



NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

MBA PROFESSIONAL REPORT

**Improving the Efficiency of Defense Auctions: Multi-Stage
Auctions as a Market Research Tool**

**By: Steven W. Vanden Bos
December 2007**

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REPORT DOCUMENTATION PAGE			<i>Form Approved OMB No. 0704-0188</i>	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE December 2007	3. REPORT TYPE AND DATES COVERED MBA Professional Report	
4. TITLE AND SUBTITLE Improving the Efficiency of Defense Auctions: Multi-Stage Auctions as a Market Research Tool			5. FUNDING NUMBERS N/A	
6. AUTHOR(S) Steven W. Vanden Bos				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited			12b. DISTRIBUTION CODE	
13. ABSTRACT (maximum 200 words) The purpose of this MBA Project was to investigate auction theory to obtain a means of coupling market-research and final contract objectives. Federal buyer's have imperfect information regarding what could best meet needs and have difficulty obtaining information. A multi-stage auction model was designed and compared to current single-stage auctions. The multi-stage auction obtains an improvement in total buyer's surplus, actual buyer's surplus and selects the ideal seller more frequently. The multi-stage auction may be implemented without major policy changes and may be used effectively in contracts for services, or in contingent environments.				
14. SUBJECT TERMS Auctions, Efficiency, Market Research, Defense, Contracts, Contracting			15. NUMBER OF PAGES 97	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UU	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18

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**IMPROVING THE EFFICIENCY OF DEFENSE AUCTIONS: MULTI-STAGE
AUCTIONS AS A MARKET RESEARCH TOOL**

Steven W. Vanden Bos, Captain, United States Air Force

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF BUSINESS ADMINISTRATION

from the

**NAVAL POSTGRADUATE SCHOOL
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The purpose of this MBA Project was to investigate auction theory to obtain a means of coupling market-research and final contract objectives. Federal buyer's have imperfect information regarding what could best meet needs and have difficulty obtaining information. A multi-stage auction model was designed and compared to current single-stage auctions. The multi-stage auction improves total buyer's surplus, actual buyer's surplus and selects the ideal seller more frequently. The multi-stage auction may be implemented without major policy changes and may be used effectively in contracts for services, or in contingent environments.

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I. INTRODUCTION

A. BACKGROUND

Ideal Federal Acquisition Regulation (FAR) auctions include an initial information gathering phase.¹ Individual agents are assigned tasks to perform. End-users are responsible for generating a cost estimate and articulating the ultimate requirement. Contracting officers are primarily responsible for conducting market research and developing an acquisition strategy. Ideally, the information gathering process translates into evaluation criteria and contract metrics that accomplish mission objectives. In practice it is not clear that the right information is learned, or that it is translated into contract and mission objectives because there is no objectively determined quality standard. The FAR mandates completion but quality is subjectively determined. Criticism leveled at the Department of Defense (DoD) acquisition community by many in the press,² government,³ and think tanks,⁴ evidences that acquisition teams may not always translate information into objectives correctly.

Complexity makes the problem more difficult for acquisition teams. There can be hundreds, or just a few, attributes that may be applicable to a particular objective. Market research is designed to address the attribute problem, however, it is difficult to gather accurate, relevant data. Additionally, acquisition teams operate within a structured regulatory process that is usually conducted sequentially. Each factor is both independent and dependent making it difficult to synchronize the information into a performable contract and achieve objectives. For example, requirement generation is an independent problem because it must be solved in-and-of itself, but dependant because it

¹ FAR Parts 7, 10, & 11.

² Renae Merle, "Problems Stall Pentagon's New Fighting Vehicle," *Washington Post*, February 7, 2007, A01; <http://www.washingtonpost.com/wp-dyn/content/article/2007/02/06/AR2007020601997.html>, Retrieved November 2007.

³ GAO-06-110, "Better Support of Weapons Systems Program Managers Needed to Improve Outcomes," November 2005, <http://www.gao.gov/new.items/d06110.pdf>, Retrieved October 2007.

⁴ Chris Edwards, "Government Schemes Cost More Than Promised," *CATO Institute, Tax and Policy Bulletin*, No. 17, September 2003, <http://www.cato.org/pubs/tbb/tbb-0309-17.pdf>, Retrieved August 2007.

has to be coupled with market research and related to eventual outcomes. At present, there is no mechanism to objectively solve and couple each independent element into final contract objectives, other than subjective judgment.

There are two primary difficulties compounding the information discovery process. First, the buyer and each individual seller are attempting to unilaterally optimize the requirement given asymmetric distribution of information. Secondly, it is difficult to know with certainty what the optimal combination of attributes is to achieve given objectives.

1. Asymmetric Information

Information is asymmetric if one or more parties to an auction have some level of private information.⁵ On the other hand, information is symmetric when it is common to all parties. In the information gathering phase each individual party has some incentive to gather as much privately held information as possible, while simultaneously protecting their own private information.

The information gathering phase is similar to a trip to a car dealership. The seller would like to reveal as little as possible about the true cost of car, to gain as much margin as possible. Contrarily the buyer has an incentive to reveal as little as possible about their true willingness to pay, to gain as good a deal as possible. Each party has private information they would prefer to protect and information they are attempting to capture.

In the case of the federal auction the buyer's private information includes the budget allotment, what attributes are most valued, and how the evaluation will be conducted. Each individual seller's private information includes their means and methods of production, the cost function, and their anticipated profits. Buyer's would prefer to protect information about their budget allotment because if revealed all seller's

⁵ Kenneth Hendricks and Robert H. Porter, "An Empirical Study of an Auction with Asymmetric Information," *The American Economic Review*, no. 78 (1988): 865-883.
<http://www.jstor.org/view/00028282/di950063/95p0032k/0?currentResult=00028282%2bdi950063%2b95p0032k%2b0%2c00&searchUrl=http%3A%2F%2Fwww.jstor.org%2Fsearch%2FAdvancedResults%3Fhp%3D25%26si%3D1%26q0%3Dasymmetric%26f0%3Dti%26c0%3DAND%26q1%3Dauction%26f1%3Dti%26c1%3DAND%26wc%3Don%26sd%3D%26ed%3D%26la%3D>, Retrieved November 2007.

with cost functions below the budget amount have an incentive to price at total budget; buyer's also prefer to gain as much information about the true cost of production and the price trade-offs from each individual seller. Sellers' prefer to guard their proprietary data to protect current and future profits while simultaneously learning as much information as possible about the true budget allotment.

There is also symmetric information within the early auction processes. Common data may, or may not, include elements such as the number of likely competitors, general industry knowledge, and on-going market trends. However, it is the asymmetric knowledge that makes gathering meaningful data related to objectives difficult.

2. Requirement Asymmetry

The problem of requirement generation is primarily a combinatorial optimization problem. It is reasonable to assume that the primary objective of the requiring agency is to achieve the maximum benefit per dollar cost. For a given objective, what's the optimal combination of reliability, sustainability, speed, weight, maneuverability, et cetera? The optimal solution may not be readily apparent because it is contextual and difficult to measure.

The buyer has private information about what they value most. There is some incentive for the buyer to reveal the optimal attribute array to ensure they achieve their objectives; however, if the buyer does not know what they want, they have an incentive to be ambiguous.

Each potential seller has private information about what combination of requirements would provide the buyer the most benefit, in that sellers are generally more familiar than the buyer with the capabilities and limitations of current and cutting-edge technologies. Each potential seller also has private information about their individual cost function, however, and is biased in favor of it. For example, if a seller's cost function affords them an advantage in producing a maneuverable aircraft, they will try to market and sell a maneuverable aircraft. Sellers' have an incentive to reveal requirement optimization information that is most advantageous to their individual profits, but not necessarily the optimal good or service from the buyer's stand-point.

3. Regulatory Environment

The FAR offers broad regulatory guidance in developing requirements and conducting market research.⁶ The level of research effort should be appropriate for the complexity of the requirement and the market. Performance or end-state requirements definitions are encouraged for use.

More complex, expensive acquisitions require a formal Acquisition Strategy Plan.⁷ The contents of the formal plan are detailed and exhaustive, requiring the acquisition team to address 29 individual issues that relate to the unique acquisition.

The information gathering regulatory system is relatively broad and instructive. The primary constraint is not necessarily within any subcategory, but in the sequencing. Agencies must complete all phases of information gathering prior to soliciting requirements.⁸ A FAR—Part 15 negotiated acquisition is processed in the following order:

- Define need
- Conduct Market research
- Acquisition planning
- Conduct auction
- Evaluate offers
- Award

It is a linear process that is easy to understand, however, its aggregate quality is dependant upon the quality and availability of information gained early. It is possible to make any number of errors in gathering and relating the information to the acquisition objectives.

⁶ FAR 10.001, 10.002, & 11.002.

⁷ FAR 7.104 & 7.105.

⁸ FAR 10.001(a)(ii).

B. COUPLING PROBLEM

Final acquisition outcomes are in part dependent upon identifying the applicable cost and attribute trade-offs in the earliest stages and making them correctly. Information is difficult to gather and evaluate in the early stages due to the existence of asymmetric information.

Any given buyer is interested in how other buyers buy and how sellers sell, i.e., how the market typically operates. The highest value information is relevant to cost and the trade-offs among the array of attributes; however, sellers have very little incentive to reveal their individual cost functions truthfully because the information is proprietary. Even if any given contractor's costs were somehow known, it would only be applicable to that individual contractor, so may not translate well into overall acquisition strategy and contract objectives.

An additional problem is the regulatory segmentation of the problem. While requirement generation and market research may be conducted simultaneously, an auction cannot begin until both processes are complete. The segmentation discounts the possibility that learning could take place intra-auction that may be advantageous to the buyer and the eventual seller(s).

C. RESEARCH OBJECTIVE

The dual objective of this research is to develop a more efficient auction model and to demonstrate that the defense acquisition process is currently less efficient than it could be. A general form model will serve to describe the more generic auction process. Secondary objectives include exploring whether a new model can withstand regulatory compliance and the impact that an efficient model may have upon defense outcomes. The impact segment will go as far downstream as possible to demonstrate the robustness the model may have upon the system. Furthermore, the contingency contracting and service contracting impacts will be discussed.

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II. LITERATURE REVIEW

A. AUCTIONS

Auctions are commonly used as a mechanism to buy and sell goods and services. They're widely used by sellers to allocate resources such as art, livestock, estates, the electromagnetic spectrum, and the wide-array of products available on eBay. Buyers may use auctions to obtain services such as home construction and land and water use rights. It is widely agreed that there are four common auction types utilized: English, Dutch, first-price sealed-bid, and second-price sealed-bid auctions.⁹

1. English Auctions

The English auction is the most widely known auction and begins when a bidder, or an auctioneer, announces an opening bid (also called open auction). The entry bid is typically on the low-range of the expected final sales price of the unit. All bidders are then invited to raise the price above the previously highest bid. The auction continues in the same sequential manner until there is only one bidder left, the highest bidder, and no one wishes to raise the bid.

There is both symmetric and asymmetric information within any particular auction. Before the bidding occurs all potential bidders have some private information—how highly they value the item. As bidding commences, the private willingness to pay of all bidders (except for the winner) is eventually revealed, as the highest price they were willing to pay, before they refused to continue. Therefore information begins asymmetric and ends more symmetrically distributed.

The fact that the bidders are able to see others' value for the same object and is conducted sequentially distinguishes it from other auctions.

⁹ Michael R. Baye, *Managerial Economics and Business Strategy*, Boston: Irwin McGraw-Hill, 2000, Note: The Auctions segment follows Chapter 12 and is generally attributed to Bate, unless noted.

2. Dutch Auctions

The Dutch auction begins when an auctioneer announces a price that is excessively high for the lot. The auctioneer continues to reduce price incrementally until one person agrees to buy at the last price announced.

Information is asymmetrically distributed through the entire auction, until the end. No individual bidder knows any other bidders' willingness to pay until the end. The seller also does not know anything about any bidder's private value. Note also that the auction is equivalent to a simultaneous auction because no individual has any information other than their individual private value until the end of the auction (at which point all private information is irrelevant).

3. First-Price Sealed Bid Auctions

The first-price auction begins when an auctioneer invites bidder to document their bid for an object and forward the price to the seller. The auction closes when the auctioneer opens the bids and grants the highest bidder the lot. The buyer pays the amount they submitted on the bidding document.

Information is asymmetrically distributed throughout the entire auction, until the end. The first-price auction is also simultaneous, similar to the Dutch auction and distinctive from the English sequential auction.

4. Second-Price Sealed Bid Auctions

The second-price auction is also called the Vickrey auction, after William Vickrey.¹⁰ It is conducted in the same fashion as the first-price sealed-bid auction. Bidders' disclose the price with a sealed bid, information is asymmetric, and the highest bid wins the auction. The distinction is that the winning bidder pays the price that second highest bidder set.

¹⁰ William Vickrey, "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, 16 (1961): 8-37.

5. The Reverse Auction

Defense auctions (or, more generally, procurement auctions) are mirror images of the auctions described above. The conditions are the same, however, rather than having one seller and many buyers, DoD reverses the roles of the actors; one buyer and many sellers. In a reverse auction, the sellers are the bidders while the buyer is the bid-taker. In addition, the winning bidder is not the one who expresses willingness to pay the most, but the one who expresses willingness to be paid the least.

6. Private, Correlated, and Common Value

Thus far, we have assumed that the bidders all valued some item uniquely in that an actors' valuation had no relationship with anyone else's valuation. There's also little reason to believe that there would be a great amount discussion among the bidders inquiring about one another's values. Even if a discussion did take place there is little reason to believe that any individual would have an incentive to change their valuation based on the discussion. Hence, each bidder's valuation of the object has been independent (not related to another's value) and private (not disclosed).

Suppose, however, that an English auction were conducted to sell an item of speculative value, such as an estate. All bidders are likely to have some information about the true value of the estate. Imagine that Fred values it for the gravel he expects to extract from the stream at \$200,000 and Barney values it for the oil he expects to extract at \$400,000. It is likely that Fred could value it more highly if he knew what Barney knows. This is a case of correlated value.

A common value auction describes a case where the overall benefit to the bidders is a fixed value common to all, e.g., oil below ground.¹¹ All potential bidders may have private information regarding their individual assessment of the amount, or expected rates of profit based on their individual cost function; however, the resource is fixed.

¹¹ Paul Klemperer, "What Really Matters in Auction Design," *Journal of Economic Perspectives*, 16, no. 1 (Winter 2002): 169-189.
<http://www.jstor.org/search/AdvancedSearch?si=1&hp=25&q0=klemperer%2C+Paul&f0=au&c0=AND&q1=&f1=&c1=AND&q2=&f2=au&c2=AND&q3=&f3=ti&wc=on&Search=Search&sd=&ed=&la=&node.Economics=1&ic=08953309>, Retrieved July 2007.

In defense auctions, it is rational to assume that sellers' behavior mirrors that of buyers' with independent private valuations. However, because the bidders are sellers they have independent private costs rather than values. They are independent because typically defense only allows one prime contractor to bid on any given requirement, therefore cost functions are by definition independent.¹² One seller's cost function is unrelated to any other. It is also reasonable to assume that any given contractor would prefer to not to reveal their individual cost function to the buyer or other sellers; therefore, sellers' costs are independent and private.

7. Optimal Strategies

When considering optimal bidding strategies assume that bidders' have independent private values. It is unnecessary to detail optimal strategies for common or correlated values because the model we are interested in is a defense auction. Further, assume that all bidders' wish to win the item.

Suppose an English auction were conducted for a horse. Suppose Bill knew with certainty that he would gain \$1,500 worth of total value from the horse working on his ranch. Further, assume that Bill is risk neutral. Prior to the auction Bill could imagine a true state of nature value for the horse. The state of nature value could be either higher or lower than his private value. If the state of nature value were higher than his private value, it would be irrational for him to pay the additional amount because he would not be able to re-capture the additional sum. If the state of nature value were lower than his private value, he would still be willing to pay up to \$1,500 for the horse and capture the difference as consumer surplus. Thus, all Bill needs to know is his individual private value. The same condition prevails for all bidders. Each individual's optimal strategy is to continue bidding for the horse until the price exceeds their private value and then stop.

Suppose the horse is now being sold in a first-price sealed-bid auction. Bidders' submit individual bids; highest bid wins and pays that amount. Bill still values the horse at \$1,500 and is risk neutral. If he were to bid above \$1,500 he could not re-capture the

¹² FAR 9.603 allows for the exception of a disclosed "partnership arrangement," however, in that case the partnership is treated as unitary.

sum, therefore he would not rationally pay above his private value. He could bid either below \$1,500 capturing some consumer surplus or at \$1,500 capturing zero surplus. The farther he bids below \$1,500 the greater the probability of not being the highest bidder, but the closer he bids to \$1,500 the lower his surplus. Bill's risk of losing the auction will also increase if more bidders are present. Imagine that there are n bidders and their private valuations are uniformly distributed between the highest possible valuation H and the lowest possible valuation L . Bill's optimal bid b is:

$$b_b = v_b - \frac{v_b - L}{n}, \text{ where } v_b = \$1,500$$

As n increases (competition) and as L increases Bill's optimal strategy becomes to bid closer to his actual value (\$1,500).¹³ Bill would not necessarily have to solve the equation for all n bidders. If Bill was certain that the next lowest bidder had a value of \$1,400 he would be able to substitute \$1,400 for L and n would equal one, thus his optimal bid would be:

$$b_b = \$1,500 - \frac{\$1,500 - \$1,400}{1} = \$1,400$$

If he were not certain, he would only know the probable value P_v of the next low bidder L_i :

$$b_b = \$1,500 - \frac{\$1,500 - P_v(L_i)}{1}$$

¹³ John G. Riley, "Expected Revenue from Open and Sealed Bid Auctions," *The Journal of Economic Perspectives*, no. 3 (1989): 41-50.
<http://www.jstor.org.libproxy.nps.edu/view/08953309/di960527/96p0008g/0?searchUrl=http%3a//www.jstor.org/search/BasicResults%3fhp%3d25%26si%3d1%26gw%3djtx%26jtxsi%3d1%26jcpsi%3d1%26arts%3d1%26Query%3dsealed%2bbid%2bauction%2bstrategy%26wc%3don&frame=noframe¤tResult=08953309%2bdi960527%2b96p0008g%2b0%2cFF07&userID=cd9b416c@nps.navy.mil/01c0a80a651b8ac116457f1ee2&dpi=3&config=jstor>, Retrieved November 2007. See Riley for a general form first-price sealed-bid optimum model.

The optimal strategy for each player is to shrink their bid by the amount equal to the difference between their value and what they perceive the next lowest value is. Note that there is no need to address an optimal strategy related to a Dutch auction because they have the exact same informational components, so the Dutch auction and the first-price auction are strategically the same.¹⁴

Suppose the horse is now being offered in a second-price auction. Bidders' submit individual bids; highest bid wins, but pays the second-highest amount. Bill values the horse at \$1,500 and remains risk neutral. Bill could bid higher than \$1,500 in the hope that the second-price was lower than \$1,500, but if that were the case, Bill would have won with a bid of \$1,500 anyway. Additionally, there is some risk that the second-price will be above \$1,500, in which case he could not re-capture the sum. He has no incentive to place a bid higher than \$1,500. Bill could also submit a bid below \$1,500 to raise his surplus, but his risk of losing the horse increases. In addition, if his below \$1,500 bid wins he will capture some level of surplus in any case. The increased risk of losing is not balanced with the equal likelihood of additional surplus (as was the case in the first-price auction) therefore he would not bid lower than \$1,500. Bill's optimal strategy in a second-price auction is to bid the exact amount of his true value (\$1,500).

A bidder's optimum strategy to gain some unit available at auction is summarized below:

¹⁴ Michael R. Baye, *Managerial Economics and Business Strategy*, Boston: Irwin McGraw-Hill, 2000: 464.

Auction Type	Optimal Strategy
<i>English</i>	Bid up to true-value and stop
<i>Dutch</i>	$b_i = v_i - \frac{v_i - L}{n}$
<i>First-price, sealed-bid</i>	$b_i = v_i - \frac{v_i - L}{n}$
<i>Second-price</i>	Bid exactly true-value

Figure 1. Optimal Bid Strategies

8. Revenue Equivalence

Suppose an auction house had procured a horse and planned to hold a sales auction. Which auction type might the auctioneer prefer if his goal were to maximize revenue, given that all bidders' have independent private values?

If using an English action the auctioneer can expect to receive the amount that has been called out last. The winner receives the horse for that amount, however, the last valuation was made by the person with the second-highest valuation (made implicitly when they self-eliminated). The individual with the highest valuation may have been willing to pay more, but the auctioneer cannot capture it. The auctioneer's expected revenue equals the second-highest valuation.

If conducting a first-price sealed-bid or Dutch auction (we have already addressed their strategic equivalence) an auctioneer can expect to receive the amount equal to the highest-bid. However, all bidders' scale down their individual bids by the amount equal to what they perceive as the next lowest valuation. Thus, the auctioneer's expected revenue equals the second-highest valuation.

If conducting a second-price auction an auctioneer can expect to receive the amount equal to the second-highest bid, which, we have shown, should also be the second-highest valuation. Thus, if all bidders' have independent private values the

expected revenue of the four common auctions is the same. Auctioneers would be expected to be indifferent among the auction forms; this principle is commonly referred to as the *revenue equivalence theorem*.

Note that the same analysis used above to prove revenue equivalence for forward auctions can be used to prove cost equivalence in a reverse auction. In other words, in an independent private value reverse auction with rational risk-neutral bidders, the bid-taker (buyer) can expect the final contract price paid to be the same no matter what form of auction is used.

B. EFFICIENCY MATTERS

If the auctioneer (in the case of defense auctions the buyer) is expected to be indifferent among the various forms of auction, how can defense auctions (first-price sealed-bid) be less than optimally efficient? What is efficiency?

There are three general types of efficiency. Allocative efficiency refers to a situation where all buyers who wish to buy—buy, and all sellers who wish to sell—sell, assuming productive efficiency is maximized.¹⁵ Productive efficiency is achieved when resources are produced at the lowest cost achievable. Dynamic efficiency is a measure of how markets achieve greater allocative and productive efficiency over time. A graphical depiction may be helpful:

¹⁵ Luis M. B. Cabral, *Introduction to Industrial Organization*, Cambridge: The MIT Press, 2000. This section is primarily a synopsis of Chapter II.

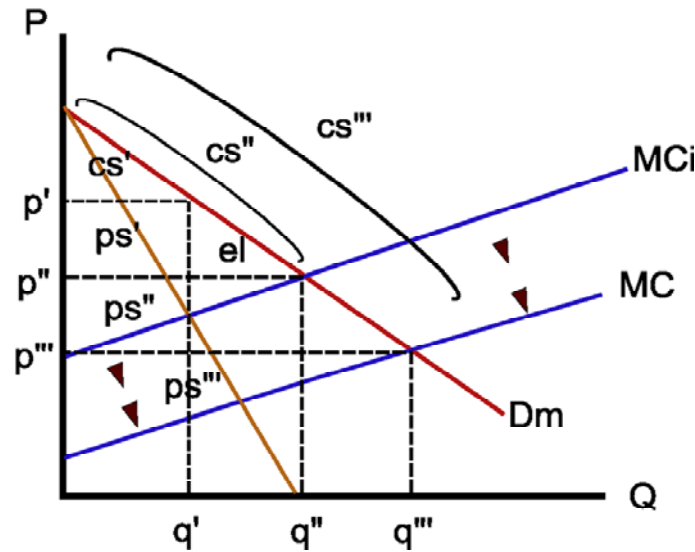


Figure 2. Efficiency Illustration

Suppose an oligopoly market in equilibrium depicted at (q', p') , where (MC_i) is the combined marginal cost function for the oligopolists. Producer surplus is depicted at (ps') and consumer surplus depicted at (cs') ; however the market is not allocatively efficient because there is an efficiency loss at (el) . The efficiency loss represents buyers that would buy and sellers that would sell if it were a perfectly competitive market; thus the loss of efficiency.

Now suppose the market were perfectly competitive with equilibrium output q'' . In this case all buyers with WTP above p'' buy, and all sellers with cost below p'' sell; the area (ps') and (el') become portions of newly attainable (cs'') or (ps'') , an allocatively efficient and productively efficient market.

Now suppose that over time productive or technological improvements are made at the oligopoly firms. The initial marginal cost curve (MC_i) shifts down and to the right to (MC) . The area (ps''') represents the gain in producer surplus (note the area is larger than ps'' , so it represents a real gain), and the area above the new price (p''') , becomes (cs''') , additional consumer surplus. Total surplus is increased, therefore there is a gain in allocative efficiency, and the marginal cost curve shift represents an increase in both productive and dynamic efficiency.

An increase in allocative efficiency means that more of a good is made and sold, at a lower price. An increase in productive efficiency (which can be a movement along), or dynamic efficiency means that more goods are bought and sold for each dollar required to make the good. Any efficiency increases result in more benefit per-dollar-cost.

Recall from the introduction section that defense's ex ante problem is two fold. There's a combinatorial optimization problem (what to ask for) and an information symmetry problem (neither party has a unilateral incentive to share information). Therefore defense may gain an improvement in allocative efficiency notwithstanding that competition may yield the same final auction price.

In particular, consumer surplus in a defense auction is equal to the value that a contracted item provides minus the price paid. Producer surplus in a defense auction is equal to the price received minus the production cost. Consequently, total surplus is maximized (and full efficiency is achieved) by maximizing the difference between the value provided the government and the production cost.

Full efficiency is difficult to achieve in the defense auction context, however, because of "incomplete information" about the buyer's utility function. In particular, neither the buyer nor any seller knows with certainty which set of requirements or weighting among performance dimensions will generate the greatest value for DoD. Each actor (buyer or seller) has some idea about what will provide the most value, but nobody knows for certain. This incomplete information condition not only makes it difficult to determine which specific product or product requirements will generate the most value but also makes it difficult to identify which seller can provide this value at the lowest production cost. Consequently, full efficiency is difficult to achieve.

C. INFORMATION MATTERS

Information has attributes, which distinguishes it from ordinary goods.¹⁶ Imagine a shop that sells two goods, information and apples. All the characteristics of an apple can be observed prior to purchase; contrarily, if the keeper revealed the characteristics of information, other than the broadest, the keeper could no longer sell it. A buyer may buy one, or more, apples; contrarily, each new piece of information is unique. The keeper can only sell each apple once (ownership transfers); contrarily, the keeper can sell the same bit of information to all possible buyers, therefore, it is relatively non-rival in consumption. Typical risks associated with information are that an optimal decision cannot be made without acquiring more and, as always, there is some chance that what an individual does know, is incorrect.

Information is not sold in shops, in most cases, it is more generally acquired, usually through search and (as we highlighted in the background section) can be perfect or imperfect. If information were perfect (symmetric) each individual would know everything there is to know. Often information is imperfect (asymmetric) which implies some level of moral hazard, or at least, an opportunity to improve efficiency.

When considering the value of information George Stigler highlighted some key principles.¹⁷ Suppose that Bill is in the market for crop insurance. If Bill were to survey the market for the same amount and level of crop insurance, he may get a price range. If he were to further survey the market a second time it is likely that the range of prices would be less varied, but the value of the information would also diminish.

¹⁶ Joseph E. Stiglitz, "The Contributions of the Economics of Information to Twentieth Century Economics," *The Quarterly Journal of Economics*, 115, no. 4. (November 2000): 1441-1478.

¹⁷ George J. Stigler, "The Economics of Learning," *The Journal of Political Economy*, 69, no. 3. (June 1961): 213-225, <http://links.jstor.org/sici?sici=0022-3808%28196106%2969%3A3%3C213%3ATEOI%3E2.0.CO%3B2-D>, Retrieved November 2007.

Continuous canvassing of the market for information has diminishing gains. Stigler computed an optimal level of search for Bill and concluded that Bill should search until the marginal benefit of the search equaled the continued marginal cost of the search.¹⁸ In addition to the time value of information, surveying the market for facts is not the only information that may be relevant.

1. Signaling

Consider that there may be additional information problems regarding Bill's crop insurance. Imagine that two types of insurance seekers' exist and that the two types are high-risk and low-risk. Bill has perfect information regarding his type. Bill's objective is to gain as much coverage as possible per dollar. An agent's objective, however, is to maximize profit and therefore has a preference for selling policies to low-risk seekers. There's some level of asymmetric information, but if Bill does nothing to communicate his type to the agent he risks not getting coverage. Given the objectives, it may be in Bill's best interest to behave in ways that signal to potential insurance agents that he's a low-risk type.¹⁹ Information such as Bill's access to water, frequency of fertilization, et cetera, would improve the likelihood that he could experience gains from trade. As a mechanism a signal is some costly activity that an individual unilaterally conducts to gain some level of benefit at some time in the future. Note that Bill's type is not directly communicated to the agent. A signal he controls is transferred to the agent and the agent then decides to offer, or not offer, a policy at some price.

2. Screening

To demonstrate the difference between signaling and screening the illustration must take slightly different form. Imagine that the two types of insurance seekers are now good-workers and poor-workers. Also, imagine that the agent's price quote depends on whether he actually, ex post, watered and fertilized the crop as he claimed he would.

¹⁸ Stigler, 217-218.

¹⁹ Michael Spence, "Job Market Signaling," *The Quarterly Journal of Economics*, 87, no. 3. (August 1973): 355-374, <http://links.jstor.org/sici?sici=0033-5533%28197308%2987%3A3%3C355%3AJMS%3E2.0.CO%3B2-3>, Retrieved November 2007.

Given this set of circumstances, Bill may have a disincentive to signal information regarding his type, i.e., if his type is poor-worker he would prefer to mask that data. In any case, he has an incentive to signal *good*, notwithstanding the true state of nature that does exist. Knowing this, the agent could discount his signal and may prefer to unilaterally require some form of screening mechanism to eliminate poor-workers, ex ante.²⁰ The insurance agency would generally be better off if it were possible to screen seekers, ex ante, provided that the cost of the research necessary to develop an effective screening policy did not exceed the benefit of screening.

Note that signaling and screening are similar but not synonymous. A signal is related to some attribute (frequency of fertilization related to risk type) and one party can pursue it independently (not strictly related to current transaction). A screening policy is generally imposed by the under-informed party as a condition to address an information asymmetry (good/poor worker) where the alternate party has a disincentive for truth-telling (moral hazard).

D. LITERATURE REVIEW SUMMARY

There are four common forms of auctions: English, Dutch, first-price sealed-bid, and second-price. English auctions begin by the (low) price announcement and continue until only one buyer remains, price is highest bid and the second-highest valuation. Optimal bidding strategy is to continue to bid unless/until the bid exceeds a player's individual valuation. A Dutch auction begins with descending price announcements and continues until someone agrees to accept, price is the last announced, but should equal second-highest valuation on average. A first-price sealed-bid auction requires all players to document their sealed-bid and submit to the auctioneer or bid-taker. Price paid equals the highest price bid, but should equal the second-highest valuation on average because the optimal strategy for each player is to shade down their individual bid until it equals the next highest valuation. A second-price auction is the same as the first-price sealed-bid auction except that the price paid is the second-highest price bid. Players' optimal

²⁰ Joseph E. Stiglitz, "The Theory of Screening, Education, and the Distribution of Income," *The American Economic Review*, 65, no. 3. (June 1975): 283-300, <http://links.jstor.org/sici?sici=0002-8282%28197506%2965%3A3%3C283%3ATTO%22EA%3E2.0.CO%3B2-0>, Retrieved November 2007.

strategy is to bid their true valuation for the object, thus, the second-highest valuation is paid. With risk-neutral bidders, auctioneers should be indifferent among the common auctions because expected revenue is the same for all.

There are three forms of efficiency: allocative, productive, and dynamic. If a market clears it is allocatively efficient. Productive efficiency means that a good is made at the lowest possible cost. Dynamic efficiency measures how much productive efficiency improves over time. Improving allocative efficiency means that more of a good is made and sold, at a lower price. An increase in productive efficiency (which can be a movement along), or dynamic efficiency, means that more goods are bought and sold for each dollar required to make the good.

Information is different from ordinary goods and has some properties of public goods. Imperfect information can prevent or reduce the efficiency of some transactions diminishing some of the gains from trade. Information search has a cost that should be equated to the benefit. Tacit information gathering, in the form of signaling, is a means to improve the likelihood that the trade will occur, or be more efficient. Screening is an explicit form of information gathering that may be employed, when one party has an incentive to cheat, to improve the likelihood that some trade will occur, or be more efficient.

III. MODELING

A. THE MODELS

Developing a general auction model must be complete before any comparison between individual auctions can be made, and the model will offer a framework for understanding deviations that may be made. Two distinct but complementary models will flow from the general form: A multiple-stage auction, and a single stage auction. The objective is to design and later compare the two models.

B. GENERAL BUYER AND SELLER INCENTIVES

Building a model of federal acquisition auctions requires understanding the initial situation and what a different model might achieve. In any auctions, a boundless array of feasible attributes that could be measured exists; however, to model all possible auctions is undesirable. It is therefore appropriate to set initial conditions equal for either model.

Imagine that the array of quality elements is limited to two components: reliability (x) and delivery schedule (y). Some tradeoff exists between the two elements of quality and can be expressed by weights placed on the two elements when determining overall quality. In particular, overall quality is given by $\alpha x + \beta y$, where α indicates the importance of or weight placed on reliability (x) while β is the importance of or weight placed on delivery schedule (y). The tradeoff between the two elements of quality is induced by making the additional assumption that $\alpha + \beta = 100$. Thus, if α is relatively high (i.e., reliability is relatively important) than β must be relatively low (i.e., delivery schedule is relatively less important), and vice versa.

The overall value (η) to the DoD is determined by subtracting price (P) from quality ($\alpha x + \beta y$). In other words, we have:

η = overall value

x, y = elements of quality

α, β = weight placed on each element of quality, where $\alpha \sim U[0,100]$ and $\beta=100-\alpha$

P = price

$$\eta = \alpha x + \beta y - P$$

The discussion so far is applicable to any type of mechanism for federal procurement. Value is a function of the magnitude and relative importance of the quality attributes, less the price. At the same point in time, prior to the auction a contractor j has a cost function which is independent of the DoD value function and which can be expressed as:

C_j = total cost from firm j

x, y = elements of quality

a_j, b_j = marginal cost parameters for each element of quality, where $a_j, b_j \sim U[0,10]$

$$C_j = a_j x^2 + b_j y^2$$

In the firm's cost function, the quality elements are quadratic because they are subject to the usual condition of increasing marginal costs. In other words, if the firm doubles output, costs go up exponentially rather than at the same rate. Thus, we now have two independent functions to describe the incentives of both the buyer and sellers for any type of or mechanism for federal procurement.

1. Imperfect Information about Buyer (DoD) Preferences

It is often the case that the buyer in any procurement (in this case, the DoD) has only imperfect information about its own preferences. In other words, the buyer is not always fully aware of all possible capabilities of available technology nor is the buyer fully aware of the precise benefits of these capabilities. Similarly, contractors may have better (or at least different) information about the capabilities of available technology, but may have only an imprecise understanding of the benefits of these capabilities for the buyer.

In the model we have presented, this uncertainty about buyer value can be captured by assuming that both the buyer (DoD) and the sellers (contractors) have imperfect information about the true value of α and β , the weights on the different elements of quality, in the buyer's value function.

To represent this imperfect information condition, we can envision the information about α and β that is held by the buyer and each seller as being provided via a series of independent draws by each player from an opaque urn containing 100 balls. In this urn, there are α black balls and β white balls (recall that $\alpha + \beta = 100$). An individual player (be it a buyer or seller) infers the true number of black and white balls in the urn (the true values of α and β) from the information they have received from their draws (the number of black and white balls).

To represent the different levels of precision in information about buyer preferences, suppose that the buyer (DoD) draws m_b balls from the urn while each seller (contractor) draws m_s balls from the urn. Note that the buyer might have more precise information than each contractor, in which case we would have $m_b > m_s$, or the buyer might have less precise information than each contractor, in which case we would have $m_b < m_s$.

Recognize that if the buyer draws B black balls and W white balls from the urn, then his ex ante estimates of the values of α and β will be given by:

$$\begin{aligned}\alpha_b &= \text{buyer's ex ante estimate of the value of } \alpha \\ \beta_b &= \text{buyer's ex ante estimate of the value of } \beta \\ \alpha_b &= \frac{B}{B+W} \times 100 = \frac{B}{m_b} \times 100 \\ \beta_b &= \frac{W}{B+W} \times 100 = \frac{W}{m_b} \times 100\end{aligned}$$

Each individual contractor j 's ex ante estimate of the value of α and β (α_j and β_j) will be determined the same way based on the individual contractor's draws from the urn.

C. A MULTIPLE STAGE AUCTION

A multiple stage auction can be viewed as a sequential process of information revelation.

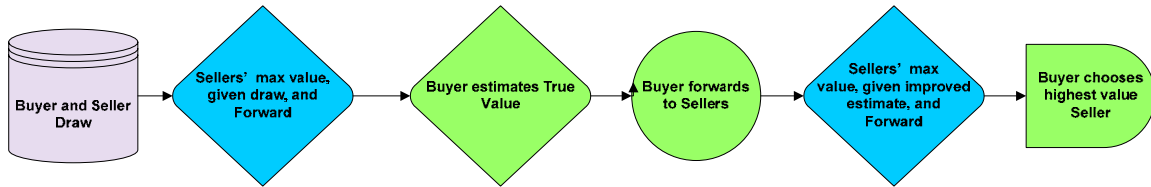


Figure 3. The Multiple Stage Auction

The buyer, the seller, or both parties learn a piece of information in each stage. Additionally, one set of players' (the array of contractors, or the DoD) have the opportunity to make a decision that maximizes their individual value.

Suppose now that we were to take DoD's value function, the contractors' cost functions, and the ball drawing game and then imagine what a multi-stage auction would look like compared to the current federal auction process, based on these initial conditions.

1. Stage One

At stage one of the game all individual players draw balls out of the urn as described above.

2. Stage Two

In stage two of the game, each contractor j will submit a bid to the buyer (DoD) which consists of a price (P_j) and two quality elements (x_j and y_j). The objective each contractor has in stage two is to decide the optimal levels of P , x , and y based on the individual contractor cost function and the information about buyer preferences from the draw in stage one. One player, the buyer, is excluded from play in stage two of this game.

An individual seller j has two crucial components of information. First, the contractor has perfect knowledge of its individual cost function:

$$C_j = a_j x_j^2 + b_j y_j^2$$

The contractor also has some information (as much as – but different from – any other individual contractor) about the true values of α and β .

Each auction stage will be conducted as a generalized multi-dimensional second-price auction with all bids from each stage being binding (i.e., the buyer can accept and announce as the “winner” any bid from any contractor in any stage of bidding). This will result in it being part of each contractor’s optimal (dominant) strategy to submit a bid in which price equals cost. Thus, each contractor j will set:

$$P_j = C_j = a_j x_j^2 + b_j y_j^2$$

In addition, the contractors whose bids generate the lowest buyer value will be eliminated from the competition after the bidding in stage two, thus it is also part of each contractor’s optimal strategy to submit an overall bid which, given the contractor’s information, maximizes the value that it can profitably provide the buyer. Thus, with P_j already dictated as indicated above, it is the objective of contractor j in stage two to:

Choose x_j and y_j to maximize $\alpha_j x_j + \beta_j y_j - P_j$

Choose x_j and y_j to maximize $\alpha_j x_j + \beta_j y_j - a_j x_j^2 - b_j y_j^2$

Given that there is no interaction between x_j and y_j in the above objective function, we can separate the objective into two independent objectives:

Choose x_j to maximize $\alpha_j x_j - a_j x_j^2$

and

Choose y_j to maximize $\beta_j y_j - b_j y_j^2$

Each contractor j ’s optimal bid can then be determined by differentiating each of the above objective functions (and setting the derivative equal to zero) to find the maximum value attainable for each function.

$$\frac{d}{dx_j}(\alpha_j x_j - a_j x_j^2) = \alpha_j - 2a_j x_j = 0$$

$$\Rightarrow x_j = \frac{\alpha_j}{2a_j}$$

and

$$\frac{d}{dy_j}(\beta_j y_j - b_j y_j^2) = \beta_j - 2b_j y_j = 0$$

$$\Rightarrow y_j = \frac{\beta_j}{2b_j}$$

Thus, in stage two, it is each contractor's optimal strategy to submit a bid (x_j, y_j, P_j) such that:

$$x_j = \frac{\alpha_j}{2a_j}$$

$$y_j = \frac{\beta_j}{2b_j}$$

$$P_j = a_j x_j^2 + b_j y_j^2$$

Stage two closes when some array of contractors submits bids revealing their individual value for (x, y, P) . The initial submission must be binding to induce truth telling. If stage two were not binding contractors would have an incentive to cheat (by submitting high value bids which the contractor cannot possibly provide profitably) in an effort to simply be selected for later rounds.

3. Stage Three

The objective of stage three is for the buyer to re-estimate the true values of α and β based on the bids submitted by contractors in stage two. The sellers do not play in stage three.

In stage three, the buyer has two components of information from which he can estimate the true values of α and β . First, the buyer knows his individual estimates from stage one (α_b and β_b). Additionally, the buyer also knows the bids (x_j, y_j, P_j) for each

contractor in stage two. However, because the individual contractors do not directly reveal their estimates of α and β from stage one, the buyer must infer each contractor j 's estimates α_j and β_j based on his bid (x_j, y_j, P_j) .

From above, we know that optimization by contractor j yields:

$$x_j = \frac{\alpha_j}{2a_j} \Rightarrow a_j = \frac{\alpha_j}{2x_j}$$

$$y_j = \frac{\beta_j}{2b_j} \Rightarrow b_j = \frac{\beta_j}{2y_j}$$

$$P_j = a_j x_j^2 + b_j y_j^2$$

Substituting the first two equations above into the third yields:

$$P_j = \left(\frac{\alpha_j}{2x_j} \right) x_j^2 + \left(\frac{\beta_j}{2y_j} \right) y_j^2$$

$$P_j = \frac{\alpha_j x_j}{2} + \frac{\beta_j y_j}{2}$$

$$2P_j = \alpha_j x_j + \beta_j y_j$$

Finally, substituting $\beta_j = 100 - \alpha_j$ into this last equation gives us:

$$2P_j = \alpha_j x_j + (100 - \alpha_j) y_j$$

$$2P_j = 100y_j + \alpha_j (x_j - y_j)$$

$$2P_j - 100y_j = \alpha_j (x_j - y_j)$$

$$\alpha_j = \frac{2P_j - 100y_j}{x_j - y_j}$$

$$\beta_j = 100 - \alpha_j = \frac{100(x_j - y_j) - 2P_j + 100y_j}{x_j - y_j} = \frac{2P_j - 100x_j}{x_j - y_j}$$

Thus, the buyer can infer the contractor's estimates of α_j and β_j from each bid (x_j, y_j, P_j) . Combining these estimates with the buyer's own estimates α_b and β_b allows the buyer to generate updated estimates of α and β as follows:

$\hat{\alpha}$ = updated estimate of α

$\hat{\beta}$ = updated estimate of β

$$\hat{\alpha} = \frac{\alpha_b m_b + m_s \sum_{j=1}^n \alpha_j}{100(m_b + nm_s)}$$

$$\hat{\beta} = \frac{\beta_b m_b + m_s \sum_{j=1}^n \beta_j}{100(m_b + nm_s)}$$

4. Stage Four

The objective of stage four is to announce the updated estimate ($\hat{\alpha}, \hat{\beta}$) of the true values of (α, β). The number of initial competitors is a key component of the auction because the larger the pool of information—the better the estimate.

Additionally at this stage, the buyer will calculate the value generated by each contractor's bid using the updated estimates of α and β . Only a subset of contractors, those whose initial bids generated the greatest value, will be allowed to continue to stage five and beyond.

5. Stage Five

The objective of stage five of the auction is for the remaining contractors to re-bid, after the buyer announces the improved estimates of α and β . Stage five ends when each remaining contractor announces their final price.

Choose x_j and y_j to maximize $\hat{\alpha}_j x_j + \hat{\beta}_j y_j - P_j$

Choose x_j and y_j to maximize $\hat{\alpha}_j x_j + \hat{\beta}_j y_j - a_j x_j^2 - b_j y_j^2$

6. Stage Six

The last stage of the auction is the award announcement. The successful seller is the firm whose bid maximizes total value; however, the auction is conducted as a second-price auction (to induce truth revelation) and therefore the winning firm is not paid its

own price bid. Instead, the winning firm is paid the highest price that it could have bid and still won the auction. In particular, suppose that the winning bid provided amounts of each quality element equal to x^* and y^* and the total value provided by the second-place bidder was v_2 . In this case, the price paid to the winning contractor (P^*) would be given by:

$$\begin{aligned}\hat{\alpha}_j x^* + \hat{\beta}_j y^* - P^* &= v_2 \\ P^* &= \hat{\alpha}_j x^* + \hat{\beta}_j y^* - v_2\end{aligned}$$

Recall from above that using a second-price auction induces the contractors to reveal their real costs and that the *revenue equivalence theorem* implies that the buyer (defense) would have paid the same price, in any case.

D. A SINGLE STAGE AUCTION

In a single stage auction, the balls are drawn from the urn in stage one. All players have some information about the true ratio. Information is extracted in the same way: each player has the same absolute amount of independent information and each individual player's private information may differ. The information is exactly the same, in all respects, as the information that players begin with notwithstanding which form of auction is conducted (multi-stage, or single stage variation).

Based on the initial draw there are several directions a possible single stage auction could take. In this case, all depend on the buyer's preference set, because the buyer (DoD) initiates the auction. First, the buyer may communicate, or not communicate, the information they gained—as an individual player—from the initial draw because they can make a unilateral publishing decision. Secondly, the buyer can set the decision criteria, therefore, can choose the outcome based on their ex ante draw, or the sellers draw, or some sub-set (e.g., average) of all known draws.

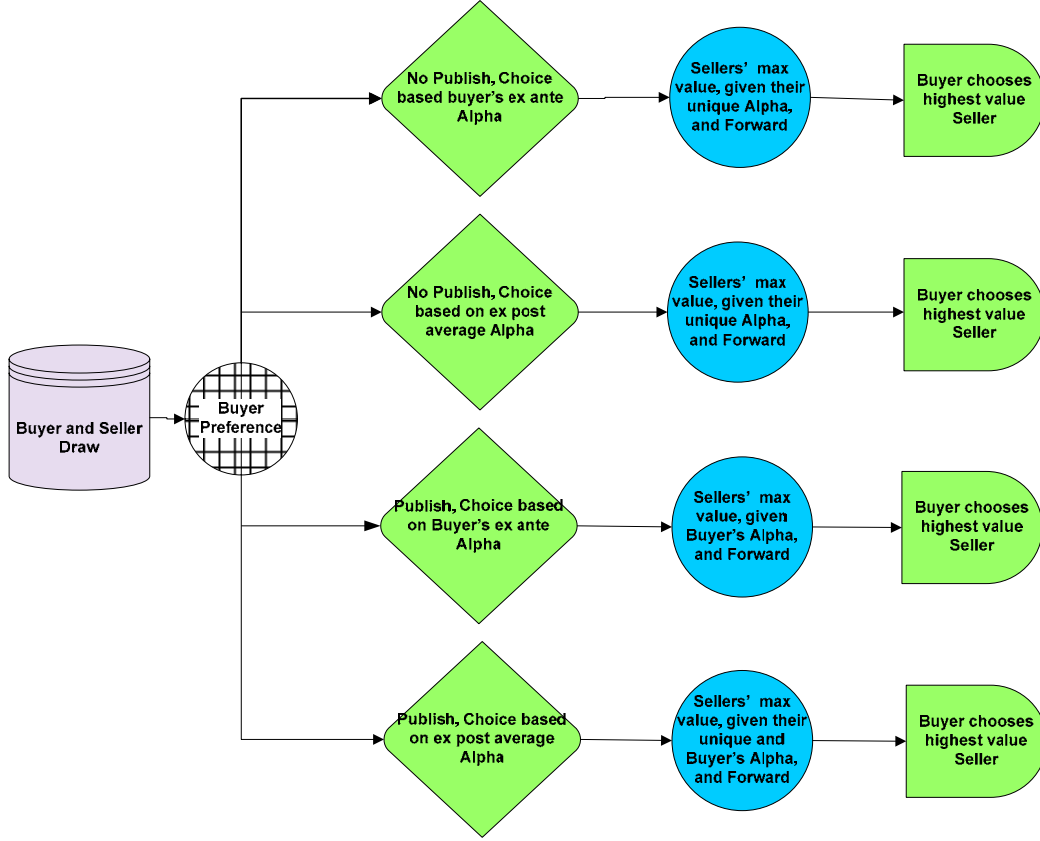


Figure 4. Single Stage Auctions

1. No Publish, Choice by Buyer Alpha

It is possible for the buyer to refrain from publishing their private information and base their decision on their individual draw. Therefore, the seller's optimal bid would be determined by their individual cost function and individual draw.

$$x_j = \frac{\alpha_j}{2a_j}$$

$$y_j = \frac{\beta_j}{2b_j}$$

$$P_j = a_j x_j^2 + b_j y_j^2$$

Moreover, the buyer's optimal value would be determined by which seller has the highest overall value.

$$\eta_{\max} = \max_j (\alpha_b x_j + \beta_b y_j - P_j)$$

This auction approximates a situation in which a contracting officer is ambiguous about the evaluation criteria during the solicitation to mitigate protest risk, but the customer is fairly certain about their desired outcome.

2. No Publish, Choice by Ex Post Weighted-Average Alpha

It is possible for the buyer to refrain from publishing their private information and base their decision on the combination of their private draw, averaged with each individual seller's draw. The sellers' optimal bid is again given by:

$$\begin{aligned} x_j &= \frac{\alpha_j}{2a_j} \\ y_j &= \frac{\beta_j}{2b_j} \\ P_j &= a_j x_j^2 + b_j y_j^2 \end{aligned}$$

And the buyer's optimal value would be determined by which seller has the highest overall value.

$\hat{\alpha}$ = estimate of optimal α

$\hat{\beta}$ = estimate of optimal β

$$\hat{\alpha} = \frac{\alpha_b m_b + m_s \sum_{j=1}^n \alpha_j}{100(m_b + n m_s)}$$

$$\hat{\beta} = \frac{\beta_b m_b + m_s \sum_{j=1}^n \beta_j}{100(m_b + n m_s)}$$

and solve for η

$$\eta_{\max} = \max_j (\hat{\alpha} x_j + \hat{\beta} y_j - P_j)$$

This auction approximates a situation in which a contracting officer is ambiguous about the evaluation criteria during the solicitation to mitigate protest risk, and the customer is uncertain about their desired outcome.

3. Publish, Choice by Buyer Alpha

It is possible for the buyer to publish their private information (initial draw) and base their decision criteria on their draw. In this case, the buyer is using its ex ante estimate (α_b, β_b) as a best representative for the true value of (α, β) . Contractors then bid according to the published α_b and β_b as well as their individual cost functions.

$$\begin{aligned}x_j &= \frac{\alpha_b}{2a_j} \\y_j &= \frac{\beta_b}{2b_j} \\P_j &= a_j x_j^2 + b_j y_j^2\end{aligned}$$

This auction represents a defense auction in which the contracting officer places no weight on the information held by contractors regarding what product type offers the best value for DoD. Contracting officers know what they intend to evaluate and are explicit regarding how the attributes are related.

4. Publish, Choice by Ex Post Weighted-Average Alpha

It is possible for the buyer to publish their private information (initial draw) and base their decision criteria on the average draw. If this were done the contractor's optimal strategy would be to calculate a weighted-average of the published α_b, β_b with their private draw and then optimize their choice of x, y , and price.

$$\begin{aligned}\hat{\alpha}_j &= \frac{\alpha_b m_b + m_s \alpha_j}{100(m_b + m_s)} \\ \hat{\beta}_j &= \frac{\beta_b m_b + m_s \beta_j}{100(m_b + m_s)} \\ x_j &= \frac{\hat{\alpha}_j}{2a_j} \\ y_j &= \frac{\hat{\beta}_j}{2b_j} \\ P_j &= a_j x_j^2 + b_j y_j^2\end{aligned}$$

As in the original multi-stage model, the buyer would then re-estimate what the true values of α and β are based on the bids submitted by contractors. The mathematics of this re-estimation are more complicated in this case, however, given that the buyer must first extract each seller's individual updated estimates $\hat{\alpha}_j$ and $\hat{\beta}_j$ and must from these extract each seller's original estimates α_j and β_j . Nonetheless, once the buyer has done so and produced aggregated re-estimated values $\hat{\alpha}$ and $\hat{\beta}$, the buyer's optimal strategy would be to choose the highest value seller using these re-estimates.

$$\eta_{\max} = \max_j (\hat{\alpha}x_j + \hat{\beta}y_j - P_j)$$

This auction matches the previous defense auction, except that during the auction the contracting officer has learned some new piece of information and would seek to apply the new information to the evaluation.

E. MODEL SUMMARY

The general model described the parameters of the auction, formalized a value function, and described ex ante information. The multiple stage auction developed a process of information revelation, such that, buyer and seller could re-maximize the two variables to obtain higher value. The single stage auction model allows the buyer to unilaterally determine the relative value of variables, varies more, and has no additional information revelation process.

The critical distinction between a multiple stage auction and a single stage auction is the number of times signals transfer from buyer to seller, or seller to buyer. In the multiple stage auction, the sellers communicate the private values of their draws and the buyer communicates their best estimate of the optimal value. In the single stage auction the buyer may, or may not, communicate a draw signal, so the sellers optimize given the information available.

The objective of modeling all variations of the auction model was to compare all possible forms. Moreover, recall that prior to holding the auction the buyer has a computational problem to solve (the correct weights to apply to the value attributes) and an asymmetric information problem to overcome. The possible efficacy gain in

allocative efficiency therefore can be expected to raise the gains from trade. Productive efficiency is already being improved by holding an auction competition. Analysis will include direct, indirect, internal, and external comparisons of all auction formats.

IV. ANALYSIS

A. SIMULATION MODEL

To test the significance of the mathematical model, the relationships addressed in Section III were entered into a standard Excel spreadsheet. The simulation mirrors precisely the model described in the modeling section. The entire model can be viewed in Appendix A, however, this section will focus on the major features, display simulation outcomes, and analyze the outcomes. The simulation model will be described by the input parameters, conduct of the auction, and how the outcomes of various auctions are measured.

1. Input Parameters

The first input parameters of the model are the seller cost curve parameters (a_j and b_j) are drawn from a uniform distribution. Secondly, draws for estimated of α and β are binomially distributed around the true value. Both inputs change in every auction simulation, for every seller and for the buyer.

Individual Draw						Contractors' Cost Functions	
Sellers	6	Round 1	100			ai	bi
Retained	2	Draws	α	Draw	α β		
Seller 1		5	2	40.0	60.0	4.38626	1.61734
Seller 2		5	3	60.0	40.0	5.80848	0.5585
Seller 3		5	2	40.0	60.0	3.49799	4.00502
Seller 4		5	4	80.0	20.0	2.46414	2.90553
Seller 5		5	4	80.0	20.0	4.24947	6.83271
Seller 6		5	0	0.0	100.0	1.97578	2.6196
Seller 7		5	2	40.0	60.0	6.64869	4.69526
Seller 8		5	4	80.0	20.0	9.28329	7.23153
Seller 9		5	4	80.0	20.0	6.29437	1.61405
Seller 10		5	4	80.0	20.0	2.39273	4.15416
Buyer		15	7	46.7	53.3		
Binomial			Actual Values				
Probability			0.6	60	40		
Revised Estimate			55.4	44.6			
Check			44.6				
			Uniform				
			Lower	0.5	0.5		
			Upper	9.5	9.5		

Figure 5. Model Input Parameters

The number of sellers is changeable so that intensity of competitions can be addressed. It is also interesting to vary the number of draws the sellers and buyers receive relative to another to answer the question: Is the multi-stage auction still an improvement over the standard auction when the buyer has more information than any individual seller at auction entry, and visa versa? The *Retained* cell allows the model to eliminate competitors after stage three of the multi-stage auction. Recall that this is necessary to induce sellers to legitimately offer the initial bids that they believe provide the buyer the best value based on their parameter estimate. The *Revised Estimate* cell calculates what the buyer would compute at stage-three of the multiple stage auction.

2. Auction Conduct

Each auction simulation will capture each contractor's x , y , P values that they would submit, assuming they're behaving optimally, given the knowledge available.

Option 3: Buyer Publishes Buyer ex ante Weights							Option 4: One Stage-Buyer Publishes & Updates Buyer ex post Weights									
X	Y	P	Perceived Gov Value	Actual Gov Value	Rank		α	β	X	Y	P	Perceived Gov Value	Actual Gov Value	Rank		
5.3	16.5	\$ 564	\$ 564	\$ 415	2	1	45.0	55.0	5.1	17.0	\$ 583	\$ 460	\$ 405	4		
4.0	47.7	\$ 1,367	\$ 1,367	\$ 784	1	1	50.0	50.0	4.3	44.8	\$ 1,227	\$ 1,009	\$ 822	1	1	
6.7	6.7	\$ 333	\$ 333	\$ 333	5		45.0	55.0	6.4	6.9	\$ 334	\$ 329	\$ 327	5		
9.5	9.2	\$ 466	\$ 466	\$ 470	4		55.0	45.0	11.2	7.7	\$ 481	\$ 482	\$ 498	3		
5.5	3.9	\$ 232	\$ 232	\$ 253	6		55.0	45.0	6.5	3.3	\$ 252	\$ 253	\$ 268	6		
11.8	10.2	\$ 547	\$ 547	\$ 569	3		35.0	65.0	8.9	12.4	\$ 558	\$ 486	\$ 469	2	1	
			\$ 1,367	\$ 784									\$ 1,009	\$ 822		
Buyer Surplus			\$ 564	\$ (19)			Buyer Surplus			\$ 486	\$ 299					
Seller Surplus			\$ 803				Seller Surplus			\$ 523						
Check			\$ 1,367				Check			\$ 1,009						

Figure 6. Auction Conduct

Recall that the government has the unilateral choice regarding how to conduct the evaluation in the single-stage auctions. Option-three simulates a FAR policy defense auction; the buyer publishes the evaluation criteria reflecting their *ex ante* weights (governments draw) and bases the decision on the buyers' bid based on these *ex ante* weights $\eta_{\max} = \alpha_b x_j + \beta_b y_j - P_j$. Option-four allows the sellers to re-estimate α and β based on the buyer's published draw; therefore, two additional columns are required;

choice is based on all the data available to all players, ex post. The *Perceived Government Value* column calculates the valuation of η for each contractor's submission given the information available with the auction. The *Actual Government Value* column calculates the valuation of η for each contractor's submission given the true (but unknown) values of α and β . Recognize that to measure and compare performance of the various auction models it is necessary to calculate the buyer's and winning seller's surplus. Recall that the sum of the buyer's and seller's surplus is the measure of efficiency.

3. Buyer's Surplus

The model will evaluate the winner of each auction within each simulation. Buyer's and seller's surplus will be measured and compared with the amount of surplus that would have been achieved had all parties had perfect information (the true weights are used to make bids and determine the winning seller). Recognize that it is possible for the buyer's surplus in any particular auction to exceed the buyer's surplus that is achieved in the perfect information auction. Note that in these cases the total surplus available does not increase, it merely depicts the share of the total surplus that is attributed to the buyer, or the seller, relative to the perfect information auction outcome. For example, suppose that the value of actual buyer's surplus in *Option 2* was equal to 125%; this simply means that the buyer's surplus achieved in *Option 2* was 25% higher than the buyer's surplus achieved in the perfect information auction. If that were the case the buyer would be better off with the imperfect information auction outcome. However, this not expected on average.

4. Measuring Outcomes

Of particular interest (because it directly measures efficiency) will be the columns (forecast variables) measuring *Perceived Buyer Surplus* (surplus the buyer thinks they are receiving given information known) and *Actual Buyer Surplus* (surplus buyer is actually receiving). The third high interest metric is *Consistency*. It measures the winner of all five auctions and compares the selected seller with the winner of the perfect information

auction; thus, it counts how frequently each auction chooses the incorrect seller. The last forecast variable of interest is the *Actual Total Surplus* column. It captures the percent of surplus that the auction achieves compared to the total surplus achieved if a perfect information auction were conducted. If likened to a pie, *Actual Total Surplus* measures the size of the pie (large, medium, small) and the *Actual Buyer Surplus* captures the portion of the pie the buyer receives.

		Perceived Buyer Surplus	Actual Buyer Surplus	Seller Surplus	Actual Total Surplus	Seller	Seller Chooses X, Y		Buyer Evaluates X, Y					Perceived Buyer Surplus	Actual Buyer Surplus	Seller Surplus	Actual Total Surplus	Consistency
							α	β	α	β	X	Y	P					
Option 1	Buyer Doesn't Publish Buyer ex ante Weights	\$ 554	\$ 146	\$ 725	\$ 871	2	60.0	40.0	46.7	53.3	5.2	35.8	\$ 871	110.24%	28.99%	196.98%	100.00%	1
Option 2	Buyer Doesn't Publish Buyer ex post Weights	\$ 432	\$ 291	\$ 580	\$ 871	2	60.0	40.0	55.4	44.6	5.2	35.8	\$ 871	85.99%	57.86%	157.55%	100.00%	1
Option 3	Buyer Publishes Buyer ex ante Weights	\$ 564	(19)	\$ 803	\$ 784	2	46.7	53.3	46.7	53.3	4.0	47.7	\$ 1,367	112.11%	-3.83%	218.11%	89.99%	1
Option 4	One Stage-Buyer Publishes Buyer ex post Weights	\$ 482	\$ 296	\$ 526	\$ 822	2	50.0	50.0	55.4	44.6	4.3	44.8	\$ 1,227	95.93%	58.80%	142.94%	94.37%	1
Option 5	Two Stage-Buyer Updates Buyer ex post Weights	\$ 483	\$ 320	\$ 541	\$ 861	2	55.4	44.6	55.4	44.6	4.8	39.9	\$ 1,023	95.95%	63.66%	146.78%	98.80%	1
	Perfect Information	\$ 503	\$ 503	\$ 368	\$ 871	2	60	40	60	40	12.2	6.9	\$ 503					

Figure 7. Auction Outcome Metrics

B. VALUABLE COMPARISONS

The comparisons that are most interesting, i.e., relevant to improving the defense auction, are: 1) How does the buyer's surplus vary between perceptions and actual, 2) How much surplus does the buyer gain between multi-stage versus single-stage auctions (especially option 3, the federal policy auction), 3) How do outcomes change as the number of draws given each player varies, 4) How do outcomes change when the number of contractors retained in the multi-stage auction vary?

The model will track actual and perceived surplus and consistency as the measures of efficiency improvement. Six simulation variations, each consisting of 25,000 individual simulations will be run to test the veracity of the model. The variations will be as follows:

	Simulation #					
	1	2	3	4	5	6
Buyer's Draws	15	15	5	5	15	15
Sellers' draws	15	15	15	15	5	5
Beginning Sellers'	10	4	10	4	10	4
Retained Sellers'	5	2	5	2	5	2

Table 1. Simulation Auction Variations

C. SIMULATION OUTCOMES

The number of simulations for each auction variation is 25,000. Data from each forecast variable (perceived buyer surplus, actual buyer surplus, consistency, and percent of total surplus) is available in Appendices B-G. Analysis will focus on the average performance of each forecast variable. Data on the average performance is provided below.

Auction Option	Simulation Variation						
	1	2	3	4	5	6	Mean
<i>No publish choice ex ante (O-1)</i>							
Mean perceived buyer surplus	97.71	94.16	103.23	95.03	89.89	85.01	94.17
Mean actual buyer surplus	80.64	81.68	63.26	62.26	70.47	68.31	71.10
Perceived less actual	17.07	12.48	39.97	32.77	19.42	16.7	23.07
Mean total surplus	89.64	90.78	84.96	86.7	81.84	81.66	85.93
Consistency	25	19	32	26	32	26	26.67
<i>No publish choice ex post (O-2)</i>							
Mean perceived surplus	95.71	94.95	95.79	95.02	88.42	86.07	92.66
Mean actual surplus	92.71	91.98	92.78	92.19	80.83	78.95	88.24
Perceived less actual	3	2.97	3.01	2.83	7.59	7.12	4.42
Mean total surplus	94.2	94.04	94.26	94	85.71	84.79	91.17
Consistency	10	8	10	8	22	19	12.83
<i>Publish choice ex ante (O-3)</i>							
Mean perceived surplus	108.27	105.62	123.36	117.2	108.09	105.53	111.35
Mean actual surplus	77.62	76.07	41.75	32.92	78.04	75.85	63.71
Perceived less actual	30.65	29.55	81.61	84.28	30.05	29.68	47.64
Mean total surplus	90.18	91.8	76.1	77.78	90.4	91.61	86.31
Consistency	20	13	26	19	19	13	18.33
<i>Publish choice ex post (O-4)</i>							
Mean perceived surplus	98.46	97.95	97.12	96.51	98.94	97.95	97.82
Mean actual surplus	95.54	94.98	94.12	93.69	91.61	91.08	93.50
Perceived less actual	2.92	2.97	3	2.82	7.33	6.87	4.32
Mean total surplus	96.64	96.76	97.11	95.28	94.49	94.71	95.83
Consistency	8	5	10	8	11	8	8.33
<i>Multi-stage auction (O-5)</i>							
Mean perceived surplus	100.83	99.7	100.87	99.77	100.1	95.93	99.53
Mean actual surplus	97.89	96.78	97.87	96.89	92.74	88.99	95.19
Perceived less actual	2.94	2.92	3	2.88	7.36	6.94	4.34
Mean total surplus	98.88	99.03	98.83	98.92	96.55	96.03	98.04
Consistency	7	5	7	7	13	12	8.50

Table 2. Mean of Forecast Variables by Auction Option and Simulation Variation

The *Mean* column provides the average performance of each forecast variable for all auction variations. It is evident that on average the multi-stage auction yields the buyer greater actual total surplus than any of the single-stage auction variations; however, the questions of interest should be examined in more detail.

1. Single-Stage versus Multi-Stage Auctions

To determine the mean performance of the single-stage auction it is necessary to average the performance of each forecast variable. To determine the level of improvement the multi-stage auction achieves it is necessary to utilize the formula:

$$\% \Delta = \frac{V_m - V_s}{V_s}$$

A table comparing the average performance of all of the single-stage auctions with the multi-stage auction is below.

	1	2	3	4	5	6	Mean
<i>Mean Single-stage Auction</i>							
Mean actual surplus	86.63	86.18	72.98	70.27	80.24	78.55	79.14
Perceived less actual	13.41	11.9925	31.8975	30.675	16.0975	15.0925	19.86
Consistency	15.75	11.25	19.5	15.25	21	16.5	16.54
Mean total surplus	92.665	93.345	88.1075	88.44	88.11	88.1925	89.81
<i>Multi-stage Delta</i>							
Mean actual surplus	0.1300	0.1230	0.3411	0.3789	0.1558	0.1329	0.2103
Perceived less actual	-0.7808	-0.7565	-0.9059	-0.9061	-0.5428	-0.5402	-0.7387
Consistency	-0.5556	-0.5556	-0.6410	-0.5410	-0.3810	-0.2727	-0.4911
Mean total surplus	0.0671	0.0609	0.1217	0.1185	0.0958	0.0889	0.0921

Table 3. Multi-stage Auctions Compared to All Single-stage Auctions

Observe that the multi-stage auction achieves 21% more actual buyer's surplus than the single-stage auction. The multi-stage auction results in the buyer choosing a different seller than they would have chosen, given perfect information, 49% less frequently than if the buyer had utilized a single-stage auction. The multi-stage auction also results in 9% more total surplus than the average single-stage auctions. Hence, on average the buyer attains a larger pie (actual total surplus) and receives 21% more of the larger pie.

Recall that perceived surplus measures what the buyer perceives as their surplus gained, given the buyer's level of knowledge. The row *Perceived less actual* computes the difference between the buyers incorrect perception and the actual surplus they

achieved (the multi-stage auction buyer is approximately 5% incorrect). On average the buyer in the single-stage auction is 20% incorrect regarding their level of surplus; the multi-stage auction corrects approximately 74% of the misperception.

Recognize, however, that auction *Option 4* is currently prohibited from use by federal contracting officers according to FAR regulations; therefore, it should be excluded so that the results are applicable to current federal auctions.²¹ A summary of the data is below.

	1	2	3	4	5	6	Mean
<i>Mean Single-stage Auction</i>							
Mean actual surplus	83.66	83.24	65.93	62.46	76.45	74.37	74.35
Perceived less actual	16.91	15.00	41.53	39.96	19.02	17.83	25.04
Consistency	18.33	13.33	22.67	17.67	24.33	19.33	19.28
Mean total surplus	91.34	92.21	85.11	86.16	85.98	86.02	87.80
<i>Multi-stage Delta</i>							
Mean actual surplus	0.1701	0.1626	0.4845	0.5513	0.2131	0.1966	0.30
Perceived less actual	-0.8396	-0.7810	-0.8676	-0.8370	-0.6975	-0.6410	-0.78
Consistency	-0.62	-0.63	-0.69	-0.60	-0.47	-0.38	-0.56
Mean total surplus	0.0825	0.0740	0.1612	0.1481	0.1229	0.1164	0.12

Table 4. Multi-Stage Auction Compared to Single-Stage Options 1-3

Observe that on average if the buyer had conducted a multi-stage auction they achieve a 12% larger pie and receive 30% more of the larger pie. The buyer also chooses the incorrect seller 56% less frequently and their perception of surplus is corrected by about 78%.

It also may be valuable to highlight the policy auction, Option 3; the buyer knows the requirement and evaluates the auction according to their *ex ante* information.²²

<i>Publish choice ex ante (O-3)</i>							
Mean actual surplus	77.62	76.07	41.75	32.92	78.04	75.85	63.71
Perceived less actual	30.65	29.55	81.61	84.28	30.05	29.68	47.64
Mean total surplus	90.18	91.8	76.1	77.78	90.4	91.61	86.31
Consistency	20	13	26	19	19	13	18.33
<i>Multi-stage Delta</i>							
Mean actual surplus	0.2611	0.2722	1.3442	1.9432	0.1884	0.1732	0.6971
Perceived less actual	-0.9041	-0.9012	-0.9632	-0.9658	-0.7551	-0.7662	-0.8759
Mean total surplus	0.0965	0.0788	0.2987	0.2718	0.0680	0.0482	0.1437
Consistency	-0.6500	-0.6154	-0.7308	-0.6316	-0.3158	-0.0769	-0.5034

Table 5. Multi-Stage Auction versus a FAR Policy Auction, Option 3

²¹ FAR 15.301—Proposal Evaluation.

²² Ibid.

Observe that on average the buyer achieves a 14% larger pie and a 70% larger apportionment utilizing a multi-stage auction; chooses the incorrect seller 50% less frequently, and corrects their perception 87% compared to the policy federal auction.

2. Comparisons when Competition and Information Vary

Recall that we also expressed interest in analyzing the single and multiple stage auctions as the level of competition varied from high to low and as the level of information varied from high to low. The means of discrimination are as follows:

<i>Information & Competiton</i>	<i>Simulation #</i>					
	1	2	3	4	5	6
Buyers' draws	15	15	5	5	15	15
Sellers' draws	15	15	15	15	5	5
Number of sellers' entering auction	<u>10</u>	4	<u>10</u>	4	<u>10</u>	4
Number of seller draws	150	60	150	60	50	20
Total number of draws	165	75	155	65	65	35

Table 6. Levels of Competition and Information

Simulations 1-3-5 include the highest level of competition (underlined), and therefore represent the relatively high competition auctions. It is also evident that simulations 1-2-3 include the three highest-information scenarios (Bold-italics), and therefore can represent the high information auctions. It is further evident that that the sellers obtained more draws (information) than the buyer in simulations 3-4 and vice versa in simulations 5-6.

It is also most appropriate to compare the multi-stage auction with the three single stage auctions that are not prohibited (Options 1-2-3). A comparison of the forecast variables across the simulations and among the auction options yields:

	1	2	3	4	5	6	Mean high-competition	Mean low-competition	Mean high-info draw	Mean low-info draw
<i>Multi-stage auction</i>										
Mean actual surplus	97.89	96.78	97.87	96.89	92.74	88.99	96.17	94.22	97.51	92.87
Perceived less actual	2.94	2.92	3.00	2.88	7.36	6.94	4.43	4.25	2.95	5.73
Mean total surplus	98.88	99.03	98.83	98.92	96.55	96.03	98.09	97.99	98.91	97.17
Consistency	7.00	5.00	7.00	7.00	13.00	12.00	9.00	8.00	6.33	10.67
<i>Mean Single-stage Auction (Opt. 1-3)</i>										
Mean actual surplus	83.66	83.24	65.93	62.46	76.45	74.37	75.34	73.36	77.61	71.09
Perceived less actual	16.91	15.00	41.53	39.96	19.02	17.83	25.82	24.26	24.48	25.60
Mean total surplus	91.34	92.21	85.11	86.16	85.98	86.02	87.48	88.13	89.55	86.05
Consistency	18.33	13.33	22.67	17.67	24.33	19.33	21.78	16.78	18.11	20.44
<i>Multi-stage auction Improvement</i>										
Mean actual surplus							0.28	0.28	0.26	0.31
Perceived less actual							-0.83	-0.82	-0.88	-0.78
Mean total surplus							0.12	0.11	0.10	0.13
Consistency							-0.59	-0.52	-0.65	-0.48

Table 7. Single versus Multiple Stage Auctions as Competition and Information Vary

Observe that the high-competition auctions are superior to the low-competition auctions at approximately the same rate; and the improvement upon the single stage auction is similar among all forecast variables except consistency. In the high-competition multi-stage auctions, the buyer chooses the incorrect seller 1% more frequently than in the low-competition environment; in the high-competition single stage auction buyers chose the incorrect seller 5% more frequently than in the low-competition environment. Note, however, that the improvement in the buyer's selection of the correct seller partially reflects the smaller pool of possible sellers (4) in the low-competition environment the buyer's mathematically less likely to choose incorrectly. Thus, we can conclude that the multiple stage auction offers about the same level of improvement upon the single stage auction in high and low competitive environments.

A slightly different story is evident in high-low information environments. A multi-stage auction improves the buyer's average actual surplus by 31% in the low-information environment (5% more than in the high-information environment) and the total average surplus by 13% (3% more than in the high-information environment). Note that again the buyer's risk of adverse selection is not as significant as in the high-information environment; however, it is evident that the multi-stage auction offers the buyer the most significant improvement over the single stage auction in information thin (scarce) markets.

It is also possible to observe changes in outcomes as the buyer has more, or fewer draws than each individual seller. Consider the average data depicted below:

	1	2	3	4	5	6	sellers' draws exceed buyer's	buyer's draws exceed sellers'
<i>Multi-stage auction</i>								
Mean actual surplus	97.89	96.78	97.87	96.89	92.74	88.99	97.38	90.87
Perceived less actual	2.94	2.92	3	2.88	7.36	6.94	2.94	7.15
Mean total surplus	98.88	99.03	98.83	98.92	96.55	96.03	98.88	96.29
Consistency	7	5	7	7	13	12	7.00	12.50
<i>Mean Single-stage Auction (Opt. 1-3)</i>								
Mean actual surplus	83.66	83.24	65.93	62.46	76.45	74.37	64.19	75.41
Perceived less actual	16.91	15.00	41.53	39.96	19.02	17.83	40.75	18.43
Mean total surplus	91.34	92.21	85.11	86.16	85.98	86.02	85.63	86.00
Consistency	18	13	23	18	24	19	20.17	21.83
<i>Multi-stage auction improvement</i>								
Mean actual surplus							0.52	0.20
Perceived less actual							-0.93	-0.61
Mean total surplus							0.15	0.12
Consistency							-0.65	-0.43

Table 8. Single versus Multiple Stage Auctions as Buyer-Seller Information Varies

Observe that if the buyer were to conduct a single stage auction they are clearly better off having more information than any individual seller (e.g., 10% average actual surplus). Contrarily, the situation is reversed if a buyer were conducting a multi-stage auction (7% improvement in average actual surplus if sellers have more information). This reflects the fact that the multi-stage auction exploits the sellers' information, while most of the single stage auctions do not. As a result, performance increases when all sellers have more information as opposed to the one buyer. Again, the buyer is absolutely better off choosing a multi-stage auction. Note that the greatest improvement over the single stage auction is when the buyer has relatively less information about what their true need is.

Generally, we may conclude that the multi-stage auction should be an especially attractive tool when the buyer has a low level of information either relative to the market, or the sellers.

D. SUMMARY OF ANALYSIS

If utilizing a single-stage auction, not explicitly prohibited by regulation, the buyer over-estimates the amount of buyer's surplus achieved by 25%; the multi-stage auction corrects about 80% of the misperception. The multi-stage auction provides the buyer both a larger pie and a larger apportionment in all cases when compared to the single-stage auction. Both the high-competition and the high-information auction are superior for the buyer when the multi-stage auction is used. The multi-stage auction is most desirable to use when information is scarce.

V. CONCLUSIONS AND RECOMMENDATIONS

A. OUTCOMES DISCUSSION

The simulation model demonstrates that a buyer should prefer the multi-stage auction over the single-stage auction. The multi-stage auction solves the computational problem (what is the optimal requirement) and overcomes the information asymmetry problem by systematically revealing objective information in the first stage. The FAR mandates a *Best Value* basis for award that also minimizes operations overhead.²³ Multiple stage auctions may accomplish both objectives, by making best value explicit, and by minimizing the amount of time that may be spent conducting market research and crafting an acquisition strategy.

1. Hypothetical Comparison

To demonstrate how this may be implemented it is useful to imagine the same requirement being auctioned with a single or multiple stage auction. Suppose an agency had requirements for waste disposal, grounds maintenance, and landscaping maturing in the same contract period. Initially some appear to be complimentary and perhaps could be consolidated, however, the appropriate combination of quality, time, and price trade-offs that create an optimal contract is not self-evident.

If the standard auction model were used it could take months to gather enough data to formulate a useful strategy. During the market research and requirement generation phases (work statement revision), there is no systematic way to ensure the outcome (metrics and probable evaluation criteria) will trade-off attributes correctly, or enable contract performance. How much overhead could be saved consolidating the requirements? What is the ideal range for grass length: two-to-three inches, or one-to-four, and what is the price difference? In any case, it is a difficult process to get correct.

²³ FAR 1.102-2—Performance Standards.

If the multi-stage auction were used, a Statement of Objectives could be developed that outlines broad acquisition goals.²⁴ Contractors could then combine the contracts in whatever manner they felt was appropriate (according to their individual cost function and individual draw). The contacting officer would have a great-deal more information from which to trade-off price, schedule, and quality and develop an optimal Statement of Work and evaluation criteria (signaling the true weights). Contractors would incorporate the information into their Final Proposal Revision, resulting in a more efficient outcome. Further, because the research burden is placed on the contractors it may significantly reduce Procurement Acquisition Lead Time (PALT).

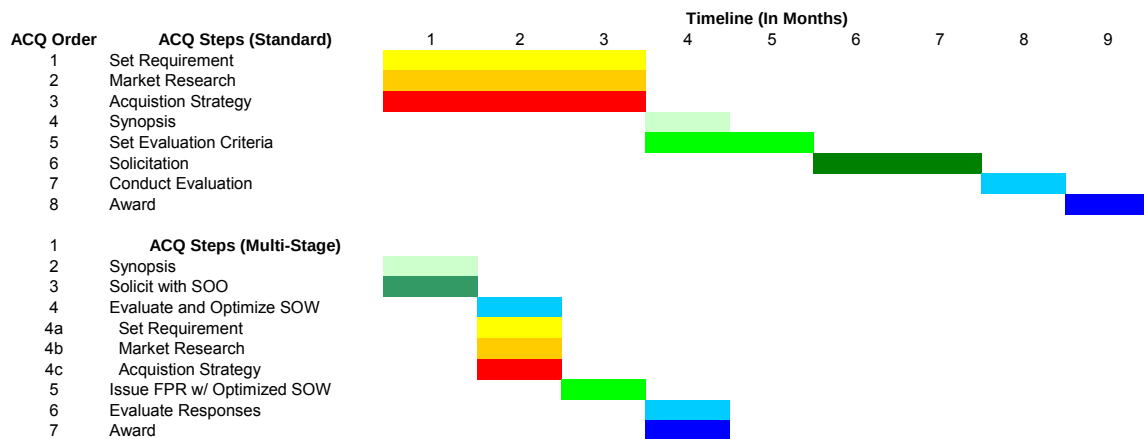


Figure 8. Hypothetical Timeline for Processing Single versus Multi-stage Auctions

Initial research would not be as critical to program success because the multi-stage auction assumes the eventual requirement is an outcome of the auction, rather than predefined.

Additional benefits of broadly defining a desired outcome *ex ante* (similar to the way DoD 5000 series determines capabilities), and optimizing during the auction process is that the trade-offs become self-evident. In the standard auction model there is no way to assess the quality of market research to ensure the correct trade-offs have been identified or made. The multi-stage auction couples both market research and optimal acquisition strategy into the auction. The effect is that the multi-stage auction will achieve the best value per-dollar-cost.

²⁴ FAR 37.602.

Contracting officers may also achieve better outcomes in information thin environments, such as contingencies. For example, suppose a field commander has some knowledge about what they ultimately want, but lacks resources to adequately prepare a definitive requirement. The standard auction requires that they define the requirement before the auction is conducted—consuming the scarcest good among all possible goods: time—and there is no guarantee that the supplies, or services, are obtainable in immature environments. However, the multi-stage auction would allow commanders to quantify and trade-off important factors, such as time versus quality, and transfer some of the research burden to the contractors: a more efficient outcome.

B. IMPLEMENTATION DISCUSSION

A possibly significant implementation barrier may be the Bona Fide need rule; however, as part of the DoD 5000 series revision “capabilities” replaced predetermined “assets” as the desired objective.²⁵ A multi-stage auction has a similar implementation path. But, inasmuch as the *bona fide* need rule is fiscal law addressing the timing of obligations relative to fiscal year needs, there is little evidence from the Government Accountability Office that establishing a “perfectly defined minimum need,” is the primary concern.²⁶

Some acquisition reformers have observed that improving the acquisition process may be zero-sum.²⁷ Others have argued that acquisition will flounder until the President, in concert with Congress, enacts sweeping reform.²⁸ Multi-stage auctions, however, may be implemented as a policy level change, or unilaterally by any contracting officer.

²⁵ Defense Acquisition University, “Defense Acquisition Guidebook,” 1st Edition, November 2006, <https://akss.dau.mil/dag/GuideBook/PDFs/GBNov2006.pdf>, Retrieved October 2007.

²⁶ GAO-04-261SP, “Principles of Federal Appropriations Law,” 3rd Edition, Volume 4, January 2004, <http://www.gao.gov/special.pubs/d04261sp.pdf>, especially Chapter V, Retrieved October 2007.

²⁷ Richard K. Sylvester and Joseph A. Ferrara, “Implementing Evolutionary Acquisition,” *Acquisition Review Quarterly*, Winter 2003, <http://www.dau.mil/pubs/arq/2003arq/arq2003.asp#winter>, Retrieved October 2007.

²⁸ Mark Cancian, “Acquisition Reform: It's Not As Easy As It Seems,” *Acquisition Review Quarterly*, (Summer 1995), <http://www.dau.mil/pubs/arq/arq95.asp>, Retrieved October 2007.

1. Policy Path

Reforming federal acquisition policy is one way to implement multiple stage auctions. Under this option, the auction is moved up-front, possibly requiring a FAR work-around, such that an auction can precede synopsis, market research and acquisition strategy. This option is the path of *most* resistance. The current FAR mandates completing these major stages prior to solicitation and that each stage is discrete. A policy change that blends them into the auction may require years of policy entrepreneurship to implement. If this opportunity were to be pursued it may be advisable to seek a blanket waiver for a limited time period, to demonstrate the possible benefits. The benefits may have to be measured and reported for several years before policy change is even considered.

2. Unilateral Path

A unilateral approach may also be conducted. Strategies that can be implemented at the micro-policy level to achieve benefits, while simultaneously avoiding any policy maker having to spend political capital, are more likely to be effectively implemented.

The unilateral option supposes that a contracting officer abbreviates the initial acquisition steps, such as market research and acquisition strategy, by arguing that: “they’re coupled into the auction.” A unilateral implementation may use a Statement of Objectives; however, the contacting officer could review offers for the optimal approach (quotes if conducted for supplies) rather than accepting the offeror’s Statement of Work, and issue an amendment seeking prices based upon the revised ideal work statement.²⁹ Note that the unilateral implementation applies to service and weapons systems acquisition; a design-build implementation could be applied to construction projects.³⁰

This is similar to selecting from a priced menu, or an array of menus. Imagine an individual dining-out using the standard auction model. She must set the requirement—chicken—decide which attributes are important—taste, atmosphere, service level—and

²⁹ FAR 37.602.

³⁰ FAR 36.3—Two Phase Design-Build Selection Procedures.

when the prices are known make a selection. If she used a multi-stage auction, she would gather the menus of all the restaurants in town and have near perfect information when making a choice. Additionally, she could make many choices, gaining contract flexibility. To equate this to defense procurements, it adds flexibility to contracts and makes evident the price-value trades required. Negotiating flexible contracts could be especially valuable if the budget is unstable.

Additionally, an agency could unilaterally revise their individual policies and procedures to conduct multi-stage auctions. Guidance may be useful to ensure that a multi-stage auction is used effectively rather than just to bypass FAR Parts 6 & 10, and could look similar to agency guidance applicable to reverse auctions.³¹

C. ADDITIONAL RESEARCH

Additions to the basic models may be conducted. A researcher could include additional attributes ($X, Y \Rightarrow A, B, C, D$). A researcher could also gather field evidence to determine whether existing contracts had been awarded to the optimal firm(s). Another path could be to run an experiment and later an implementation test. The experiment would use a stylized procurement situation in a laboratory setting while an implementation test would involve one region running a traditional auction while one runs multi-stage auctions, then compare.

D. FINAL COMMENTS

Auction theory can be utilized to achieve more efficient defense auctions, improving value and possibly decreasing administrative work-load (fewer ex ante steps). The process demonstrated may be implemented by an individual contracting officer, or at all departmental levels, with or without formal policy changes. The multi-stage auction may improve service and contingency contracting outcomes because it is likely to get better offers in information thin environments and does not anchor on a pre-set requirement.

³¹ Susan L. Turley, "Wielding the Virtual Gavel--DoD Moves Forward with Reverse Auctions," *Air Force Institute of Technology (Thesis)*, CI02-91, 15-26, (2002).

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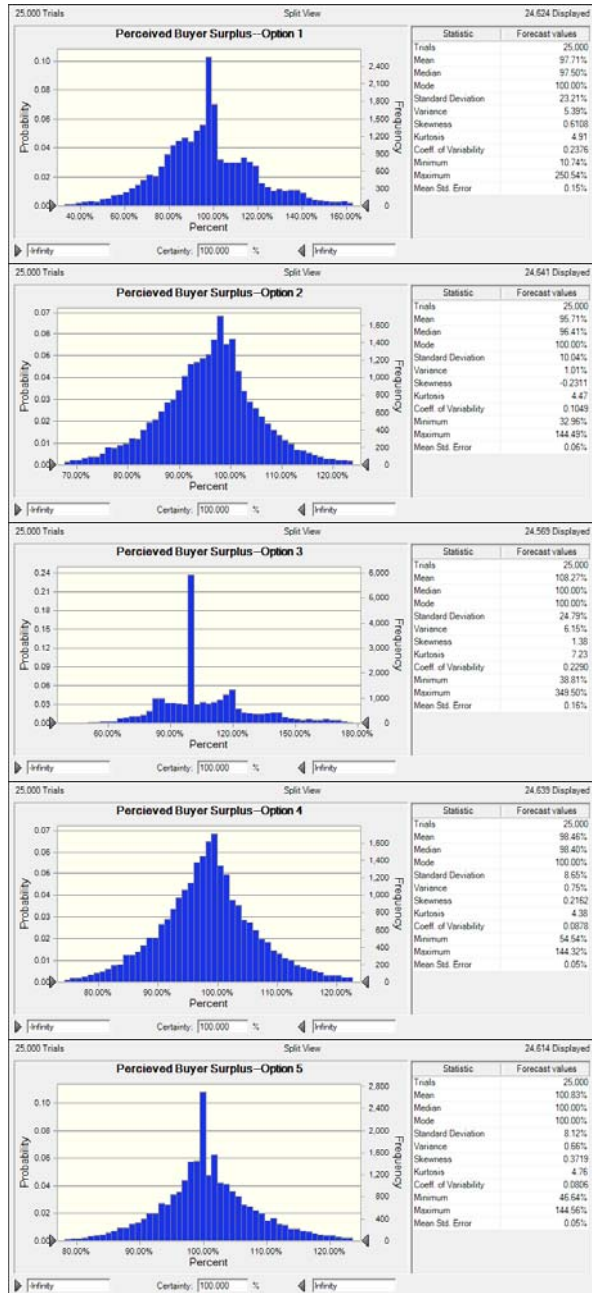
APPENDIX A: SIMULATION MODEL

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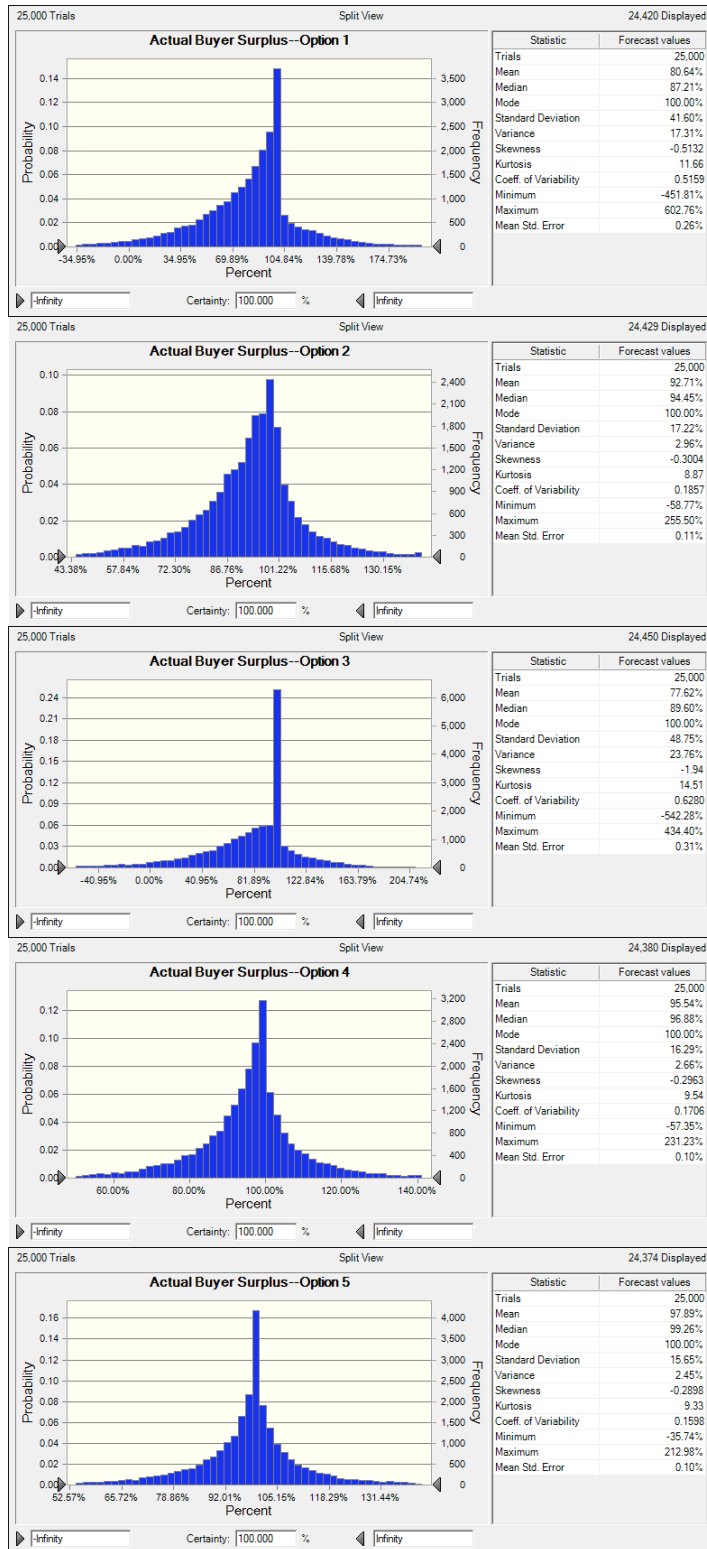
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APPENDIX B: SIMULATION ONE

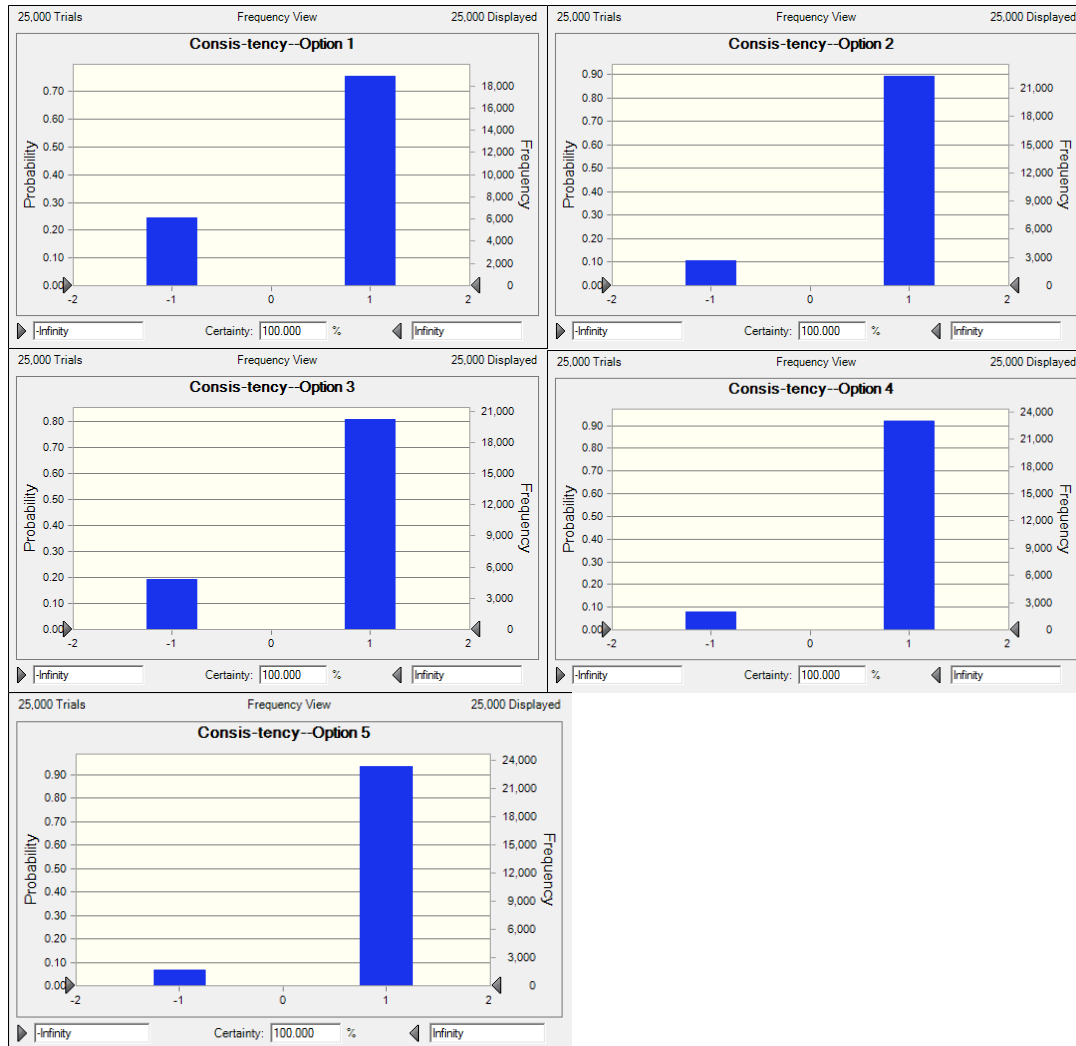
A. PERCEIVED BUYER SURPLUS



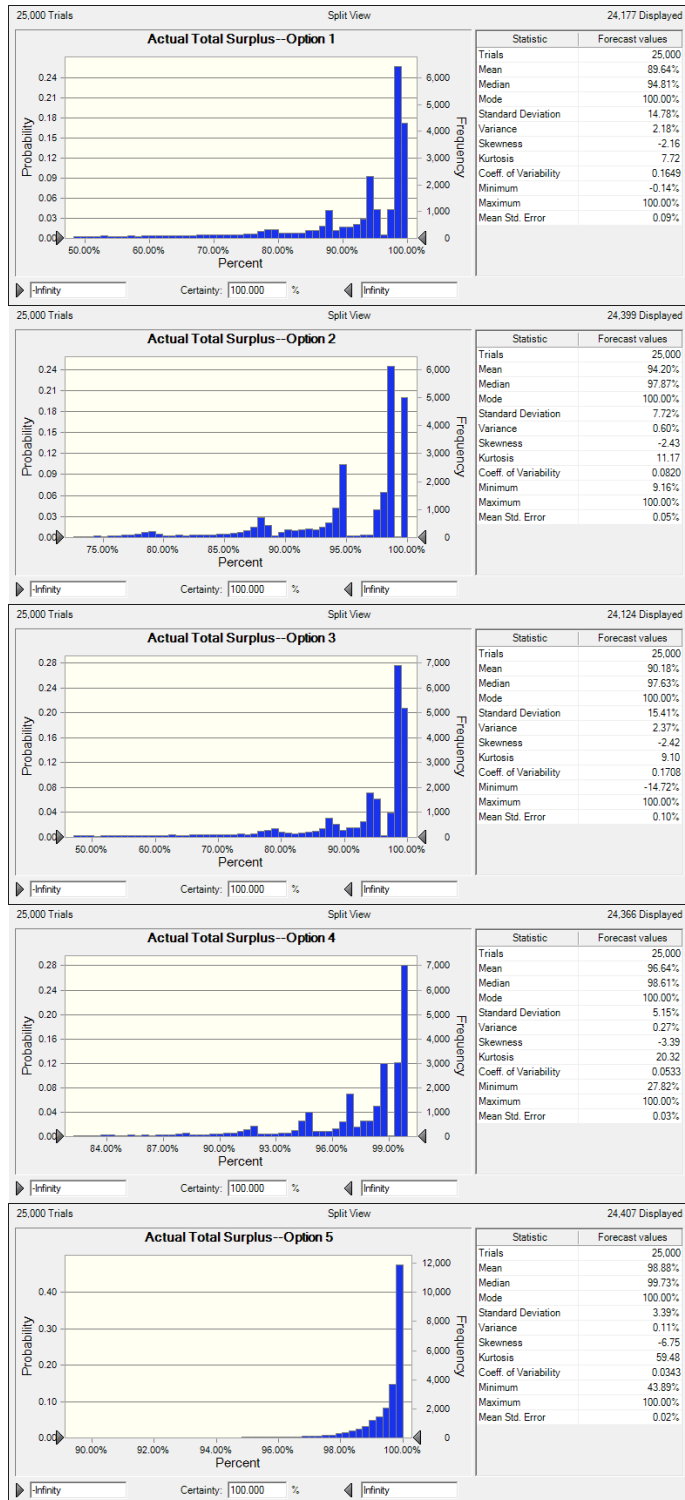
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C. CONSISTENCY

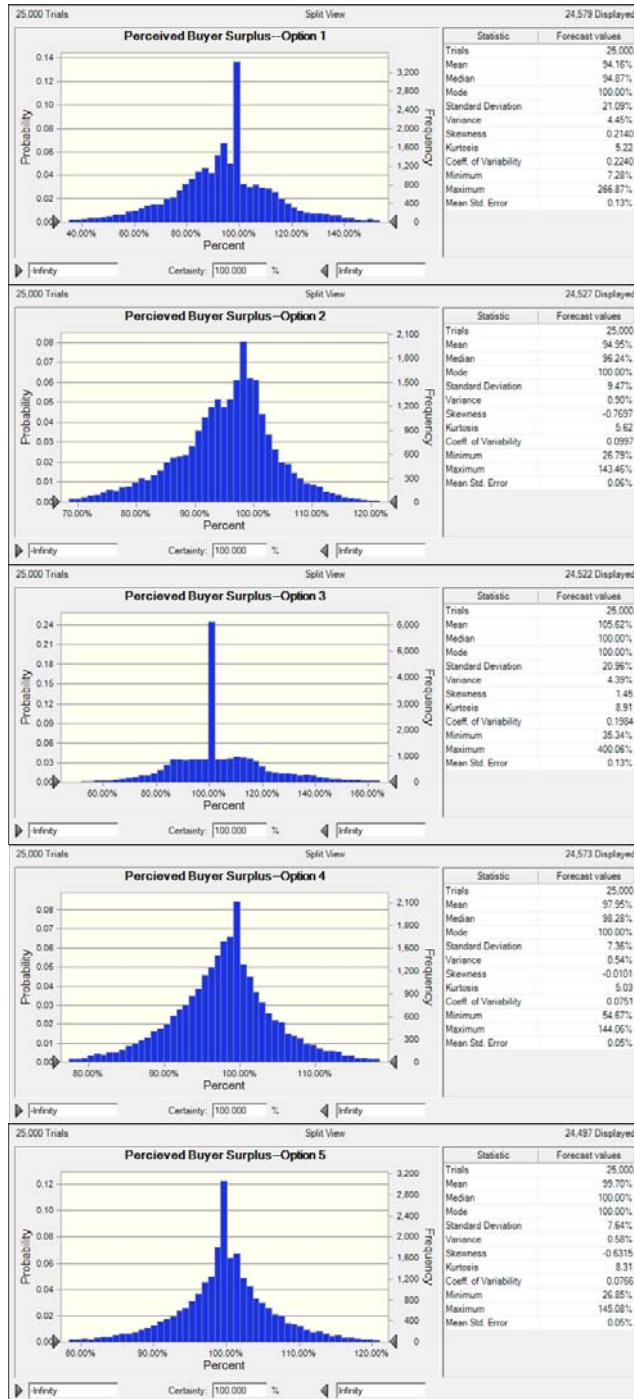


D. ACTUAL TOTAL SURPLUS

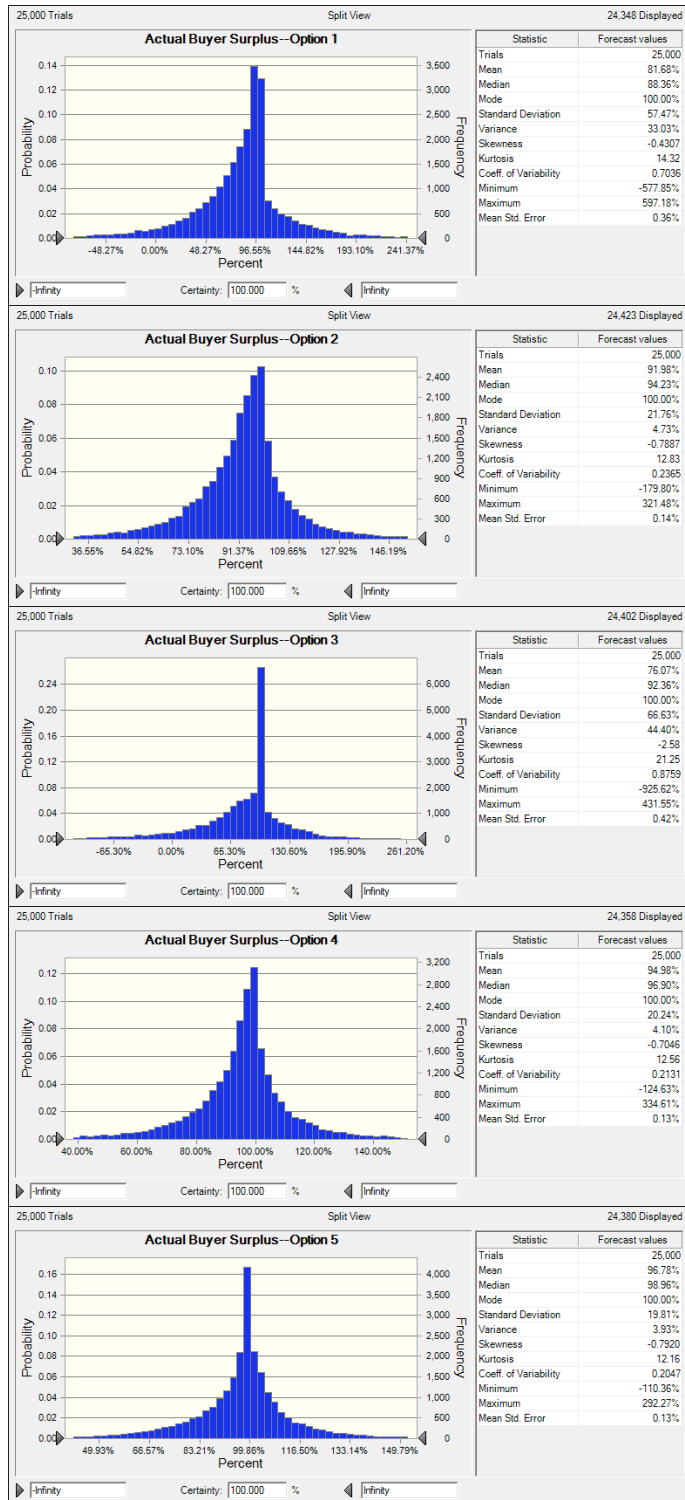


APPENDIX C: SIMULATION TWO

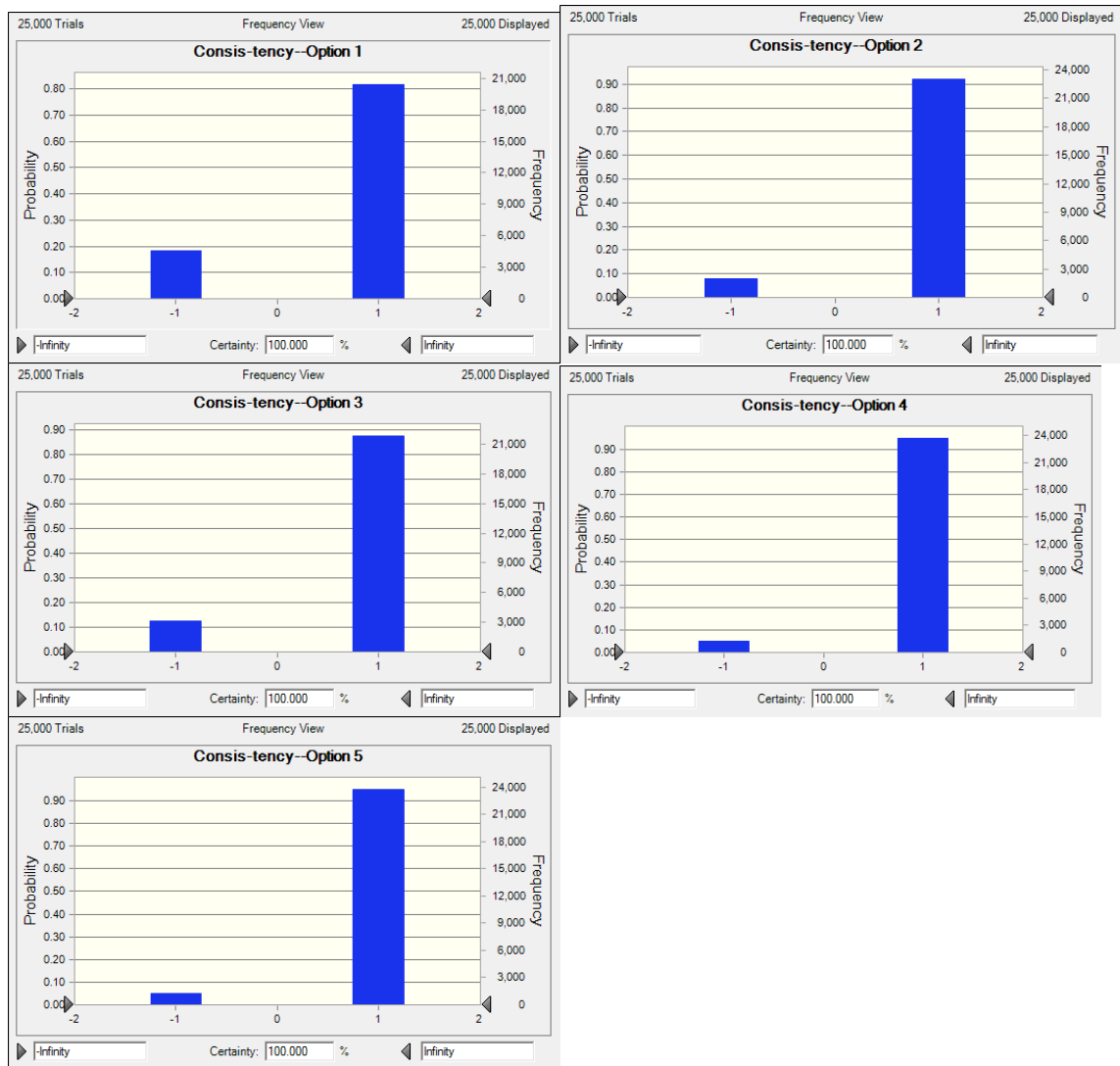
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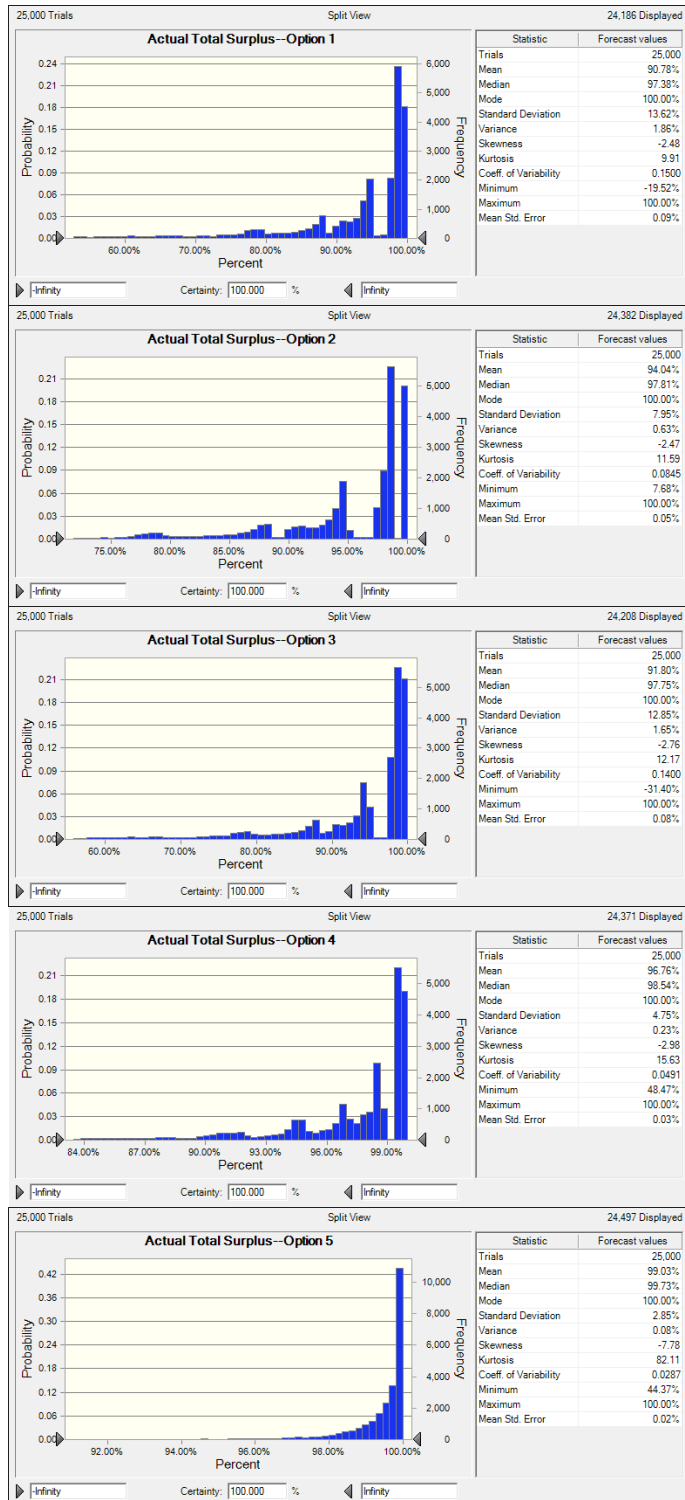
B. ACTUAL BUYER SURPLUS



C. CONSISTENCY

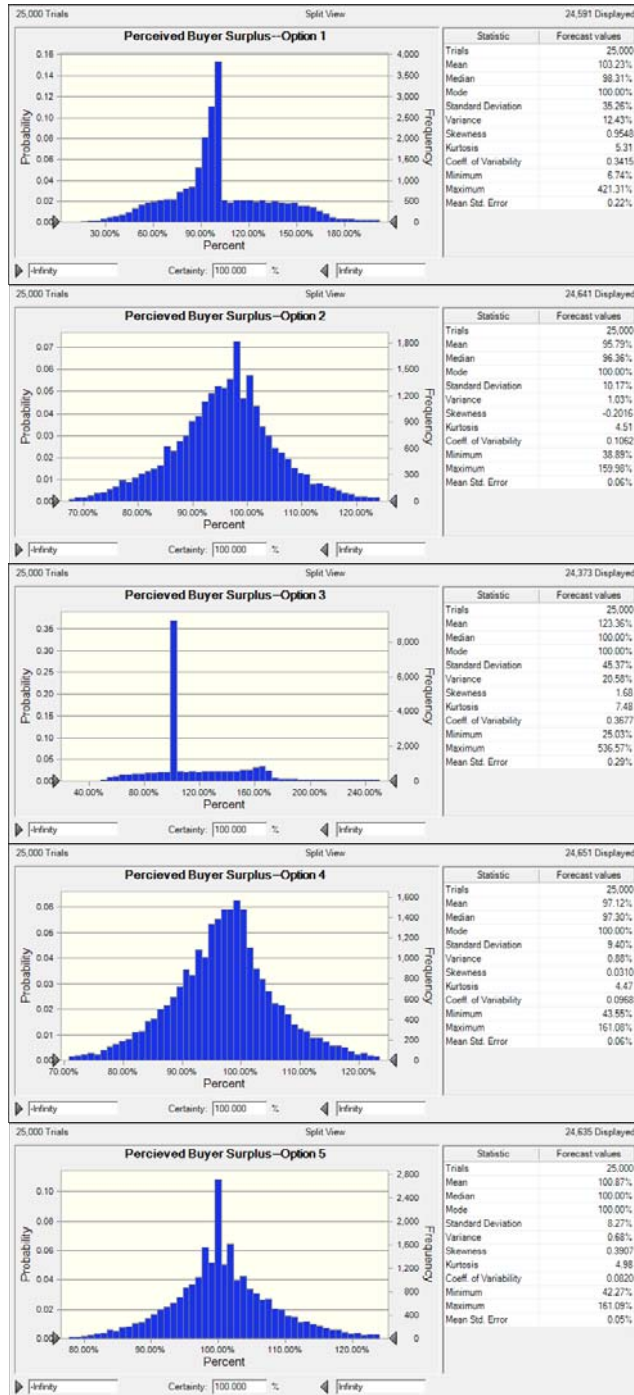


D. ACTUAL TOTAL SURPLUS

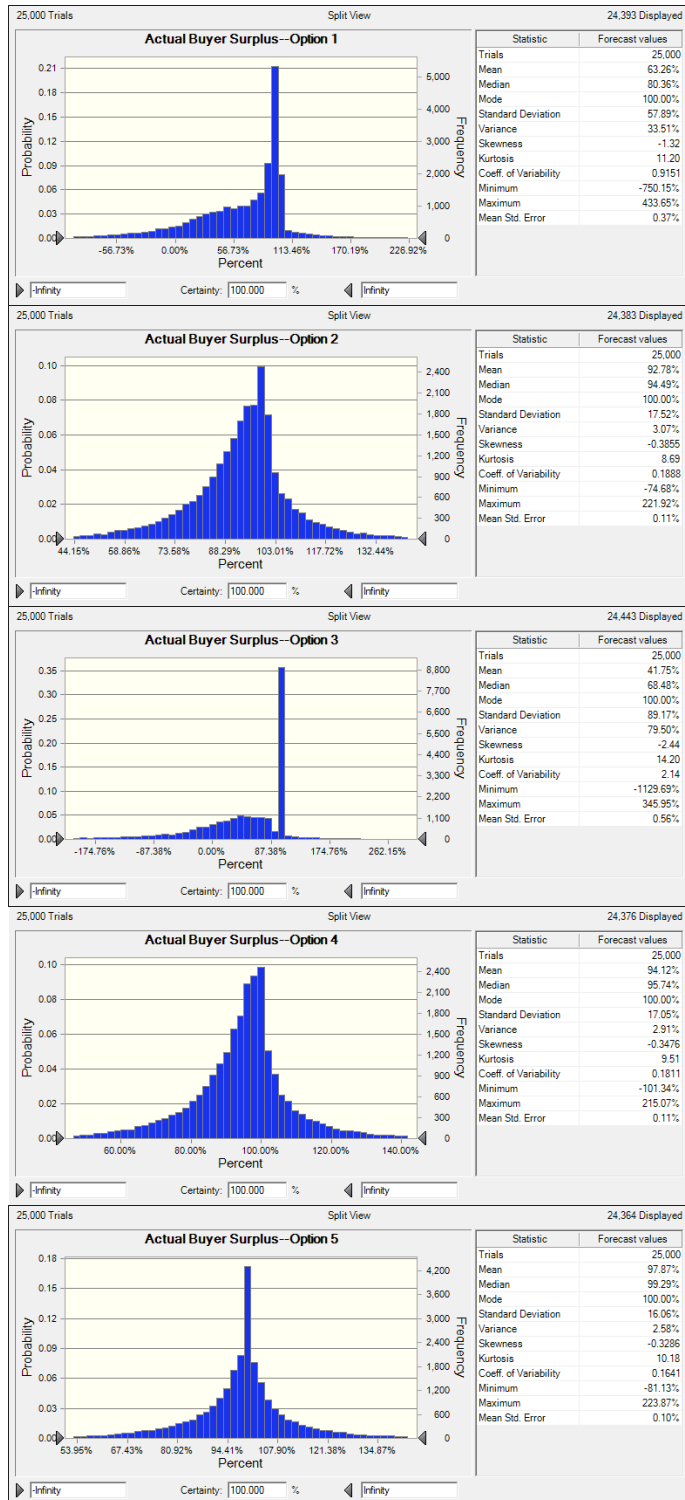


APPENDIX D: SIMULATION THREE

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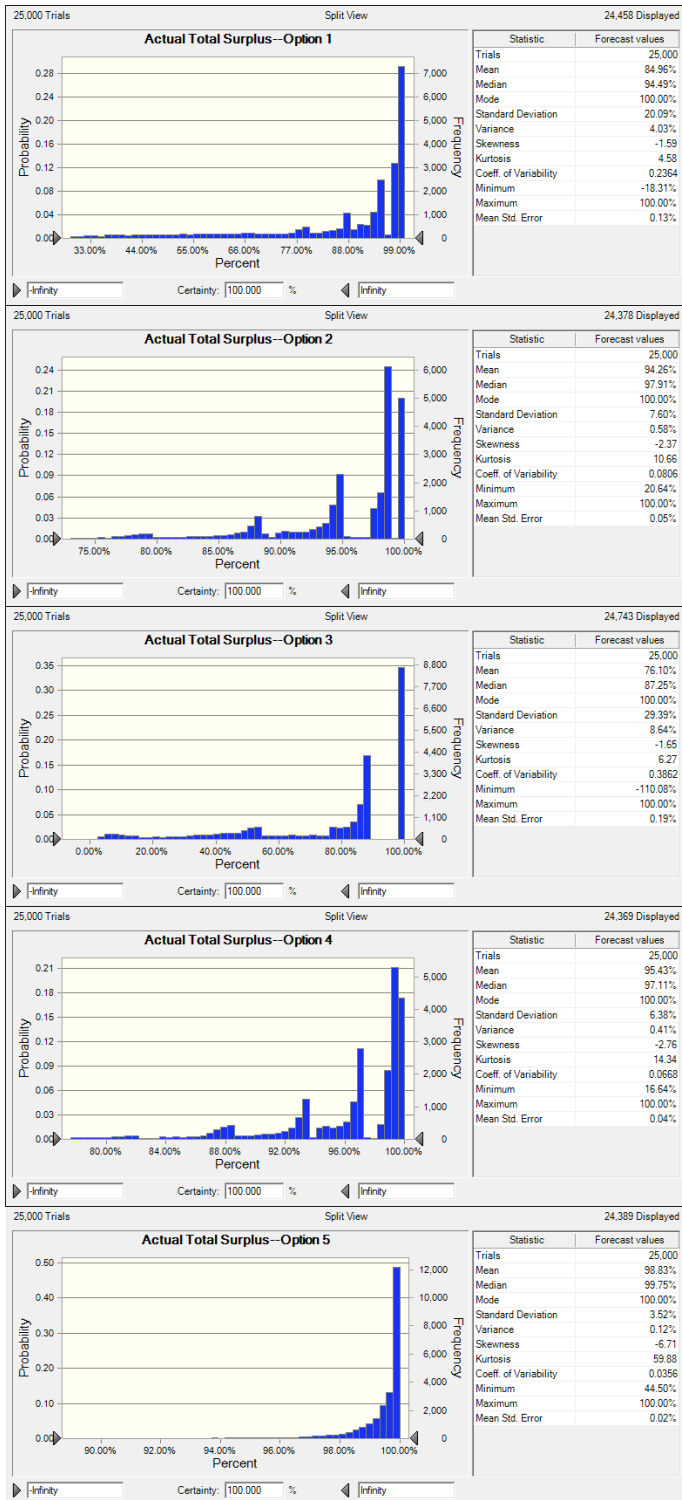
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C. CONSISTENCY

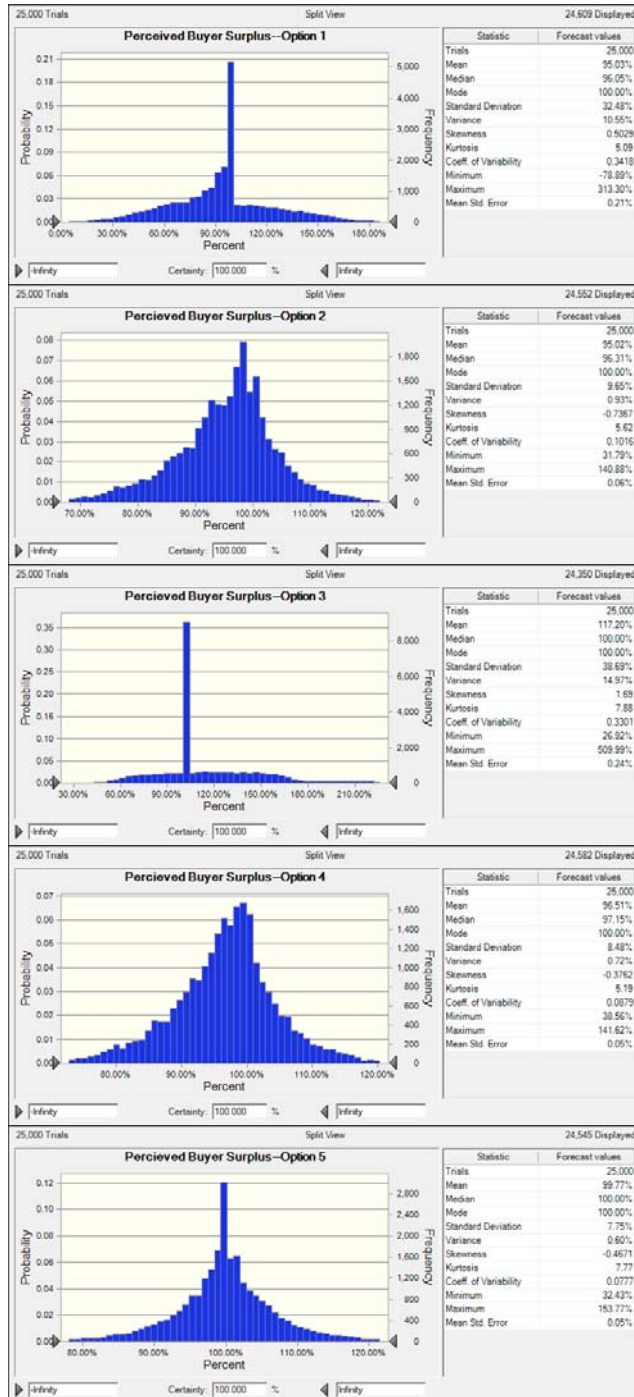


D. ACTUAL TOTAL SURPLUS

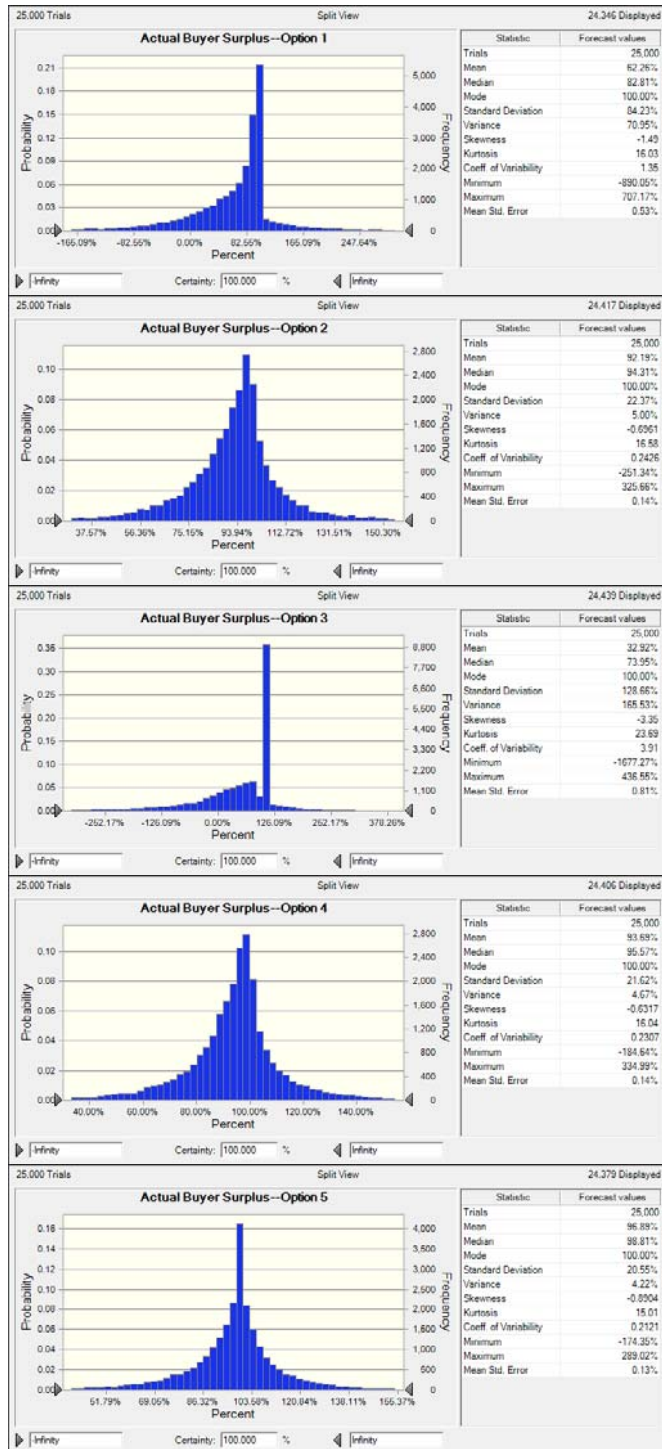


APPENDIX E: SIMULATION FOUR

A. PERCEIVED BUYER SURPLUS



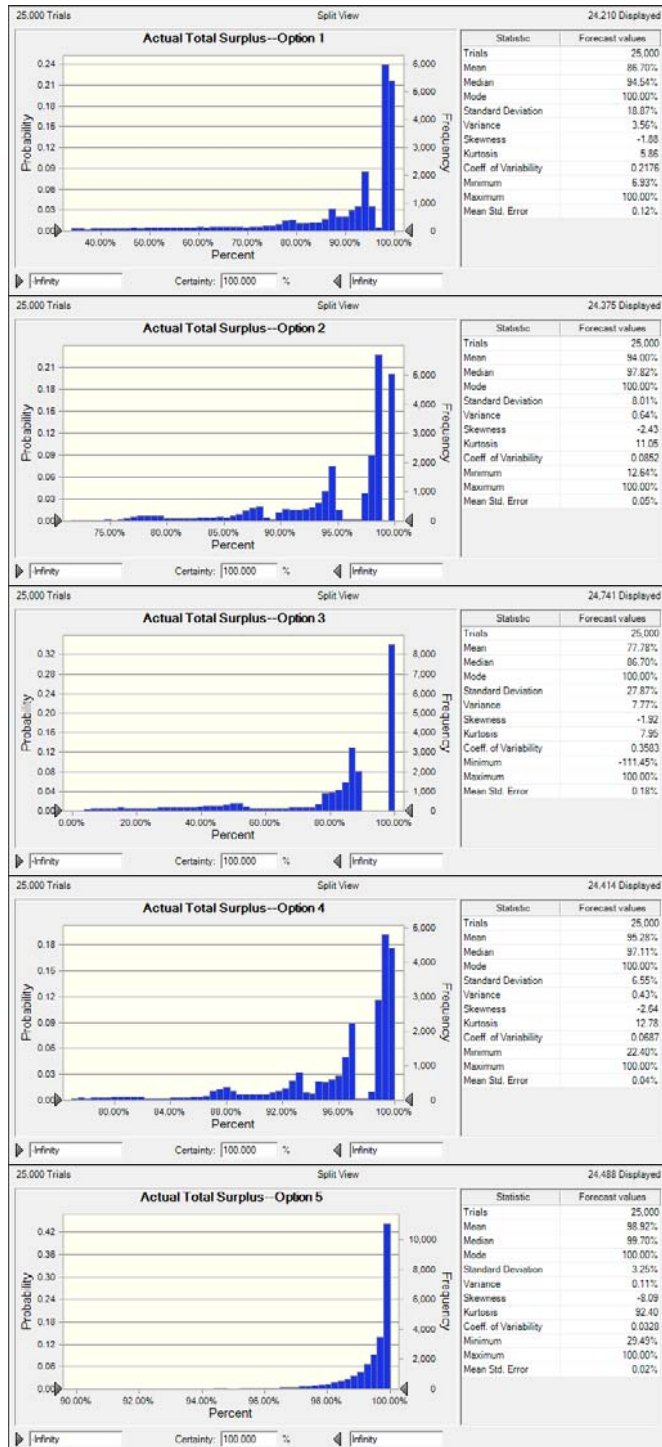
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C. CONSISTENCY

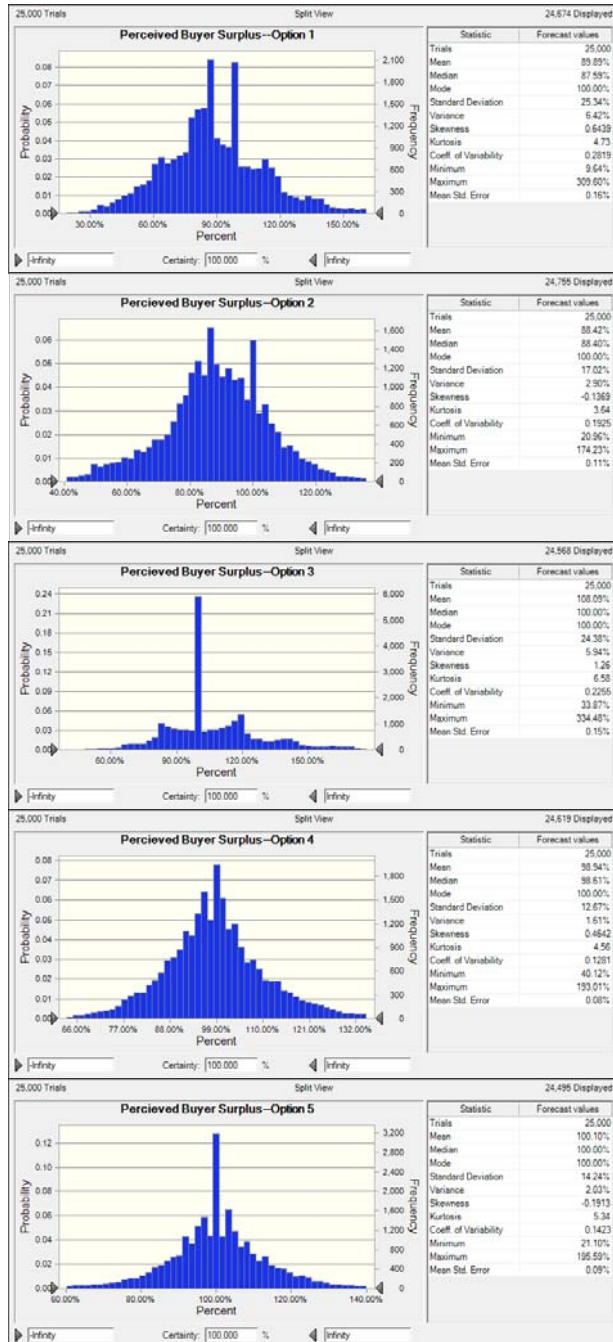


D. ACTUAL TOTAL SURPLUS

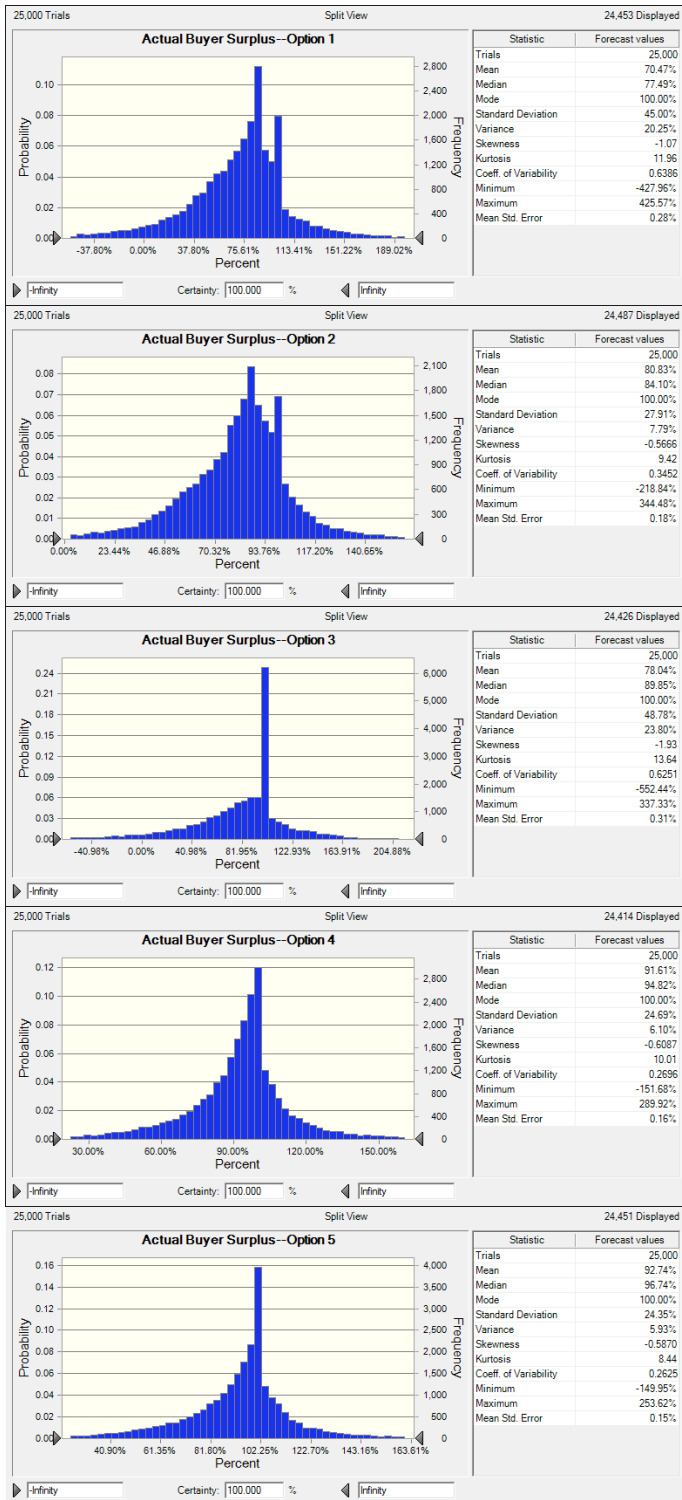


APPENDIX F: SIMULATION FIVE

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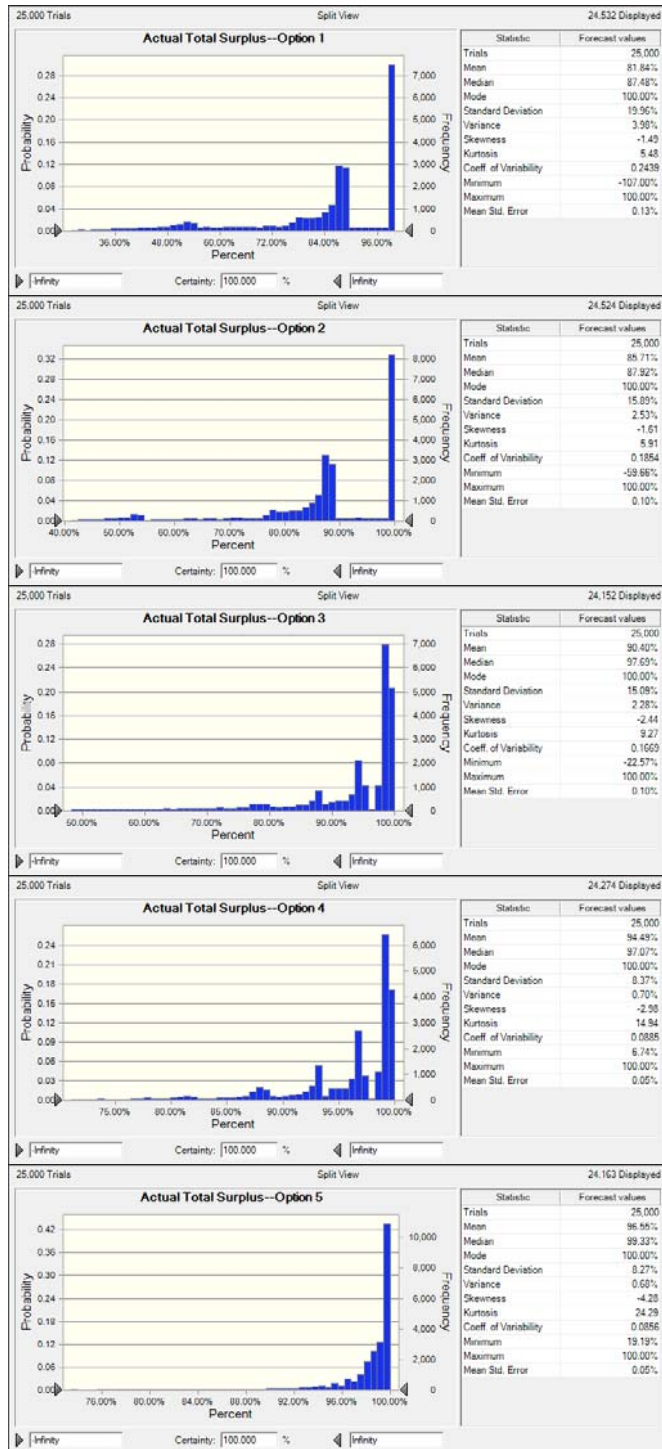
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C. CONSISTENCY

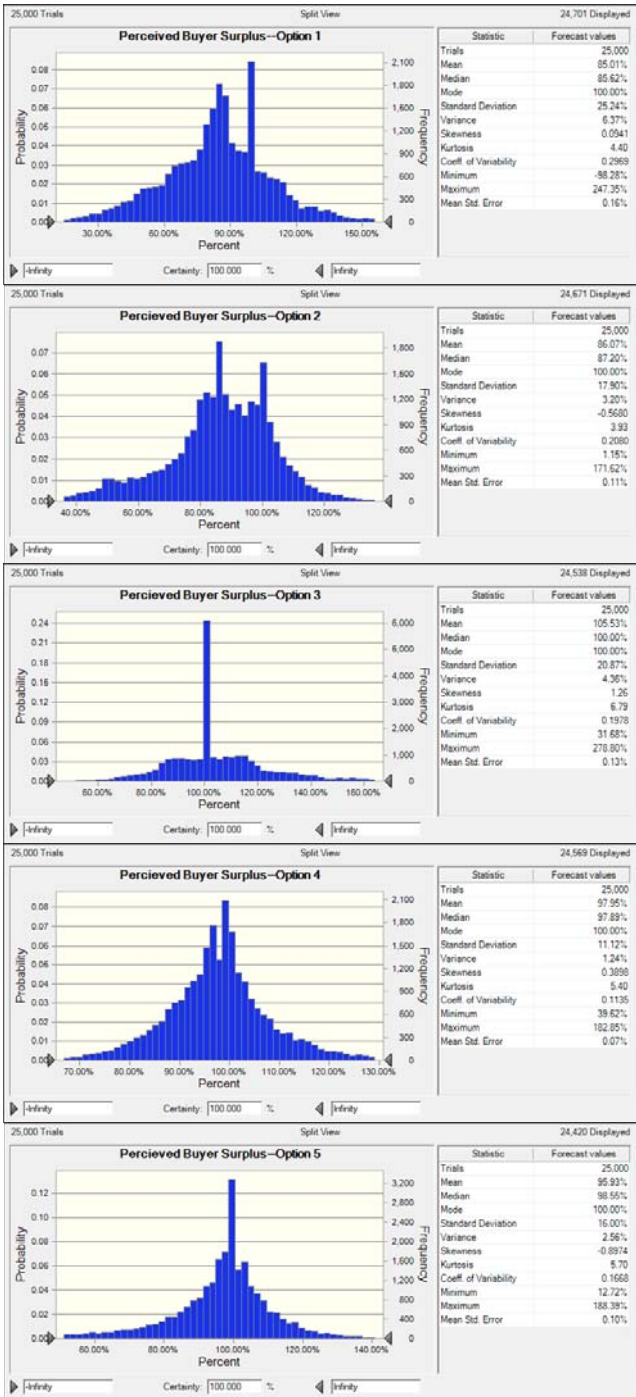


D. ACTUAL TOTAL SURPLUS

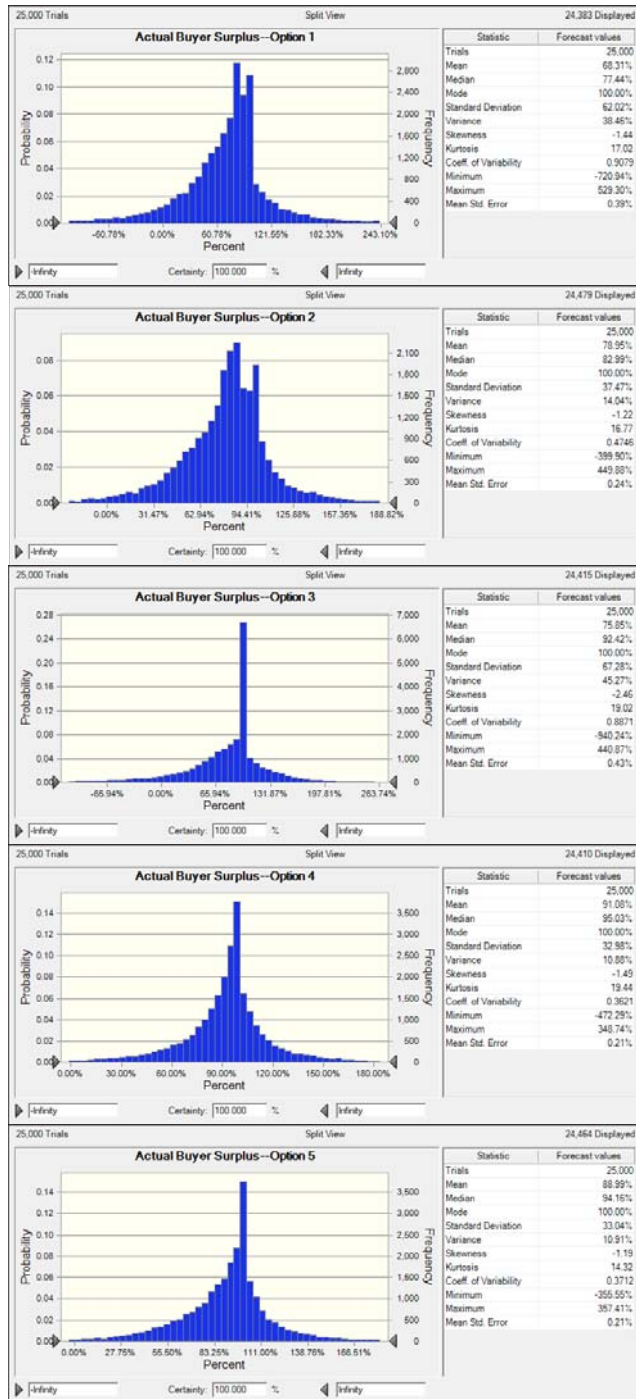


APPENDIX G: SIMULATION SIX

A. PERCEIVED BUYER SURPLUS



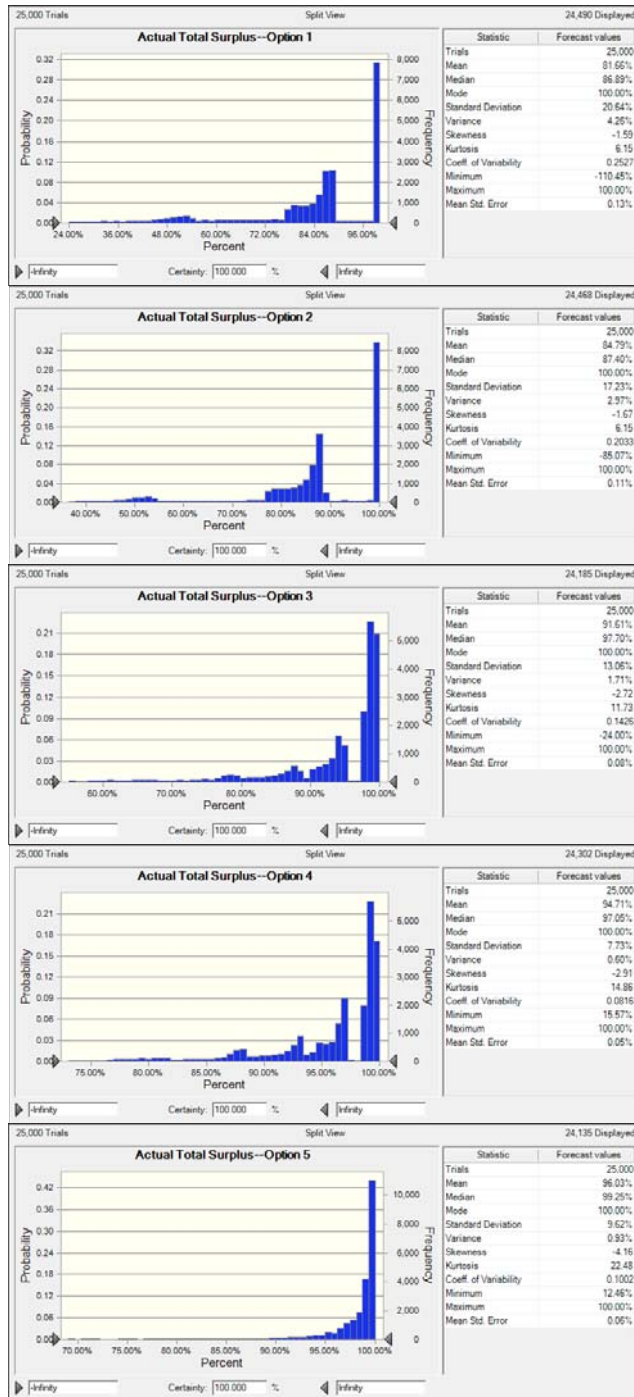
B. ACTUAL BUYER SURPLUS



C. CONSISTENCY



D. ACTUAL TOTAL SURPLUS



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