

ENERGY AND LOCATION OPTIMIZATION FOR RELAY NETWORKS WITH DIFFERENTIAL MODULATION

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ABSTRACT

In wireless networks, relay transmissions can enable cooperative diversity by forming virtual antenna arrays. The optimum resource allocation in such relay networks is critical to enhance their performance and efficiency. However, existing works on resource optimization only consider single-relay systems and focus on the power allocation. In this paper, we consider both the power optimization and the location optimization for systems with arbitrary number of relays. Equally attractive is that our investigation is tailored for differential modulations, which bypass the channel estimation at the receiver and are particularly suitable for wireless relay networks. We first derive an upper bound of the error performance. Based on this bound, we then develop the optimum energy and distance allocation schemes that minimize the average system error. Analytical and simulated comparisons confirm that the optimized systems provide considerable improvement over un-optimized ones. In addition, we show that location optimization may be more critical than energy optimization.

1. INTRODUCTION

Recent studies show that networks consisting of unmanned robotic and tele-operated aerial and ground vehicles etc. serving as sensors, communications relays and weapons systems are enablers of the C4ISR capabilities [14]. In particular, relay networks provide diversity gains by forming virtual antenna arrays with distributed network nodes in wireless communications. By exploiting the cooperative diversity, the antenna packing limitations can be eliminated and the spatial diversity gain can be achieved [1, 7, 9]. A majority of existing works on relay networks focuses on coherent demodulation based on the availability of the channel state information (CSI) at both the relays and the destination node (see e.g., [7, 9, 10]). Accurate estimation of the CSI, however, can induce considerable communication overhead and transceiver complexity, which increase with the number of relay nodes employed. In addition, CSI estimation may not be feasible when the channel is rapidly time-varying. To bypass channel estimation, cooperative diversity schemes obviating CSI have been recently introduced. These relay systems rely on

noncoherent or differential modulations, including conventional frequency-shift keying (FSK) and differential phase-shift keying (DPSK) [3, 4, 13], as well as space-time coding (STC) based ones [5, 11].

To improve the error performance and enhance the energy efficiency of relay networks, optimum resource allocation recently emerges as an important problem attracting increasing research interests (see e.g., [2, 8, 12]). These works are based on different relaying protocols (amplify-and-forward, decode-and-forward and block Markov coding), under various optimization criteria (signal-to-noise ratio (SNR) gain, SNR outage probability, energy efficiency and capacity), and with different levels of CSI (instantaneous CSI and channel statistics). However, all of them only consider the power allocation and mostly focus on a single-relay setup.

In this paper, we consider a relay network with arbitrary number of relays. More importantly, we treat the resource allocation as two optimization problems: the optimization of the energy (power) distribution and the optimization of the relay location. Equally attractive is that our analysis is tailored for relay systems with differential modulation, which is known to reduce the receiver complexity by bypassing channel estimation [3, 4, 5, 13]. To enable the resource optimization, we first derive an upper bound of the overall symbol error rate (SER) performance for relay networks employing the decode-and-forward (DF) protocol. The energy and location optimization will then be carried out based on this performance bound. We show that under the constraints of the total energy per symbol and the source and destination distance, the optimum SER performance can be achieved through the joint energy and location optimization. Interestingly, location optimization may be more critical than energy optimization. In other words, near-optimum performance can be achieved by location optimization alone, but not by the energy optimization alone.

The rest of this paper is organized as follows. The system model, including the relay protocol, the differential (de-)modulation and the diversity combining rules for the relay transmission, is introduced in Section 2. In Section 3, an SER upper bound is established for a relay setup with arbi-

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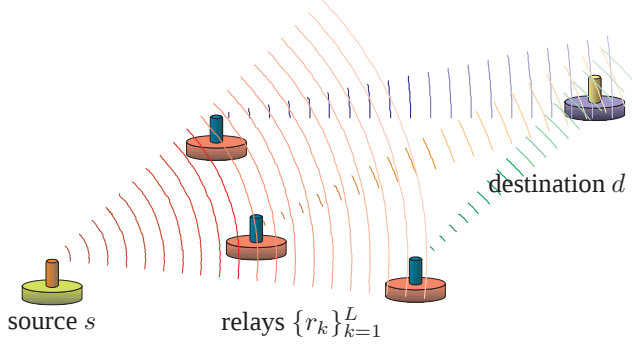


Fig. 1. Setup of the relay network.

bitrary number of relays. The optimum energy and distance allocation is derived in Section 4. Simulations, comparisons and discussions are presented in Section 5. Summarizing remarks are given in Section 6.

Notation: We use $(\cdot)^*$ for conjugate, $\mathbb{E}[\cdot]$ for expectation, $\Re\{\cdot\}$ for the real part, and $:=$ for “is defined as”. $\mathcal{CN}(\mu, \sigma^2)$ represents the complex Gaussian distribution with mean μ and variance σ^2 .

2. SYSTEM MODEL

Consider a network setup with one source node s , L relay nodes $\{r_k\}_{k=1}^L$ and one destination node d , as depicted in Fig. 1. Each node is equipped with a switch that controls its transmit/receive mode to enable half-duplex communications. Multiplexing among the network nodes can be achieved via frequency-division, time-division or code-division techniques. For notational convenience, we will consider the time-division multiplexing (TDM). However, the presented analysis and results are readily applicable to frequency-division multiplexing (FDM) and code-division multiplexing (CDM).

2.1. Relaying Protocol and Channel Modeling

We consider the decode-and-forward (DF) relaying protocol, in which the relay nodes de-modulate the signal from the source node, then re-modulate and forward to the destination node. During the first segment of each symbol duration, the source node broadcasts the first symbol to all relay nodes. Next, each relay transmits the remodulated signal to the destination during their distinct segments within the rest of the symbol duration. As a result, the total of L source-to-relay ($s-r$) links share a common channel, whereas the L relay-to-destination ($r-d$) links have mutually orthogonal channels. With TDM, the orthogonality is ensured by assigning distinct time segments to each relay. This setup is widely adopted in relay networks (see e.g., [2, 4, 9]).

As we mentioned in the introduction, differential modulation is considered at both the source and relay nodes to

bypass channel estimation. Specifically, with the n th phase-shift keying (PSK) symbol being denoted as $s_n = e^{j2\pi c_n/M}$, $c_n \in \{0, 1, \dots, M-1\}$, the corresponding transmitted signal from the source is:

$$x_n^s = \begin{cases} x_{n-1}^s s_n, & n \geq 1 \\ 1, & n = 0. \end{cases} \quad (1)$$

The encoded signal is broadcast via a common channel, and the received signal at the k th relay is given by

$$y_n^{r_k,s} = \sqrt{\mathcal{E}_s} h_n^{r_k,s} x_n^s + z_n^{r_k}, \quad k = 1, 2, \dots, L, \quad (2)$$

where $\mathcal{E}_s$ is the energy per symbol at the source node, the fading coefficient of the channel between s and r_k during the n th symbol duration is $h_n^{r_k,s} \sim \mathcal{CN}(0, \sigma_{h_{r_k,s}}^2)$, and the noise component $z_n^{r_k} \sim \mathcal{CN}(0, \mathcal{N}_{r_k})$. This signal is differentially demodulated and remodulated independently at each relay r_k . The demodulation step generates an estimate $\hat{s}_n^{r_k}$ from $y_n^{r_k,s}$ in Eq. (2), using the decision rule that we will present in the next subsection. The remodulation step is carried out as in Eq. (1), but with s_n replaced by its estimate and x_n^s replaced by $x_n^{r_k}$. Then, the received signal at the destination corresponding to each relay node is given by

$$y_n^{d,r_k} = \sqrt{\mathcal{E}_{r_k}} h_n^{d,r_k} x_n^{r_k} + z_n^d, \quad k = 1, 2, \dots, L, \quad (3)$$

where $\mathcal{E}_{r_k}$ is the energy per symbol at the k th relay node, the fading coefficient of the channel between r_k and d during the n th symbol duration is $h_n^{d,r_k} \sim \mathcal{CN}(0, \sigma_{h_{d,r_k}}^2)$, and the noise component is $z_n^d \sim \mathcal{CN}(0, \mathcal{N}_d)$. Accordingly, we can find the received instantaneous signal-to-noise ratio (SNR) between the transmitter j and the receiver i as

$$\gamma_{i,j} = \frac{|h_n^{i,j}|^2 \mathcal{E}_j}{\mathcal{N}_i}, \quad i, j \in \{s, r_k, d\}.$$

Then, the average received SNR is $\bar{\gamma}_{i,j} = (\sigma_{h_{i,j}}^2 \mathcal{E}_j) / \mathcal{N}_i$.

2.2. Differential Demodulation and Decision Rules

As mentioned before, differential demodulation is performed at the relay and destination nodes. To derive the demodulation, decision and diversity combining rules, let us begin with the received signal at the relay or the destination node $y_n = h_n x_n + z_n$, which is extracted from Eqs. (2) and (3) by dropping the superscripts. Using the differential encoding in Eq. (1), the received signal can be re-expressed as:

$$y_n = h_n (x_{n-1} s_n) + z_n = y_{n-1} s_n + z'_n,$$

where $z'_n = z_n - z_{n-1} s_n$. For M-ary PSK symbols, it follows that $\mathbb{E}[s_n^* s_n] = 1$. Hence, the conditional distribution of y_n is complex Gaussian with mean $y_{n-1} s_n$ and variance $2\mathcal{N}_i$. As a result, we obtain the log likelihood function (LLF) of y_n as:

$$l_m^{i,j}(y_n) := \ln p_{y_n|s_n}(y_n|I_m) = \Re\{(y_n)^* y_{n-1} I_m\}, \quad (4)$$

where $i, j \in \{s, r_k, d\}$, $I_m = e^{j2\pi m/M}$ and $m \in \{0, 1, \dots, M-1\}$.

At the k th relay node, the differential demodulator is then straightforward:

$$\begin{aligned} \hat{s}_n^{r_k} &= e^{j2\pi m'/M} : m' = \arg \max_m l_m^{r_k, s}(y_n^{r_k, s}) \\ &= \arg \max_m \Re\{(y_n^{r_k, s})^* y_{n-1}^{r_k, s} I_m\}. \end{aligned}$$

At the destination node, however, there are L different LLF's corresponding to the L transmitted signals from the relays:

$$l_m^{d, r_k}(y_n) = \Re\{(y_n^{d, r_k})^* y_{n-1}^{d, r_k} I_m\}, \quad k = 1, 2, \dots, L. \quad (5)$$

If the channel state information (full or partial) is known at the relays and the destination node, then it is possible to combine the LLF's by capturing the detection error at the relay node according to the so-termed transition probability (see e.g., [3]). However, keeping in mind that differential modulation is considered in the first place because of its capability of bypassing channel estimation, we will focus on the scenario where *no channel state information is available*. Under this circumstance, the LLF's in Eq. (5) have to be combined with equal weights. Accordingly, the decision rule at the destination node can be readily obtained as:

$$\hat{s}_n^d = e^{j2\pi m'/M} : m' = \arg \max_m \sum_{k=1}^L \Re\{(y_n^{d, r_k})^* y_{n-1}^{d, r_k} I_m\}.$$

With no channel information assumed at either the relays or the destination node, this decision rule turns out to be the differential detection with postdetection equal gain combining (EGC) [17, Chapter 6.6].

3. ERROR PERFORMANCE ANALYSIS

To facilitate our resource optimization, we will derive the analytical expression of the error performance for the cooperative system described in the preceding section. Symbol error probability of cooperative networks with relay transmissions has been derived in [9] for coherent detection, and in [13] for a differential scheme with a single relay, both employing the amplify-and-forward (AF) protocol. Here, we consider a general L -relay setup under the DF protocol, with differential (de-)modulation and diversity combining assuming no CSI.

Let us denote the average symbol error rate (SER) at the k th relay node as P_{e, r_k} . For differential M -ary PSK (DMPSK) signaling, the $s - r_k$ link SER P_{e, r_k} can be obtained as [16, Chapter 8.2.5]

$$P_{e, r_k} = \frac{\sqrt{g_{PSK}}}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{M_{\gamma_s}(-[1 - \sqrt{1 - g_{PSK}} \cos \theta])}{1 - \sqrt{1 - g_{PSK}} \cos \theta} d\theta, \quad (6)$$

where $M_{\gamma}(x) = 1/(1 - x\bar{\gamma})$, $\forall x > 0$, and $\bar{\gamma}$ represents the average signal-to-noise ratio (SNR). In particular, for $M = 2$ (DBPSK), Eq. (6) can be simplified as

$$P_{e, r_k} = \frac{1}{2(1 + \bar{\gamma}_{r_k, s})}. \quad (7)$$

At the destination, the signals from the L relays are combined to make a decision. Conditioned on that the symbol s_n is correctly demodulated and remodulated at all relay nodes, the conditional SER $P_{e, d}$ can be obtained by applying the results for L -diversity branch reception of M -phase signals in [15, Appendix C] as:

$$\begin{aligned} P_{e, d} &= \frac{(-1)^{L-1}(1-\mu^2)^L}{\pi(L-1)!} \left(\frac{\partial^{L-1}}{\partial b^{L-1}} \left\{ \frac{1}{b-\mu^2} \left[\frac{\pi}{M} (M-1) \right. \right. \right. \\ &\quad \left. \left. \left. - \frac{\mu \sin(\pi/M)}{\sqrt{b-\mu^2 \cos^2(\pi/M)}} \cot^{-1} \frac{-\mu \cos(\pi/M)}{\sqrt{b-\mu^2 \cos^2(\pi/M)}} \right] \right\} \right)_{b=1}, \quad (8) \end{aligned}$$

where $\mu = \bar{\gamma}_{d, r_k}/(1 + \bar{\gamma}_{d, r_k})$. For DBPSK, Eq. (8) can be simplified as

$$P_{e, d} = \frac{1}{2} \left[1 - \mu \sum_{k=0}^{L-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4} \right)^k \right]. \quad (9)$$

Using the unconditional SER P_{e, r_k} at the relays and the conditional SER $P_{e, d}$ at the destination, we formulate an upper bound on the overall average error performance, namely the unconditional SER P_e at the destination, as follows:

Proposition 1 With P_{e, r_k} and $P_{e, d}$ given by Eqs. (6) and (8), respectively, an upper bound on P_e can be found as:

$$P_e \leq \bar{P}_e = 1 - \prod_{k=1}^L (1 - P_{e, r_k})(1 - P_{e, d}). \quad (10)$$

Proof: To prove that Eq. (10) provides an upper bound on the exact SER P_e , let us start with the probability of correct detection $P_c = 1 - P_e$. Counting the events that lead to the correct detection, P_c can be obtained as

$$\begin{aligned} P_c &= \Pr\{[(\hat{s}_n^{r_k} = s_n, \forall r_k) \cap (\hat{s}_n^d = s_n)] \\ &\quad \cup [(\hat{s}_n^{r_k} \neq s_n, \text{ for some } k) \cap (\hat{s}_n^d = s_n)]\} \\ &= \Pr\{\hat{s}_n^d = s_n | \hat{s}_n^{r_k} = s_n, \forall r_k\} \cdot \Pr\{\hat{s}_n^{r_k} = s_n, \forall r_k\} \\ &\quad + \Pr\{\hat{s}_n^d = s_n | \hat{s}_n^{r_k} \neq s_n, \text{ for some } k\} \cdot \Pr\{\hat{s}_n^{r_k} \neq s_n, \text{ for some } k\}. \end{aligned} \quad (11)$$

where $\hat{s}_n^{r_k}$ and $\hat{s}_n^d$ are the symbol estimates formed at the relay r_k and the destination d , respectively. The first summand in Eq. (11) turns out to be $\prod_{k=1}^L (1 - P_{e, r_k})(1 - P_{e, d})$, which leads to the upper bound in Eq. (10).

Several remarks are due here on the second summand in Eq. (11), which corresponds to the gap between the true SER and its upper bound $\Delta P_e := \bar{P}_e - P_e$ and determines the

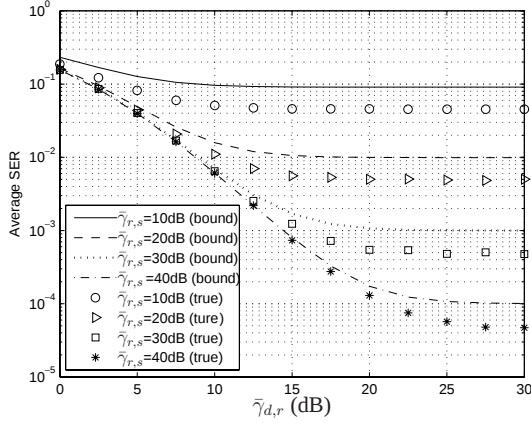


Fig. 2. SER at different $\bar{\gamma}_{r,s}$ values ($L = 2, M = 2$).

tightness of the error bound in Proposition 1. For DBPSK with a single relay ($L = 1, M = 2$), this gap can be easily obtained as:

$$\Delta P_e = P_{e,r} P_{e,d}. \quad (12)$$

For practical $P_{e,r}$ and $P_{e,d}$ values (e.g., $< 10^{-3}$), ΔP_e is negligible compared with $P_e = P_{e,r} + P_{e,d} - 2P_{e,r}P_{e,d}$. However, for $L \geq 2$, all possible errors have to be considered for both the $s - r$ and $r - d$ links, which renders ΔP_e analytically untractable. But intuitively, as the L increases, ΔP_e also increases since there is an increasing chance that detection errors at the relay nodes do not lead to a detection error at the destination node. In addition to this effect, the performance bound $\bar{P}_e$ and the gap ΔP_e also depends on the quality of the $s - r$ and $r - d$ links.

These effects are evident from the simulated examples in Figs. 2 and 3, where a relay network with $L = 2$ relay nodes using DBPSK signaling is considered at various $\bar{\gamma}_{r,s}$ and $\bar{\gamma}_{d,r}$ levels. In these simulated examples, the channels between the source and all relays have identical powers $\sigma_{h_{r_k,s}}^2 = \sigma_{h_{r,s}}^2$, $\forall k$, which implies that $\bar{\gamma}_{r_k,s} = \bar{\gamma}_{r,s}$, $\forall k$. Accordingly, we have $P_{e,r_k} = P_{e,r}$, $\forall k$, and $P_e \leq \bar{P}_e = 1 - (1 - P_{e,r})^L (1 - P_{e,d})$ from Proposition 1. Likewise, the SNR between all the relay nodes and the destination have the same power, $\bar{\gamma}_{d,r_k} = \bar{\gamma}_{d,r}$, $\forall k$. In both Figs. 2 and 3, the bound $\bar{P}_e$ closely captures the dependency of the system SER on the SNR levels $\bar{\gamma}_{r,s}$ and $\bar{\gamma}_{d,r}$. Specifically, we have the following observations:

- Fig. 2 reveals that, at any given value of $\bar{\gamma}_{r,s}$, the system SER exhibits an error floor as $\bar{\gamma}_{d,r}$ increases. Intuitively, this error floor comes from the detection error at the relays, which heavily relies on the $s - r$ link quality $\bar{\gamma}_{r,s}$ and can only be reduced by imposing sufficiently high $\bar{\gamma}_{r,s}$.
- On the contrary, Fig. 3 shows that, at medium-to-high $\bar{\gamma}_{d,r}$ levels, the overall SER can always be reduced by

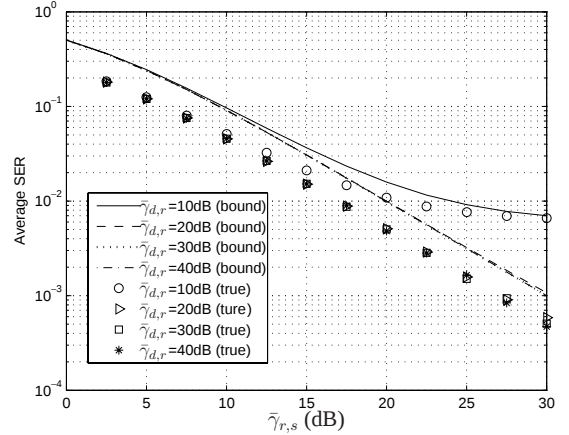


Fig. 3. SER at different $\bar{\gamma}_{d,r}$ values ($L = 2, M = 2$).

increasing the SNR of the $s - r$ link $\bar{\gamma}_{r,s}$ and does not exhibit any error floor.

4. OPTIMUM RESOURCE ALLOCATION

In the preceding section, we derived a simple upper bound on the overall SER of the relay network. Our simulated and numerical examples indicate that the $s - r$ link SNR and the $r - d$ link SNR play unbalanced roles in determining the system SER. In this section, we will investigate the effects of resource allocation on the SER performance. We will show that an optimum allocation of the limited resource is possible, and it achieves the optimum system error performance.

For analytical tractability, we consider an idealized L -relay system with all relay nodes located at the same distance from the source and the destination nodes; that is, $D_{s,r_k} = D_{s,r}$ and $D_{r_k,d} = D_{r,d}$, $\forall k$. It is then reasonable to assign equal energies at all relay nodes $\mathcal{E}_{r_k} = \mathcal{E}_r$, $\forall k$. To carry out the optimization in the ensuing subsections, we will also make use of the relationship between the average power of channel fading coefficient $\sigma_{h_{i,j}}^2$ and the inter-node distance $D_{j,i}$ as follows:

$$\sigma_{h_{i,j}}^2 = C \cdot D_{j,i}^{-\nu}, \quad i, j \in \{s, r, d\}, \quad (13)$$

where ν is the path loss exponent of the wireless channel and C is a constant which we henceforth set to 1 without loss of generality.

4.1. Optimum energy allocation

Problem Statement 1 For any given source, relay and destination node locations ($D_{s,r}$ and $D_{r,d}$, or equivalently $\sigma_{h_{r,s}}^2$ and $\sigma_{h_{d,r}}^2$), and the total energy per symbol $\mathcal{E}$, determine the

optimum energy allocation $\mathcal{E}_s$ and $\mathcal{E}_r$ which minimize $\bar{P}_e$ in Eq. (10) while satisfying:

$$\mathcal{E}_s + \sum_{k=1}^L \mathcal{E}_{r_k} = \mathcal{E}_s + L\mathcal{E}_r = \mathcal{E}. \quad (14)$$

Without loss of generality, assuming that all noise components are independent and identically distributed (i.i.d) with $\mathcal{N}_{r_k} = \mathcal{N}_d = \mathcal{N}_0$. Then, by defining the total SNR, $\rho := \mathcal{E}/\mathcal{N}_0$, the transmit SNR at the source node $\rho_s := \mathcal{E}_s/\mathcal{N}_0$ and the transmit SNR at the relay nodes $\rho_r := \mathcal{E}_r/\mathcal{N}_0$, the energy constraint can be re-expressed as the SNR constraint :

$$\rho = \rho_s + L\rho_r. \quad (15)$$

Using Eq. (13), the average received SNR's at the relay and destination nodes can be expressed in terms of the transmit SNR's as:

$$\begin{aligned} \bar{\gamma}_{r,s} &= \rho_s \sigma_{h_{r,s}}^2 = \rho_s D_{s,r}^{-\nu} \\ \text{and } \bar{\gamma}_{d,r} &= \rho_r \sigma_{h_{d,r}}^2 = \rho_r D_{r,d}^{-\nu}. \end{aligned} \quad (16)$$

As a result, the total energy constraint (15) can be further rewritten as

$$\rho = \bar{\gamma}_{r,s}/\sigma_{h_{r,s}}^2 + L\bar{\gamma}_{d,r}/\sigma_{h_{d,r}}^2 = \bar{\gamma}_{r,s}D_{s,r}^\nu + L\bar{\gamma}_{d,r}D_{r,d}^\nu. \quad (17)$$

To gain some insights, we start from a single-relay setup and establish the following result:

Proposition 2 For a single-relay setup with $L = 1$, at given $s - r$ and $r - d$ distances $D_{s,r}$ and $D_{r,d}$, and under the total energy constraint in Eq. (14), the optimum energy allocation $\mathcal{E}_s$ should satisfy :

$$\rho_s = \frac{D_{r,d}^{-\nu/2}}{D_{s,r}^{-\nu/2} + D_{r,d}^{-\nu/2}} \cdot \rho \Leftrightarrow \mathcal{E}_s = \frac{D_{r,d}^{-\nu/2}}{D_{s,r}^{-\nu/2} + D_{r,d}^{-\nu/2}} \cdot \mathcal{E}. \quad (18)$$

and correspondingly, $\mathcal{E}_r = \mathcal{E} - \mathcal{E}_s$.

This solution is achieved by solving the first order conditions under medium-to-high SNR values. Treating the SER bound $\bar{P}_e$ as a function of $\bar{\gamma}_{r,s}$ and $\bar{\gamma}_{d,r}$, we have the first order conditions for the optimum solution

$$\frac{\partial \bar{P}_e}{\partial \bar{\gamma}_{r,s}} - \lambda D_{s,r}^\nu = 0, \quad (19)$$

$$\frac{\partial \bar{P}_e}{\partial \bar{\gamma}_{d,r}} - \lambda D_{r,d}^\nu = 0, \quad (20)$$

$$\rho - (\bar{\gamma}_{r,s}D_{s,r}^\nu + \bar{\gamma}_{d,r}D_{r,d}^\nu) = 0, \quad (21)$$

where λ is the Lagrange multiplier. From Eqs. (19) and (20), we can achieve the expression of $\bar{\gamma}_{d,r}$. Then, by substituting $\bar{\gamma}_{d,r}$ into Eq. (21), the following exact solution (see Eq. (22)) for ρ_s is achieved. By applying high SNR approximation, we

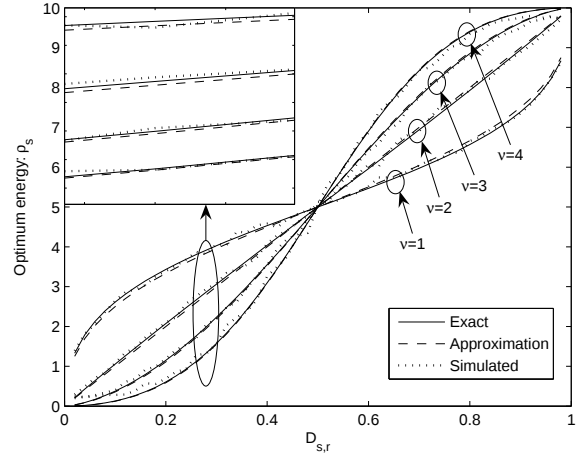


Fig. 4. Optimum energy distribution ($\rho = 10\text{dB}$, $L = 1$, DBPSK).

can find ρ_s as in Eq. (18). The detailed proof of the exact solution and its simplification for the optimum energy allocation can be found in [6]. From Eq. (18), it readily follows that the energy allocation ratio between the source and the relay nodes is

$$\frac{\mathcal{E}_s}{\mathcal{E}_r} = \left(\frac{D_{s,r}}{D_{r,d}} \right)^{\nu/2}. \quad (23)$$

Eq. (23) reveals explicitly that the optimum energy allocation heavily hinges upon the inter-node distances. In addition, the path loss exponent of the wireless channel, ν , also affects the optimum energy allocation. Interestingly, the $\mathcal{E}_s/\mathcal{E}_r$ ratio is linear in $D_{s,r}/D_{r,d}$ only when $\nu = 2$.

Fig. 4 depicts the transmit SNR ρ_s obtained from the optimum energy allocation. A one-dimensional setup is considered; that is, $D_{s,r} + D_{r,d} = D_{s,d}$. The system parameters are: $\rho = 10\text{dB}$, $L = 1$, $D_{s,d} = 1$ and $\nu = (1, 2, 3, 4)$. In Fig. 4, the simulated optimum ρ_s is plotted, together with the exact analytical value in Eq. (22) and the approximation in Eq. (18). Although the approximate expressions Eq. (18) is obtained under the high SNR assumption, they remain very accurate even at the medium SNR of 10dB.

From Fig. 4, we also observe that, for all ν values, the source node energy $\mathcal{E}_s$ increases as the relay moves towards the destination node. With $\nu = 2$, $\mathcal{E}_s$ increases linearly with $D_{s,r}$. At higher values of the path loss exponent, $\nu > 2$, we observe that

$$\frac{\rho_s}{\rho} \begin{cases} < D_{s,r}/D_{s,d}, & \text{when } D_{s,r} < D_{s,d}/2, \\ > D_{s,r}/D_{s,d}, & \text{when } D_{s,r} > D_{s,d}/2. \end{cases} \quad (24)$$

In other words, the optimum energy allocation favors the link with larger inter-node distance. When the path loss exponent

$$\frac{\mathcal{E}_s}{\mathcal{N}_0} = \rho_s = \sqrt{\frac{2D_{r,d}^{-2\nu} + D_{s,r}^{-\nu}D_{r,d}^{-\nu}(6D_{r,d}^{-\nu}\rho + 5) + 2D_{s,r}^{-2\nu}(2D_{r,d}^{-2\nu}\rho^2 + 3D_{r,d}^{-\nu}\rho + 1)}{4D_{s,r}^{-\nu}D_{r,d}^{-\nu}(D_{s,r}^{-\nu} - D_{r,d}^{-\nu})^2}} - \frac{2D_{r,d}^{-\nu}\rho + 3}{2(D_{s,r}^{-\nu} - D_{r,d}^{-\nu})}, \quad (22)$$

$\nu = 1$, Fig. 4 shows the opposite of Eq. (24).

So far, we have been focusing on the single-relay case, where an analytical solution in Eq. (22) can be obtained and a very accurate and insightful approximation is available under the high SNR assumption. For $L \geq 2$, however, the first order conditions obtained by differentiating the SER bound $\bar{P}_e$ have complicated forms, which render analytical solutions impossible. Fortunately, the SER bound $\bar{P}_e$ as in Proposition 1 still allows for a *numerical search*, as opposed to Monte Carlo simulations needed otherwise.

For example, with DBPSK and $L = 2$, we have

$$P_{e,d} = \frac{1}{2} - \frac{3}{4} \left(\frac{\bar{\gamma}_{d,r}}{1 + \bar{\gamma}_{d,r}} \right) + \frac{1}{4} \left(\frac{\bar{\gamma}_{d,r}}{1 + \bar{\gamma}_{d,r}} \right)^3, \quad (25)$$

and, accordingly, the SER bound is given by

$$\bar{P}_e = 1 - \frac{\bar{\gamma}_{r,s}^2(2 + \bar{\gamma}_{d,r})(1 + 2\bar{\gamma}_{d,r})^2}{4(1 + \bar{\gamma}_{r,s})^2(1 + \bar{\gamma}_{d,r})^3}. \quad (26)$$

By using the first order conditions in Eq. (19) and the high SNR approximation, the optimum $\bar{\gamma}_{r,s}$ and $\bar{\gamma}_{d,r}$ should satisfy

$$\frac{4(1 + \bar{\gamma}_{d,r})(2 + \bar{\gamma}_{d,r})(1 + 2\bar{\gamma}_{d,r})}{3\bar{\gamma}_{r,s}(1 + \bar{\gamma}_{r,s})} = \frac{D_{r,d}^{-\nu}}{D_{s,r}^{-\nu}}. \quad (27)$$

Although an analytical solution is not readily available, one can resort to the numerical search.

4.2. Optimum distance allocation

Consider now a one-dimensional relay setup with the distance between the source and the destination being $D_{s,d}$. If the transmit energies at the source and relays are preset, where is the optimum location to place the relays? To answer this question, we treat the distance $D_{s,d}$ as a fixed resource and formulate an optimization problem as follows:

Problem Statement 2 *For any given transmit energies at the source and relay nodes ($\mathcal{E}_s$ and $\mathcal{E}_r$, or equivalently ρ_s and ρ_r), and the path loss exponent ν of the wireless channel, determine the optimal location of the relays, $D_{s,r}$, which minimizes $\bar{P}_e$ in Eq. (10) while satisfying $0 < D_{s,r} < D_{s,d}$.*

Starting with the single-relay ($L = 1$) setup and applying the high-SNR approximation, we establish the following result:

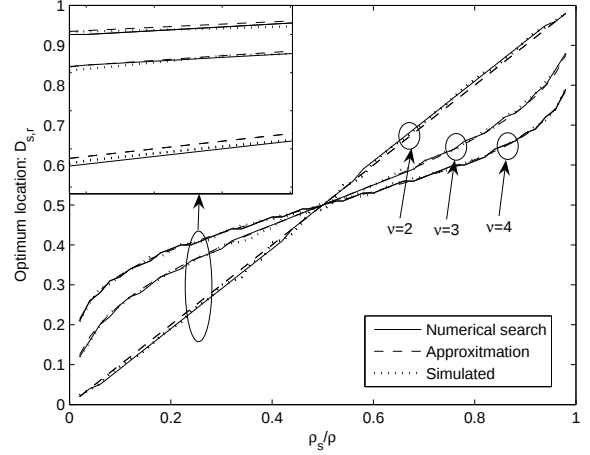


Fig. 5. Optimum location distribution ($\rho = 10\text{dB}$, $L = 1$, DBPSK).

Proposition 3 *For a single-relay setup with $L = 1$ and the source-destination distance $D_{s,d}$, and let $\mathcal{E}_s$ and $\mathcal{E}_r$ denote the prescribed transmit energy levels at the source and relay nodes, respectively, the optimum location of the relay is*

$$D_{s,r} = \frac{\rho_s^{1/(\nu-1)}}{\rho_s^{1/(\nu-1)} + \rho_r^{1/(\nu-1)}} \cdot D_{s,d}, \quad (28)$$

and, accordingly, $D_{r,d} = D_{s,d} - D_{s,r}$.

Similar to the energy optimization case, the solution can be achieved by solving the first order conditions, in which we treat the SER bound $\bar{P}_e$ as a function of $D_{s,r}$ and $D_{r,d}$. We omit detailed proof due to the space limit. With the one-dimensional setup, Proposition 3 can be represented as

$$\frac{D_{s,r}}{D_{r,d}} = \left(\frac{\rho_s}{\rho_r} \right)^{1/(\nu-1)} = \left(\frac{\mathcal{E}_s}{\mathcal{E}_r} \right)^{1/(\nu-1)}. \quad (29)$$

Interestingly, Eq. (29) bears a very similar form as its counterpart for the optimum energy allocation in Eq. (23). In fact, when the path loss exponent $\nu = 2$, Eq. (29) is essentially identical to Eq. (23). For general ν values, however, these two relationships are quite different. Such a discrepancy is actually very reasonable, because Eqs. (23) and (29) result from two distinct optimization problems: the former is obtained for arbitrary distances $D_{s,r}$ and $D_{r,d}$ under a total energy constraint; whereas the latter is obtained for prescribed $\mathcal{E}_s$ and $\mathcal{E}_r$ under a total distance constraint. With the SER bound $\bar{P}_e$ being a two-dimensional function, the energy and

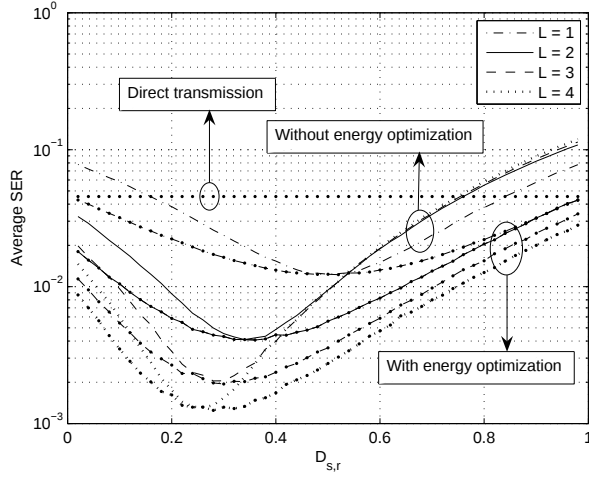


Fig. 6. SER comparison between relay systems with and without energy optimization ($\rho = 10\text{dB}$, $\nu = 4$, DBPSK).

location optimizations are carried out on uncorrelated dimensions.

For general L values, the optimum location can be determined in the similar manner as we discussed in the preceding subsection. Essentially, the path loss exponent ν renders it impossible to derive the analytical solution to the optimum location problem, even with the high SNR approximation. One can resort to the numerical search using the SER bound in Proposition 1. In Fig. 5, the optimum distances obtained from the numerical search and the simulations are compared for different ν values, at total SNR $\rho = 10\text{dB}$ and with $L = 1$ relay node. Notice that, as its counterpart in Fig. 4, the optimum distance allocation is linear in $\mathcal{E}_s/\mathcal{E}$ only when $\nu = 2$.

Summarizing, we established the optimum energy and distance allocation rules which minimize the SER of the relay system. The analytical solutions were derived for $L = 1$, and numerical search can be performed for $L \geq 2$. To verify the performance improvement resulted from the optimization, we will present extensive comparisons, generalizations and further discussions in the next section.

5. SIMULATIONS AND DISCUSSIONS

Next, we will discuss the performance of the DF based cooperative system combined with differential demodulation and optimum resource allocation.

To verify the advantage of the optimum energy allocation, we plot in Fig. 6 the SER of the relay system with and without energy optimization, at $\rho = 10\text{dB}$ and with various numbers of relays $L = (1, 2, 3, 4)$. As a benchmark, we also plot the SER of a direct transmission using all available energy. In the

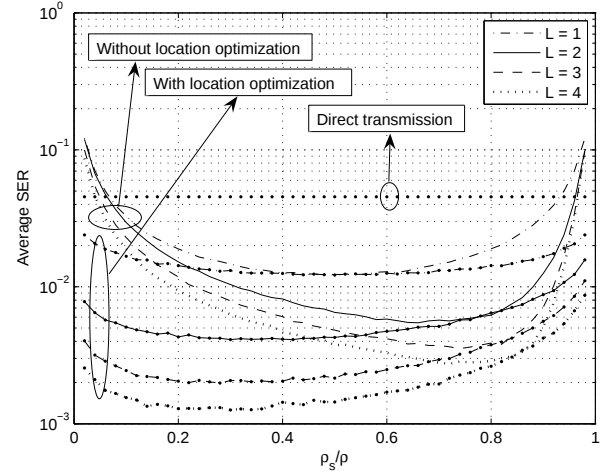


Fig. 7. SER comparison between relay systems with and without location optimization ($\rho = 10\text{dB}$, $\nu = 4$, DBPSK).

system without energy optimization, a uniform energy allocation is employed; that is, $\rho_s = \rho_r = \rho/(L + 1)$ at any $D_{s,r}$. From Fig. 6, we observe that: i) the un-optimized system may even under-perform the direct transmission, regardless of the number of relays L ; ii) as L increases, the SER performance can get even worse unless the energy optimization is performed; iii) the energy-optimized system universally outperforms the direct transmission with the same total energy; and iv) the energy-optimized system universally outperforms the un-optimized system. These observations confirm our discussions in preceding sections.

Interestingly, notice that the minima of the energy-optimized SER curves almost coincide with those of the un-optimized ones. This implies that the near-optimum SER can be achieved even with the uniform energy allocation across the source and relay nodes, provided that the relay location is carefully selected.

In Fig. 7, we verify the advantage of the optimum distance allocation by comparing the SER with and without location optimization. Again, the SER of a direct transmission using all available energy is plotted as the benchmark. In the system without location optimization, the relays are placed at the midpoint of the source-destination link, as suggested in [9, 13]. Similar to the energy optimization case, Fig. 7 confirms the advantages of the location optimization: i) the un-optimized system may even under-perform the direct transmission, regardless of the number of relays L ; ii) the location-optimized system universally outperforms the direct transmission with the same total energy; and iii) the location-optimized system universally outperforms the un-optimized system. Different from the energy optimization case, however, as L increases, the SER performance always improves even without any location optimization.

The curves in Fig. 7 also exhibit more flatness compared with the ones in Fig. 6. This implies that the system SER is more sensitive to the location distribution than to the energy distribution. In addition, the minima of the location-optimized SER curves are far from those of the un-optimized ones, except for the $L = 1$ case (see Fig. 7). This indicates that placing the relay nodes at the midpoint *cannot* achieve the minimum SER even with careful allocation of the source and relay energies, for any $L > 1$. Notice that, even with optimum energy allocations in Fig. 7, the midpoint relay location still *cannot* achieve the minimum SER, unless $L = 1$. This is to be distinguished from the uniform energy case depicted in Fig. 6, as well as from the coherent relay systems in [9].

6. CONCLUSIONS

In this paper, we investigated the optimum energy distribution and optimum location of relays in a system with arbitrary number of relays employing differential demodulation. The base of our optimization is an upper bound on the average SER, which we derived for the decode-and-forward cooperative protocol. Our simulations and numerical examples confirm that both the energy and location optimizations provide considerable SER advantages. Without optimization, the system with more relays may at times underperform the system with less relays or the direct transmission. We have also shown that the location optimization may be more critical than the energy optimization. In other words, the differential relay system with uniform energy distribution can achieve near-optimum SER by appropriately choosing the relay location; while a system with relays sitting at the midpoint between the source and the destination cannot approach the optimum SER even with optimized energy distribution.

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