

AN ENSEMBLE OF ULTRA-STABLE QUARTZ OSCILLATORS TO IMPROVE SPACECRAFT ONBOARD FREQUENCY STABILITY

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Abstract

In the past years, we have reported on continuous improvement in the capability of our Time and Frequency Laboratory at the Johns Hopkins University Applied Physics Laboratory and its contribution to the TAI. We have discussed our ensemble of hydrogen maser and cesium beam atomic clocks into a timescale that enables UTC (APL) to be steered within ± 20 nanoseconds per month of UTC. A significant element to the control of UTC (APL) has been the use of our autonomous timescale algorithm, reported in 2005 at the combined PTTI and IEEE FCS.

In our laboratory report this year, we will discuss experimentation using this timescale algorithm to investigate the timekeeping capability of an ensemble of three quartz ultra-stable oscillators (USO). This experimental ensemble could represent the frequency references of a system of spacecraft, similar to those proposed in deep space missions of long duration. Although these spacecraft would likely be in regular communication with the ground segment controllers, it may be necessary that they operate autonomously for periods of time ranging from a few days to several weeks. To make this possible, we are considering improvement in the frequency stability of the onboard timing reference that will act as a fly-wheel when frequency corrections from mission controllers have been temporarily interrupted. We present experimental data and examine the compliance of the ensemble of three USOs to a frequency stability of better than $\pm 1 \times 10^{-11}$ per week.

I. MOTIVATION FOR EXPERIMENT

With expanding interest in both human and robotic exploration of the Moon, Mars, and other deep space destinations, there is a need for reliable and accurate timekeeping and navigation at those locations. Those destinations are well beyond the reaches of Earth-based or Earth-orbiting timekeeping and navigation systems, so other space-based systems are under consideration. One could envision an orbiting satellite system such as the Global Positioning System (GPS), Galileo, or GLONASS around the Moon and Mars, but we are proposing the possibility of a timekeeping and navigation system based on a frequency reference with greater reliability and less frequent interaction with Earthbound resources.

Quartz-based oscillators have evolved over many years to address a wide range of frequency control applications. The most stable quartz oscillators are known as Ultra-Stable Oscillators (USOs) and find space flight use on a limited number of scientific and military satellites. While USOs do not have the long-term frequency stability of atomic clocks, the USO reliability and lifetime are superior, and their in-flight drift characteristics easily predicted once their environments stabilize. Fig. 1 shows measured performance of a Johns Hopkins University Applied Physics Laboratory (JHU/APL) USO in space flight

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use in orbit around Mars. The frequency performance is very well behaved, with a drift rate that gradually declined to a near asymptotic state, suggesting that future performance is predictable and, therefore, correctable over discrete time intervals spanning several days.

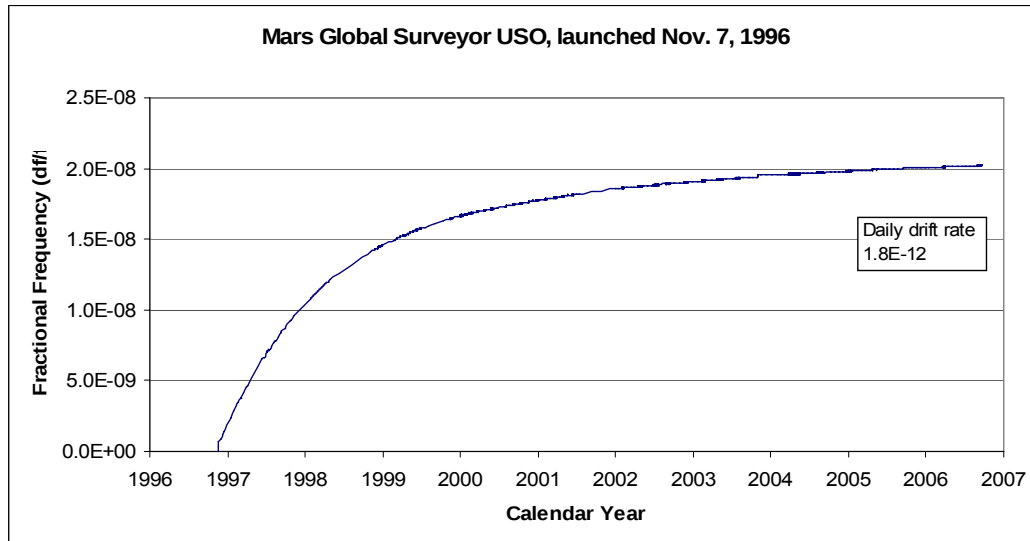


Figure 1. In-flight frequency performance of USO in orbit at Mars.

We have performed a series of experiments on several USOs in a laboratory vacuum environment to explore the viability of disciplining an ensemble of USOs with only occasional interaction from “ground truth” or master clock reference. In our experimental case, that means a steering correction is applied based on time comparison with our atomic clock facility. While GPS satellite clocks are steered on a 12-hour basis, we expanded that interval to 1 week to investigate a potential constraint associated with deep space operation. The 1-week interval is purely notional and could certainly vary depending on mission accuracy needs, and Earth contact availability.

Another aspect under consideration is how to mathematically derive the steering correction value based on predicted ensemble performance. Several algorithms have been developed for ground-based atomic clock facilities, but these algorithms are not necessarily optimized for quartz-based timekeeping, and so are being modified for this application.

II. EXPERIMENT COMPONENTS FROM RECENT APL DEVELOPMENTS

Over the last several years, JHU/APL has undertaken research in the use of our quartz USO in simulated space conditions to establish their feasibility for frequency disciplining with sparse access to a remotely located reference. The primary interest of this research is the predictability of the USO after characterization using Allan deviation. Coordinately, our research has studied the need for updating the estimation and drift removal process with the focus on establishing the period for which the disciplined USO could be expected to run autonomously [1].

At the 2005 Joint Conference of the PTTI and the IEEE Frequency Control Symposium, we reported on the study of a JHU/APL USO characterized over 70 days of operation in space-simulated conditions. The performance of this USO is shown in Fig. 2. The smooth decreasing curve shows the change in USO frequency in fractional frequency units of 1.0×10^{-10} scaled on the left-hand Y axis. The peaked irregular curve shows the two-sample, Allan deviation at 1-day time intervals over the same 70-day operational period, scaled on the right-hand Y axis in fractional frequency units of 2×10^{-12} . Notice the correlation between the frequency drift or slope of the frequency curve and the Allan deviation [2].

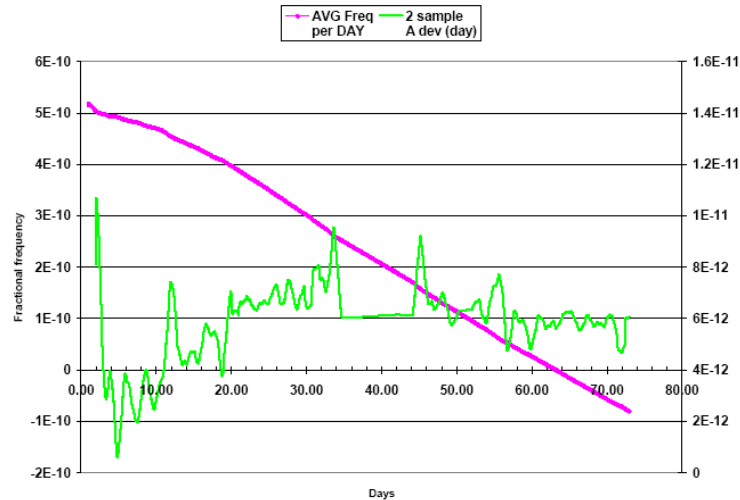


Figure 2. 70-day frequency history of JHU/APL USO reported in 2005.

The characterization of the dynamic frequency shown in Fig. 2 found that the nature of this free-running USO, acquired in a space simulated environment, was similar to the performance recovered from USOs in actual space missions. Also, the frequency character of this USO proved sufficiently well behaved to attempt drift removal through optimal estimation. In fact, the time-dependent drift process measured slow enough to consider frequency holdover operation for periods of 30 days without an update to the estimation of frequency character. All sudden frequency changes due to environmental changes or abrupt jumps were not found severe, even though vacuum-level intermittent changes and temperature transients between $+12^{\circ}\text{C}$ and $+25^{\circ}\text{C}$ were experienced by the USO over the 70-day testing period. To conduct our drift estimation and removal technique, we chose to include the performance of the USO referenced in Fig. 2 into an experimental version of our autonomous timescale algorithm used in the operation of our Time and Frequency Laboratory.

Two aspects of a USO's frequency character were found important regarding its use in the experimental autonomous timescale formulation. First, the overall rate or syntonization error is several orders of magnitude greater than that of either the cesium beam standards or the masers that comprise the ensemble of our Time and Frequency Laboratory. Second, the USO has a complex drift process not well estimated by a quadratic, since the drift character is neither constant nor monotonic. We were concerned that these USO aspects might corrupt the stability of the timescale, since their complexity is well outside the normal characterization of cesium beam standards (dominated by rate) and hydrogen masers (mostly constant drift).

Fortunately, such corruption was not observed when the autonomous algorithm formulated the experimental timescale. As shown in Fig. 3, the USO's complex frequency character is seen as an undulating line against the almost purely linear plots of the experimental timescale and the other six

atomic clocks. The USO only skews the resulting experimental timescale, due to its less than optimal characterization, by less than 100 nanoseconds over the 36 days of interpolated time as compared to the operational timescale of UTC (APL).

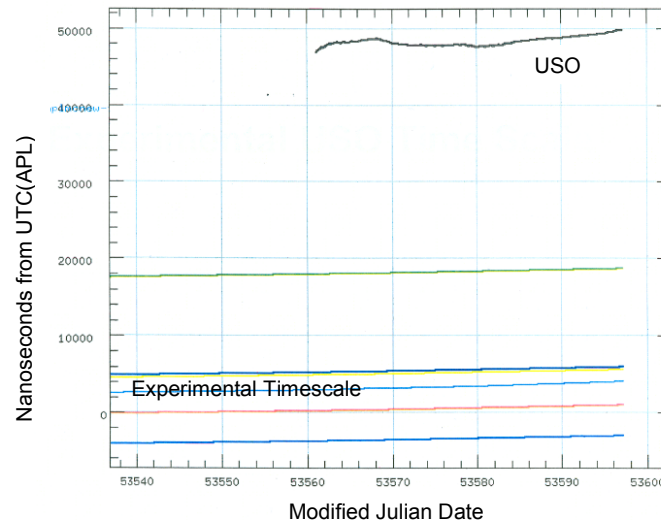


Figure 3. Influence of USO in autonomous timescale with six atomic clocks.

The result found from the experimental timescale demonstrates the ability to maintain stability of a small number of clocks with disparate frequency character. This reinforces our supposition that JHU/APL USOs, in the complexity of space environment, could be disciplined under an update rate approaching 30 days. In fact, a rough approximation from Fig. 3 indicates that this stability should be at less than $\pm 2 \mu\text{s}$ over 30 days, which represents nearly three orders of magnitude improvement over the uncharacterized USO time stability.

The timekeeping performance of the JHU/APL autonomous timescale algorithm using a USO and six atomic clocks advanced our confidence that an ensemble of three or more USOs systems might provide improved frequency stability to remote space mission applications. In our notion, the optimized frequency character of the USO timekeeping ensemble would be used to form a local timescale that could discriminate any sudden jump in frequency or newly emergent drift character in one of the constituent clocks. This discriminator could then provide a request for service from the ground operations or provide an alert to dependent users that an anomalous event has occurred. We have been encouraged from our laboratory work with USOs, as well as our analysis of the long-term frequency performance of continuously working flight systems, such as the Mars Global Surveyor in Fig. 1, that frequency jumps or sudden changes in drift character are not likely without some perturbing event. The local timescale could also provide a backup means for estimation and frequency correction in the event that a remote upload from the ground control was unavailable.

III. EXPERIMENT DESIGN

The goal of the experiment conducted this year was to characterize and improve the frequency stability of a USO, operating in a simulated space environment by periodically correcting its local frequency to optimally remove the effect of aging drift. In our approach, the frequency of the USO is corrected by changing a synthesized output of the reference rather than directly frequency tuning the oscillator. As stated earlier, it has been demonstrated that USOs experience highly predictable frequency and drift

characteristics over long periods of time when operating in space, and that with their operational availability is measured in decades. This robust character, along with low mass and low power consumption, makes them ideal frequency references for spacecraft designed for long duration missions. However, due to their continuous frequency drift, the USOs of the onboard mission clocks diverge from the ground-based mission master clock and any data time tagged to the spacecraft clock must be post-processed and referenced to the ground clock. Therefore, improved frequency stability could improve the onboard mission time and mitigate the need for post-processing.

Figure 4 illustrates our experimental setup consisting of three USOs enclosed in a vacuum chamber maintained below 1×10^{-6} torr to simulate deep space conditions. The frequency output of each USO was converted to 5 MHz and sent to a phase-comparator-based data acquisition system referenced to the 5 MHz RF output of a hydrogen maser in the JHU/APL Time and Frequency Laboratory. The 5 MHz output of the best performing USO was split before the input to the phase-comparator measurement system and used as the reference to a micro-phase stepper. In actual practice, the micro-phase stepper would be replaced with a very fine step, frequency synthesizer. The 5 MHz output of the micro-phase stepper was then placed into the phase-comparator measurement system, again, referenced to the hydrogen maser.

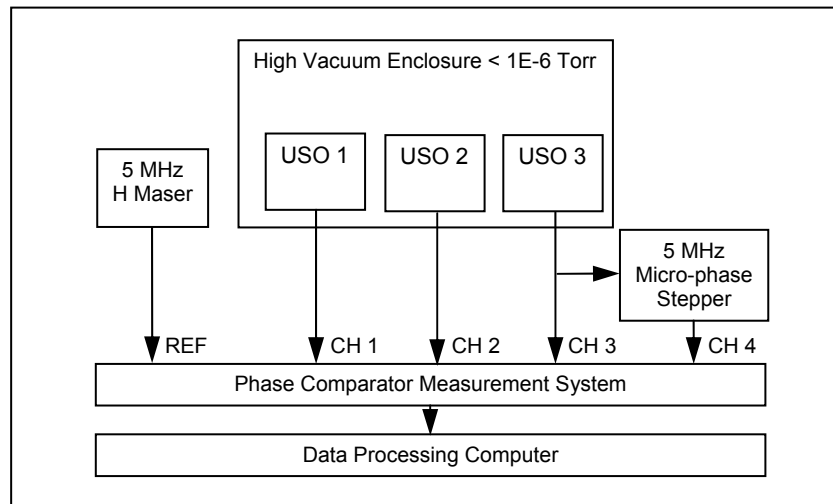


Figure 4. Block diagram of USO ensemble experiment configuration.

Only hourly readings were recorded and converted from nanosecond to microsecond phase values due to the large rate (frequency offset) between the maser and the USOs. The four measured phase comparisons were the “maser minus the micro-phase stepper” and the “maser minus the each of the three USOs.” To simulate the spacecraft onboard system, another set of measurements were created which measured each of the three USOs against the micro-phase stepper. A modified version of the APL autonomous timescale algorithm was created to calculate an ensemble of these three USOs into a mean frequency referenced to the micro-phase stepper. The improved frequency stability provided by the mean of this ensemble was then used to detect frequency changes in the USO reference of the micro-phase stepper during several periods of experimental autonomous operation.

In our experiment, the hydrogen maser represents the “truth” of the ground control and the output of the micro-phase stepper designated the onboard master frequency reference of the ensemble. The objective was to periodically syntonize the frequency output of the micro-phase stepper to the frequency output of the maser with minimal intervention. Therefore, it was decided that the micro-phase stepper adjustments

would be limited to once per week. If this were an actual mission, we would expect to receive frequency data during normal downloads. These data could be processed daily and weekly adjustments could be uploaded to the spacecraft. In our experiment, the weekly upload consisted of passing three parameters:

- 1.) a new micro-phase stepper setting,
- 2.) the expected new frequency offset,
- 3.) the computed frequency drift.

These three parameters provide the minimally necessary information for the maintenance of autonomous operation.

Due to the large frequency drift created by the USO reference of the micro-phase stepper and the 1-week period between micro-phase stepper frequency settings, the autonomous frequency control algorithm must overcompensate and take advantage of the natural drift of the USO to keep the frequency output of the micro-phase stepper syntonized to the maser. Consequently, a phase plot of the maser compared to the micro-phase stepper displayed the pattern in Fig 5.

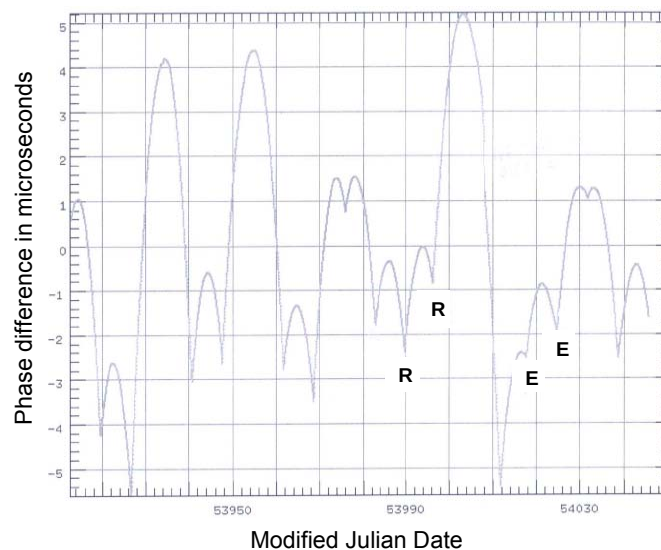


Figure 5. Phase of syntonized micro-stepper to H maser reference.

The operational decision process for autonomous operation followed two rules, if a ground-determined update could not be accessed.

If, after 1 week, no adjustment was given from the ground control, the uploaded frequency offset and the drift were used by the onboard processor to compute an estimate of the current frequency offset of the micro-stepper. The estimate is corrected for any frequency drift determined in the USO reference of the micro-stepper of the USO by comparing to the onboard frequency mean of the ensemble during the previous week. This was done by first removing any drift and differencing the beginning of the week frequency and the end of week frequency. An onboard frequency adjustment algorithm, identical to the one used by ground control, would then determine and implement a new micro-phase stepper setting. The use of this rule was demonstrated for two time epochs in the experiment, designated “E” in the phase plot of Fig. 5

If no adjustment was received from ground control and the estimation of the reference USO of the micro-phase stepper was determined sufficiently out of character from a stored history of uploaded parameters, a repeat of the previous frequency adjustment was then used for the micro-phase stepper. This assumed that the previous adjustment was based on ground truth-determined data and also assumed that there were no frequency deviations of the USO. Once this rule is used, the estimation method cannot be used again until ground control uploaded an updated set of values based on real data. Consequently, this rule of using the previous frequency adjustment would continue until ground operations is able to regain control. The use of this rule was demonstrated for two time epochs in the experiment, designated “R” in the phase plot of Fig. 5.

IV. STABILITY OF THE USO ENSEMBLE

Table 1 lists the sequence of control operations conducted in the experiment and the residual phase, frequency difference, and drift of the micro-phase stepper output measured against the maser reference. Through the course of the experiment, most of the adjustment operations for the micro-phase stepper were performed in a simulated ground control “upload” with the weekly overcompensated steer determined from the observed phase data. The introduction of these adjustments corresponds to the points of sudden phase reversal (discontinuous frequency), as shown in Fig. 5. Table 1 also provides a comparison of the measured stability of the micro-phase stepper (and coordinately its USO reference) to that estimated by the steering algorithm. The missing entries under “Measured Drift” and “Estimated Frequency Offset” in the first several weekly operating periods are due to the amount of data acquisition required to initialize these parameters in the system.

Table 1. Operational control log for USO ensemble experiment.

MJD	Measured ¹			Estimated ²	Adjustment	Decision Type
	Phase Residual μs	Freq. Offset μs/day	Drift μs/day ²	Freq. Offset μs/day	Freq. Step μs/day	
53912.680	+ 0.5	-11.20			-12.00	upload
53919.524	- 4.3	- 1.80			- 3.00	upload
53926.527	- 5.6	- 1.30			- 4.00	upload
53940.502	- 3.0	- 2.00			- 3.50	upload
53947.512	- 2.6	- 1.22	- 0.30		- 3.30	upload
53961.511	- 2.7	- 2.09	- 0.30	- 2.08	- 3.14	upload
53968.515	- 3.4	- 1.08	- 0.27	- 1.01	- 2.03	upload
53975.756	+ 0.8	- 0.63	- 0.35	- 0.98	- 1.34	upload
53982.640	- 1.8	- 1.43	- 0.31	- 1.66	- 2.38	upload
53989.594	(- 2.5)	(- 1.09)	(- 0.30)	- 1.02	- 2.38	repeat
53996.615	(- 0.8)	(- 0.63)	(- 0.29)	- 0.75	- 2.38	repeat
54003.518	+ 5.2	- 0.25	- 0.26	- 0.54	- 0.08	upload
54011.616	- 5.4	- 2.23	- 0.28	- 2.24	- 3.61	upload
54017.556	(- 2.5)	(- 0.20)	(- 0.24)	- 0.24	- 1.18	estimate
54024.604	(- 2.0)	(- 0.54)	(- 0.19)	- 1.07	- 2.10	estimate
54031.545	+ 1.0	- 0.34	- 0.23	- 0.88	- 0.69	upload
54038.557	- 2.5	- 1.36	- 0.25	- 1.23	- 2.31	upload
54045.547	- 1.7	- 0.83	- 0.29	- 0.77	- 1.80	upload

1. maser-micro-phase stepper; values in parenthesis were determined post-process for result comparison

2. estimated by steering algorithm propagated from previous ground upload

3. μs/day $\approx 1.15 \times 10^{-11}$ fractional frequency

As stated earlier, the experiment also included two consecutive sequences of both operational control “repeat” and “estimate” rules to simulate the condition of the ensemble system being unable to receive ground update information. The implementation of these rules would necessarily place gaps in Table 1 because of unrecoverable data loss in a real mission scenario. However, our experiment allows the use of post-processing to determine the actual performance during the use of the “repeat” and “estimate” rules, and this information is provided in Table 1, inside parentheses. In either case of the “repeat” or “estimate” rule, the result of the second implementation was an advance in the run of the residual phase of the micro-phase stepper against the maser, indicative of too large a frequency step in the steering adjustment.

The effect of these errors in steering adjustment can be better resolved in the fractional frequency offset of the micro-phase stepper against the maser reference, as shown in Fig. 6. For the “repeat” rule, the frequency adjustment of $-2.38 \mu\text{s/day}$ was implemented twice, accumulating a positive frequency offset of about $0.17 \mu\text{s/day}$ for each “repeat” adjustment. From Table 1, this accumulated offset can be most directly attributed to a decrease in the measured drift of the USO reference for the micro-phase stepper from that determined by the “upload” at MJD 53982.6.

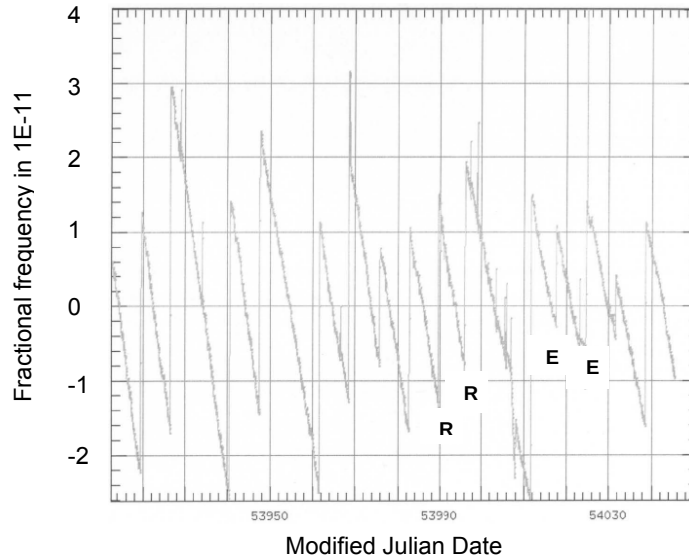


Figure 6. Frequency offset of micro-phase stepper to H maser reference.

Similarly, the change in USO drift during the implementation of the “estimate” rule for applying the steering adjustment imparted about $0.43 \mu\text{s/day}$ positive frequency offset between the estimate and the actual measured frequency offset from the maser. This decrease in frequency drift is easily noticed in the frequency trend of Fig. 6 between MJD 54017.5 and MJD 54024.6. Such sudden changes in the drift character of USOs are not typically observed over weekly operating intervals, and our algorithm did not incorporate a method for incrementing drift, rather just reusing the drift obtained from the “upload” at MJD 54011.6.

The timekeeping error caused by drift changes in the USO reference of the micro-phase stepper during the use of the “repeat” and “estimate” rules led us to examine the quality of the timescale to determine changes in the frequency character of USOs during periods without ground support. To investigate the ability for the timescale to discriminate sudden frequency changes, we introduced a single 1×10^{-10} offset

into the frequency comparison data between the micro-phase stepper and the ensemble timescale and each of the two USO's, which do not serve as the reference to the micro-phase stepper, over a previously observed weekly measurement interval from MJD 54052.8 to MJD 54059.8. The net effect of this incursion to the data set simulates the instance of a frequency jump in the USO reference for the micro-phase stepper; as such a jump would not be resolved in the frequency comparison data of the micro-phase stepper and its reference USO.

Figure 7 shows the effect of the 1×10^{-10} frequency offset in the frequency comparisons between the USO reference of the micro-phase stepper (USO 3 in Fig. 4) against the ensemble timescale and each of the two USOs that do not serve as a reference to the micro-phase stepper (USO 1 and USO 2 respectively in Fig. 4). In each of these three frequency comparisons of Fig. 7, the 1×10^{-10} frequency offset is noticed at MJD 54057. More interestingly, the close correlation of the first two frequency comparisons indicate that the frequency stability of the ensemble timescale is highly influenced by the performance of one of the USOs that do not serve as the micro-phase stepper reference (USO 1 in Fig. 4).

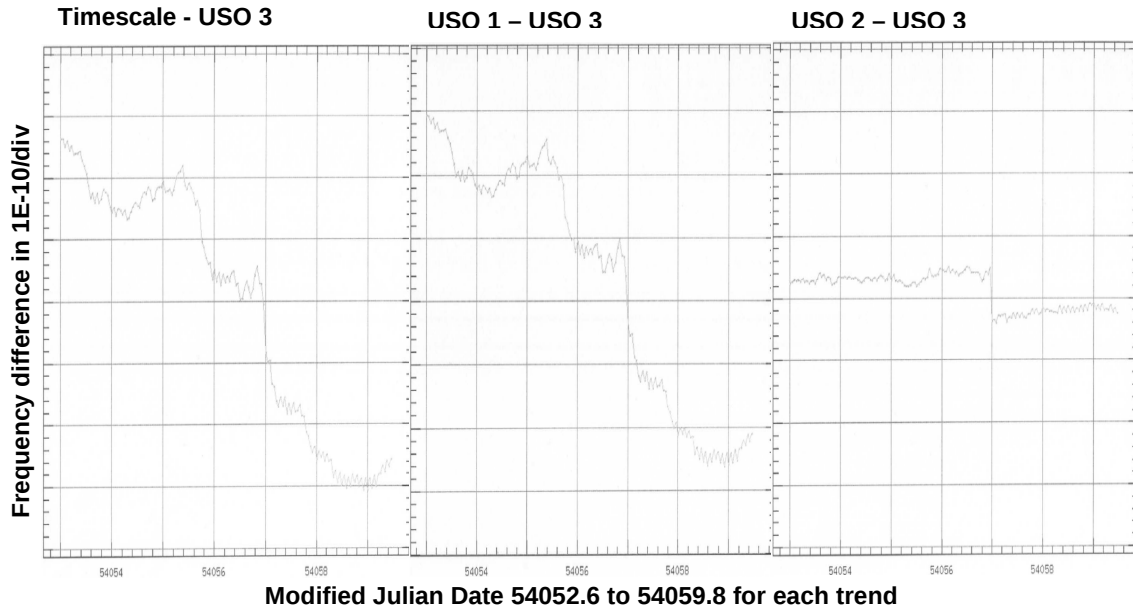


Figure 7. Frequency comparison among USOs of ensemble to resolve 1×10^{-10} offset effect.

The apparent strong dependence of the ensemble timescale to USO 1 is generally unwanted and is typically remedied in application through the use of weighting to diminish its contribution or, in some instances, would be eliminated from the ensemble. However, since our ensemble was limited to only three members, we chose to keep USO 1 in the experiment, as we believe it is realistic to an actual situation that such performance in a USO may emerge once deployed in space application. Also, although the performance of the experimental timescale would have clearly been more useful to the estimation process of the steering algorithm had the performance of USO 1 been similar to the other two, the ability of the timescale to be used as a discriminator for change in the frequency character of the USO reference to the micro-phase stepper was demonstrated.

V. FURTHER STUDY AND IMPROVEMENTS

The results of our experiment showed significant confidence that JHU/APL USOs could be maintained to a frequency stability approaching $\pm 1 \times 10^{-11}$ for extended periods of autonomous “flywheel” operation from 7 to nearly 20 days. The timekeeping of our USO experiment compared to the H maser reference demonstrated $\pm 5 \mu\text{s}$ accuracy for over 100 days using weekly steering intervals that included the use of four autonomous decisions. This performance was accomplished while encountering several instances of data loss and mishaps in the laboratory environment that would be unlikely under the vigilance of space mission operations.

Figure 8 shows the Allan deviation determined for the USO reference of the micro-phase stepper (USO 3 in Fig. 4) and Fig. 9 shows the resultant Allan deviation for the steered output of the micro-phase stepper. The comparison of these results demonstrates the effective weekly disciplining of the experimental USO clock system, with an improvement in frequency stability of two magnitudes to about 1×10^{-12} for 1000-hour time intervals. The statistics shown in Figs. 8 and 9 also indicate that the frequency stability of a USO clock system could be kept better than $\pm 1 \times 10^{-11}$, perhaps as good as $\pm 5 \times 10^{-12}$, if the update provided by the ground support system would be provided daily. Such an update interval would not be unrealistic to deep space mission operations.

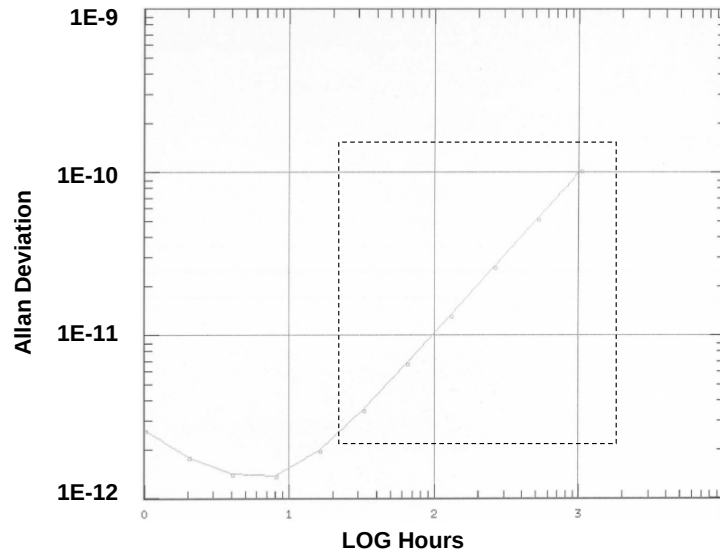


Figure 8. Allan deviation of USO micro-phase stepper reference; dashed inset box highlights drift-dominated character from frequency noise.

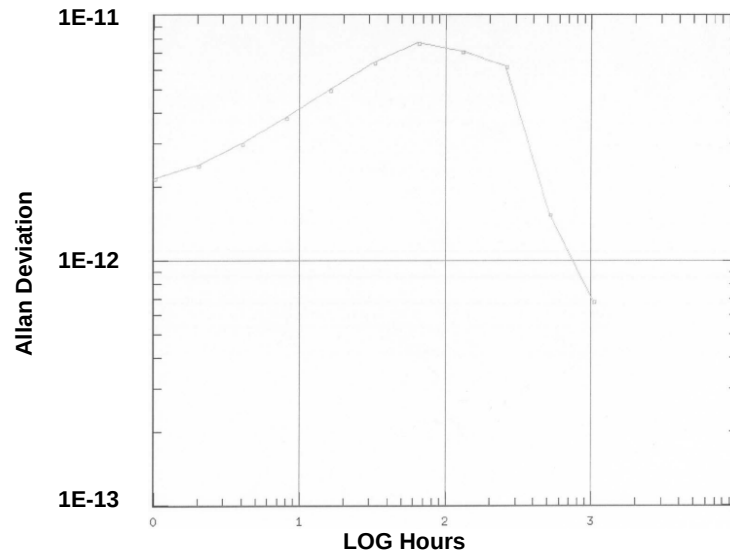


Figure 9. Allan deviation of steered micro-phase stepper.

Further improvement in the autonomy of a remote timekeeping system using an ensemble of USOs will likely require a method for the *in-situ* determination of change in frequency drift character during periods of operation when ground support is not available. The appearance of a short-term change in frequency drift in our experiment, as described in Fig. 6 during the second implementation of the “estimate” rule, was indicative of possible performance events in space, either intrinsic to the USO or caused by external influences such as a fluctuation in local ionizing radiation. Recently, important work has been achieved in the development of the “dynamic Allan variance” and determining the diffusion coefficients in the three-state clock model [3,4]. It is well known that a three-way pair-wise comparison can resolve the Allan variance performance of each ensemble member [5]. We believe that an enhancement to our steering algorithm that monitors the dynamic Allan variance of the USO clock can adjust the “estimate” parameters within certain bounded limits. This ability to discriminate a change in frequency character could also alert dependent users and call for service from the ground control, when available. The stark contrast in the Allan deviation of the USO in Fig. 8 between the drift process, characterized by “ $c\tau/\sqrt{2}$,” and the noise-driven frequency character of the USO promotes interest in further study of USO ensembles and the use of the dynamic Allan variance for robust deep space timekeeping.

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