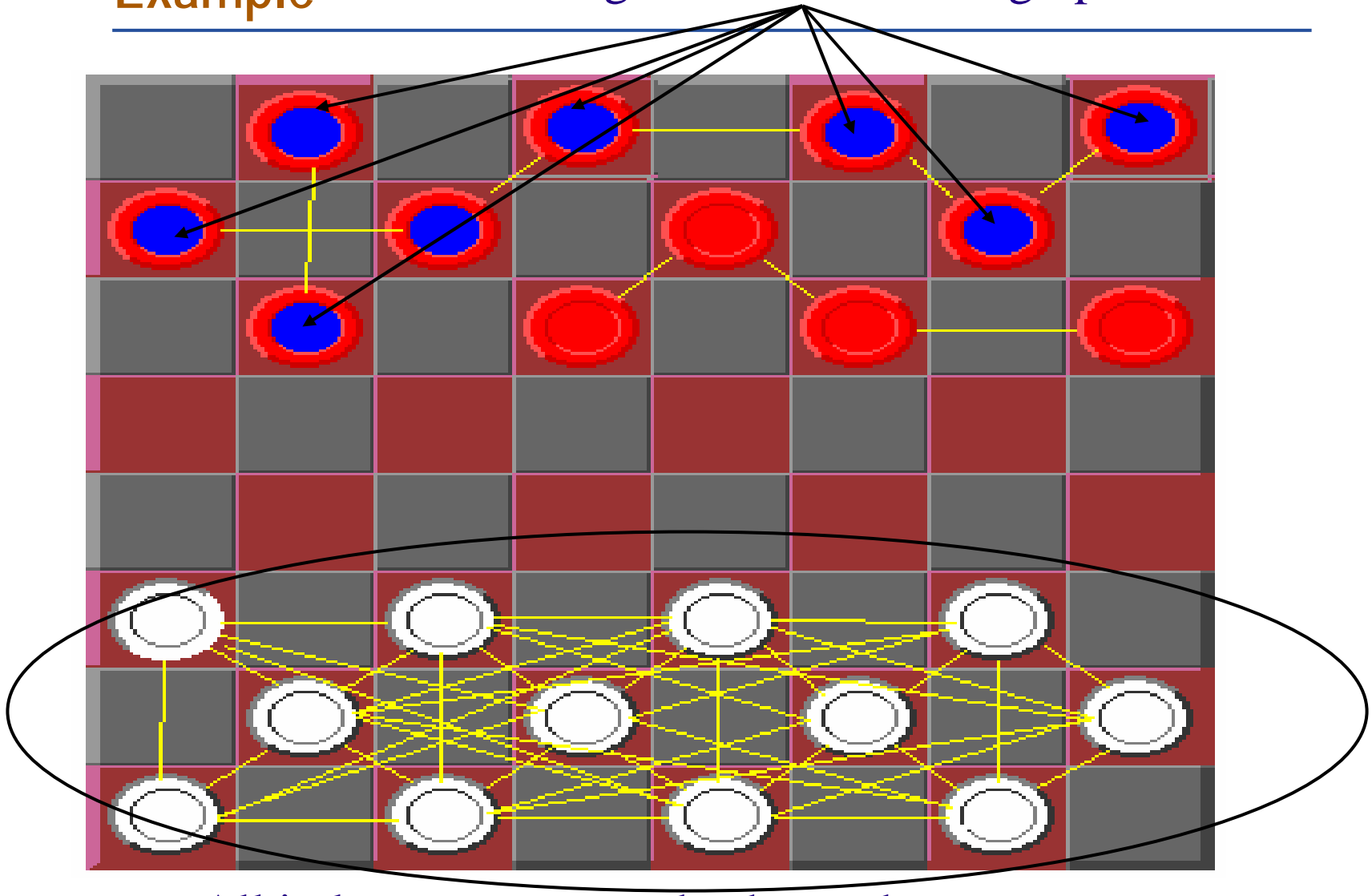
## An example in an imperfect information game

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- Network checkers
  - Network vulnerability through dynamic games.
  - Pieces connected by a network of varying topology.
  - Only pieces in the *largest connected sub-graph* exhibit mobility, isolated pieces don't.
  - Each side aware of materiel and the largest sub-graph size.
  - Network details hidden (distribution of degree etc. ).

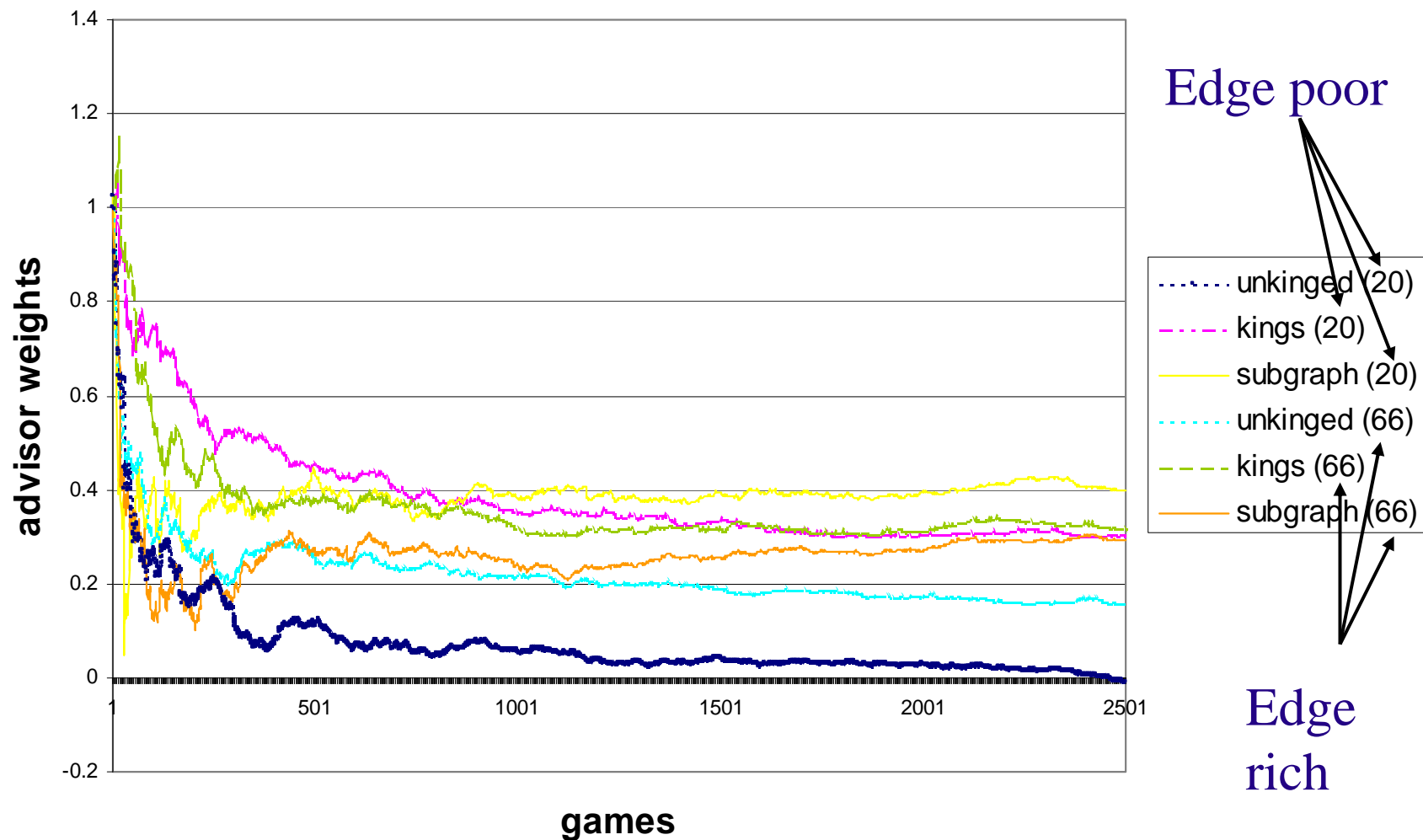
## Example

## Largest connected sub-graph



All in largest connected sub-graph

# Results of Learning Advisor Weights



## Hidden Spatial States

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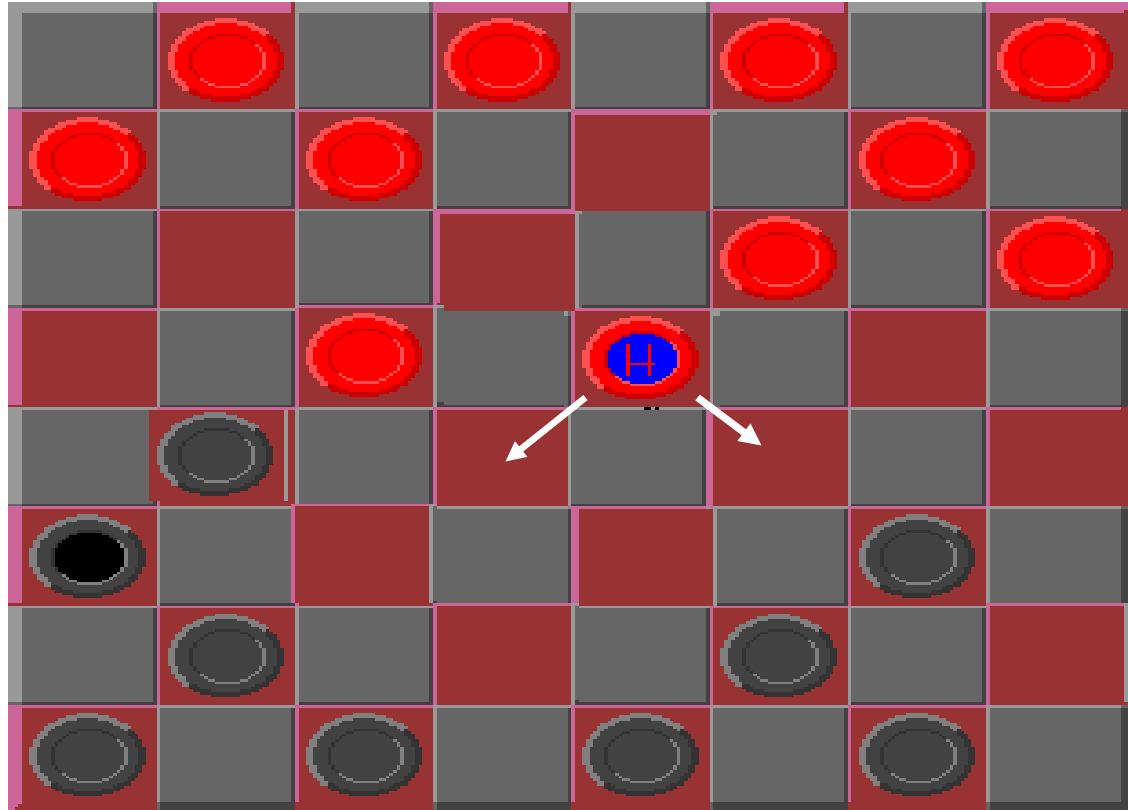
- Begun research onto hidden spatial states.
- Difficult for the following reasons
  - Vastly increased branching factor of possible states. For example, suppose we have one invisible piece. Positive prob. Of being in  $j$  squares  
$$\text{New branching factor} = \text{Old branching factor} * j$$
  - Have to construct a distribution of opponent's probable states.  $\Pr(\text{opponent state is } s) > 0$
  - Pruning of the estimated states risky (opponent could exploit this).

## Our approach

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- Start with a small number of invisible pieces.
- Use theorem of total probability and conditioning on events to develop a Markov chain for the probability of hidden pieces in some state. Generate  $\Pr(\text{board}_1), \dots, \Pr(\text{board}_n)$
- Really a non-Markov problem (opponent's strategies will be history dependent).
- Know when pieces are taken – including hidden piece.
- If you run into a hidden piece you lose a turn (but gain information on the hidden pieces location).
- If you try to take a hidden piece and it's not there you also lose a turn, but gain information on location.

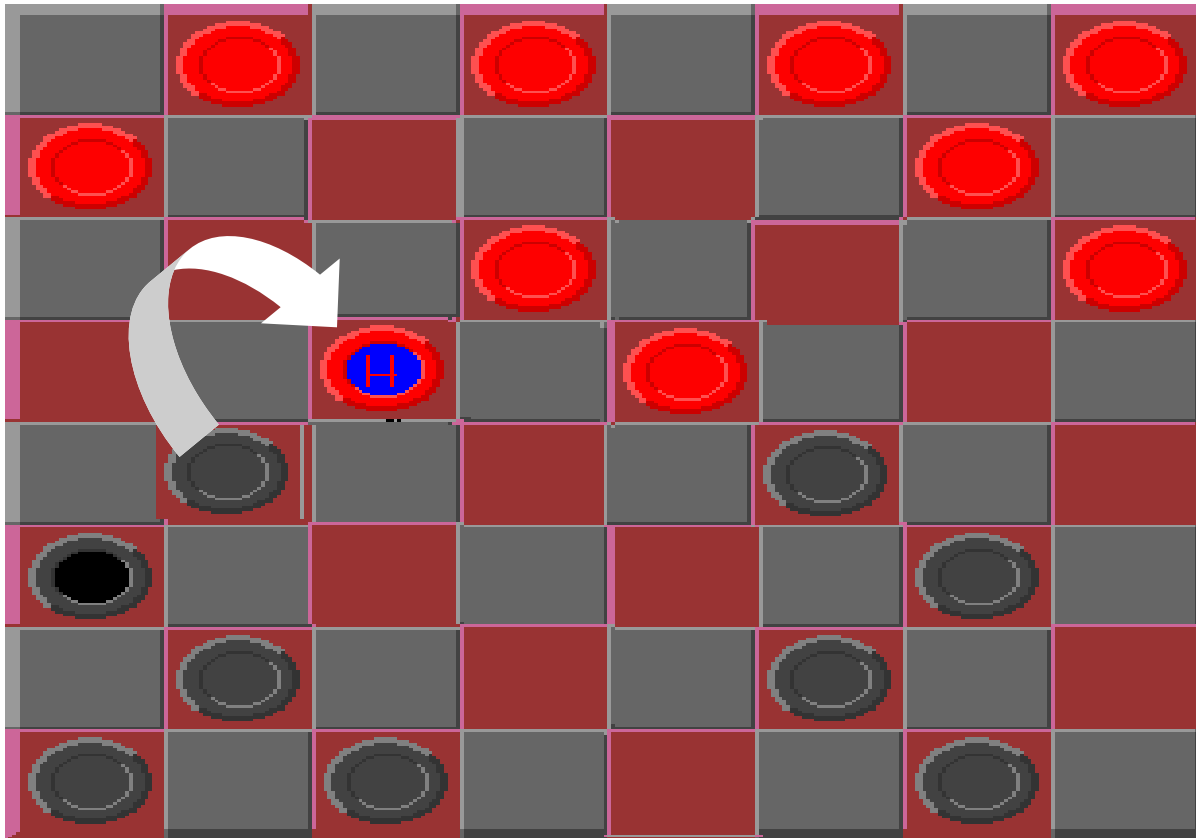
## Estimation example: null move by opponent.



$$\Pr_{\text{new}}(\text{hidden at } (3,3)) = 1 / 2 \Pr_{\text{old}}(\text{hidden at } (4,4))$$

$$\Pr_{\text{new}}(\text{hidden at } (3,5)) = 1 / 2 \Pr_{\text{old}}(\text{hidden at } (4,4))$$

## Estimation example: reconnaissance move by self.



$$\Pr_{\text{new}} (\text{hidden at } \mathbf{x}) =$$

$$\Pr_{\text{old}} (\text{hidden at } \mathbf{x} \mid \text{not at } (4,2))$$

## Information theoretic advisor

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- Opponent's movement of hidden pieces increases uncertainty of state.
- Reconnaissance moves decrease uncertainty.
- Entropy the best way to model this.
- If opponent's states have probability  $p_1, p_2, \dots; p_n$
- Then the entropy is  $H = -\sum p_i \log_2 p_i$ .
- We incorporate the value of reconnaissance moves through a term in the evaluation function that takes into account the entropy.

## Evaluation of Moves

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- Have to calculate the expectation over the possible board states. Consider moves that are legal for a particular opposition board state (with probability  $>0$ ) or a reconnaissance move.
- Reconnaissance move- find out where the opponent is or isn't.
- Intend to use temporal difference learning to find the advisor weights.
- Depth of search nearly impossible, since have to carry on the same estimation/evaluation cycle.

## Current research questions

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- If you don't see an opponent move
  - Was it a reconnaissance move made or a hidden move?
  - Have to learn this through Bayesian methods.
- Want to look at entropy balance
  - We therefore have to estimate our opponents estimate of our state probabilities.
- Pruning
  - What happens when we prune boards with extremely low probability?
- Are there fast and frugal heuristics to generate strategies equal or better than the computationally expensive way?

Questions?