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Recursive Bayesian method for tracking a magnetic target with a gradiometer

Marius Birsan

Defence R&D Canada – Atlantic

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Abstract

This report describes a numerical method that may be used to efficiently locate and track magnetic targets with a tensor gradiometer. A target containing ferromagnetic material can be adequately modeled at a distance by an equivalent magnetic dipole. This magnetic target can be observed by means of a magnetic gradiometer that measures a symmetric, traceless gradient tensor as a function of time. Of interest is the inverse problem of the determination of the magnetic parameters of the target, and its position and velocity relative to the sensor at each time step. The previous method of direct inversion of the non-linear equations of the magnetic gradient tensor provided multiple solutions, and the results can be highly sensitive to noise in data.

In this study, the determination of target magnetic moment, position and velocity is formulated as an optimal stochastic estimation problem, which could be solved using a sequential Monte Carlo based approach known as the 'particle filter'. In addition to the conventional particle filter, the proposed tracking and classification algorithm uses the unscented Kalman filter (UKF) to generate the prior distribution of the unknown parameters.

The proposed method is then demonstrated by using it to locate and track an automobile over a period of time using real data collected with a magnetic gradiometer. Two cases are investigated: (i) the observation contains only gradiometer data when a double solution exists, and (ii) magnetic field components are added to the previous case and a unique solution is obtained. The automobile was moving either on a straight or a curved track.

Résumé

Le présent rapport décrit une méthode numérique qui peut être utilisée pour localiser et poursuivre avec efficacité des cibles magnétiques au moyen d'un gradiomètre de tenseur. Une cible contenant un matériau ferromagnétique peut être modelée à distance de façon satisfaisante au moyen d'un doublet magnétique équivalent. Cette cible magnétique peut être observée à l'aide d'un gradiomètre magnétique qui mesure un tenseur de gradient symétrique en fonction du temps. Ce qui nous intéresse c'est le problème inverse de la détermination des paramètres magnétiques de la cible et de ses position et vitesse par rapport au capteur à chaque intervalle de temps. L'ancienne méthode d'inversion directe des équations non linéaires du tenseur de gradient magnétique donnait lieu à des solutions multiples, et les résultats peuvent être très sensibles au bruit contenu dans les données.

Dans la présente étude, la détermination du moment magnétique, de la position et de la vitesse de la cible est formulée comme un problème d'estimation stochastique optimale, qui pourrait être résolu à l'aide d'une méthode séquentielle de Monte Carlo appelée « filtre particulaire ». En plus du filtre particulaire classique, l'algorithme de poursuite et de classification proposé fait appel à l'estimateur de Julier et Uhlmann pour générer la distribution a priori des paramètres inconnus.

La méthode proposée fait ensuite l'objet d'une démonstration pour localiser et poursuivre une automobile pendant un certain temps à l'aide de données réelles collectées au moyen d'un gradiomètre magnétique. Deux cas sont étudiés : (i) l'observation est basée sur des données de gradiomètre seulement, ce qui donne lieu à deux solutions; (ii) des

composantes du champ magnétique sont ajoutées au cas précédent, ce qui produit une solution unique. L'automobile se déplaçait soit sur une piste rectiligne, soit sur une piste courbe.

Executive summary

Introduction

A requirement exists to improve the Navy's capability to conduct intelligence, surveillance and reconnaissance (ISR) operations. Precise determination of target motion parameters, i.e. position, velocity, and target classification, are primary concerns in automated surveillance systems. A moving target containing ferromagnetic material can be observed by means of a magnetic tensor gradiometer that measure a symmetric gradient tensor as a function of time. Of interest is the inverse problem of the determination of the position and magnetic parameters of the target at a given time step from its magnetic signature collected up to and including that time step. The previous method of direct inversion of the non-linear equations of the magnetic gradient tensor provided multiple solutions in addition to the physical one, and the results can be highly sensitive to noise in the data.

Work description and results

This report describes a novel numerical method that may be used to efficiently locate and track magnetic targets with a tensor gradiometer. The determination of target magnetic moment, position and velocity is formulated as an optimal stochastic estimation problem, which could be solved using a sequential Monte Carlo based approach known as the 'particle filter'. In addition to the conventional particle filter, the proposed tracking and classification algorithm uses the unscented Kalman filter (UKF) to generate the prior distribution of the unknown parameters.

To analyze the dynamic system of a moving target one needs two models: (i) a model describing the evolution of the state with time, and (ii) a model relating the noisy measurements to the state. It is demonstrated that, if the system dynamics is imposed (e.g. rectilinear motion) and only the gradient data are used, one obtains the physical solution together with a second solution representing the reflection through the origin. This study then shows that the rotationally invariant quantities associated with the gradient tensor have powerful properties for target localization.

Finally, using real data collected with a magnetic gradient sensor, the ability of the algorithm to locate and track an automobile moving either on a straight or a curved path is demonstrated. Two cases are analyzed: (i) the observation contains only gradiometer data when a double solution given by a scaled state vector exists, and (ii) magnetic field components are added to the previous case and all target parameters are uniquely determined.

Significance and future work

The need for tracking under realistic conditions has motivated a series of trials where magnetic signals from various targets were recorded. The proposed method of recursive Bayesian estimation proved to be accurate and applicable to various situations (straight or curved paths). The validation of the presented algorithms on experimental data indicates the possibility to incorporate it into automated surveillance systems.

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Sommaire

Introduction

Il existe un besoin d'améliorer la capacité des forces navales pour mener des opérations de renseignement, de surveillance et de reconnaissance (RSR). La détermination avec précision des paramètres de déplacement de la cible, c.-à-d. la position, la vitesse et la classification de la cible, est une préoccupation principale dans les systèmes de surveillance automatisés. On peut observer une cible mobile contenant du matériau ferromagnétique au moyen d'un gradiomètre de tenseur magnétique qui mesure un tenseur de gradient symétrique en fonction du temps. Ce qui nous intéresse c'est le problème inverse de la détermination de la position et des paramètres magnétiques de la cible à un intervalle de temps donné à partir de sa signature magnétique collectée jusqu'à cet intervalle de temps. L'ancienne méthode d'inversion directe des équations non linéaires du tenseur de gradient magnétique donnait lieu à des solutions multiples en plus de la solution physique, et les résultats peuvent être très sensibles au bruit contenu dans les données.

Description des recherches et résultats

Le présent rapport décrit une méthode numérique novatrice qui peut être utilisée pour localiser et poursuivre avec efficacité des cibles magnétiques au moyen d'un gradiomètre de tenseur. La détermination du moment magnétique, de la position et de la vitesse de la cible est formulée comme un problème d'estimation stochastique optimale, qui pourrait être résolu à l'aide d'une méthode séquentielle de Monte Carlo appelée « filtre particulaire ». En plus du filtre particulaire classique, l'algorithme de poursuite et de classification proposé fait appel à l'estimateur de Julier et Uhlmann pour générer la distribution a priori des paramètres inconnus.

Pour analyser le système dynamique d'une cible mobile, il faut deux modèles : (i) un modèle décrivant l'évolution de l'état dans le temps; (ii) un modèle mettant en correspondance les mesures bruitées et l'état. Il est démontré que, si la dynamique du système est imposée (p. ex. un mouvement rectiligne) et que seules les données de gradient soient utilisées, on obtient la solution physique ainsi qu'une deuxième solution représentant la réflexion passant par l'origine. Cette étude démontre donc que les grandeurs invariantes en rotation associées au tenseur de gradient ont des propriétés puissantes pour la localisation des cibles.

Finalement, on fait la démonstration de la capacité de l'algorithme à localiser et à poursuivre, à l'aide de données réelles collectées au moyen d'un tenseur de gradient magnétique, une automobile se déplaçant selon une trajectoire rectiligne ou courbe. Deux cas sont analysés : (i) l'observation est basée sur des données de gradiomètre seulement, ce qui donne lieu à deux solutions obtenues d'un vecteur d'état mis à l'échelle; (ii) des composantes du champ magnétique sont ajoutées au cas précédent, et tous les paramètres de cible sont déterminés de façon univoque.

Portée et recherches futures

Le besoin d’effectuer la poursuite dans des conditions réalistes a donné lieu à une série d’essais dans lesquels on a enregistré les signatures magnétiques de diverses cibles. La méthode proposée faisant appel à la solution bayésienne récursive s’est avérée exacte et applicable à diverses situations (trajectoires rectilignes ou courbes). La validation des algorithmes présentés au moyen de données expérimentales indique la possibilité de les incorporer dans des systèmes de surveillance automatisés.

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1. Introduction

Precise determination of target motion parameters, i.e. position, velocity, and target classification, are primary concerns in automated surveillance systems. A target containing ferromagnetic material can be adequately modeled at a distance by an equivalent magnetic dipole moment. This target in motion can be observed by means of a magnetic tensor gradiometer that measure a symmetric, traceless gradient tensor as a function of time as it passes. Of interest is the inverse problem of the determination of the position and magnetic parameters of the target at time step k from its magnetic signature collected up to and including time k.

Wynn [1] demonstrated by numerical methods that, using the measurements from one magnetic gradiometer only, the inversion solution is given by two scaled vectors representing the source-to-sensor bearing vector, and the magnetic moment orientation vector. The range and the magnetic moment magnitude cannot be resolved with this method. Moreover, there were 4 solutions for bearing vector and scaled moment vector, with one being the physical solution, the second being called the 'ghost' solution, and two more solutions obtained by reflecting the first two through the origin centered on the measurement point. If an additional five measured quantities are provided given by the rate of change of the gradient tensor [2, 8], the solution represented by the three scaled vectors (position, velocity and moment) is unique except for the reflection through the origin. However, the results obtained by the direct inversion of the non-linear equations can be highly sensitive to noise in data. For this reason, novel signal processing techniques more robust against noise in data were developed based on the statistical approach of finding the best correlation to possible signal. Such methods are matched-field processing, linear statistical analysis, and Monte Carlo simulation.

In this report, we will concentrate to a sequential Monte Carlo method for target tracking known as the "particle filter". The approach makes use of the recursive Bayesian estimation (filtering) technique and of the assumption that the states follow a first order Markov process. Thus, difference equations are used to model the evolution of the system with time and measurements are assumed to be available at discrete times. Using the state-space approach to modeling dynamic systems and the discrete-time formulation of the problem, the aim is to estimate the hidden state process from the measurements.

Let define $\mathbf{x}_k$ as the state of the system at time step k, and $\mathbf{z}_{1:k} = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_k\}$ as the observation (measurements) history of a system from time 1 to k. The state variables are the position, velocity and magnetic moment of the target. Because of either noise in the state evolution process or uncertainty as to the exact nature of the process itself, the state vector $\mathbf{x}_k$ is generally regarded as a random variable. In the Bayesian filtering technique, one attempts to construct an estimate of the posterior probability density function (pdf), $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$. Since all information provided by $\mathbf{z}_{1:k}$ is conveyed by the posterior density, it may be said to be the complete solution to the estimation problem. A recursive Bayesian algorithm imposes the constrain that the estimate of $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ should be generated solely from the previous posterior density, $p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1})$, and the most recent measurement $\mathbf{z}_k$. In this way, it is not necessary to store the complete data set or to reprocess existing data when a new measurement becomes available.

The recursive propagation of the posterior density is a conceptual solution only that can be determined analytically only in a restrictive set of cases. When the analytical solution is intractable, a Monte Carlo based approach to recursive Bayesian filtering, called the particle filter, is one method that approximates the optimal Bayesian solution. In the Monte Carlo method, a set of random samples (particles) are drawn from a target distribution, such as $p(\mathbf{x} \mid \mathbf{z})$. In general, this distribution is not known. We will use $q(\mathbf{x}_k \mid \mathbf{z}_{1:k}) \neq p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ to denote a proposal distribution from which samples can be drawn. The main drawback of the conventional particle filter is that it uses transition prior, $p(\mathbf{x}_k \mid \mathbf{x}_{k-1})$, as the proposal distribution. The transition prior does not take into account current observation data. To overcome this difficulty, the unscented Kalman filter (UKF) was proposed to generate better proposal distributions by taking into consideration the most recent observation.

To analyze the dynamic system of a moving target, one needs two models: (i) a model describing the evolution of the state with time, and (ii) a model relating the noisy measurements to the state. It is shown in this study that, if the system dynamics is imposed (e.g. rectilinear motion) and only the gradient data are used, one obtains the same solution as the one obtained by Wynn using 10 measurements from the gradient and gradient-rate tensors. If the magnetic field measurements are usable simultaneously with the gradient data, we obtain three additional measurement points for a total of eight, allowing us to resolve both the range-moment ambiguity and the ghost problem.

In this report, the theory of target localization with a gradiometer-magnetometer array is first reviewed and the target localization properties of the rotation invariant quantities associated with the gradient tensor are investigated. Then, using real data collected by a gradiometer sensor, the ability of the recursive Bayesian algorithm to track an automobile over a period of time is demonstrated. Two cases are analyzed: (i) the observation contains only gradiometer data when a double solution given by the scaled state vector exists, and (ii) magnetic field components are added to the previous case when all target parameters are uniquely determined. It will be shown that the operational range of the gradiometer is limited by the actual noise level that depends on both the gradiometer construction and the system noise.

2. Locating a magnetic target with a gradiometer

The mathematical model used for the target is a moving magnetic dipole. The target is fully characterized by its position and the value of the magnetic dipole moment. Let consider that **r** is the position vector from the source to the point of observation in meters, **m** is the magnetic moment vector of the dipole, and **V** is the velocity vector in m/sec. The magnetic flux density vector **B** at a given point due to a magnetic dipole is given by the formula:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left[\frac{3\langle \mathbf{r},\mathbf{m}\rangle \mathbf{r}}{|\mathbf{r}|^5} - \frac{\mathbf{m}}{|\mathbf{r}|^3}\right] \tag{1}$$

where μ_0 is the permeability of the air (= $4\pi 10^{-7}$), and $\langle \bullet,\bullet \rangle$ is the inner product operator. In the above formula, all vectors are defined in the same coordinate system, which normally is the sensor coordinates. The three spatial derivatives of the three components of the magnetic flux density vector **B** generate the magnetic field gradient tensor:

$$G_{ij} = \frac{\partial B_i}{\partial r_j} = \frac{\mu_0}{4\pi}\left[-\frac{15\langle \mathbf{r},\mathbf{m}\rangle r_i r_j}{|\mathbf{r}|^7} + \frac{3(m_i r_j + m_j r_i + \langle \mathbf{r},\mathbf{m}\rangle \delta_{ij}}{|\mathbf{r}|^5}\right] \tag{2}$$

where δ_{ij} is the Kronecker's delta. There are only five independent components of the gradient tensor (2) because $\nabla \cdot \mathbf{B}$ and $\nabla \times \mathbf{B}$ are equal to zero in source-free lossless regions. However, this is substantially more than the three data points provided by a vector magnetometer.

Equation (2) was inverted by Wynn [1] to obtain five parameters for the scaled moment vector, $\mathbf{M} = 3\mu_0 \mathbf{m} / 4\pi r^4$, and source-to-sensor bearing (orientation) vector, $\mathbf{R} = \mathbf{r} / r$, in terms of the components of the gradient tensor. One orientation parameter is not independent because the sum of the squared directional cosines equals one. Of course, the sixth unknown is the length of the vector **r** that cannot be resolved. This is called the "range-moment ambiguity". A further complication arises from the fact that the equations to be solved involve the fourth power of **r**. Wynn [1] demonstrated by numerical methods that, using the five independent magnetic gradient measurements only, there were 4 solutions of the quadratic equations for bearing vector and scaled moment vector, with one being the physical solution, the second being called the 'ghost' solution, and two more solutions obtained by reflecting the first two through the origin centered on the measurement point. It was also demonstrated by Wynn [2] that, if an additional five measured quantities are provided given by the rate of change of the gradient tensor, $G'_{ij} = dG_{ij} / dt$, the solution represented by the three scaled vectors (moment, position, and velocity, $\mathbf{W} = \mathbf{V} / r$) is unique except for the reflection through the origin. This degeneracy can easily be dealt with in practical situations.

If the magnetic field measurement is usable, we obtain three additional measurements for a total of eight, allowing us to resolve both the range-moment ambiguity and the ghost problem.

Despite the difficulties, there are advantages in using magnetic gradiometry for locating targets. One key advantage is the suppression of many sources of noise. The gradient is obtained by spatial differentiation of the magnetic field. Thus, while the field from a dipole decays as $1/r^3$, its gradient decays as $1/r^4$. This extra power makes the effect of distant noise sources diminish rapidly. Of course, the signal from the target decays with the same functional form, but gradiometry can provide a net increase in signal to noise ratio, and hence a net increase in detection range.

In addition to noise reduction, tensor gradiometry provides a useful quantity that has a unique property in the sense that it varies directly with the proximity of the magnetic dipole. Such quantity must be invariant to a rotation of the coordinate system, but this necessary condition is not sufficient. For example, the scalar representing the total field magnitude:

$$B = |\mathbf{B}| = \left[B_X^2 + B_Y^2 + B_Z^2\right]^{1/2} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}}{|\mathbf{r}|^3}\sqrt{1+3\left[\frac{\langle \mathbf{m},\mathbf{r}\rangle}{|\mathbf{m}||\mathbf{r}|}\right]^2} \tag{3}$$

is invariant to rotation, but it can either grow or shrink with proximity, depending on the relative orientation of the target dipole, and the measurement location.

Target localization using the magnetic field is the traditional method used, for example, in underwater mine algorithms. The moving target is at the closest point of approach (CPA) to the observation point when the position and velocity vectors are perpendicular, $\langle\mathbf{r},\mathbf{V}\rangle = 0$. It is known that the point of maximum magnetic flux density is not directly related to the closest position of the target. This can be easily seen by taken the derivative of equation (3) with respect to time and equating to zero to obtain the time at which the magnetic flux density attains its peak value:

$$\frac{d|\mathbf{B}|}{dt} = A_0 t^5 + A_1 t^4 + A_2 t^3 + A_3 t^2 + A_4 t + A_5 = 0 \tag{4}$$

The solution of this equation consists of one real root and two conjugate pairs of complex roots. The real root is a complicated function of the three angles between the vectors **r** (arbitrary point on target trajectory), **V** and **m**:

$$\begin{gathered} t_{peak}(\varphi_{rv},\varphi_{rm},\varphi_{vm}) = \frac{-3e^2a + e^2b^2 - ebg + g^2}{3\eta eg} \\ \varphi_{rv} = \cos^{-1}\frac{\langle \mathbf{r},\mathbf{V}\rangle}{|\mathbf{r}||\mathbf{V}|}, \quad \varphi_{rm} = \cos^{-1}\frac{\langle \mathbf{r},\mathbf{m}\rangle}{|\mathbf{r}||\mathbf{m}|}, \quad \varphi_{vm} = \cos^{-1}\frac{\langle \mathbf{V},\mathbf{m}\rangle}{|\mathbf{V}||\mathbf{m}|} \end{gathered} \tag{5}$$

$$
\begin{aligned}
a &= \left(2\cos^2\varphi_{rv} + 4\cos^2\varphi_{rm} - \cos^2\varphi_{vm} + 6\cos\varphi_{rv}\cos\varphi_{rm}\cos\varphi_{vm} + 1\right)/\,d \\
b &= \left(2\cos\varphi_{rv}\cos^2\varphi_{vm} + 7\cos\varphi_{rm}\cos\varphi_{vm} + 3\cos\varphi_{rv}\right)/\,d \\
c &= \left(4\cos\varphi_{rv}\cos^2\varphi_{rm} - \cos\varphi_{rm}\cos\varphi_{vm} + \cos\varphi_{rv}\right)/\,d \\
d &= 3\cos^2\varphi_{vm} + 1 \\
e &= \sqrt[3]{2}\,\sqrt[6]{3} \\
f &= -a^2b^2 + 4b^3c + 4a^3 - 18abc + 27c^2 \\
g &= \left(9\sqrt{f} - \sqrt{3}\left(2b^3 - 9ab + 27c\right)\right)^{1/3} \\
\eta &= |\mathbf{V}|/|\mathbf{r}|
\end{aligned}
\qquad (6)
$$

One can see that, by letting $\mathbf{r} = \mathbf{r}_{CPA}$, only the terms in $\cos(\varphi_{rv})$ equal zero so that the peak time (5) is not zero in the general case, and the corresponding value of the magnitude of the magnetic flux is not the maximum value.

Rotation invariant quantities associated with the gradient tensor become apparent when one attempts to find its eigenvalues and eigenvectors. Using standard procedures, Pedersen and Rasmussen [3] have shown that the invariants are given by:

$$
\begin{aligned}
T &= trace(\mathbf{G}) = \nabla\cdot\mathbf{B} = 0 \\
Q &= -\frac{1}{2}trace\left(\mathbf{G}^2\right) = -\frac{9m_Z^2}{|\mathbf{r}|^{10}}\left(|\mathbf{r}|^2 + 2r_Z^2\right) \\
D &= \det\left(\mathbf{G}\right) = -\frac{27m_Z^3}{|\mathbf{r}|^{15}}r_Z\left(|\mathbf{r}|^2 + r_Z^2\right)
\end{aligned}
\qquad (7)
$$

Both Q and D require all 5 independent components of the tensor **G**. Q has only quadratic as opposed to cubic terms. The above equations are valid in a coordinate system obtained after a rotation such that a magnetic dipole of arbitrary orientation in the original reference frame becomes aligned with the Z-axis of the new reference frame. The quantity Q depends directly on the proximity of the magnetic object and it is often used to determine detection thresholds and to asses performance of the system. It is important to know these quantities because the measurement platforms are seldom stable.

In comparison to equations (4-6), the rotation invariant quantity Q in equation (7) is a simple function of target position. One can see that both quantities, D and Q, have the minima at $|\mathbf{r}| = r_Z$, i.e. $r_X = r_Y = 0$, but D is zero along the lines $r_Z = 0$. Equations (7) were written in a rotated coordinate system where the new Z-axis coincides with the direction of the target magnetic moment. If the target position in the old system is given by the vector **r**, in the new system:

$$
r_{Z'}^2 = r_X^2 + r_Y^2 + r_Z^2 \qquad (8)
$$

and, for a target moving in the horizontal plane (r_Z = constant), the minimum of Q occurs at the minimum value of $r_X^2 + r_Y^2 = r_{CPA}^2$, so that it corresponds exactly to the CPA position of the target relative to the observation point.

Another quantity often used in magnetic data interpretation is the analytic signal, which is a vector encompassing the horizontal derivatives of a harmonic quantity (with zero Laplacian) and their Hilbert transform. The amplitude of the analytic signal can be used as an edge-detection tool, particularly when the magnetic sources of interest are relatively close to the sensor. Without entering into the details [4], one can define the total field magnitude (3) as the amplitude of the analytic signal derived from the magnetic potential field. In a similar way, the amplitude of the analytic signal derived from the magnetic field (**B**) is the Pythagorean sum of the nine components of the gradient tensor (tensor magnitude):

$$G = \left[G_{XX}^2 + G_{XY}^2 + G_{XZ}^2 + G_{YX}^2 + G_{YY}^2 + G_{YZ}^2 + G_{ZX}^2 + G_{ZY}^2 + G_{ZZ}^2\right]^{1/2} \qquad (9)$$

In summary, there are specific advantages of using the magnetic tensor gradiometry for target localization:

1. Common mode rejection of geomagnetic variations.
2. Redundancy of tensor components gives inherent error correction and noise estimates.
3. Allows direct determination of 3-D analytic signal (defines source outlines).
4. Rotationally invariant quantities exist that appear to have higher resolving power than the analytic signal [3] because of their faster fall-off rate with distance.

3. Tracking algorithm

3.1 Recursive Bayesian estimation

The tracking problem requires estimation of the state vector (target co-ordinates, velocity, and magnetic moment) of a system that changes over time using a sequence of noisy measurements (observations) made on the system. For the specific application regarding this study, the target dynamics (the system model) is described by a linear equation, f(•), while the system observation (the measurement model) equation, h(•), is highly non-linear. We assume that these models are available in a probabilistic form:

$$p(\mathbf{x}_k \mid \mathbf{x}_{k-1}): \qquad \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}) \tag{10}$$

$$p(\mathbf{z}_k \mid \mathbf{x}_k): \qquad \mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k) \tag{11}$$

where $\mathbf{x}_k$ is the n_x-dimensional state vector of the system at time step k, $\mathbf{z}_k$ is the n_z-dimensional observation vector, and $\mathbf{v}_k$ and $\mathbf{w}_k$ are vectors representing the process and measurement noise, respectively. They have the dimensions n_v and n_w. It is assumed that the noise vectors are independent, identically distributed (i.i.d.) random samples and they are independent of current and past states.

From the Bayesian perspective, it is required to estimate $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ assuming that the pdf at time (k-1), $p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1})$, is available. The first step in this process is called prediction and makes use of equation (10), which is assumed to describe a Markov process of order one:

$$p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1})\, d\mathbf{x}_{k-1} \tag{12}$$

The second step, the measurement update, uses the most recent observation to produce the desired pdf via Bayes' rule:

$$\begin{gathered} p(\mathbf{x}_k \mid \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k \mid \mathbf{x}_k)\, p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1})}{p(\mathbf{z}_k \mid \mathbf{z}_{1:k-1})} \\ p(\mathbf{z}_k \mid \mathbf{z}_{1:k-1}) = \int p(\mathbf{z}_k \mid \mathbf{x}_k)\, p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1})\, d\mathbf{x}_k \end{gathered} \tag{13}$$

where the second equation is the normalization constant. Once the posterior pdf is determined, it is straightforward conceptually to produce any desired statistic of $\mathbf{x}_k$. For instance, the minimum mean-square error (MMSE) estimate of the current state could be found by computing the conditional mean:

$$\hat{\mathbf{x}}_k = \int \mathbf{x}_k \; p(\mathbf{x}_k \mid \mathbf{z}_{1:k})\, d\mathbf{x}_k \tag{14}$$

The conditional covariance matrix is obtained in a similar way:

$$\mathbf{P}_k = \int \left(\mathbf{x}_k - \hat{\mathbf{x}}_k\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_k\right)^T \; p(\mathbf{x}_k \mid \mathbf{z}_{1:k})\, d\mathbf{x}_k \qquad (15)$$

In general, the recursive propagation of the posterior density cannot be determined analytically because the integrals in (12) and (13) do not have closed-form solutions. Solutions do exist in a restrictive set of cases. For example, if f(•) and h(•) are linear functions and if Gaussian distributions are assumed for **x**, **v**, and **w**, the estimation of states is reduced to the well-known Kalman filter.

The problem of tracking a magnetic dipole does not satisfy the original Kalman filter requirements because the system observation is non-linear. Moreover, because the target can approach the sensors from any direction and can maneuver at any time, the true posterior density is multi-modal and a Gaussian description will be inaccurate.

In order to deal with non-linear systems and/or non-Gaussian reality, two categories of techniques have been developed: parametric and non-parametric. The parametric techniques are based on improvements of the Kalman filter. These filters (for example, extended and unscented Kalman [5] filters) can handle non-linear equations, but they implicitly approximate the posterior density as Gaussian. The non-parametric techniques are based on Monte Carlo simulations and are the subject of the present study. These filters assume no functional form, but instead use a set of random samples (particles) to estimate the posteriors. The advantage is that the particle filters can accommodate simultaneous alternative hypotheses that can describe a multi-modal distribution well.

3.2 Particle filter implementation

The basic idea of the Monte Carlo based approach to an intractable Bayesian filtering case is to approximate an unknown distribution, p, by a set of properly weighted particles drawn from a known distribution q. In this way, the difficult problem of distribution estimation is converted to an easy problem of weight estimation. The exact form of the proposal distribution q is a critical issue in designing the particle filter and is usually approximated to facilitate easy sampling.

A numerical approximation to the recursive Bayesian filtering method given by the equations (12) and (13) is the following algorithm [6]:

1. **Initialization:** sample N particles $\mathbf{x}_k^{(i)}$, i = 1, 2, …, N, from the proposal distribution. The proposal distribution can be the transition prior as used in the conventional particle filters, or more advanced distributions like the one used in this study.
2. **Measurement update:** update the importance weights. The Bayesian sequential importance sampling (SIS) procedure gives a recursive calculation of the normalized weight [6]:

$$w_k^{(i)} = w_{k-1}^{(i)} \frac{p(\mathbf{z}_k \mid \mathbf{x}_k^{(i)})\; p(\mathbf{x}_k^{(i)} \mid \mathbf{x}_{k-1}^{(i)})}{q(\mathbf{x}_k^{(i)} \mid \mathbf{z}_{1:k})}$$

$$w_k^{(i)} = \frac{w_k^{(i)}}{\sum\limits_{i=1,N} w_k^{(i)}} \qquad (16)$$

As an approximation to (14) take:

$$\hat{\mathbf{x}}_k \approx \sum_{i=1,N} w_k^{(i)} \ \mathbf{x}_k^{(i)}$$

3. **Re-sampling** is a necessary step introduced in particle filtering algorithms to reduce the degeneration of samples. In practice it was noticed that, after a few iterations, one of the importance weights tends to one, while the others become zero. To avoid the degeneracy, the sampling importance re-sampling (SIR) method selects N samples with replacement from the set $\mathbf{x}_k^{(i)}$, where the probability to take sample 'i' is $\mathbf{w}_k^{(i)}$. Then set $\mathbf{w}_k^{(i)} = 1/N$, i = 1, 2, …, N.
4. **Prediction:** assuming that the probability of the process noise is known, use equation (6) to simulate $\mathbf{x}_{k+1}^{(i)}$, i = 1, 2, …, N.
5. Set k = k +1, and iterate to item 2.

3.3 The unscented Kalman filter

As mentioned, the deficiency of the sequential importance sampling (SIS) approximation is that the proposal distribution may be very different from the posterior distribution, especially if using the transition prior as the proposal distribution. An improved proposal distribution must incorporate the current observation data with the optimal Gaussian approximation of the state.

In a previous study [7] on the magnetic dipole tracking application, it was shown that the unscented Kalman filter (UKF) is the best Kalman filter for the non-linear systems. The UKF is so named because it implements the Kalman recursion using the sample points provided by the unscented transform. The unscented transform deterministically generates a set of points that have a certain mean and sample covariance. The non-linear function is then applied to each of the sample points, yielding a transformed sample from which the predicted mean and covariance are calculated. The estimate of the conditional mean provided by the UKF is shown to be correct up to the second order of its Taylor series expansion. Reference [5] gives the implementation of UKF algorithm, which is presented in the Annex.

Because the UKF is the best to accurately propagate the mean and covariance of the Gaussian approximation to the state distribution, it can be used to generate the proposal distribution for the particle filter. In this way, one obtains a parametric/non-parametric hybrid filter called the unscented particle filter (UPF).

4. Modeling the target dynamics

For a full characterization of the target, the entire system at time step k can be represented by the state vector:

$$\mathbf{x}_k = \left(r_X \ r_Y \ r_Z \ V_X \ V_Y \ m_X \ m_Y \ m_Z\right)^T \tag{17}$$

The only simplifying assumption made in equation (17) is that the target is moving horizontally.

As mentioned before, the recursive Bayesian estimation method is based on the assumption that the state variables follow a first order Markov process. Thus, the discrete equations of the state evolution of the target are obtained using the piece-wise approximation:

$$\begin{aligned} r_X(k) &= r_X(k-1) + \Delta t \, V_X(k-1) \\ V_X(k) &= V_X(k-1) \\ m_X(k) &= m_X(k-1) \end{aligned} \tag{18}$$

where Δt is the time increment between the data samples in seconds, and similar relations exist for the Y and Z components ($V_Z = 0$).

In addition to the robustness against the noise in data, the proposed statistical recursive method has another advantage in comparison to the direct inversion in [1]. We know [1] that, using the five independent magnetic gradient measurements only, there is a 'ghost' solution for bearing vector and scaled moment vector in addition to the physical solution. Two more solutions are obtained by reflecting the first two through the origin centered on the measurement point. In this paper, we demonstrate that the imposed evolution of the state vector in equation (18) can resolve the 'ghost' ambiguity of solutions while using only magnetic gradient information. Let us assume that at time step (k-1) the state vector does not represent the physical scaled solution, **R, W** and **M**, but the 'ghost' solution. Even so, the estimated value of the gradient at time (k-1) would correspond to the measured value. Using equations (18) and the Taylor's series expansion, one can write to the first approximation the gradient at time k:

$$\begin{aligned} G_{ij}\left(\mathbf{R}_{(k)}\right) &= G_{ij}\left(\mathbf{R}_{(k-1)} + \mathbf{W}_{(k-1)}\,\Delta t\right) \approx G_{ij}\left(\mathbf{R}_{(k-1)}\right) + \frac{\partial G_{ij}}{\partial \mathbf{R}}\Big|_{\mathbf{R}_{(k-1)}} \mathbf{W}_{(k-1)}\,\Delta t = \\ &= G_{ij}\left(\mathbf{R}_{(k-1)}\right) + \frac{\partial G_{ij}}{\partial t}\Big|_{(t=k-1)} \Delta t \end{aligned} \tag{19}$$

As shown before [2], the 'ghost' solution can produce the correct gradient, but it cannot produce the correct gradient rate, irrespective of the value of the scaled velocity. Consequently, from equation (19), the gradient value at time (t = k) will differ from the measured value. This means that the proposed state transitions and measurement model (five gradient components), to be able to reproduce the measurements, must accept only the physical solution (and the reflection). It is easily seen in equation (2) that, if **m** and **r** are replaced by –**m** and –**r**, respectively, the value of **G** does not change. Necessarily, the scaled velocity $\mathbf{W}_{(k-1)}$ will produce the observed gradient rates.

To obtain an un-ambiguous magnetic tracking one needs three additional independent measurements of the magnetic field for a total of eight data points [8]. The use of a stationary gradiometer and magnetometer resolves both the 'ghost' problem and the range-moment ambiguity. In this case, the solution is unique because the replacement of **m** and **r** by –**m** and –**r**, respectively, in equation (1) changes the sign of **B**.

In the following, both these two possibilities will be analyzed. In the first case, the measurement vector at time k has only gradiometer data:

$$\mathbf{z}_k = \left(G_{XX}\ G_{YX}\ G_{XZ}\ G_{XY}\ G_{YY}\ G_{ZY}\ G_{XZ}\ G_{YZ}\ G_{ZZ}\right)^T \tag{20.a}$$

Only five components are independent, but because the data acquisition chanels may not be identical all nine components will be used in the localization. Gradients alone will give the bearing vector to the target and the orientation of its moment.

For the second case, the measurement vector will be a combination of magnetic gradient and magnetic field components:

$$\mathbf{z}_k = \left(G_{XX}\ G_{YX}\ G_{XZ}\ G_{XY}\ G_{YY}\ G_{ZY}\ B_X\ B_Y\ B_Z\right)^T \tag{20.b}$$

This approach resolves the range-moment ambiguity and eliminates 'ghosts'.

The equations (17) and (18) are the process and measurement equations, respectively. As one can see, the process function f(•) in equation (14) is linear, and the measurement function h(•) in equations (20), (2) and (1) is highly non-linear.

5. Experimental results

Having outlined the background to the problem and suggested how the tracking of a magnetic dipole target in real time might be carried out, these ideas are next applied to a set of real data collected with a gradiometer sensor. The experiment took place in Kejimkujik National Park, Nova Scotia, where the ambient magnetic activity due to external sources, such as power lines and traffic, is very low. On the satellite image of the experiment area (Fig.1), the cross indicates the position of the gradiometer and the double-headed parallel arrows approximately represent the tracks used in the experiment. Each track has a number attached that represents the perpendicular distance (CPA) in meters from the gradiometer to that track. The target was an automobile that was driven on the marked (5, 10, 14, 40 and 45 meters) staight paths towards North-West and South-East, and on several curved paths not shown in the picture.

The gradiometer sensor is placed at the center of a Cartesian coordinate system with the X-axis oriented towards the magnetic North, Y-axis towards West, and the Z-axis oriented upwards with zero at the ground level. Consequently, the target coordinates will be given in this reference frame. Note that on the satellite picture the tracks are drawn in the geographical coordinates. A yellow arrow in Fig.1 with the label 'Nm' approximately indicates the magnetic North in the experiment area (where the declination is 21°), so that the actual angle between the X-axis and the staight tracks is about 45°.

The gradiometer sensor was built with 4 tri-axial Bartington fluxgate magnetometers, one placed in the origin, and the other three on the orthogonal axes at 0.65m from origin. Estimates of the gradient tensor components are made by forward differencing the corresponding magnetic field components. The three independent magnetic field components used in this study additional to the gradiometer data are measured at the magnetometer sensor placed in the origin. All data were acquired at a sampling rate of 10Hz. The maximum measured noise level of this system was detected in the G_{ZZ} channel and was 90 pT/m rms/√Hz in the 0-5Hz bandpass. The individual sensor noise level measured at the central magnetometer was 40 pT rms/√Hz (the specification for the Bartington magnetometer alone is <12 pT rms/√Hz at 1Hz). The measured noise depends on both the gradiometer construction [8] and the instrumental noise. Due to this relatively high level of noise, this system exhibits rather restricted range, but it provided a full test for the tracking algorithm against a real dipole source. For comparison, the superconducting magnetic gradiometer used for the tracking experiment in [8] had a noise level of about 0.1 pT/m rms/√Hz at 1Hz.

In applying the Bayesian filtering technique to the system, the initial conditions and the noise covariance matrixes need to be specified. In the initialization step, the particles should be drawn from an unknown proposal distribution. The basic assumption is that the target can approach the sensors from any horizontal direction. Therefore, the filter must accommodate simultaneous alternative hypotheses until they can remove the ambiguity by future measurements. A reasonable initial estimate of the horizontal position is an approximate circle around the gradiometer sensor with a radius of about 100m from where the magnetic signal becomes sizable. In the present study, 36 particles were used with the horizontal positions spread over a circle every 10° from 0° to 350°. Because we have no a priori information about the vertical position, the magnitude of velocity, and magnetic dipole moments, a good initial estimate of these parameters is merely the null vector.

The initial covariance matrix, **P**(0|0), gives a measure of belief in the initial state estimate. It is assumed that initially all the states are un-correlated, so that the matrix is diagonal. This matrix is not known and has to be sufficiently large, but the initial **P**(0|0) is forgotten as more data is processed. The measurement noise covariance matrix can be estimated directly from the actual data and, once calculated, it does not change during the filter run.

The process noise covariance is zero for a deterministic process. However, it was practically proved to be a good idea to introduce random perturbations in the target position and velocity. These small perturbations account for the target maneuvers and prevent divergence, so that the process noise covariance may be regarded as a tuning parameter of the filter.

To demonstrate the tracking algorithm, we use the data collected with the target moving straight towards North-West on the path situated at 10 meters CPA from the sensor. All 9 components of the magnetic tensor gradient are plotted in Fig.2 in nano-T/m. Due to the instrumentation errors the symmetry of the tensor is not perfectly preserved (see, for example, that G_{XZ} is slightly different from G_{ZX}). The geometry of the experiment is shown in Fig.3 in the sensor coordinate system with X-axis towards the magnetic North. The first case analyzed is when the measurement vector has only gradiometer data (20.a). Figure 4 shows the bearing vector that proves to follow very well (dotted line) the target in motion. As discussed above, this solution is not unique and the filtering method could produce as well the reflection of the bearing vector through the origin. The other state variables, velocity and magnetic moment, are obtained as scaled parameters, so that they cannot characterize the target.

A unique solution in absolute units is obtained when, in addition to the 5 independent gradient components, the magnetic field components measured at the magnetometer placed in the origin of the coordinate system are included in the observation vector (20.b). Figures 5 to 8 present the evolution in time of several system state variables that characterize the target in motion. One can see that the trajectory of the target follows very closely the 10m track represented by the blue arrow in Fig.5. The vertical position of target is between zero and 5m. Other parameters, such as the target velocity (~ 3m/sec) and Z-magnetic moment (-0.5kA-m^2, oriented downwards as expected at this latitude), are not known a priori in this experiment, but their estimated values are reasonable for this type of target. The other two components of the magnetic moment are practically zero.

Finally, an interesting test of the algorithm makes use of data obtained for a curved track. The test shows the capability of the algorithm to adapt to the target maneuvers. Even if the target motion is assumed rectilinear in equations (18), the small perturbations introduced by the process noise allows for a variable velocity which, in turn, modify the trajectory from rectilinear to curve. If the trajectory is curved, one expects modifications in the behavior of the rotationally invariant quantities Q and B. For a rectilinear movement there is one CPA point so that Q has only one minima (-Q and B have a maxima) presented in Fig.9 using the previous data recorded with the target moving straight on the 10m track. Also Fig.9 presents the estimated inverse value of the target position vector magnitude for comparison. There is a delay of about 0.2 sec (2 data samples) between the CPA time determined by Q (or B) and the one estimated by 1/r, possibly because the filter requires a certain amount of data to correct the system dynamics.

Anticipating a bit the results, the same quantities, -Q, B and estimates of 1/r, are plotted in Fig.10 for the target moving on the curved path. The target came closer to sensor twice, but not at the same distance. The CPA point given by the estimated values of 1/r corresponds to the highest peak of –Q. This is not the case for the maximum of B. The plot clearly indicates that the quantity Q accurately determines the variations in the target proximity.

The rest of figures from 11 to 14 present the most important parameters of the target when moving on the curved track as estimated by the unscented particle filter. The blue arrow in Fig.11 indicates the straight 10m path. The target trajectory in Fig.11 is assumed to be correctly estimated because there is no tracking information collected during this experiment. However, the target was detected to move on the z = 0 horizontal plane (Fig.12) and can be characterized by its Z-component of magnetic moment that was estimated, as previously, to be -0.5kA-m^2.

Similar results as above were obtained for the target moving on the 5 and 14 meters tracks. The value of the magnetic moment explains why there was no signal recorded by the gradiometer when the automobile moved on the 40 and 45 meters tracks. Considering the decay with the fourth power of the distance, the magnitude of the signal when the target is at 40m from the sensor would be of about 20 pT/m, which is well below the level of the environmental noise for this system.

Figure 1. *Satellite view of the Keji park layout of tracking experiment.*

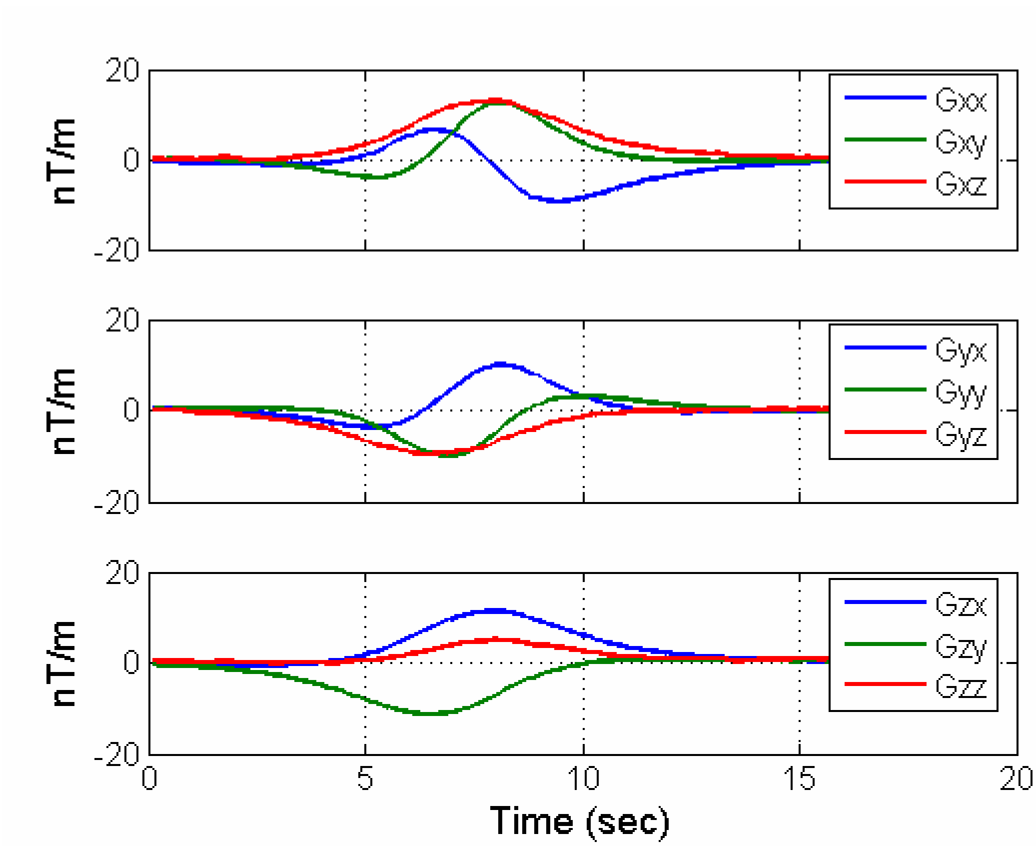

Figure 2. *All 9 magnetic gradient components for the target on the 10m track.*

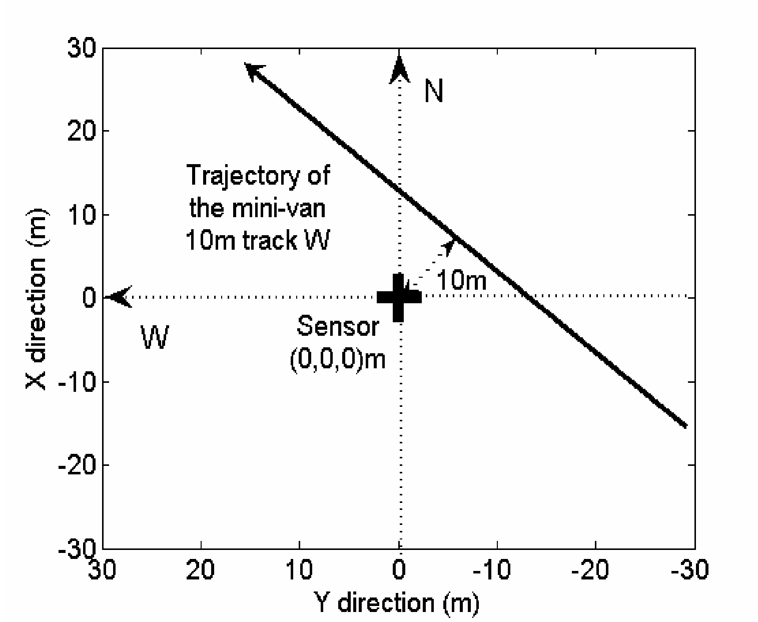

Figure 3. *Experiment coordinate system and the 10m track.*

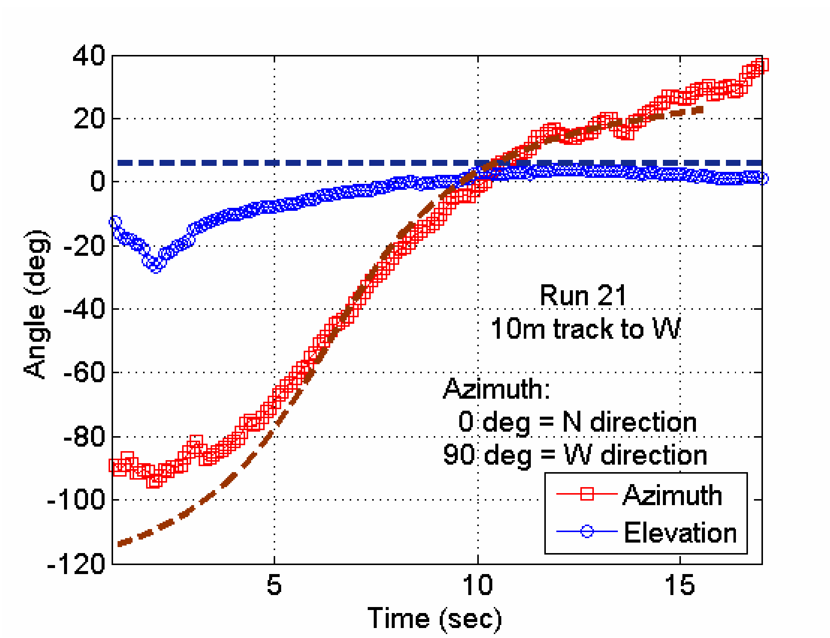

Figure 4. *The estimated and true bearing vector of target on the 10m track.*

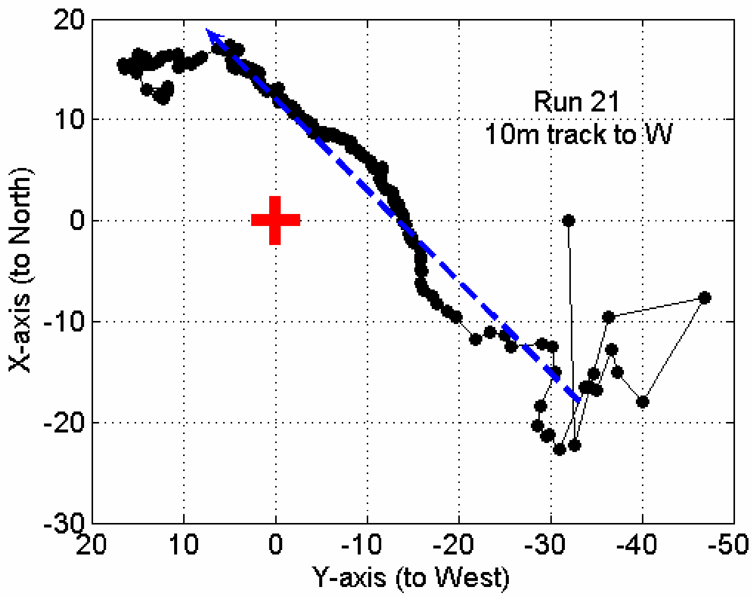

Figure 5. *X-Y estimates of target position (10m path).*

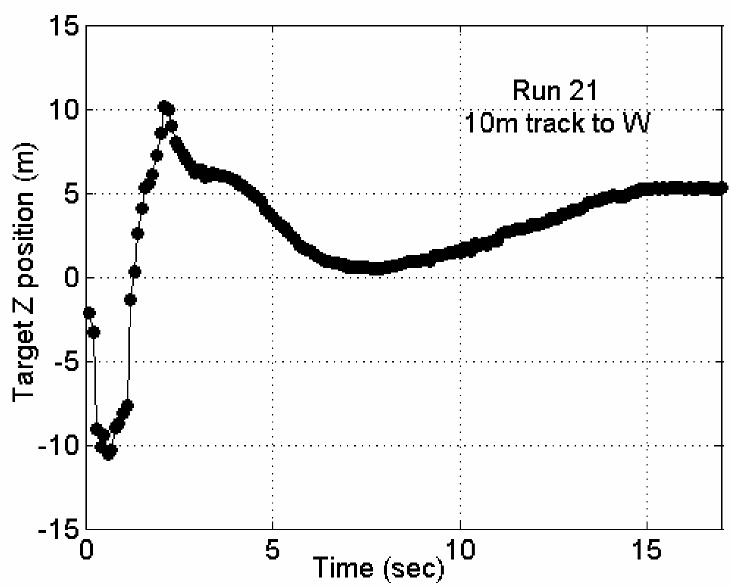

Figure 6. *Z estimates of target position (10m path).*

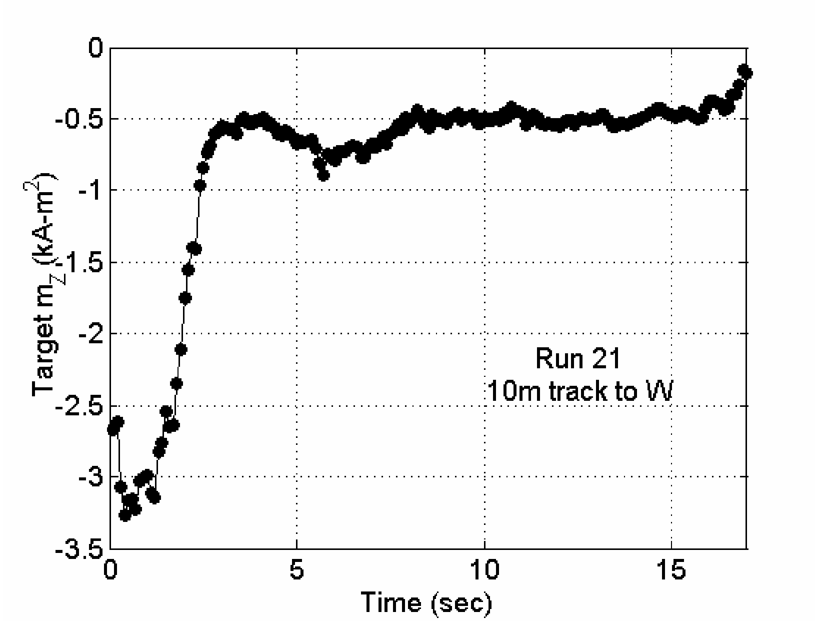

Figure 7. *Estimated target Z-magnetic moment component (10m path).*

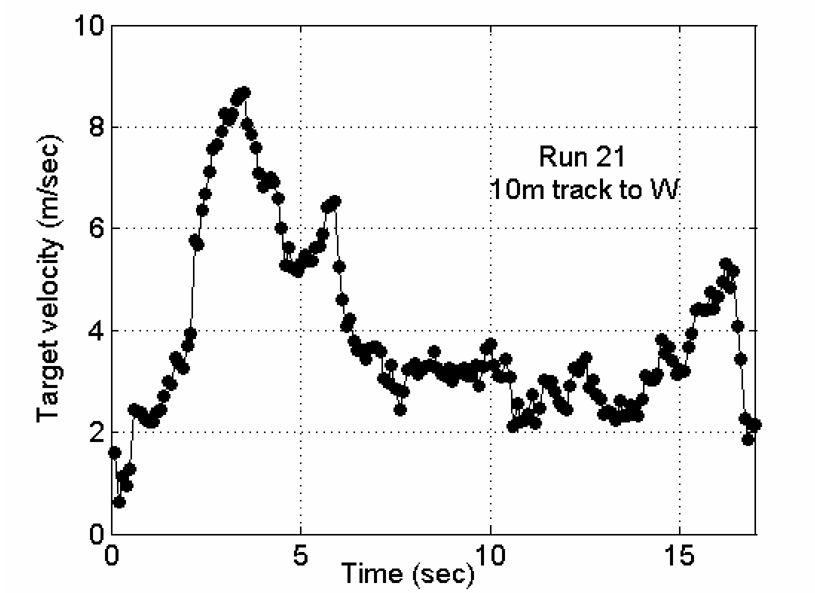

Figure 8. *Estimated magnitude of target velocity (10m path).*

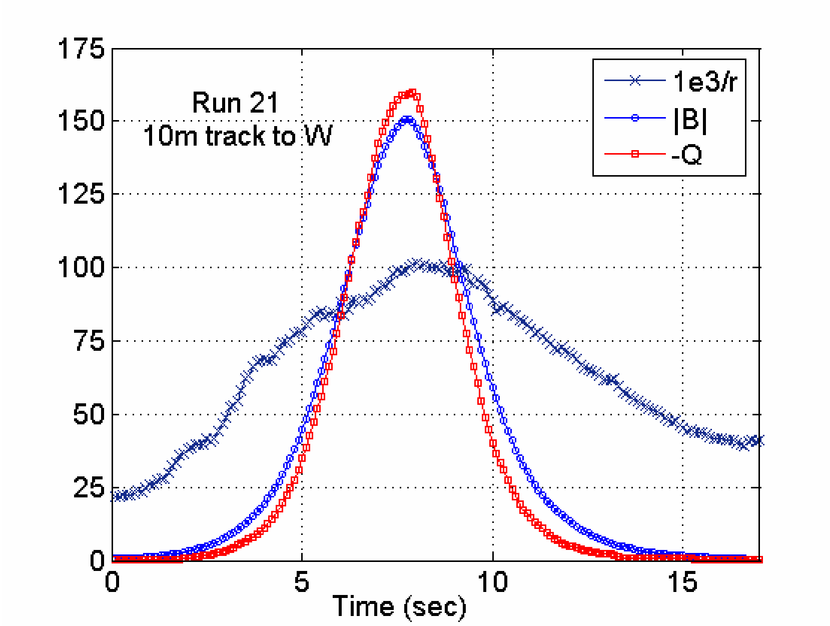

Figure 9. *Plot of –Q, B and estimated 10^3/|**r**| for the target on 10m track.*

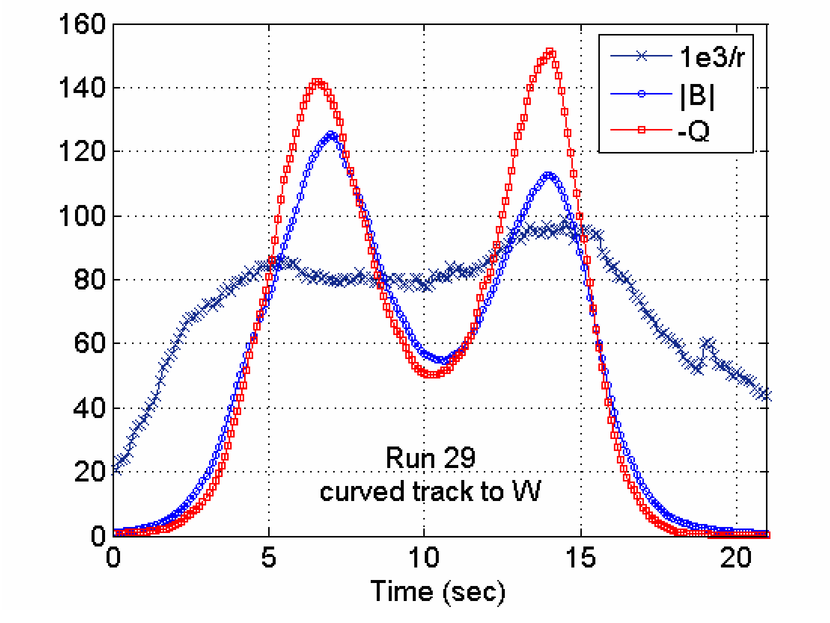

Figure 10. *Plot of –Q, B and estimated 10^3/|**r**| for the target on the curved track.*

6. Conclusions

This report investigates the possibility of using the unscented particle filter (UPF) for tracking and classification of a magnetic dipole with a gradiometer. A fluxgate gradiometer-magnetometer array was constructed and used to demonstrate the feasibility of the processing technique against a real target. It was shown that the effective range of operation for this system is given by the actual noise level that depends on gradiometer construction and instrumental noise.

The magnetic gradient measurements offer some advantages over the magnetic field. In addition to noise reduction, tensor gradiometry provides a useful quantity (Q) that has a unique property in the sense that it varies directly with the proximity of the magnetic dipole. This quantity is invariant to a rotation of the coordinate system, which is another important property because the measurement platforms are seldom stable. This report shows that the quantity Q appears to have higher resolving power than the magnitude of the magnetic field, which is also rotationally invariant.

The report investigates the inverse problem of the determination of the position, velocity and magnetic parameters of the target at a given time step from its magnetic signature collected up to and including that time step. Two cases are separately analyzed: (i) the observation contains only gradiometer data when a double solution given by scaled state vectors exists, and (ii) magnetic field components are added to the previous case and all target parameters are uniquely determined.

In the first case, the determination of the azimuth and the elevation of the target relative to the gradiometer sensor represents substantially more information than provided by a vector magnetometer. However, the solution is a scaled vector (range-velocity-moment ambiguity) and it is accompanied by its reflection through the origin.

The situation is greatly improved if the magnetic field measurement is usable. In this case, one obtains three additional independent measurements for a total of eight, resolving both the ambiguity and the solution uniqueness problem. Compared to the non-recursive solution of the dipole field equations yielding position and dipole moment due to Wynn [2, 8], the present statistical recursive method is easier to implement and is robust against the noise in the data, which may be a serious problem when direct inversion is used.

The filtering technique applied on the experimental data allows all the target parameters to be estimated with a good level of accuracy. The proposed method of recursive Bayesian estimation proved to be applicable to various situations (straight or curved tracks). The validation of the presented algorithms on experimental data indicates the possibility to incorporate it into automated surveillance systems.

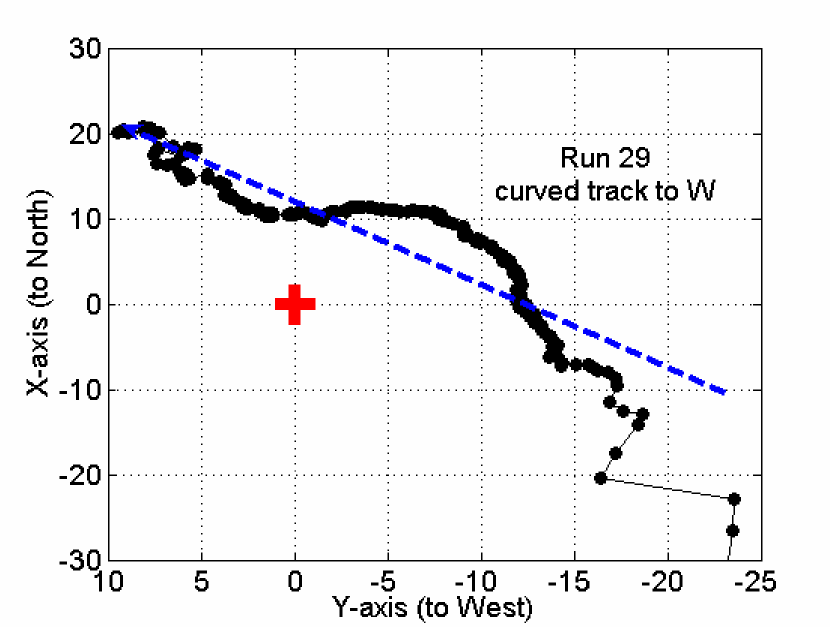

Figure 11. *X-Y estimates of target position (curved path).*

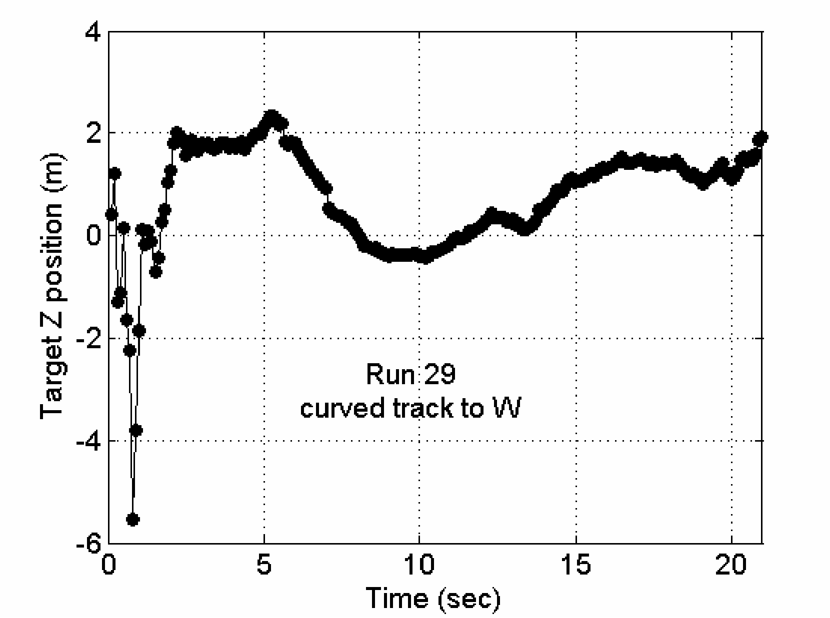

Figure 12. *Z estimates of target position (curved path).*

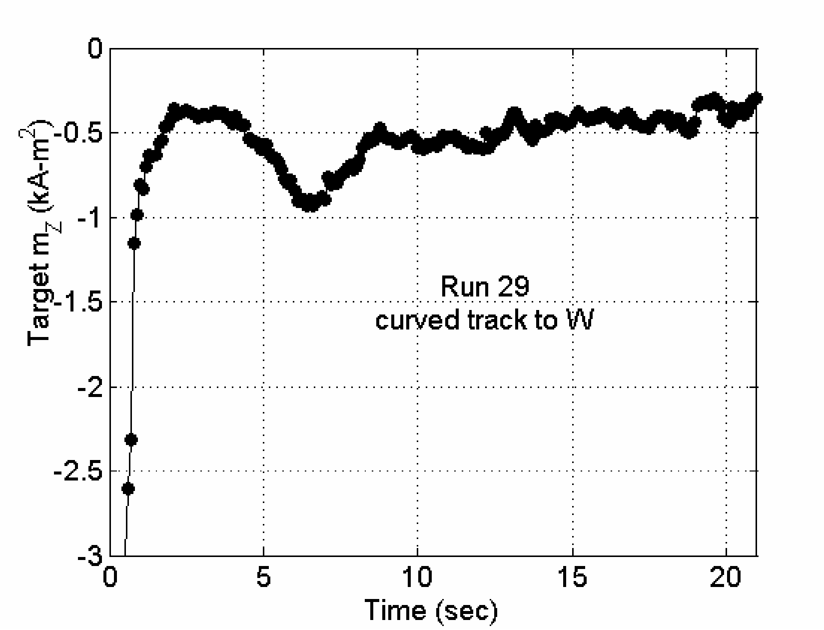

Figure 13. *Estimated target Z-magnetic moment component (curved path).*

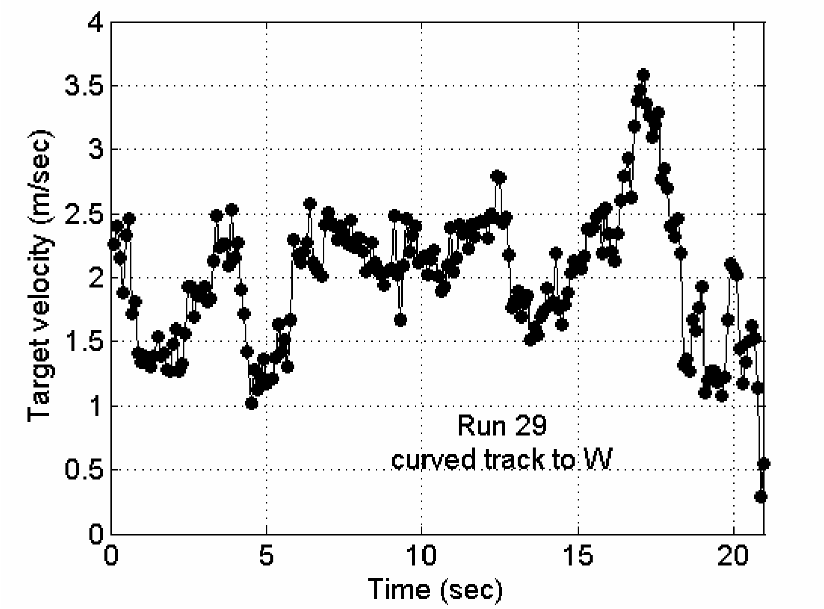

Figure 14. *Estimated magnitude of target velocity (curved path).*

7. References

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Implementing the unscented Kalman filter

The unscented transformation uses a set of weighted samples or ‘sigma points’, S_i = {W_i, $\boldsymbol{\chi}_i$}, to completely capture the true mean and covariance of the random variable **x** and then propagates the sigma points through the non-linear function. We use the following notation: **K** is the Kalman gain, W_i are the weights, λ is the scaling parameter, $n_a=n_x+n_v+n_w$, and

$$\mathbf{x}^a = \left[\mathbf{x}^T \ \mathbf{v}^T \ \mathbf{w}^T\right]^T, \quad \aleph^{\mathbf{a}} = \left[\left(\aleph^{\mathbf{x}}\right)^T \ \left(\aleph^{\mathbf{v}}\right)^T \ \left(\aleph^{\mathbf{w}}\right)^T\right]^T$$

1. Initialize with:

$$\overline{\mathbf{x}}_{\mathbf{0}} = E[\mathbf{x}_{\mathbf{0}}]; \quad \mathbf{P}_{\mathbf{0}} = E\left[(\mathbf{x}_{\mathbf{0}} - \overline{\mathbf{x}}_{\mathbf{0}})(\mathbf{x}_{\mathbf{0}} - \overline{\mathbf{x}}_{\mathbf{0}})^T\right]; \quad \overline{\mathbf{x}}_{\mathbf{0}}^{\mathbf{a}} = E\left[\mathbf{x}^{\mathbf{a}}\right] = \left[\overline{\mathbf{x}}_{\mathbf{0}}^{\mathbf{T}} \ \mathbf{0} \ \mathbf{0}\right]^T$$

$$\mathbf{P}_{\mathbf{0}}^{\mathbf{a}} = E\left[\left(\mathbf{x}_{\mathbf{0}}^{\mathbf{a}} - \overline{\mathbf{x}}_{\mathbf{0}}^{\mathbf{a}}\right)\left(\mathbf{x}_{\mathbf{0}}^{\mathbf{a}} - \overline{\mathbf{x}}_{\mathbf{0}}^{\mathbf{a}}\right)^T\right] = \begin{bmatrix} \mathbf{P_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}$$

2. For discrete time samples, k = 1, 2, …, K,

(a) Calculate sigma points:

$$\aleph_{\mathbf{k-1}}^{\mathbf{a}} = \left[\overline{\mathbf{x}}_{\mathbf{k-1}}^{\mathbf{a}} \quad \overline{\mathbf{x}}_{\mathbf{k-1}}^{\mathbf{a}} \pm \sqrt{(n_a + \lambda)\,\mathbf{P}_{\mathbf{k-1}}^{\mathbf{a}}}\right]$$

(b) Time update:

$$\aleph_{\mathbf{k|k-1}}^{\mathbf{a}} = \mathbf{f}\left(\aleph_{\mathbf{k-1}}^{\mathbf{x}}, \aleph_{\mathbf{k-1}}^{\mathbf{v}}\right)$$

$$\overline{\mathbf{x}}_{\mathbf{k|k-1}} = \sum_{i=0}^{2n_a} W_i^{(m)} \ \aleph_{i,k|k-1}^{x}$$

$$\mathbf{P}_{\mathbf{k|k-1}} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[\aleph_{\mathbf{i,k|k-1}}^{\mathbf{x}} - \overline{\mathbf{x}}_{\mathbf{k|k-1}}\right]\left[\aleph_{\mathbf{i,k|k-1}}^{\mathbf{x}} - \overline{\mathbf{x}}_{\mathbf{k|k-1}}\right]^T$$

$$\mathbf{Z}_{\mathbf{k|k-1}} = \mathbf{h}\left(\aleph^{\mathbf{x}}_{\mathbf{k|k-1}}, \aleph^{\mathbf{w}}_{\mathbf{k-1}}\right)$$

$$\overline{\mathbf{z}}_{\mathbf{k|k-1}} = \sum_{i=0}^{2n_a} W_i^{(m)} \; Z_{i,k|k-1}$$

(c) Measurement update equations:

$$\mathbf{P}_{k|k}^{ZZ} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[Z_{i,k|k-1} - \overline{\mathbf{z}}_{\mathbf{k|k-1}}\right]\left[Z_{i,k|k-1} - \overline{\mathbf{z}}_{\mathbf{k|k-1}}\right]^T$$

$$\mathbf{P}_{k|k}^{XZ} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[\aleph_{i,k|k-1} - \overline{\mathbf{x}}_{\mathbf{k|k-1}}\right]\left[Z_{i,k|k-1} - \overline{\mathbf{z}}_{\mathbf{k|k-1}}\right]^T$$

Where $\mathbf{P}^{\mathbf{zz}}$, $\mathbf{P}^{\mathbf{xz}}$ are respectively the covariance matrix of the measurement and the cross-covariance of the measurement and the state variable. Next:

$$\mathbf{K}_{\mathbf{k}} = \mathbf{P}_{k|k}^{XZ} \left[\mathbf{P}_{k|k}^{ZZ}\right]^{-1}$$

$$\overline{\mathbf{x}}_{\mathbf{k}} = \overline{\mathbf{x}}_{\mathbf{k|k-1}} + \mathbf{K}_{\mathbf{k}} \left(\mathbf{z}_{\mathbf{k}} - \overline{\mathbf{z}}_{\mathbf{k|k-1}}\right)$$

$$\mathbf{P}_{\mathbf{k}} = \mathbf{P}_{\mathbf{k|k-1}} - \mathbf{K}_{\mathbf{k}} \, \mathbf{P}_{k|k}^{ZZ} \, \mathbf{K}_{\mathbf{k}}^{\mathbf{T}}$$

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Recursive Bayesian method for tracking a magnetic target with a gradiometer

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This report describes a numerical method that may be used to efficiently locate and track magnetic targets with a tensor gradiometer. A target containing ferromagnetic material can be adequately modeled at a distance by an equivalent magnetic dipole. This magnetic target can be observed by means of a magnetic gradiometer that measures a symmetric, traceless gradient tensor as a function of time. Of interest is the inverse problem of the determination of the magnetic parameters of the target, and its position and velocity relative to the sensor at each time step. The previous method of direct inversion of the non-linear equations of the magnetic gradient tensor provided multiple solutions, and the results can be highly sensitive to noise in data.

In this study, the determination of target magnetic moment, position and velocity is formulated as an optimal stochastic estimation problem, which could be solved using a sequential Monte Carlo based approach known as the ‘particle filter’. In addition to the conventional particle filter, the proposed tracking and classification algorithm uses the unscented Kalman filter (UKF) to generate the prior distribution of the unknown parameters.

The proposed method is then demonstrated by using it to locate and track an automobile over a period of time using real data collected with a magnetic gradiometer. Two cases are investigated: (i) the observation contains only gradiometer data when a double solution exists, and (ii) magnetic field components are added to the previous case and a unique solution is obtained. The automobile was moving either on a straight or a curved track.

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Particle filter, tracking algorithm, magnetic gradient, gradiometer

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