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Power-Aperture Product, Efficiency, Signal to Noise Ratio and Search Function Time of Weighted Phased Arrays

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Electronic Warfare and Radar Division
Defence Science and Technology Organisation

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ABSTRACT

The power-aperture product and efficiency of weighted phased arrays is examined for various windows. The impact on the signal to noise ratio is also investigated for the use of windows on the transmit and receive modes of operation of the array and we examine the dependence of the search function time on the array windows. In determining the search function time to conduct surveillance over a given region of space it is necessary to obtain the surveillance solid angle which we obtain via an integral solution.

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EXECUTIVE SUMMARY

The power-aperture product and efficiency of weighted phased arrays is examined for various windows. The impact on the signal to noise ratio is also investigated for the use of windows on the transmit and receive modes of operation of the array and we examine the dependence of the search function time on the array windows. In determining the search function time to conduct surveillance over a given region of space it is necessary to obtain the surveillance solid angle which we obtain via an integral solution.

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1. Introduction

Partial performance of phased array radar relies on its power-aperture product (z) which is the product of the average radiated power and the effective array aperture area. Ideally, we would like to increase both the power and aperture without any limit, however, as these parameters increase so does the need for more resources which inevitably limit the radar performance. In this paper we examine this and other issues by considering weighted arrays consisting of N equally spaced elements with inter-element phase shift α and radiation angle θ from the plane of the array. For even and odd elements in the array we can express the array factor as

$$f(\psi) = 2 \sum_{m=1}^{N/2} \tilde{a}_m \cos\left[\left(m - \frac{1}{2}\right)\psi\right] \quad (1)$$

and

$$f(\psi) = \tilde{a}_0 + 2 \sum_{m=1}^{(N-1)/2} \tilde{a}_m \cos[m\psi] \quad (2)$$

respectively, where $\psi = (2\pi d / \lambda) \cos(\theta) + \alpha$ and d is the element spacing while λ is the wavelength. If all the amplitudes are unity ($\tilde{a}_m = 1$) then we obtain a far field radiation pattern that is the well known sinc-function ($\text{sinc}(x) = \sin(x)/x$). In what follows we examine the window functions (\tilde{a}_m) in (1) and (2) which are then compared to an unweighted array.

2. The Array Windows

We begin by considering Chebyshev weights [1] which we obtain using series expansions. For an even number of elements, $M = N/2$, so we have

$$\tilde{a}_n = \sum_{q=n}^M \frac{(2M-1)(q+M-2)!}{(q-n)!(q+n-1)!(M-q)!} (-1)^{M-q} z_0^{2q-1} \quad (3)$$

for $n = 1, 2, 3, \dots, M$. Alternatively, for an odd number of elements $M = (N-1)/2$, so we obtain the expansion

$$\tilde{a}_n = \sum_{q=n}^{M+1} \frac{2M(q+M-2)!}{(q-n)!(q+n-2)!(M-q+1)!} (-1)^{M-q+1} z_0^{2(q-1)} \quad (4)$$

with $n = 1, 2, 3, \dots, M+1$. It should be noted that the series expansion (4) is the correct version for an odd number of elements. Balanis [2] includes a factor ε_n in the denominator of his series expansion where he takes:

$$\varepsilon_n = \begin{cases} 2, & n = 1 \\ 1, & n \neq 1 \end{cases} \quad (5)$$

Equation (5) gives the wrong values for the window coefficients and should in fact be $\varepsilon_n = 1 \forall n$. Finally we define z_0 to be

$$z_0 = \frac{1}{2} \left[\left(R + \sqrt{R^2 - 1} \right)^{\frac{1}{(N-1)}} + \left(R - \sqrt{R^2 - 1} \right)^{\frac{1}{(N-1)}} \right] \quad (6)$$

where R is the ratio of the side lobe level. The next window we use is the Kaiser window [3] which is obtained by

$$\tilde{a}_n = I_0 \left(\beta \sqrt{\frac{1 - 4n^2}{(N-1)^2}} \right) / I_0(\beta) \quad (7)$$

where $-(N-1)/2 \leq n \leq (N-1)/2$, β controls the amplitude for the sidelobes and $I_0(x)$ is the modified zeroth-order Bessel function. If γ is the amplitude dB ratio of the main lobe to the largest sidelobe then we can determine β by

$$\beta = \begin{cases} 0, & \gamma \leq 13.26 \\ 0.76609(\gamma - 13.26)^{0.4} + 0.09834(\gamma - 13.26), & 13.26 < \gamma < 60 \\ 0.12438(\gamma + 6.3), & 60 \leq \gamma < 120 \end{cases} \quad (8)$$

For the Blackman-Harris window [4] we have

$$\tilde{a}_{k+1} = \bar{a}_0 - \bar{a}_1 \cos\left(2\pi \frac{k}{N-1}\right) + \bar{a}_2 \cos\left(4\pi \frac{k}{N-1}\right) - \bar{a}_3 \cos\left(6\pi \frac{k}{N-1}\right) \quad (9)$$

where $0 \leq k \leq (N-1)$ and the coefficients are $\bar{a}_0 = 0.35875$, $\bar{a}_1 = 0.48824$, $\bar{a}_2 = 0.14128$ and $\bar{a}_3 = 0.01168$. We note the fact that the original paper by Harris [4] contained significant errors for the window coefficients in (9) which were accurately obtained a few years later by Nuttall [5]. The Hamming window [6] is calculated using

$$\tilde{a}_{k+1} = 0.54 - 0.46 \cos\left(2\pi \frac{k}{N-1}\right) \quad (10)$$

with $k = 0, \dots, N-1$. Finally we make use of the Hann window

$$\tilde{a}_{k+1} = 0.5 \left(1 - \cos\left(2\pi \frac{k}{N-1}\right) \right) \quad (11)$$

for $k = 0, \dots, N-1$. Equations (9)-(11) are the generalised cosine windows with symmetric sampling parameters. In the next sections we will consider the effects of these weightings on the power-aperture product and efficiency of arrays as well as the signal to noise ratio. We note here that in what follows, we consider the low signal to noise ratio (SNR) since any increase of the power-aperture product beyond a certain level implies that the clutter to noise ratio (CNR) dominates instead.

3. The Power-Aperture Product and Efficiency of a Weighted Array

When it comes to antenna synthesis it is important to consider what performance factors are needed because it is not possible to optimise everything in antenna

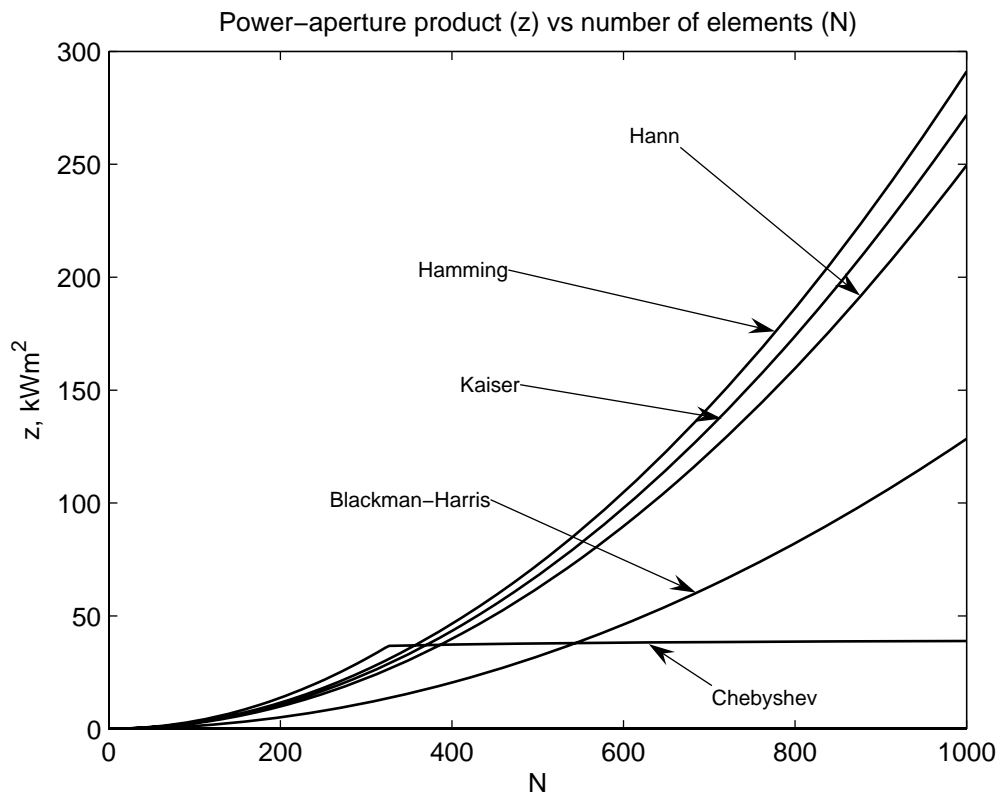


Figure 1. The total transmitted power-aperture product (z) is shown for $N=1000$ elements. For the Chebyshev and Kaiser weights, a side lobe level of $SLL=-40$ dB was chosen. We have assumed that $p_{element}a_{element}=1 \text{ Wm}^2$

design. For instance the beam pattern of a Chebyshev weighted array, formed using an odd number of radiating elements, gives greater directivity (or gain) but broader main beamwidth compared to when an even number of elements is used. When we consider such arrays in terms of the power-aperture product (z) and efficiency (ε) relative to an unweighted array the opposite of what we expect occurs. For example it turns out that for Chebyshev arrays the optimum efficiency of the arrays is greatest for an even number of elements even though we have a reduced gain as a result. It is thus also necessary to look at the power-aperture product and efficiency of various weighted arrays and compare them to the unweighted version in order to see what implications exist in array performance. Let the power-aperture product of a weighted array be given as $z = p_{array} a_{array}$ where we define p_{array} to be the average power of the array and a_{array} to be the effective area of the array. Then we have

$$z = \left[\sum_{i=1}^N \tilde{a}_{t,i} \tilde{a}_{r,i} \right]^2 p_{element} a_{element} \quad (12)$$

with $p_{element}$ as the average power of the unweighted element (assumed identical across the array), $\tilde{a}_{t,i}$ are the normalised amplitude weights applied on transmission to element i , $\tilde{a}_{r,i}$ are the normalised amplitude weights applied on reception to element i . Where the element normalisation is such that $\max(\tilde{a}_i) = 1$, it reflects the practical issue that the antenna weighting is normally achieved by attenuating an element. For an unweighted antenna $\tilde{a}_{t,i} = 1$ and $\tilde{a}_{r,i} = 1$, reducing the previous

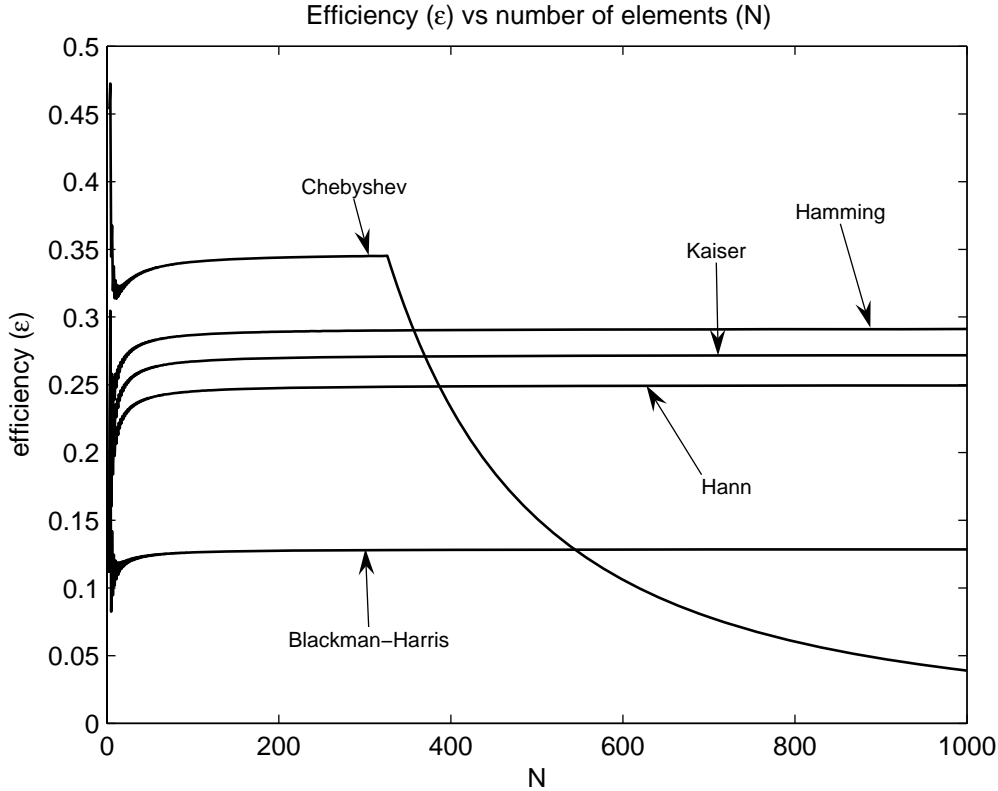


Figure 2. The efficiency of arrays using different windows is compared as a function of the number of elements. The Chebyshev and Kaiser windows were calculated using a SLL=-40 dB.

result to the well known $z = N^2 p_{element} a_{element}$. The relative efficiency of the weighted array to the unweighted array is then given by

$$\epsilon = \frac{1}{N^2} \left[\sum_{i=1}^N \tilde{a}_{t,i} \tilde{a}_{r,i} \right]^2 \frac{p_{element} g_{element}}{P_{element} g_{element}} = \frac{1}{N^2} \left[\sum_{i=1}^N \tilde{a}_{t,i} \tilde{a}_{r,i} \right]^2 \quad (13)$$

where $g_{element}$ is the gain of the unweighted element. Figure 1 shows the power-aperture product (z) as a function of the number of elements. Here the transmitted mode is weighted while the reception is unweighted (or vice-versa). It is apparent that for any given window the power-aperture product increases as the number of elements is increased except for the Chebyshev window which increases slightly beyond a critical point as the number of elements is increased. The reason for this is due to the fact that once the array increases beyond a certain size the Chebyshev weights must transfer power from the main lobe to the sidelobes in order to maintain the specified constant sidelobe ratio. The same behaviour is seen in Figure 2 which shows the efficiency of the arrays for different windows. For all windows the efficiency ‘saturates’ for large arrays except for the Chebyshev weights which drops significantly. Figure 3 examines the issue of whether for a given two-way sidelobe level it is more efficient to weight both transmission and reception or just the one- which is usually reception because of interference rejection considerations. Windows with superscript “1” represent an unweighted transmission and weighted



Figure 3. A comparison of the efficiency vs number of elements is made of arrays with different weights for both transmit (T_x) and receive (R_x) modes. For windows with superscript (1) the transmitted power is unweighted but the reception is weighted. The side lobe level is chosen to be $SLL=-40$ dB for the Chebyshev and Kaiser windows. For windows with superscript (2) both the transmit and receive modes are weighted and here $SLL=-20$ dB for the Chebyshev and Kaiser windows.

reception of -40 dB (for Chebyshev and Kaiser). On the other hand windows with superscript "2" represent the case of both transmit and receive modes being weighted and with -20 dB sidelobes for the Chebyshev and Kaiser windows. Broadly speaking, the efficiency is greatest for all cases when the transmit mode is unweighted and the receive mode is weighted except for the Chebyshev window applied to small arrays where weights on both T_x and R_x are the most efficient. Having said this, the Kaiser windows exhibit the complete opposite behaviour when compared to the other weights. For Kaiser windows it is always more efficient to apply windows on transmit and receive at the same time while it is less efficient if transmission is unweighted and reception is weighted.

4. The Impact on the Signal to Noise Ratio of a Weighted Array

In addition to the impact on the signal levels by the weighting function applied, the noise levels must also be considered by examining the resultant signal to noise ratio (SNR). If we assume that the noise at each receiver is independent and of the same mean power level, then the expected value of the system noise after weighting will be,

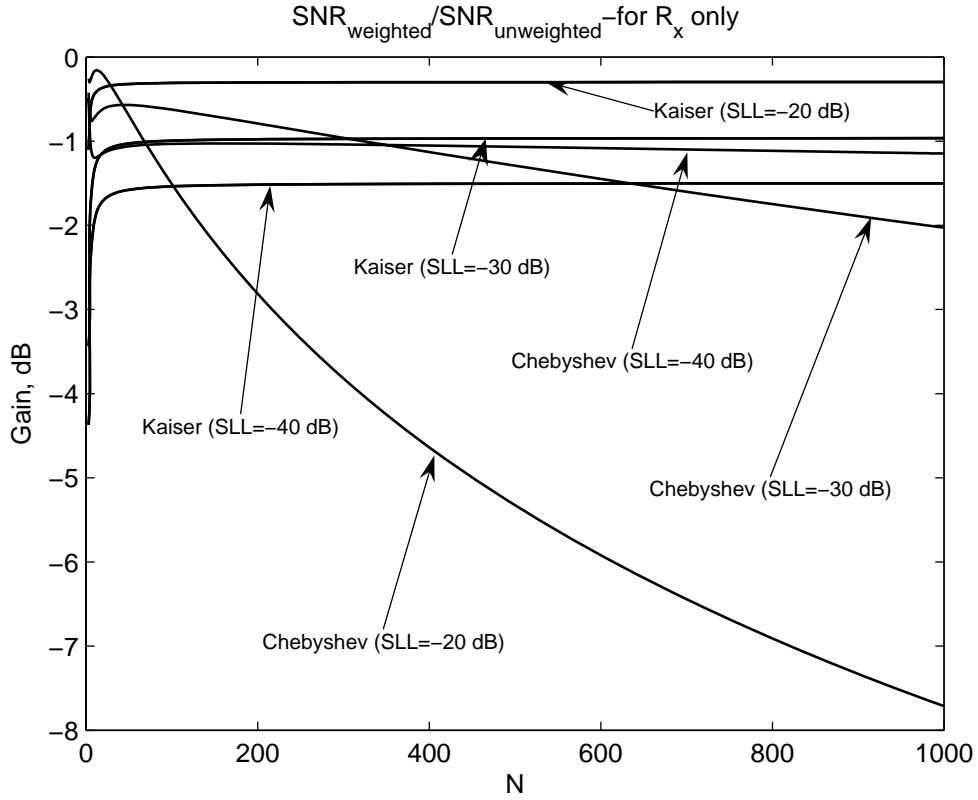


Figure 4. Impact upon the SNR of Chebyshev and Kaiser weights applied on reception only.

$$n_{array} = \left[\sum_{i=1}^N \tilde{a}_{r,i}^2 \right] n_{element} \quad (14)$$

where n_{array} is the average noise power of the array, $n_{element}$ is the average noise power of the unweighted element (assumed identical across the array), $\tilde{a}_{r,i}$ are the normalised amplitude weights applied on reception to element i . The signal to noise ratio is

$$(SNR)_{weighted} = \frac{p_{array}}{n_{array}} = \frac{\left[\sum_{i=1}^N \tilde{a}_{r,i} \tilde{a}_{r,i} \right]^2}{\left[\sum_{i=1}^N \tilde{a}_{r,i}^2 \right]} (SNR)_{element} \quad (15)$$

where $(SNR)_{element} = p_{element} / n_{element}$. An array that is unweighted on both transmit and receive gives $(SNR)_{unweighted} = N(SNR)_{element}$. Hence the impact upon the SNR of weighting the array is given by

$$\frac{(SNR)_{weighted}}{(SNR)_{unweighted}} = \frac{1}{N} \frac{\left[\sum_{i=1}^N \tilde{a}_{r,i} \tilde{a}_{r,i} \right]^2}{\left[\sum_{i=1}^N \tilde{a}_{r,i}^2 \right]} \quad (16)$$

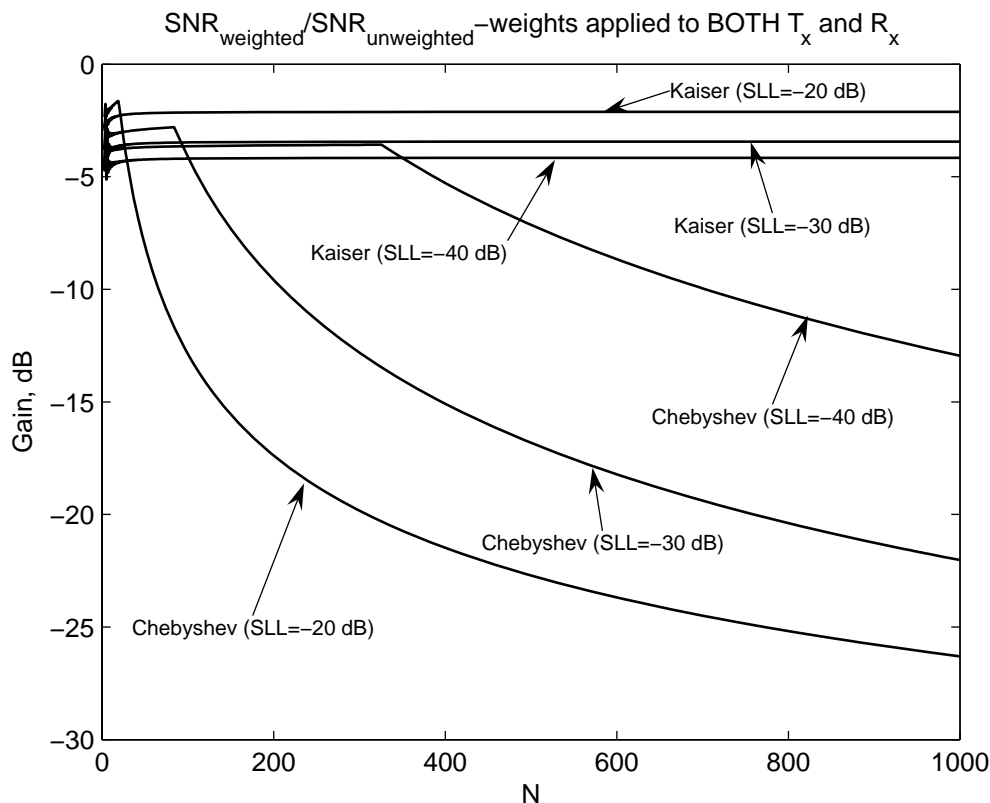


Figure 5. The impact of the SNR of Chebyshev and Kaiser weights applied both on transmit and reception.

Figure 4 shows the SNR for an unweighted transmit mode and a weighted receive mode for Chebyshev and Kaiser windows. The sidelobe levels vary from 20-40 dB. The reverse behaviour is shown for the two different windows with the Chebyshev having the greatest SNR loss at -20 dB sidelobes while for the same sidelobe level the Kaiser shows minimum loss. The losses for the Chebyshev window in particular shown here match those reported in the literature [7] provided that the array is sufficiently small, otherwise the literature understates the losses that are experienced. Figure 5 shows the results for the Chebyshev and Kaiser windows when both transmit and receive weighting is in operation.

5. Impact of Weighted Arrays on the Surveillance Search Time

We can obtain a qualitative feel for the search function time as a function of the number of array faces for weighted arrays if we consider that the search function time τ is related to the power aperture product z :

$$\tau = \frac{k\psi_s}{z} \quad (17)$$

where k is the system constant which varies depending on the radar type and contains parameters such as the maximum range, signal-to-noise ratio and so on. The search area is obtained by determining the search solid angle ψ_s which we obtain from

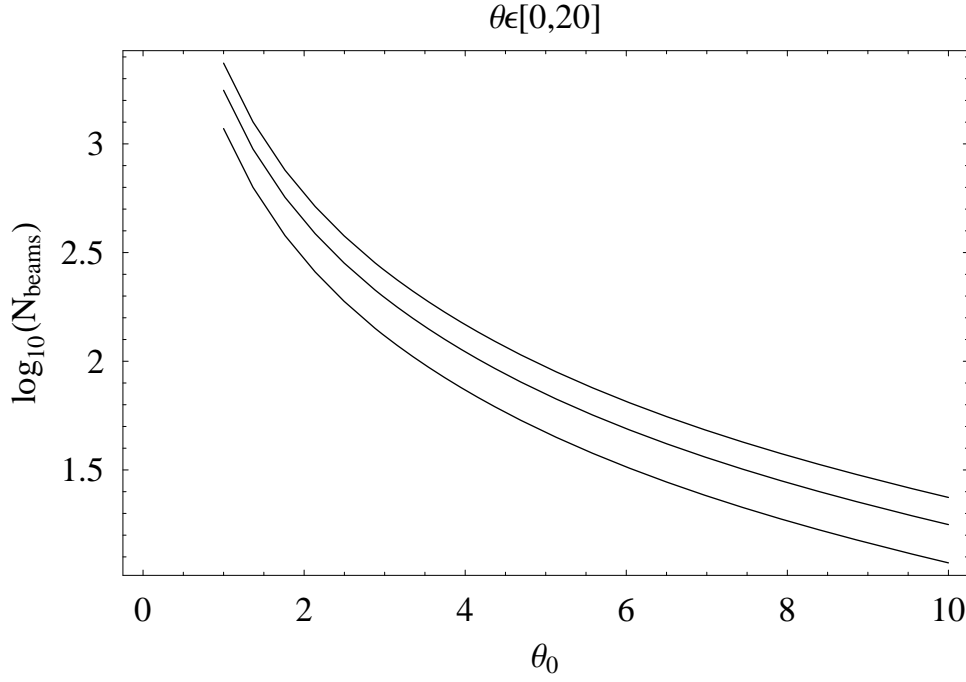


Figure 6. The number of beams for arrays with elevation angles in the range 0-20 degrees as a function of the beam overlap angle θ_0 in degrees. The top curve represents the case for $N_F = 3$, the middle curve is for $N_F = 4$ and the bottom curve is the $N_F = 6$ case. The array tilt angle is taken to be $\alpha = 0^\circ$.

$$\psi_S = \iint_S r^{-2} \hat{n} \cdot d\bar{a} = - \int_a d\bar{a} \cdot \nabla r^{-1} \quad (18)$$

In (18), we can evaluate the Nabla using spherical coordinates which simplifies to

$$\psi_S = \iiint_S \sin(\frac{1}{2}\pi - \theta - \alpha) d\theta d\phi \quad (19)$$

where α is the array tilt angle. If we let the elevation angles bounding the surveillance region be defined as θ_1 and θ_2 then the azimuth angles must dictate the maximum extent that one array face can have in azimuth for surveillance purposes. Thus the azimuth angles must depend on the number of faces considered, ie, $N_F = \pm\pi / \phi_{1,2}$ so that the surveillance solid angle becomes

$$\psi_S = \int_{-\pi/N_F}^{\pi/N_F} \int_{\theta_1}^{\theta_2} \sin(\frac{1}{2}\pi - \theta - \alpha) d\theta d\phi \equiv \frac{2\pi}{N_F} \int_{\theta_1}^{\theta_2} \sin(\frac{1}{2}\pi - \theta - \alpha) d\theta \quad (20)$$

Equation (20) finally reduces to the analytic expression

$$\psi_S = \frac{2\pi}{N_F} \left\{ \cos\left(\frac{\pi}{2} - \theta_2 - \alpha\right) - \cos\left(\frac{\pi}{2} - \theta_1 - \alpha\right) \right\} \quad (21)$$

From the expression for the search time (17) we have

$$\tau = \frac{2\pi k}{N_F} \left[\sum_{i=1}^N \tilde{a}_{t,i} \tilde{a}_{r,i} \right]^{-2} \left\{ \cos\left(\frac{\pi}{2} - \theta_2 - \alpha\right) - \cos\left(\frac{\pi}{2} - \theta_1 - \alpha\right) \right\} \quad (22)$$

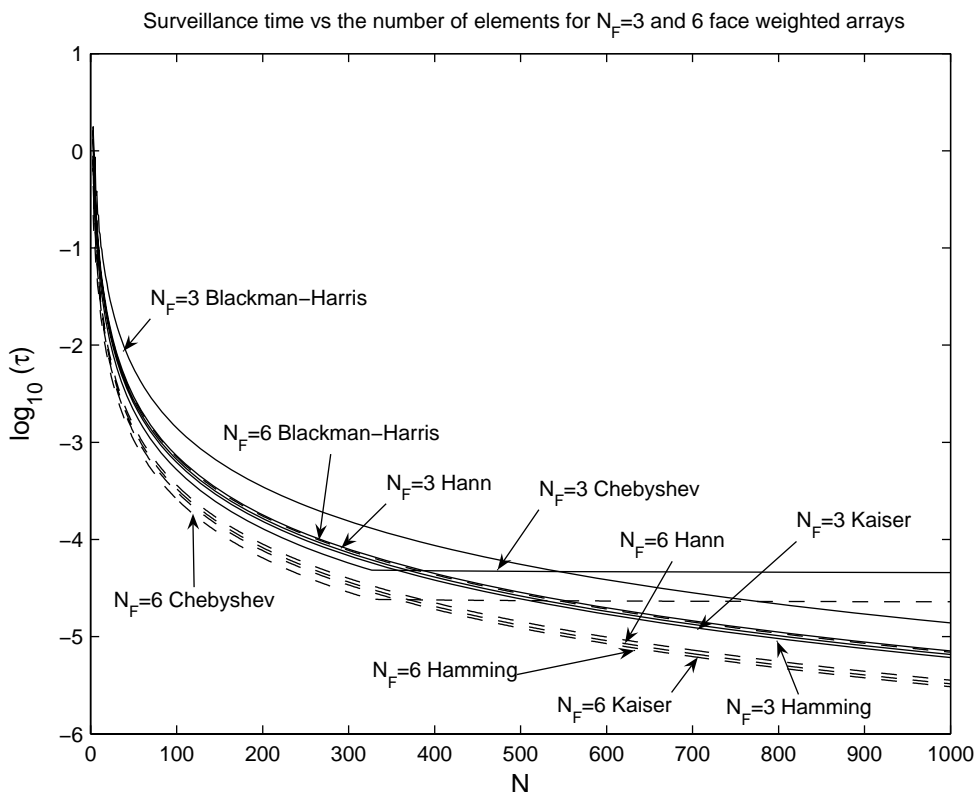


Figure 7. The relative dependence of the search function time τ to cover different regions of surveillance as given by the number of faces N_F (corresponding to different azimuth angles). The results are for T_x unweighted and R_x weighted with the Chebyshev and Kaiser windows at a SLL of -40 dB. As expected, the search time drops dramatically as the number of elements in the array increase with the smallest search times in this case occurring mostly for the $N_F = 6$ case-dashed curves. Note that the $N_F = 3$ Hann, $N_F = 6$ Blackman-Harris and $N_F = 3$ Kaiser windows give approximately the same surveillance time. Furthermore, for large N the surveillance time is reversed for some weights, eg, $N_F = 3$ Blackman-Harris compared to the $N_F = 3$ and $N_F = 6$ Chebyshev. The array tile angle is taken to be $\alpha = 0^\circ$.

where $p_{element}$ and $a_{element}$ have been absorbed in the system constant k . We note that when it comes to hemispherical surveillance, (22) is for one active array face in simultaneous use. Alternatively if we require the search time τ' to cover 360 degrees surveillance in terms of different number of active array faces in simultaneous use (N_{use}), then we have $\tau' = N_{use} \tau$ where τ is given by (22). From (22) we see that for an unweighted array the search function time τ has the same form except that

$$\left[\sum_{i=1}^N \tilde{a}_{t,i} \tilde{a}_{r,i} \right]^2 \equiv N^2 \quad (23)$$

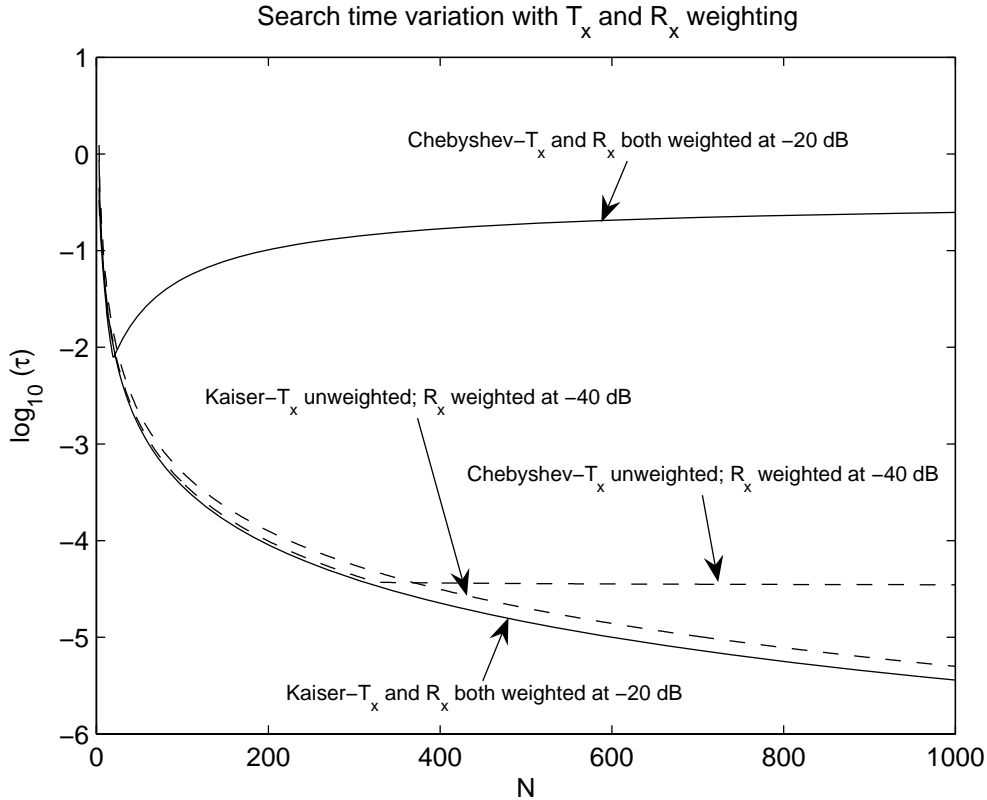


Figure 8. For a $N_F = 4$ face array, we examine the variation of the surveillance search time for the case where the T_x are unweighted but the R_x are weighted (dashed curves). On the other hand the solid curves represent the case where both T_x and R_x are weighted as shown.

From (19) we can also obtain the solid angle ψ_{beam} of the beams that overlap at an arbitrary angle θ_0 , which can be the half-power beamwidth for example, so

$$\psi_{beam} = \int_0^{\theta_0} \int_0^{\theta_0} \sin(\frac{1}{2}\pi - \theta) d\theta d\phi \tag{24}$$

The solution of (24) is simple and with the use of (21) we can obtain an analytic expression for the number of beams:

$$N_{beams} = \frac{2\pi [\cos(\frac{1}{2}\pi - \theta_2 - \alpha) - \cos(\frac{1}{2}\pi - \theta_1 - \alpha)]}{N_F \theta_0 \cos(\frac{1}{2}\pi - \theta_0)} \tag{25}$$

Equation (25) gives exactly the same results as those by Billam [8] for the number of beams but Billam derives his results by transforming the problem to the more complicated uv -space coordinate system. In Figure 6 the number of beams have been plotted as a function of the overlap angle θ_0 for $N_F = 3, 4, 6$ array faces for elevation angles in the range 0-20 degrees. In Figure 7 we show the search time for $N_F = 3$ and 6-face weighted arrays with tilt angle of $\alpha = 0^0$. The results are for angle limits of $\theta_1 = 0^0$ and $\theta_2 = 60^0$ respectively. The side lobe level (SLL) is chosen as -40 dB for the Chebyshev and Kaiser windows. Since a 360 degree surveillance is determined by the number of faces N_F which is related to the maximum azimuth extent for one array face, Figure 7 shows that in general, the smallest search times come from

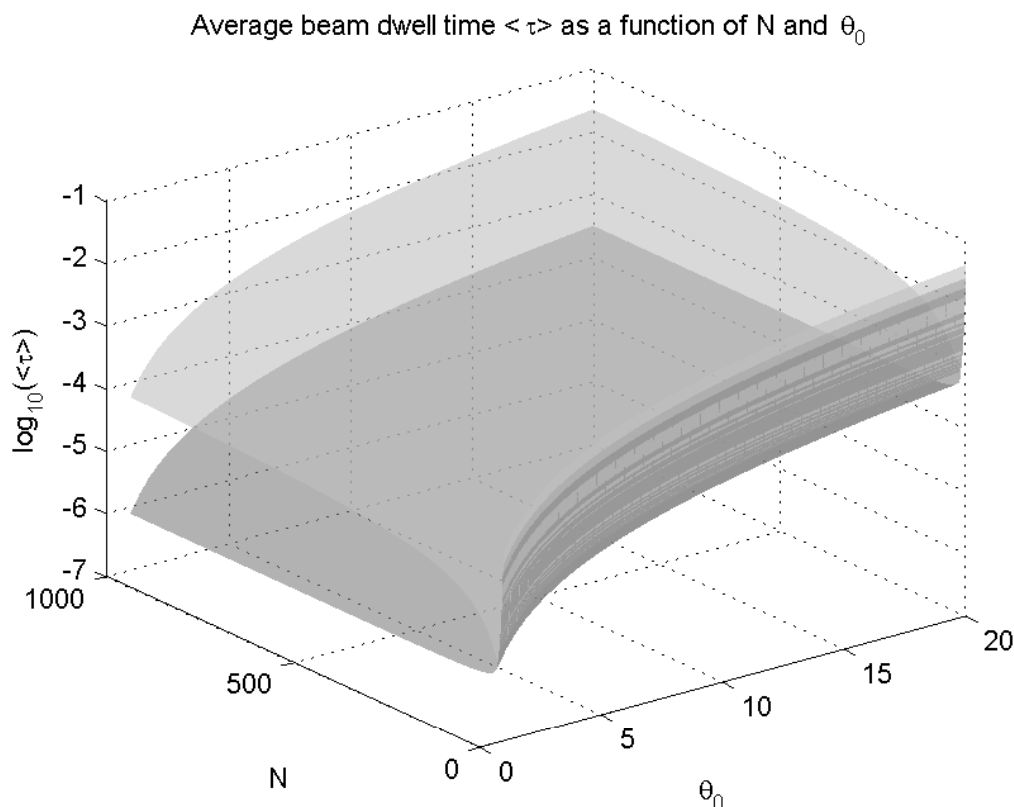


Figure 9. The average beam dwell time vs the number of elements and the beam overlap angle (in degrees). The lighter (top) surface has T_x and R_x weighted at SLL=-20 dB while the darker (bottom) surface is for T_x unweighted and R_x weighted at SLL=-20 dB. The plots are for Chebyshev arrays.

$N_F = 6$ face arrays (dashed curves) but note that windows for the $N_F = 3$ Hann, $N_F = 6$ Blackman-Harris and $N_F = 3$ Kaiser cases give approximately the same surveillance search time. This is somewhat true for the window functions $N_F = 6$ Hamming, Kaiser and Hann. As can be seen, the search time to cover a given region can be changed depending on the selection of the weighting functions. For example, the $N_F = 3$ Blackman-Harris weights give a search time that is greater than that of the $N_F = 3$ and $N_F = 6$ Chebyshev cases but when the number of elements increases beyond a certain point the reverse holds. Figure 8 shows the changes in the surveillance time depending on whether one or both of the transmit and receive modes are in operation. The odd dip and subsequent increase of the search time in the case of the Chebyshev curve with T_x and R_x both weighted at -20 dB can be explained via Figure 3 (see the Chebyshev² curve). As the efficiency increases the search time drops in Figure 8 but as the efficiency starts to rapidly decline the search time increases in Figure 8. Thus, the search time for each region depends on the window function (efficiency) and is furthermore affected by the total area under surveillance which has direct impact on τ . More precisely, we can show that the ratio of the weighted surveillance time τ_w to that of the unweighted surveillance time τ_U is highly dependent on the array weights. It is easy to show that they are related by the inverse of the array efficiency ε ,

$$\frac{\tau_w}{\tau_U} = \frac{1}{\varepsilon} \quad (26)$$

where ε is given by (13)-see also Figure 2 and 3. Another very important quantity of interest is the determination of the average beam dwell time $\langle \tau \rangle$ as a function of the scan angles. Multiplying the average beam dwell time with the total number of beams gives the surveillance search time τ for a subregion or frame time for a region, ie, $\tau = \langle \tau \rangle N_{beams}$. From this we can obtain, using (22) and (25),

$$\langle \tau \rangle = k \theta_0 \cos\left(\frac{1}{2}\pi - \theta_0\right) \left[\sum_{i=1}^N \tilde{a}_{t,i} \tilde{a}_{r,i} \right]^{-2} \quad (27)$$

where throughout we have taken the value of the system constant k to be unity without loss of generality. In Figure 9 we show the variation of the average beam dwell time as a function of the number of elements N and overlap angle θ_0 for Chebyshev weighted arrays with a SLL of -20 dB in all cases. The smallest surveillance times occur for the case where we have unweighted transmission and weighted reception. However, we also observe that the beam overlap angle plays a significant role in increasing the average beam dwell time, especially for large arrays.

6. Conclusions

Using various window functions we have obtained the results for the power-aperture product against the number of elements for arrays. The efficiency of such arrays has been discussed and the effect this has on the signal-to-noise ratio has also been investigated. Finally we have looked at the way weighted arrays impact the values for the surveillance search time and have derived a general expression for this.

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