HUMAN-GOAL-BASED METRICS FOR MODELS OF URBAN AND NATURAL TERRAIN AND FOR APPROXIMATION THEORY

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ABSTRACT

In virtually all approximation theory, classical mathematical metrics, especially Sobolev norms, on linear spaces of smooth functions have been used as the measures of approximation. However, the assumption of smoothness is not generically applicable and most of the classical metrics used to measure how well one function approximates another do not provide information that is consistent with human perception and goals. In this paper, we construct metrics for approximation based on human goals. For urban and natural terrain, we introduce a difference-of-visibility or "DV" metric. For the computational experiments, we use two univariate data sets that include "discontinuities" (representing, for example, the sides of buildings). Computational results for observers at 595 positions indicate that, in the DV metric, the coarse-grid linear spline and the cubic L_1 spline produce roughly equally accurate regions of visibility and that they are better approximants of real terrain than the conventional cubic spline, which has extraneous oscillation that leads to inaccurate visibility. Extensions of the DV metric involving weighting and extensions for measuring false negative and false positive error are described. A global difference-of-visibility metric is described. The approach in this paper is not limited to visibility and extends to many other situations. In all of these situations, the metric for the properties of a phenomenon in the context of a human goal should be the quantitative formulation of that human goal itself, not a metric that is adopted from unrelated mathematical concepts or other areas of applications.

1. INTRODUCTION

In virtually all approximation theory, classical mathematical metrics, especially Sobolev norms, have been used as the measures of approximation (Bezhaev and Vasilenko, 2001; de Boor, 2001; Schumaker, 1981). In this theory, smoothness of the underlying function being approximated is typically needed to obtain estimates of accuracy. The assumption of global or piecewise smoothness and the measurement of accuracy using classical mathematical metrics on linear function spaces have been quite successful for design of esthetically pleasing surfaces (Farin, 2001) and in classical mathematical physics and engineering, including but not limited to solid and fluid materials, electromagnetics, acoustics, thermodynamics, diffusion and phase transitions.

Over the past three decades, approximation problems in urban and natural terrain, geology, geography, human modeling, biological modeling, communication networks, social science, economics, finance, images and other information-based and human-based areas have been analyzed under an assumption of sufficient smoothness. In these areas, however, the assumption of smoothness is often not applicable. While there are many situations in geometric modeling in which global or piecewise smoothness is a property of the function being approximated, smoothness is not generic in the real world. Natural and urban terrain, geology, humans, animals, plants, clothing, economic and financial phenomena, images in general and "most" of what we wish to model are inherently nonsmooth.

The classical metrics (norms) used to measure how well a function f and its derivatives approximate a function g and its derivatives are often based on the L_p metrics. In the basic L_p metric, the difference between two functions f and g is measured by

$$\left\{ \int_D |f-g|^p \, \mathrm{d}D \right\}^{1/p} \tag{1}$$

when $1 \le p < \infty$ and by

$$\sup_{\bar{D}} |f-g| \tag{2}$$

when $p = \infty$. The widely used "rms metric" is a discrete version of the L_2 metric. Unfortunately, neither the L_2 metric nor, in general, the L_p metrics measure approximation in the way that human perception measures approximation. Consider a flat surface g such as

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depicted in Fig. 1. Let *f* be the same flat surface in most places but have a long thin ridge, as depicted in Fig. 2. For thin ridges, the L_p metric, $1 \le p < \infty$, of the difference between *f* and *g* will be small. Nevertheless, in human perception, *f* is generally not considered a good approximation of *g*. One important aspect of human perception is visibility. The visibility between observer-target pairs on a surface *f* with a long thin ridge is very much different from the unobstructed visibility on the flat surface *g*. For $1 \le p < \infty$, L_p metrics for the difference between *f* and *g* can thus be small when the human-perceived difference between *f* and *g* is large. When *g* is a flat surface and *f* is the same surface with additional small-amplitude oscillation, the L_∞ metric of the difference between *f* and *g* is small when the human-perceived difference is large. The L_p metric, $1 \le p \le \infty$, of the difference can also be large when the human-perceived difference is small, for example, when *g* is a Heaviside function (one value on one side of a line or curve and some other value on the other side) and *f* is the same function but with a slight shift in the location of the discontinuity. Heaviside functions represent sides of buildings and cliffs and are common in urban and natural terrain. The L_p metrics and all commonly used classical metrics give equal weight to equal amounts of undershoot and overshoot. However, this equal weighting does not match human goals well because "too high" (overshoot) and "too low" (undershoot) are not opposites of each other when visibility, drainage, communication (radio, optical, etc.) and many other human goals are under consideration. Neither the L_p metrics nor most of the other classical metrics used to measure how well one function approximates another provide information that is consistent with human goals.

A few researchers have used metrics other than L_p metrics to measure approximation in geometric modeling. Petukhov carried out analysis in the Hausdorff metric (the maximum of the minimum distances between the surfaces), which has a potential advantage because it is "non-directional" (Petukhov, 2002). DeVore has described important nonlinear approaches, including greedy algorithms and optimal basis selection (DeVore, 1998). In these approaches the approximants of a function come not from linear spaces but rather from nonlinear manifolds. While, in this approach, the difference between two functions can be measured by any metric, including L_p metrics, the rate of nonlinear approximation is characterized by smoothness conditions that are significantly weaker than are required in classical linear-space theory. The recent report *Illuminating the Path: The Research and Development Agenda for Visual Analytics* (Thomas and Cook, 2005) emphasizes integration of visualization and analytical reasoning. New human-goal-based metrics are part of this integration. There are many different human goals, including accuracy of visibility, that could be bases for measuring the difference between two surfaces.

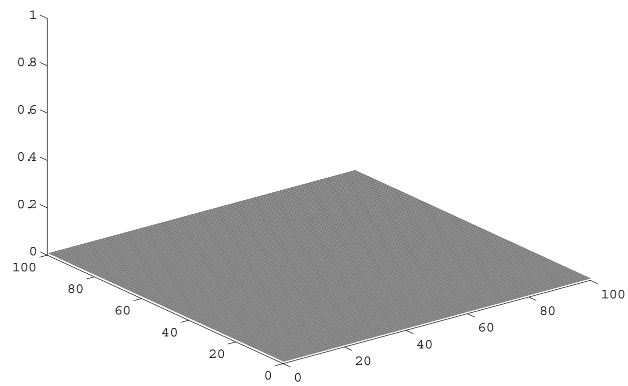

Fig. 1. Flat surface

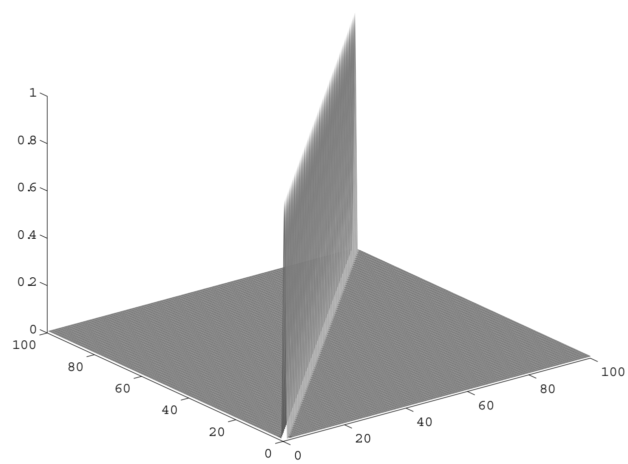

Fig. 2. Flat surface with long thin ridge

The premise of the present paper is that, if a metric for the difference between two functions is intended to be part of a human decision-making process, then this metric should be based on human goals. While the metrics currently in widespread use are known not to express what is important in human goals, metrics that express what is important in human goals have not yet been developed. It is the purpose of this paper to construct human-goal-based metrics for approximation and to do so without the smoothness assumptions required by previous approximation theory.

2. A METRIC BASED ON VISIBILITY

The extent to which the visibility regions (line-of-sight regions) of two surfaces coincide is an issue related to a human goal that is important in many situations. In this section, we define a visibility function and two metrics based on visibility.

Let there be given a height function $\varphi(x,y)$ for (x,y) in some 2D domain D. Let there also be given two 3D domains, O and T, at the points of which "observers" and "targets," respectively, are located. For example, when observers and targets are humans, unmanned ground vehicles and unmanned aeriel vehicles that are always on or above φ and always at or below a certain height H, the domains O and T would both be

$$\{(x,y,z) \mid (x,y) \in D,\ \varphi(x,y) \le z \le H\} \tag{3}$$

A target at a point $(\hat{x},\hat{y},\hat{z})$ in T is visible to an observer at a point (x,y,z) in O if the (open) line segment from (x,y,z) to $(\hat{x},\hat{y},\hat{z})$ is always above the surface φ, that is,

$$z + t(\hat{z}-z) > \varphi(x + t(\hat{x}-x), y + t(\hat{y}-y)), \quad 0 < t < 1 \tag{4}$$

In this case, we say that there is "line of sight" from (x,y,z) to $(\hat{x},\hat{y},\hat{z})$.

Define a point-to-point visibility function

$$\mathrm{vis}_\varphi(x,y,z;\hat{x},\hat{y},\hat{z}) = \begin{cases} 1 & \text{if point } (\hat{x},\hat{y},\hat{z}) \text{ is visible from point } (x,y,z) \\ 0 & \text{if point } (\hat{x},\hat{y},\hat{z}) \text{ is not visible from point } (x,y,z) \end{cases} \tag{5}$$

Based on this point-to-point visibility function, we can define a visibility metric V for the surface φ and for an observer at (x,y,z) in O to be the integral of vis_φ over T, that is,

$$\mathrm{V}_{\varphi;T}(x,y,z) = \iiint_T \mathrm{vis}_\varphi(x,y,z;\hat{x},\hat{y},\hat{z})\, \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z} \tag{6}$$

Let there be given two height functions, a model $f(x,y)$ and "ground truth" ("real terrain") $g(x,y)$. For an observer at (x,y,z), we define the difference-of-visibility or "DV" metric between functions f and g to be

$$\mathrm{DV}_{f,g;T}(x,y,z) = \iiint_T \left| \begin{matrix} \mathrm{vis}_f(x,y,z;\hat{x},\hat{y},\hat{z}) \\ -\,\mathrm{vis}_g(x,y,z;\hat{x},\hat{y},\hat{z}) \end{matrix} \right| \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z} \tag{7}$$

If the DV metric is small (large), visibility on f does (does not) closely approximate visibility on g. In practice, the ground truth g that appears in vis_g in the DV metric is rarely known completely. Only data points on and perhaps some other information (derivatives at some points, monotonicity, drainage patterns, etc.) about g are known. Hence, an exact DV metric will typically have to be replaced by an approximate DV metric. We use such an approach to carry out the computational experiments in the next section.

3. ACCURACY OF VARIOUS SPLINES IN THE DV METRIC

Computationally efficient codes for calculating DV metrics for bivariate functions $f(x,y)$ and $g(x,y)$ in 3D space are not yet available. However, codes for univariate functions $f(x)$ and $g(x)$ in 2D space are available. For such univariate functions, one defines a DV metric by simply omitting the dependence on y and $\hat{y}$ in the visibility functions, the domains O and T and the integral in (7).

For the computational experiments to be reported in this section, we use a 21-point data set representing a Heaviside function (constant heights 1 and 2 separated by a discontinuity) and the following 49-point artificial urban terrain data set.

(0,2),
(0.124354995,2.015564437),
(0.255341921,2.061552813),
(0.291456795,2.610076627),
(0.380506377,2.692582404),
(0.558599315,2.358495283),
(0.643501109,2.5),
(0.785398164,2.828427125),
(1.107148718,2.236067977),
(1.325817664,2.061552813),
(1.570796327,2),
(1.665748034,1.054751155),
(1.951302704,1.346291202),
(2.158798931,1.802775638),
(2.279422599,2.304886114),
(2.312065376,2.5),
(2.617993878,2.5),
(2.879793266,2.5),
(3.141592654,2.5),
(3.228859117,2.5),
(3.665191430,2.5),
(3.926990817,1),
(4.014257281,1.25),

(4.101523742,1),
(4.188790204,1.5),
(4.276056667,1),
(4.363323131,2),
(4.712388980,1),
(4.974188368,1.258819045),
(5.235987756,1.5),
(5.497787144,1.707106781),
(5.759586532,1.866025404),
(6.021385919,1.965925826),
(6.283185308,2),
(6.4,1),
(6.5,1),
(7,1),
(7.4,1.1),
(7.7,1.2),
(8,1.3),
(8.1,1.3),
(8.4,1.4),
(8.7,1.5),
(9,1.6),
(9.1,1.6),
(9.4,1.7),
(9.7,1.8),
(9.85,1.85),
(10,1.9)

Without the point (9.85,1.85), this 49-point data set was used for the L_1 and L_2 splines presented in Sec. 4 of (Lavery, 2006). The first 34 points of this data set were used for the L_1 and L_2 splines in Sec. 4 of (Lavery, 2002). In the figures below, the data are represented by the symbol $\times$.

We represent the ground truth function g by a linear spline, that is, by straight line segments connecting the data points on the original "fine" grid, as shown in Figs. 3 and 7. For each data set, we calculate three approximants f by interpolating every second data point in the data sets, that is, interpolating only the even-numbered data points (with numbering starting from 0, a total of 11 points for the 21-point Heaviside data set and a total of 25 points for the 49-point urban terrain data set) and ignoring all of the odd-numbered points. These three functions f are the linear spline, the cubic L_1 spline (Lavery, 2000a, 2000b, 2001, 2006) and the conventional cubic spline ("cubic L_2 spline") (Lavery 2000a, 2001, 2006) all constructed on a "coarse grid" using every second data point. For the 21-point Heaviside data set, these three approximants are plotted in Figs. 4, 5 and 6 and, for the 49-point urban terrain data set, they are plotted in Figs. 8, 9 and 10. We choose the domain T to be $\{(x,z) \mid 0 \le x \le 10,\ g(x) \le z \le 3\}$. The observers were at all of the positions (x,z), $x = 0.05, 0.15, 0.25, \ldots, 9.95$, $z = 1.5, 2.0, 2.5, 3.0$, that are located on or above the terrain g. In the figures, the positions of the observers are represented by the symbol "o". The integral in the DV metric (a double integral for this univariate situation) was discretized by the midpoint rule using 100 equal intervals in the $\hat{x}$ direction and 100 equal intervals in the $\hat{z}$ direction.

For the 21-point Heaviside data set with observers at 307 positions, the minimum, median, average and maximum DV metrics are given in Table 1. For the 49-point urban terrain data set with observers at 288 positions, the minimum, median, average and maximum DV metrics are given in Table 2. These results indicate that, in the DV metric, the coarse-grid linear spline and the coarse-grid cubic L_1 spline are roughly equally accurate approximants of g and that they are more accurate approximants of g than the coarse-grid conventional cubic spline, which has extraneous oscillation that leads to inaccurate visibility.

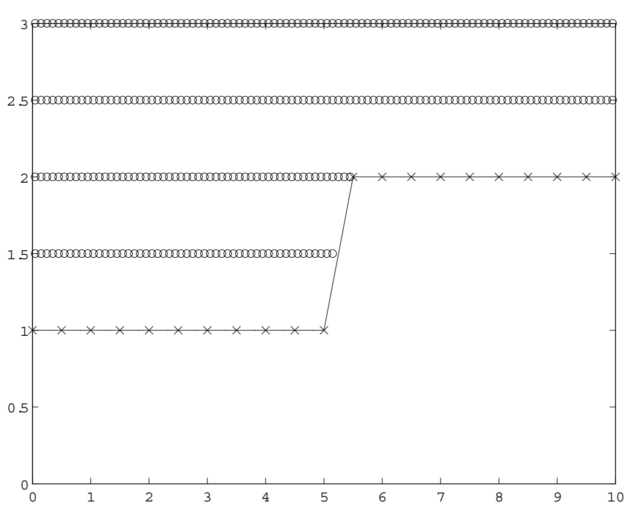

Fig. 3. Fine-grid linear spline g for 21-point Heaviside data set

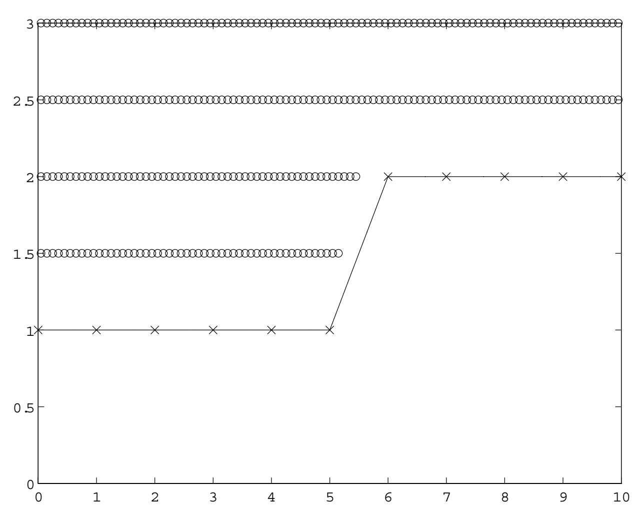

Fig. 4. Coarse-grid linear spline f for 21-point Heaviside data set

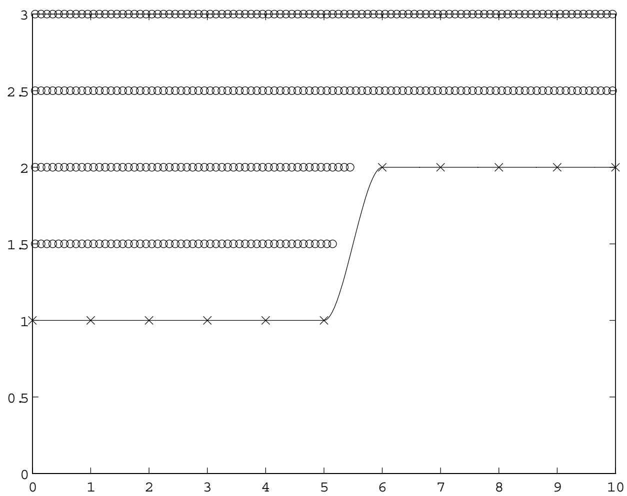

Fig. 5. Coarse-grid cubic L_1 spline f for 21-point Heaviside data set

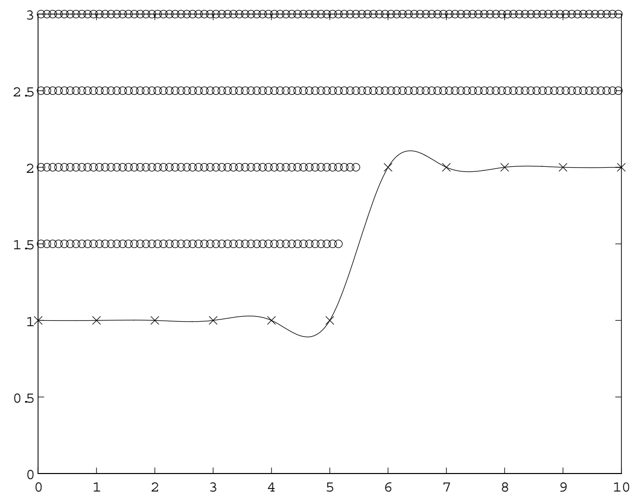

Fig. 6. Coarse-grid conventional cubic spline f for 21-point Heaviside data set

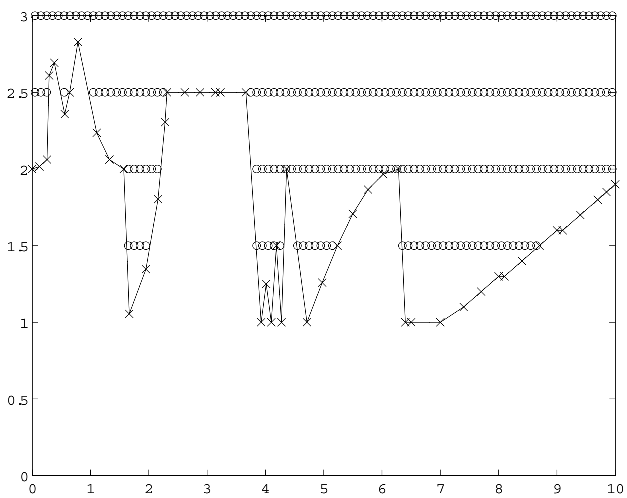

Fig. 7. Fine-grid linear spline g for 49-point artificial urban terrain data set

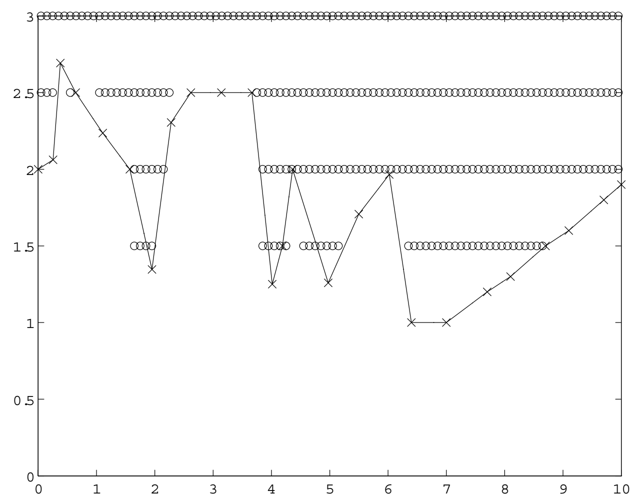

Fig. 8. Coarse-grid linear spline f for 49-point artificial urban terrain data set

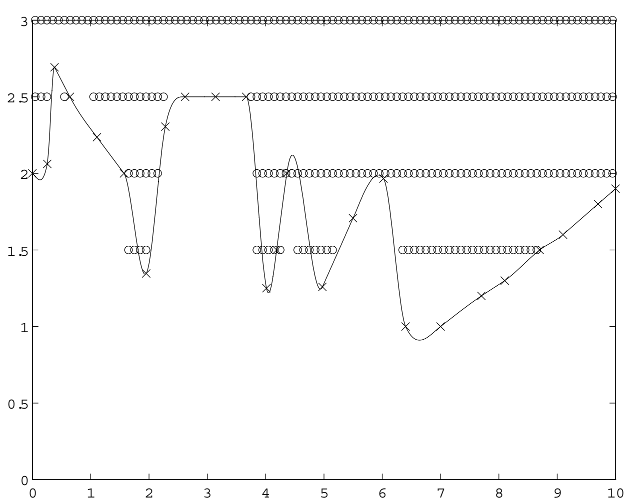

Fig. 9. Coarse-grid cubic L_1 spline f for 49-point artificial urban terrain data set

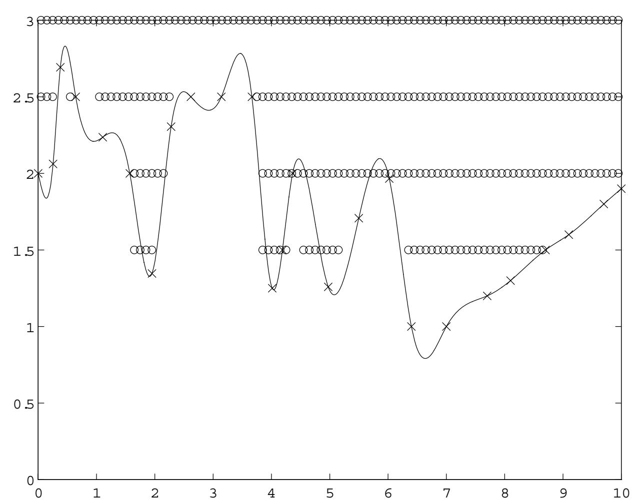

Fig. 10. Coarse-grid conventional cubic spline f for 49-point artificial urban terrain data set

Table 1. Minimum, Median, Average and Maximum DV Metrics for 21-point Heaviside Data Set

	Min	Median	Average	Max
Linear spline	0.255	0.255	0.501	1.269
Cubic L_1 spline	0.255	0.255	0.477	1.191
Conventional cubic spline	0.120	0.474	0.640	1.542

Table 2. Minimum, Median, Average and Maximum DV Metrics for 49-point Artificial Urban Terrain Data Set

	Min	Median	Average	Max
Linear spline	0.015	0.608	0.691	4.287
Cubic L_1 spline	0.018	0.771	0.913	7.314
Conventional cubic spline	0.066	1.572	1.993	7.647

4. ADDITIONAL HUMAN-GOAL-BASED METRICS

There are many extensions. The point-to-point visibility function vis_φ assumes that all visible targets are equally visible, no matter how distant they are from the observer. With telescopic lenses, this assumption is reasonable in many circumstances. However, when telescopic lenses are not available (for example, in low-cost sensors) and when visibility decreases with distance (for example, due to haze or smog in the atmosphere), it is appropriate to introduce in the DV metric a weighting function w that decreases as the distance of the target from the observer increases, perhaps at different rates in different regions and directions. In this case, the point-to-point visibility function $\text{vis}_\varphi(x, y, z; \hat{x}, \hat{y}, \hat{z})$ for a surface φ can be defined to be

$$\text{vis}_\varphi(x, y, z; \hat{x}, \hat{y}, \hat{z}) = \begin{cases} w(x, y, z; \hat{x}, \hat{y}, \hat{z}) & \text{if point } (\hat{x}, \hat{y}, \hat{z}) \text{ is visible from point } (x,y,z) \\ 0 & \text{if point } (\hat{x}, \hat{y}, \hat{z}) \text{ is not visible from point } (x,y,z) \end{cases} \quad (8)$$

When a telescopic lense is not available but the atmosphere is clear, one could choose

$$w(x, y, z; \hat{x}, \hat{y}, \hat{z}) = \frac{1}{(x-\hat{x})^2 + (y-\hat{y})^2 + (z-\hat{z})^2} \quad (9)$$

to express the fact that the visually recorded area of the target as seen by the observer is inversely proportional to the square of the distance from the observer to the target.

When the observer position is variable, it may be best to use "global" metrics. One can define a global visibility metric GV for a function φ to be the integral of V over O, that is,

$$\text{GV}_{\varphi;O,T} = \iiint_O \left[\iiint_T \text{vis}_\varphi(x,y,z;\hat{x},\hat{y},\hat{z})\, d\hat{x}\, d\hat{y}\, d\hat{z} \right] \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \quad (10)$$

One can define a global difference-of-visibility metric GDV between functions f and g to be the integral of DV over O, that is,

$$\text{GDV}_{f,g;O,T} = \iiint_O \left[\iiint_T \left| \begin{matrix} \text{vis}_f(x,y,z;\hat{x},\hat{y},\hat{z}) \\ -\text{vis}_g(x,y,z;\hat{x},\hat{y},\hat{z}) \end{matrix} \right| \mathrm{d}\hat{x}\, \mathrm{d}\hat{y}\, \mathrm{d}\hat{z} \right] \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \quad (11)$$

As defined in (7) and (11), the DV and GDV metrics weight all observer and target points equally. In many cases, users may wish to weight some regions in O and T differently from other regions to reflect the importance of accurate modeling in those regions. This can be done by introducing weight functions in the integrands of the DV and GDV metrics. Analogous adjustments can, of course, be made in the integrands of the visibility metric V of (6) and the global visibility metric GV of (10).

The difference-of-visibility metrics DV and GDV treat false negatives (f predicts no visibility when g has visibility) and false positives (f predicts visibility when g does not have visibility) equally and provide sums of these two types of error. However, there are many circumstances in which one wishes to measure only one of these types of error, not both together. For example, for concealing unsightly civilian assets such as garbage dumps and for concealing many military assets, false negatives are serious while false positives are less so. On the other hand, for emplacing optical communication nodes under friendly conditions, false negatives are generally not serious. They merely increase costs marginally by reducing the options in the planning stage. However, false positives are serious because they result in constructing nodes in locations where the system can not function and thereby increase costs dramatically. Difference-of-visibility functionals that measure only false negative error or only false positive error are easily constructed. For example, a difference-of-visibility metric for false negative error is

$$\iiint_T \max\left\{ \begin{array}{l} \text{vis}_g(x,y,z;x,y,z) \\ -\text{vis}_f(x,y,z;x,y,z) \end{array}, 0 \right\} \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z} \qquad (12)$$

and a difference-of-visibility metric for false positive error is

$$\iiint_T \max\left\{ \begin{array}{l} \text{vis}_f(x,y,z;x,y,z) \\ -\text{vis}_g(x,y,z;x,y,z) \end{array}, 0 \right\} \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z} \qquad (13)$$

The sum of these two errors is the total difference-of-visibility error DV.

In this paper, we have worked with functions $f(x,y)$ and $g(x,y)$ that represent univalent height fields or “terrain skins.” The extension of the techniques described here to fully 3D terrain (with, for example, space underneath bridges, interiors of buildings, subway tunnels, etc.) is computationally intensive but conceptually straightforward.

The emphasis in this paper has been on visibility as a metric because 1) measuring visibility is a common and important military and human goal and 2) quantitative metrics for visibility can be defined in ways, such as those described above, that human beings recognize are directly connected with the human goal. However, the approach of this paper is not limited to visibility and extends to many other situations. Discovering the metrics by which human beings judge similarities and differences in all areas of human interest, the metrics by which they extract features and the metrics by which they understand perceptual cues will require long-term interdisciplinary research by mathematicians, statisticians, cognitive scientists and domain experts.

While classical mathematical metrics may on occasion be found to be appropriate, it is likely that most of the metrics related to human goals will be quite different from the metrics that have been commonly used in the past. Human beings never assess visibility or other situations using the L_2 (“rms”) metric, that is, using an “arithmetic average” of spatial or temporal experience. In spite of this fact, the L_2 metric has been in widespread use in terrain representation and image analysis, in large part because better metrics were not known. New metrics and a new analysis based on these new metrics need to be developed.

5. GENERATION OF OPTIMAL MODELS

Besides being ways of determining how accurate the visibility of a given model f is, the DV and GDV metrics are potential bases for generation of optimal models f, that is, of functions f that minimize the DV and GDV metrics over an appropriate function space or manifold. This minimization will be challenging, since the DV and GDV metrics are only C^0 continuous with respect to f even when f varies smoothly. Numerous algorithms, including genetic algorithms and simulated annealing, may be applicable for carrying out this minimization. However, generic algorithms such as these need to be applied with care, if at all. Utilizing the structure of terrain in whatever algorithm is chosen will likely be essential for computational efficiency.

6. CONCLUSION

The contribution of this paper is not merely the identification of new visibility metrics for measuring the accuracy of geometric models in civilian and military applications but also the demonstration that, in a set of computational experiments, linear and L_1 splines are, in the DV metric, significantly more accurate than conventional splines, consistent with the judgment of most human observers.

The DV and GDV metrics are computationally intensive. Whether one can find computational procedures for calculating these metrics that have acceptably low computational cost is an open question. The DV and GDV metrics for determining the difference in visibility between exact terrain and an approximate model will generally have to be computed in some approximate manner, as was done in the computational experiments described in Sec. 3 in this paper, because terrain is typically known only through the point clouds of data that represent it and is virtually never known exactly everywhere.

In cases where the computational cost of the metric is too large, perhaps because of the complexity of the nonlinearities involved, the question of approximate ersatz metrics will arise. This question in turn leads to a question of meta-metrics, that is, “metrics of metrics” that measure how well one metric approximates another metric. The results in Sec. 3 of this present paper indicate that calculation of the linear spline and calculation of the L_1 spline (by minimization of L_1 norm of the second derivative) are candidate ersatzes for optimization of the DV metric. Whatever metrics or ersatz metrics are proposed should be justified on the basis of human goals, not on the basis of tradition, convenience or mathematical beauty.

In the past, smoothness has been virtually the only pattern used in approximation theory. However, there are equally strong patterns—ones not related to smoothness—in many classes of nonsmooth functions. Human beings instinctively recognize cities and areas of cities by these patterns. These patterns are potential guideposts on the way to developing a fully nonlinear approximation theory with nonsmooth metrics of nonsmooth functions on nonsmooth manifolds that is applicable to terrain and many other modern areas of interest.

The human-goal-based metrics for visibility described above in this paper are bases on which to commence an investigation of practically useful metrics in all military and civilian areas in which human goals play a role, that is, in most areas of military and civilian interest. A shift from a framework based on classical metrics, smoothness and linearity to a framework based on direct linkage with human goals without a priori assumptions of smoothness or linearity is urgently required to support human sense making and decision making. In all of these situations, the metric for the properties of a phenomenon in the context of a human goal should be the quantitative formulation of that human goal itself, not a metric that is adopted from unrelated mathematical concepts or other areas of applications.

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HUMAN-GOAL-BASED METRICS FOR MODELS OF URBAN AND NATURAL TERRAIN AND FOR APPROXIMATION THEORY

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CLASSICAL METRICS FOR TERRAIN

Classical L_p metrics, especially L_2 metric, commonly used to measure how well terrain model approximates real terrain

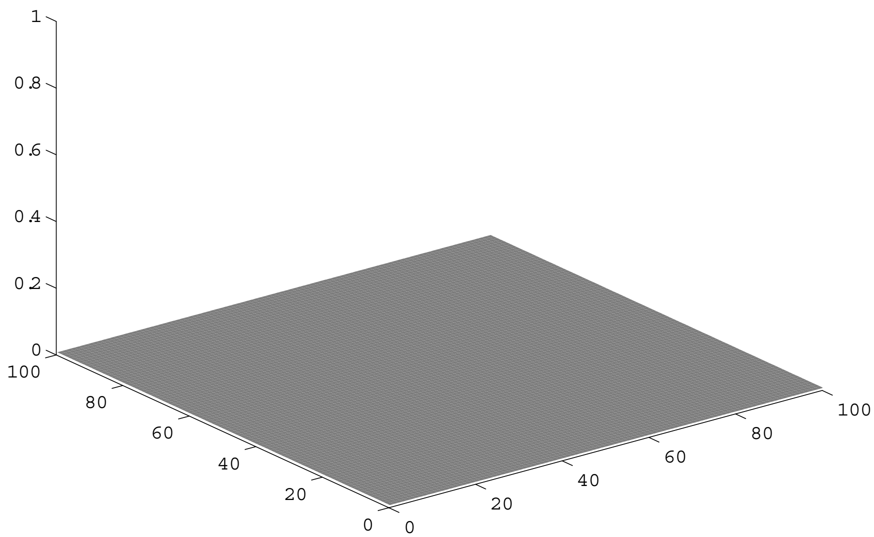
L_p metric for difference between functions f and g:

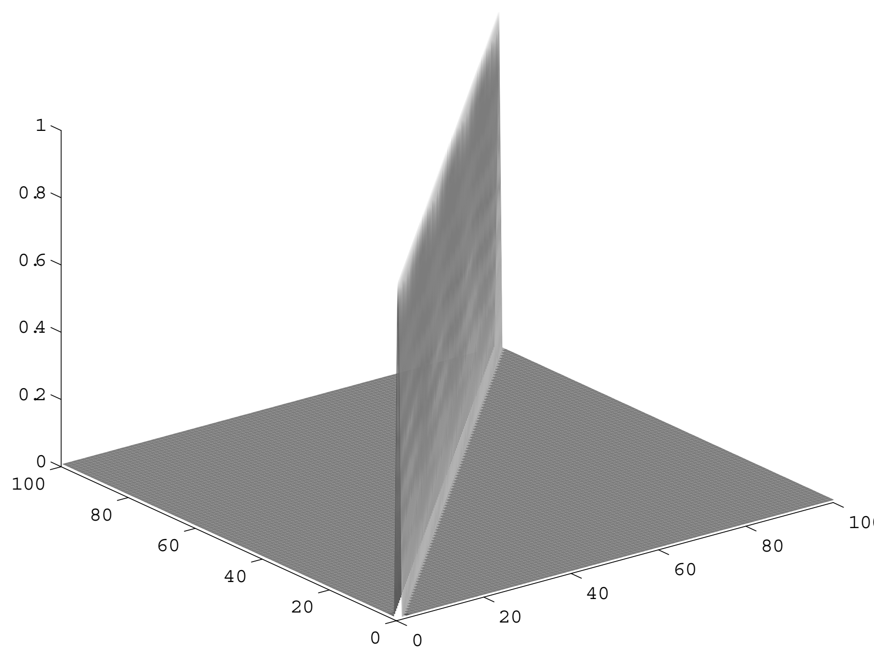
$$\left\{ \int_D |f-g|^p \, \mathrm{d}D \right\}^{1/p} \qquad \text{for} \quad 1 \le p < \infty$$

$$\sup_{\overline{D}} |f-g| \qquad \text{for} \quad p = \infty$$

L_p METRICS ARE POOR MEASURES FOR ACCURACY OF TERRAIN

In L_p metric ($1 \leq p << \infty$), the following 2 surfaces are "nearly the same":





A METRIC BASED ON VISIBILITY

Observer at (x, y, z) in region *O*

Target at $(\hat{x}, \hat{y}, \hat{z})$ in region *T*

Point-to-point visibility function for surface *φ* (telescopic lense, clear atmosphere)

$$\begin{aligned}\mathrm{vis}_\varphi(x, y, z; \hat{x}, \hat{y}, \hat{z}) &= 1 \text{ if point } (\hat{x}, \hat{y}, \hat{z}) \text{ is visible} \\ &\qquad \text{from point } (x, y, z) \\ &= 0 \text{ if point } (\hat{x}, \hat{y}, \hat{z}) \text{ is not visible} \\ &\qquad \text{from point } (x, y, z)\end{aligned}$$

Visibility metric for surface *φ*

$$\mathrm{V}_{\varphi;T}(x, y, z) = \iiint_T \mathrm{vis}_\varphi(x, y, z; \hat{x}, \hat{y}, \hat{z})\, \mathrm{d}\hat{x}\, \mathrm{d}\hat{y}\, \mathrm{d}\hat{z}$$

A METRIC BASED ON VISIBILITY

Model *f*

"Ground truth" *g*

Difference-of-visibility metric between *f* and *g*

$$\mathrm{DV}_{f,g;T}(x,y,z) = \iiint_T \left| \begin{array}{l} \mathrm{vis}_f(x,y,z;\hat{x},\hat{y},\hat{z}) \\ -\mathrm{vis}_g(x,y,z;\hat{x},\hat{y},\hat{z}) \end{array} \right| \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z}$$

- Integral discretized in computational experiments

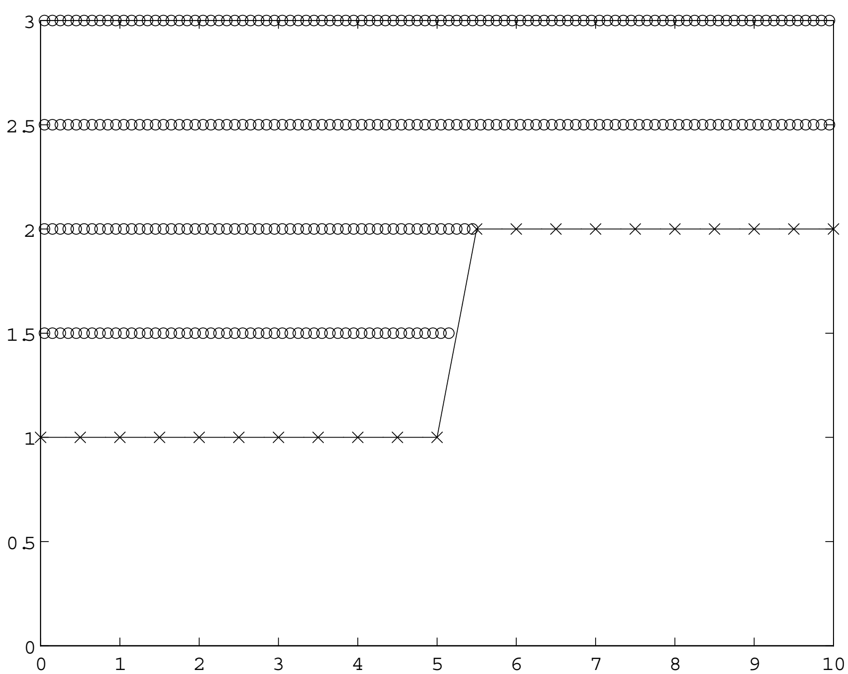
“GROUND TRUTH” AND VARIOUS SPLINE MODELS

- Univariate instead of bivariate for computational tests
- “Ground truth function” g = fine-grid linear spline (straight line segments connecting data points)
- Models (on coarse grid consisting of every 2nd point)
  - Linear spline
  - Cubic L_1 spline (minimizes $\int_D |z''|\,dx$)
  - Conventional cubic spline (minimizes $\int_D (z'')^2 dx$)

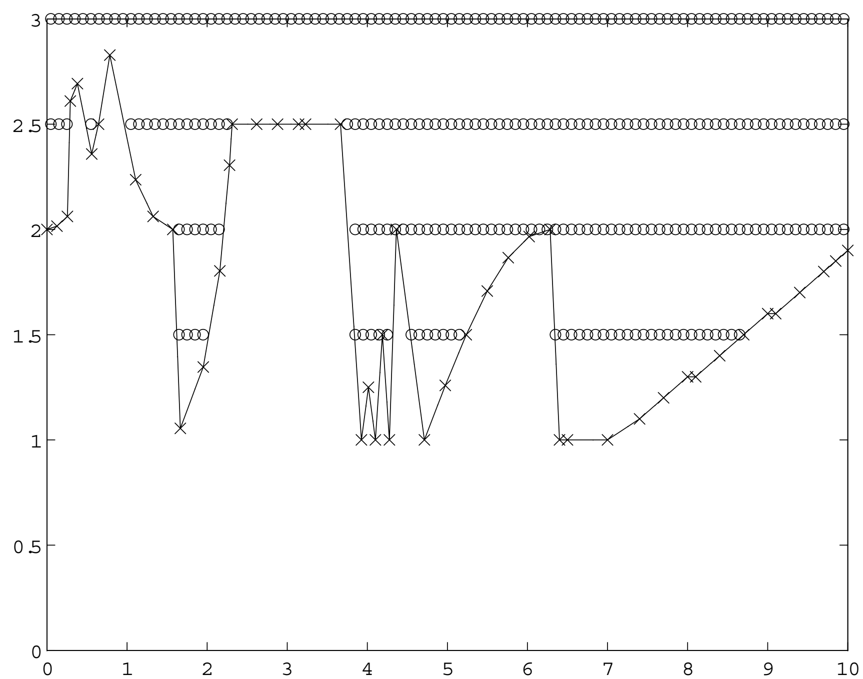
"GROUND TRUTH" TERRAIN (FINE-GRID LINEAR SPLINE)

Positions of observers denoted by "o"

21-point Heaviside data set

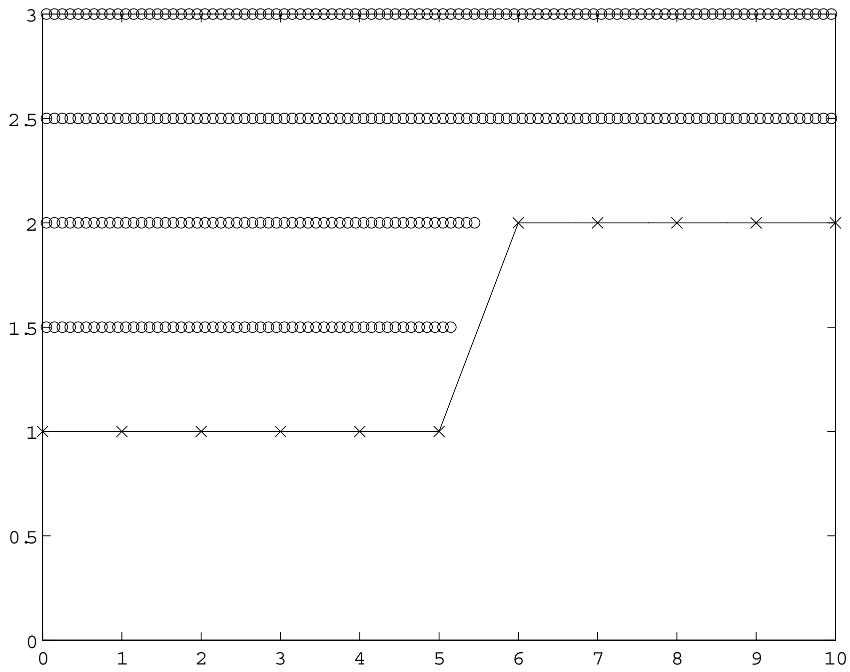


49-point artificial urban terrain data set

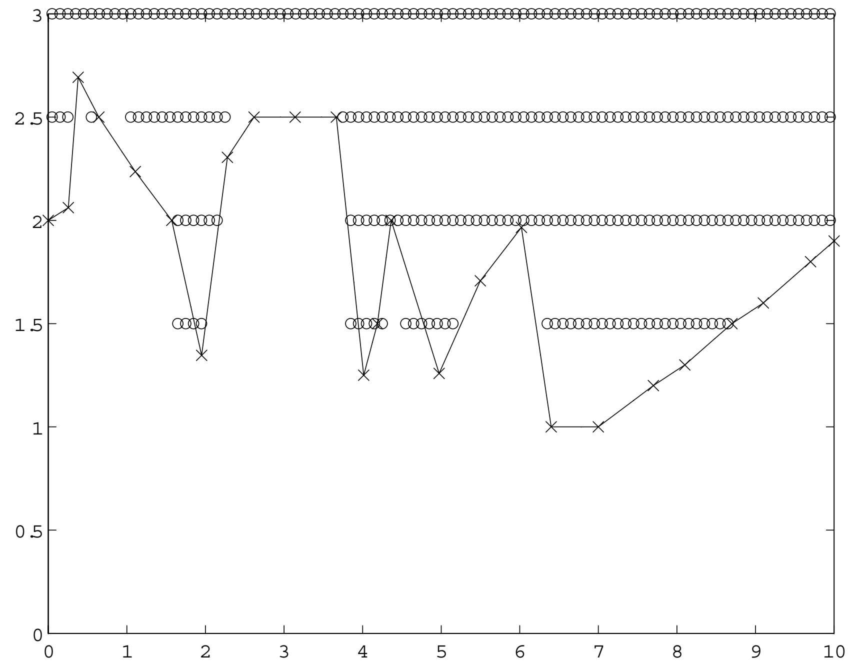


COARSE-GRID LINEAR SPLINES

21-point Heaviside data set

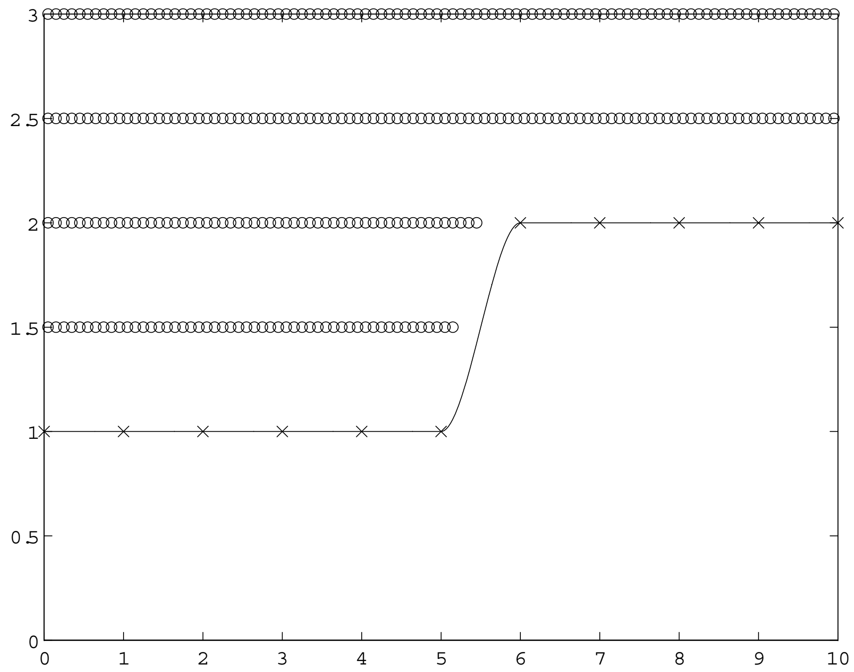


49-point artificial urban terrain data set

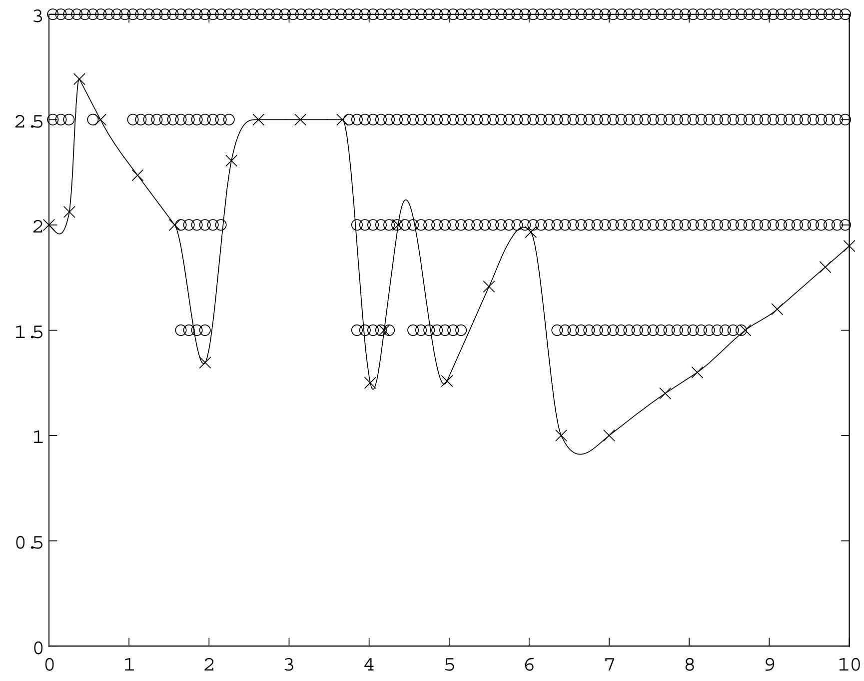


COARSE-GRID L_1 SPLINES

21-point Heaviside data set

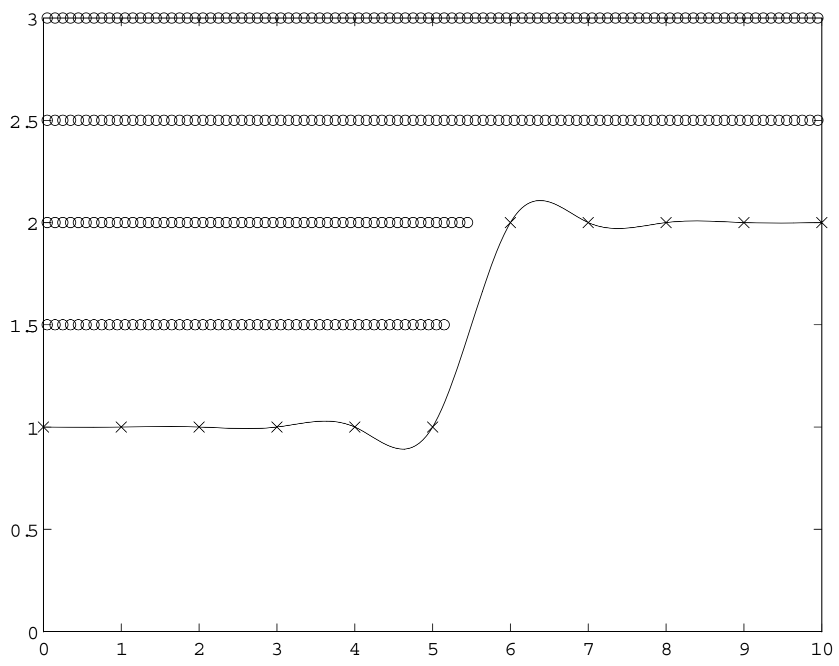


49-point artificial urban terrain data set

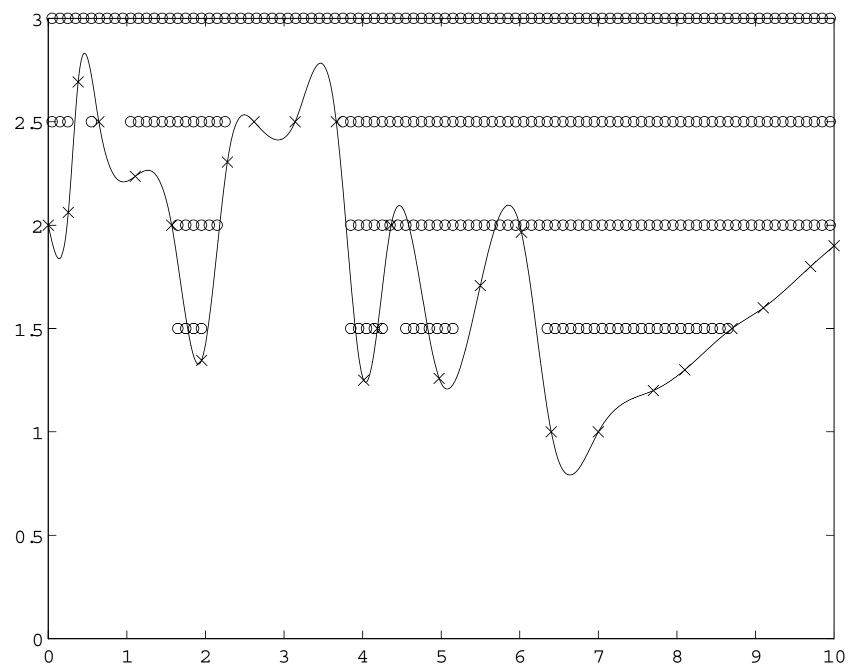


COARSE-GRID CONVENTIONAL SPLINES

21-point Heaviside data set



49-point artificial urban terrain data set



ACCURACY OF VARIOUS SPLINES IN DV METRIC

Minimum, median, average and maximum DV metrics

For 21-point Heaviside data set

	Min	Median	Average	Max
Linear spline	0.255	0.255	0.501	1.269
L_1 spline	0.255	0.255	0.477	1.191
Conventional spline	0.120	0.474	0.640	1.542

For 49-point artificial terrain data set

	Min	Median	Average	Max
Linear spline	0.015	0.608	0.691	4.287
L_1 spline	0.018	0.771	0.913	7.314
Conventional spline	0.066	1.572	1.993	7.647

SUMMARY OF COMPUTATIONAL RESULTS

In DV metric,

- coarse-grid linear spline and coarse-grid cubic L_1 spline are roughly equally accurate

  and

- they are more accurate than coarse-grid conventional cubic spline (which has extraneous oscillation that leads to inaccurate visibility)

ADDITIONAL HUMAN-GOAL-BASED METRICS

Visibility function that reflects current conditions:

$$\begin{aligned}\mathrm{vis}_{\varphi}(x, y, z; \hat{x}, \hat{y}, \hat{z}) &= w(x, y, z; \hat{x}, \hat{y}, \hat{z}) \text{ if point } (\hat{x}, \hat{y}, \hat{z}) \\ &\qquad \text{is visible from point } (x, y, z) \\ &= 0 \text{ if point } (\hat{x}, \hat{y}, \hat{z}) \text{ is not visible} \\ &\qquad \text{from point } (x, y, z)\end{aligned}$$

For no telescopic lense but clear atmosphere,

$$w(x, y, z; \hat{x}, \hat{y}, \hat{z}) = \frac{1}{(x-\hat{x})^2 + (y-\hat{y})^2 + (z-\hat{z})^2}$$

$$\mathrm{DV}_{f,g;T}(x, y, z) = \iiint_T \left| \begin{array}{l} \mathrm{vis}_f(x, y, z; \hat{x}, \hat{y}, \hat{z}) \\ -\,\mathrm{vis}_g(x, y, z; \hat{x}, \hat{y}, \hat{z}) \end{array} \right| \mathrm{d}\hat{x}\, \mathrm{d}\hat{y}\, \mathrm{d}\hat{z}$$

ADDITIONAL HUMAN-GOAL-BASED METRICS

Metric for false negatives (for concealing garbage dumps and military assets)

$$\iiint_T \max\left\{ \begin{array}{l} \mathrm{vis}_g(x,y,z;x,y,z) \\ -\mathrm{vis}_f(x,y,z;x,y,z) \end{array}, 0 \right\} \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z}$$

Metric for false positives (emplacing optical comm nodes under friendly circumstances)

$$\iiint_T \max\left\{ \begin{array}{l} \mathrm{vis}_f(x,y,z;x,y,z) \\ -\mathrm{vis}_g(x,y,z;x,y,z) \end{array}, 0 \right\} \mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}\hat{z}$$

GENERATION OF OPTIMAL MODELS

- Optimal model = model that minimizes difference-of-visibility metric between model and "ground truth"

- Minimization challenging because DV metric and other metrics likely to be used are only C^0 continuous with respect to f even when f varies smoothly.

FURTHER INVESTIGATION

- Develop fully nonlinear analysis based on DV and/or other metrics for nonsmooth functions on nonsmooth manifolds
- Develop computationally efficient procedures for new metrics
- Develop ersatz metrics when desired metric too expensive (linear spline and L_1 spline = candidate ersatz optimizers)
- Justify metrics on basis of human goals, not tradition, convenience or mathematical beauty
- Investigate practically useful metrics in all areas in which human goals play a role