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An Alternative Method of Computing Altitude Adjustment Corrected Geomagnetic Coordinates as Applied to IGRF Epoch 2005

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14. ABSTRACT
An updated set of altitude adjusted corrected geomagnetic coordinates has been calculated. The coordinates are based on the 10th generation IGRF model coefficients for Epoch 2005. The corrected geomagnetic coordinates at ground level are available as a lookup table, and a corresponding set of spherical harmonic coefficients for use in computing altitude adjusted corrected geomagnetic coordinates at various altitudes (after Baker and Wing [1998]) are also available. Both products are geared towards replacing the tables and coefficients used in Hein and Bhavnani [1996]. An alternative algorithm is introduced that assists in solving discontinuity problems found near the South Atlantic Anomaly and the dip equator when calculating geomagnetic coordinates from a geographic coordinate basis.

15. SUBJECT TERMS
IGRF, Corrected geomagnetic coordinates, South Atlantic Anomaly, Spherical harmonic coefficients

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1. INTRODUCTION

Observations of the dynamics of charged particles that occur in the Earth's ionosphere and magnetosphere are often analyzed within the context of a geomagnetic, rather than geographic, coordinate system. This is somewhat analogous to the use of pressure or log-pressure as a vertical coordinate when describing Earth's atmospheric phenomena within the context of geophysical fluid dynamics. The change in coordinate systems often makes a mathematical description of the phenomena more tractable or the analysis of observed data more easily understood. The pressure-based coordinate system of meteorology is still tied to geographic coordinates. While one may convert the vertical coordinate from pressure to altitude (or vice versa), the horizontal coordinates of longitude and latitude remain untouched. The places on the globe that we consider to be the North and South Poles, or any other location on the globe for that matter, are invariant within this geographic context. As such, a geographic latitude and longitude coordinate pair will mean the same location today as it was ten years ago, or one hundred years in the future. In the context of a geomagnetic framework, however, this is not the case. Earth's geomagnetic field changes with time, so the North and South geomagnetic poles will not be in the same location today as they may be a decade or a century from now. To complicate matters, the true North and South geomagnetic poles are not opposite one another, as we commonly describe the North and South geographic poles to be, and in space the geomagnetic field is actually a sum of magnetic fields generated by planets other than Earth. All of this has to be taken into consideration if one wants to describe a three-dimensional geomagnetic coordinate system to a good degree of accuracy.

Over the years there have been many approaches of how to formulate the "best" geomagnetic coordinate system. The simplest method is the dipole approximation, where one rotates a spherical coordinate system (geographic, for example) such that its Z axis coincides with an estimated dipole axis of Earth's magnetic field. A better approximation can be made if one translates, as well as rotates, these axes; for well known examples of this "eccentric dipole" system see Schmidt [1934] or Bartels [1936]. As more precise measurements of the ionosphere and magnetosphere became possible, mainly due to the research activities of the Space Age, more sophisticated methods were called for that could depict the actual geomagnetic field with greater accuracy.

In 1839, Carl Friedrich Gauss put forth the idea that the main geomagnetic field of the Earth can be described by a spherical harmonic series. More specifically, the geomagnetic field can be described as the gradient of a scalar potential function *V*, where *V* (a function of radius, latitude, and longitude) is expressed as an orthogonal expansion in spherical harmonics (Equation 1):

$$V(r,\theta,\varphi) = R_e \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{R_e}{r} \right)^{n+1} \left(g_n^m(t) \cos m\varphi + h_n^m(t) \sin m\varphi \right) P_n^m(\cos\theta) \tag{1}$$

where R_e is the Earth's radius, g_n^m and h_n^m are spherical harmonic coefficients, and P_n^m are Schmidt semi-normalized associated Legendre functions. This forms the basis of most geomagnetic field models used today. The International Association for Geomagnetism and Aeronomy (IAGA) periodically updates and publishes a reference set of coefficients used in the spherical harmonic expansion of *V*, also known as the International Geomagnetic Reference

Field, or IGRF. Because the terrestrial geomagnetic field is time-variant, and therefore so are the IGRF model coefficients, periodic updates of this geomagnetic coordinate system must be made. The IAGA has published coefficients for past epoch years of the reference model as far back as 1900, and offer a forecast set of coefficients (of lesser order) that are valid from the current epoch to five years in the future (2010 as of the time of this writing).

The origins of the work presented in this paper date back to the 1950s. Hultqvist [1958a; 1958b] defined a "corrected" magnetic coordinate system that took some of the higher order terms of the spherical harmonic expansion of the IGRF Epoch 1945 model into account. Hultqvist's methodology involved tracing geomagnetic field lines from points on the Earth's surface to a centered dipole equator. Each point is then defined to be equivalent to a line trace along a centered dipole field. The latitude and longitude of each point in dipole coordinates were defined as the corrected geomagnetic (CGM) coordinates.

Hakura [1965] also used the higher order terms of a subsequent magnetic field model, and he used these to compute tables and maps of corrected geomagnetic coordinates. The lookup tables were used in lieu of actual field line trace calculations that were cumbersome to computers of the day. Gustafsson [1970] later updated these tables for IGRF 1965, and provided new tables for IGRF 1980 [1984] and later IGRF 1990 [1992]. It should be noted that these tables are valid *only* at the Earth's surface. There are areas on the Earth's surface where magnetic field lines traces never reach the dipole equator plane, and these areas increase with altitude. Gustafsson [1984] described a number of interpolation methods to try to fill out points in these areas.

True CGM coordinates are defined only at ground level, but a method is needed to provide coordinates at all altitudes. This report describes the latest version of such a method. As stated in Hein and Bhavnani [1996], the main precursor of this work, this method should be properly referred to as "altitude adjusted" rather than "altitude dependent." The conversion of points on the ground and at arbitrary altitudes are both referred to as "corrected geomagnetic coordinates." Since the same procedure of field line tracing to the Earth centered dipole equator applies, all points along a field line (on one side of the magnetic equator) have the same corrected geomagnetic coordinates. In practice, for non-zero altitudes, the approach taken is to trace down to a point at zero altitude, and then look up conventional CGM coordinates and interpolate using the tables printed in the above references. For past epochs, the lookup and interpolation procedures have been automated in routines that have been referred to as CGLALO (Corrected Geomagnetic **La**titude and **Lo**ngitude).

For non-zero altitudes at or near the magnetic equator, the field lines trace down to higher geomagnetic latitudes. The higher the altitude, the greater is the separation of the foot of the field line from the dip (dipole) equator, such that low CGM latitudes do not exist for non-zero altitudes, and a significant discontinuity in latitude is present.

Baker and Wing [1989] computed corrected geomagnetic coordinates using a different methodology. Using Gustafsson's tables for IGRF 1980 as a starting point, they used the IGRF 1985 magnetic field model along with its secular variation estimates (the five year "forecast" coefficients) and produced updated coordinates valid for 1 January 1988. In their work, they evaluated the X, Y, and Z components of a unit vector that were obtained using a set of fourth order spherical harmonic functions. They computed spherical harmonic coefficients that represented the three unit vector components in magnetic dipole coordinates at various altitudes (0, 150, 300, 450 km) for the transformation from geocentric to corrected geomagnetic as well as the inverse. They also developed an interpolation and extrapolation scheme for computing the spherical harmonic coefficients at arbitrary altitudes valid between 0 and 600 km. Baker and Wing were primarily concerned with middle to high CGM latitudes and therefore the features of the South Atlantic Anomaly and the equatorial region were not well represented (in fact, Gustafsson [1980] did not extend his calculations below 24 degrees geographic latitude, and there were undefined areas near the magnetic poles). In Baker and Wing [1989], the spherical harmonic computations were performed in dipole magnetic coordinates, and the conversions between geographic and dipole coordinates were accomplished using rotation matrices.

Bhavnani and Hein [1994] took a similar approach to that of Baker and Wing [1989] but were able to provide a better representation of the equatorial area and the South Atlantic Anomaly. Spherical harmonic fits for direct and inverse transformations were performed for 0, 300, and 1200 km, and the altitude dependence of the spherical harmonic coefficients was expressed as a quadratic interpolation and extrapolation to the individual coefficients using a normalized altitude (altitude/1200 km) as the independent variable. The quadratic fit is uniquely determined by the values at the three altitudes, and the representation of the spherical harmonic coefficients by the fit at those altitudes is exact. Later, Hein and Bhavnani [1996] expanded the altitude range covered in their previous work from 2000 km to 7200 km.

The last definitive revision of the IGRF model was done in 2005. Up until Epoch 2000, the IGRF model was a tenth order model (n = 10), and its precision was quoted at 1 nanoTesla. The current standard model, however, has been revised to be of order thirteen (n = 13), and the precision has increased an order of magnitude to 0.1 nanoTesla. In this work, we attempt to update Hein and Bhavnani's work by using the IGRF 10th generation model coefficients for Epoch 2005[1]. We did not attempt to increase the order of the spherical harmonic coefficients used by the existing codes from ten to thirteen. However, in the process, we have developed an alternative to Hein and Bhavnani's usage of a spline routine to solve the equatorial discontinuity problems found near the South Atlantic Anomaly when transforming from geographic to geomagnetic coordinates.

2. BASIC METHODOLOGY

For the analysis of observations for satellites in circular or near circular orbits, the use of CGM tables and interpolation methods for a fixed altitude, such as found in Hein and Bhavnani's CGLALO 1995 code, are well suited. For satellites in more eccentric orbits, and other applications at non-uniform altitudes, a functional representation in terms of a spherical harmonic expansion, such as implemented by Baker and Wing [1989] is more appropriate, because a single routine can inherently interpolate smoothly over the entire region of space of interest. The equatorial discontinuity problem is handled by using an auxiliary coordinate system

[1] See http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html for updated model coefficients.

(magnetic dipole coordinates at altitude) to compute the spherical harmonic coefficients which are incorporated into the code. A simple mapping is used to transform to and from dipole coordinates at altitude and dipole coordinates at 0 km altitude.

In using a spherical harmonic representation of a function defined on a spherical surface, where the function is initially specified by a table of values, there must be sufficiently dense data in the table, and the order of the spherical harmonic expansion must be chosen to adequately represent the function at the tabulated values.

The spherical harmonic coefficients for a function *f*, $a_{l,m}$, are usually computed from the following integrals (Equation 2):

$$a_{l,m} = \int_{\Omega} f(\theta,\phi) Y_{l,m}(\theta,\phi) d\Omega \tag{2}$$

To compute these integrals a completely defined uniform grid is necessary. Hein and Bhavnani [1994] found that a table of values between -88 and 88 degrees latitude at 2 degree intervals, and 0 – 350 degrees in longitude at 10 degree intervals is adequate for a tenth order spherical harmonic expansion. All the computations described here were made with such a coordinate grid, and for the remainder of this paper, we shall refer to it as the standard coordinate grid.

A significant aspect of spherical harmonic fitting is the problem of convergence. In the theory of Fourier series, there is a problem with convergence of the partial sums of the Fourier expansion for a function in the vicinity of a discontinuity. A typical example is the case of a step function, in which the partial sums oscillate in the vicinity of the discontinuity. Similar, but less pronounced behavior occurs when the function to be represented is continuous but has a discontinuous first derivative.

To avoid this problem, also known as Gibbs' phenomenon, the functions chosen for the spherical harmonic expansion must be periodic in longitude (or its equivalent) and have no discontinuities. For that reason, it is not practical to use the longitude variable itself. The simplest reasonable choice of functions are the complex exponentials $e^{i\theta}$ and $e^{i\phi}$ (or their real two-dimensional vector equivalents) where θ and ϕ are the co-latitude and longitude, respectively. However, using this approach, problems arise with the quality of the spherical harmonic expansion fit to the actual data near the magnetic poles. Hein and Bhavnani [1996] choose the unit vector approach used by Baker and Wing because the spherical harmonic expansion fit did not exhibit any anomalous behavior in the vicinity of the magnetic poles.

Since CGM values for non-zero altitudes have a discontinuity near the magnetic equator, it is not practical to use a ground-based dipole coordinate system for either computing the spherical harmonic coefficients or the spherical harmonic expansion. Hein and Bhavnani [1996] used an at-altitude dipole coordinate system at each of twenty selected altitudes between 0 and 7200 km to perform these computations. The altitude dependent mapping described above to transform between the actual CGM latitude λ_{CGM} and an at-altitude dipole latitude λ_{dipole} is given by Equation 3:

$$\cos^2 \lambda_{dipole} = \left(1 + \frac{altitude[km]}{6371.2}\right) \cos^2 \lambda_{CGM} \tag{3}$$

and is identical to that used in previous versions of the code. The use of the at-altitude dipole coordinate system given by the above transformation and its inverse "closes" the discontinuity, permitting the calculation of the spherical harmonic coefficients at the selected reference altitudes.

We compute the spherical harmonic expansion at twenty reference altitudes between 0 and 7200 km following the methodology of Hein and Bhavnani [1996]. Sets of spherical harmonic coefficients for the selected altitudes were fit to a fourth order polynomial fit, using altitude normalized by 7200 km as the independent variable.

For the computation of the geographic to CGM coordinates, the procedure was as follows:

1. Compute the altitude dependent spherical harmonic coefficients as needed (if new altitude was different than the last).
2. Compute the spherical harmonic expansion for the X, Y, Z components of the unit vector describing the orientation of the transformed point in the altitude dependent dipole coordinate system.
3. Compute latitude and longitude of point, and apply the altitude transformation to the at-altitude dipole latitude. Return the computed corrected latitude and longitude values.

For the inverse computation, the procedure used was as follows:

1. Compute the altitude dependent spherical harmonic coefficients for the inverse calculation as needed (if new altitude was different than the last).
2. Transform the CGM input latitude to the at-altitude dipole latitude. Set error return flag and default return value if input latitude was invalid.
3. Compute the spherical harmonic expansion for the X, Y, Z components of the unit vector describing the orientation of the transformed point in geocentric coordinate system. Return the computed geocentric latitude and longitude values.

3. GENERATION OF SPHERICAL HARMONIC COEFFICIENTS

The spherical harmonic coefficients were computed for the components of a unit vector in the target coordinate systems, as defined by the direction cosines for each of the selected altitudes. For each altitude, the coefficients were computed using standard coordinate grid tables using the standard formulas for computing spherical harmonic coefficients.

3.1. Geographic to CGM Transformation

We first create an initial set of geomagnetic grid points based upon the standard grid of geographic points as well as a complimentary set of geographic points based upon the standard grid of geomagnetic points. These sets of horizontal grids are calculated at twenty different latitudes ranging from 0 to 7200 kilometers; the altitudes are the same as those used in Hein and

Bhavnani [1996]. These points are generated using a modified version of the GEOCGM99[2] subroutine, along with parts of Nikolai Tsyganenko's GEOPACK library and other associated "helper" routines, which we will refer to collectively as GEOCGM 2005. The package uses different methods to compute points in both the equatorial and non-equatorial regions, in addition to having been updated with the new 10th generation IGRF model coefficients.

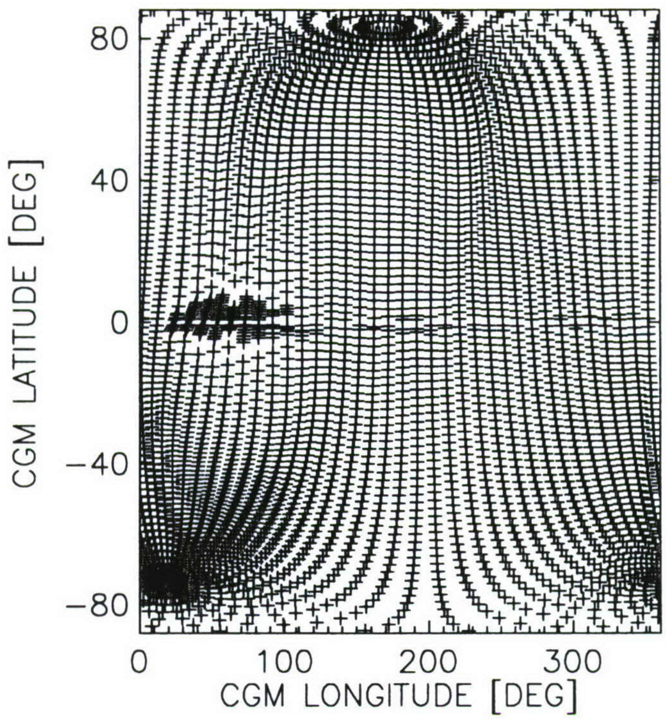

Figure 1. Corrected Geomagnetic Coordinate Grid from typical GEOCGM generated CGLALO tables for an array of CGM coordinates spaced in 2 degrees geographic latitude and 10 degrees geographic longitude. The gap near the geomagnetic equator and the area of the South Atlantic Anomaly are clearly visible.

The resulting set of points computed by GEOCGM 2005 at 0 km altitude can be seen in Figure 1. A small gap is evident near 0 degrees geomagnetic latitude; this is due to the inability of the code to compute points close to the geomagnetic equator. More evident is the South Atlantic Anomaly, which is the feature that can be seen in the region between roughly 20 and 120 degrees geomagnetic longitude. This region of points does not transform well into spectral space when trying to compute spectral harmonic transforms, so our endeavor is to somehow smooth out these points, as well as the points near the geomagnetic equator, before transforming the grid set into spectral harmonic coefficients.

For each geographic longitude in the grid, a separate code was used to compute the geographic latitude of the dip equator, such that the dip equator becomes the actual equator for the CGM

[2] Last found at http://science.msfc.nasa.gov/ssl/pad/sppb/workshop7/geomagne/geo_cgm/geo_cgm.for. See also the GEOPACK library at http://nssdcftp.gsfc.nasa.gov/models/magnetospheric/tsyganenko/Geopack_2005.for.

coordinate system. Note that the dip equator does not lie in a plane when viewed in geographic coordinates.

In Hein and Bhavnani [1996], the table of CGM coordinates that was created from the standard grid of geographic coordinates at 0 km altitude was altered by deleting the entries within a band of 15 degrees around the magnetic dip equator, and replacing those entries by using a spline fit, with the added constraint that the spline curve passes through the coordinates of the dip equator. The modified table was then incorporated into their IGRF 1995 version of the subroutine CGLALO (CGLALO 1995), which used a lookup table and interpolation procedure to compute CGM coordinates for arbitrary points at 0 km altitude. The results of this step are shown in Figure 2.

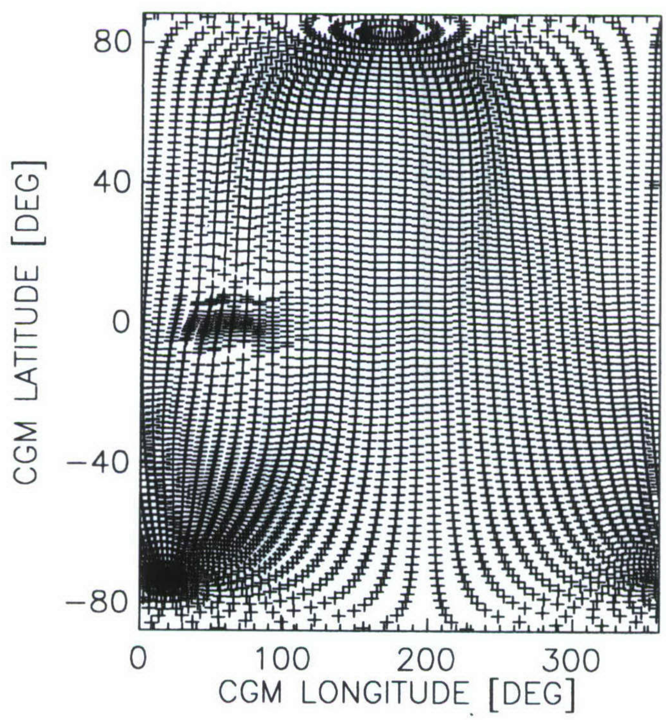

Figure 2. The points created by CGLALO 1995 as shown in Figure 1 after being processed with Hein and Bhavnani's original spline fitting routine.

While we continued the use of the lookup table and interpolation procedure, a problem that became evident while revisiting this process was that the spline fitted points often caused irregularities themselves, or missed areas outside the 15 degree band that should have been smoothed out. Simply extending the band around the dip equator where points were fitted would not have eliminated the creation of artificial irregularities in the grid, so we opted to devise a different method altogether.

We felt that by smoothing the spatial derivatives of the grid, rather than the grid points themselves, we would find a better set of initial grid points from which to create the spherical harmonic coefficients than those previously used, which had the effect of producing spherical harmonic transforms that often introduced large amounts of error when transforming from geographic coordinates to CGM coordinates and back again.

We decided to use arc length as a metric to compute smoothed curves of meridional derivatives. We scaled the great circle arc length between two points in CGM space with the great circle arc length between their two corresponding points in geographic space. Note that the arc length between these two geographic points is directly proportional to the constant two degree separation between latitude points; therefore the arc length in this case will be constant.

Given Points 1 and 2 on a sphere, each with a longitude and latitude coordinate, we calculate the arc length as

$$ds = R_e \, \Delta\sigma$$

where

$$\Delta\sigma = \tan^{-1} \frac{\sqrt{(\cos\phi_1 \sin\Delta\lambda)^2 + (\cos\phi_2 \sin\phi_1 - \sin\phi_2 \cos\phi_1 \cos\Delta\lambda)^2}}{\sin\phi_1 \sin\phi_2 - \cos\phi_1 \cos\phi_2 \cos\Delta\lambda}$$

and $\Delta\lambda = \lambda_2 - \lambda_1$ is the difference in longitude between Point 1 and Point 2, and ϕ_1 and ϕ_2 are the latitudes of Point 1 and Point 2, respectively. These arc length "derivatives" are depicted as a surface in Figure 3 in order to allow comparison between one constant longitude and the next. Wild variations of the arc length derivatives are clearly evident in the region of the South Atlantic Anomaly; less pronounced but still visible are variations near the dip equator.

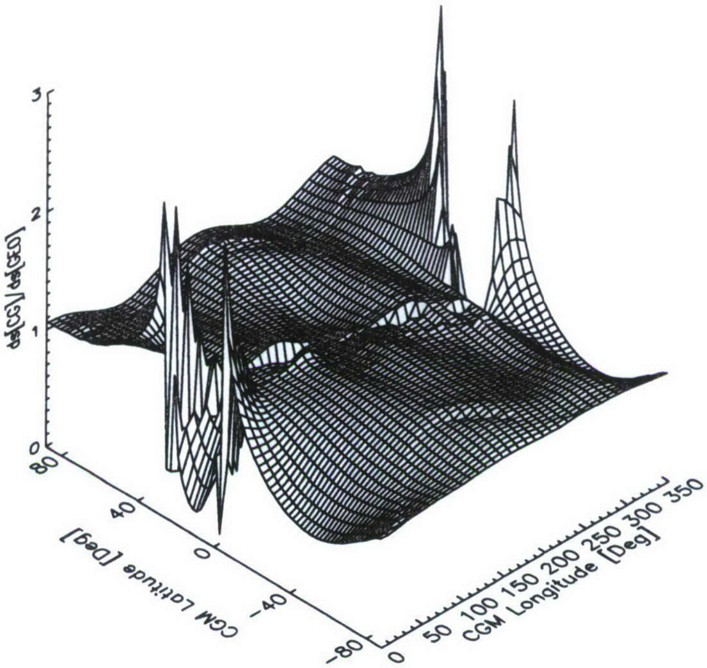

Figure 3. Rate of change of the arc length between two geomagnetic grid points and the arc length between two corresponding geographic grid points. Grid points are from 0 km altitude as generated from the original (1995) GEOCGM routine.

Each constant longitude arc length derivative curve is inspected, and the start and end points of the region of strong derivative variation is recorded. The derivative curves are then smoothed via second or third order polynomial line fit between the start and end points. Later, the original set of coordinate points along a constant longitude line is redistributed using the corresponding derivative curve as a guide, taking care to not alter the position of the dip equator in the process. The resulting set of points (for 0 km altitude) is then included as part of CGLALO 2005. The process is repeated for each altitude, although the variations tend to decrease with height and the task becomes much easier.

A surface plot of arc length derivatives computed after this smoothing process can be seen in Figure 4. The derivative curves crossing the equator have now been smoothed out with no real variation remaining. The derivatives near the South Atlantic Anomaly region have also greatly improved. Care is taken to not smooth out the features of the anomaly altogether as we feel this would be too unrealistic. Indeed, Hein and Bhavnani [1994] carefully considered the representation of this region when crafting their treatment of the area, as their aim was to not entirely remove the anomaly either. It should be noted that arc length derivative curves were computed using coordinates found in the lookup tables for past epochs (1980, 1995) to gain a sense of what correct derivative curves should look like.

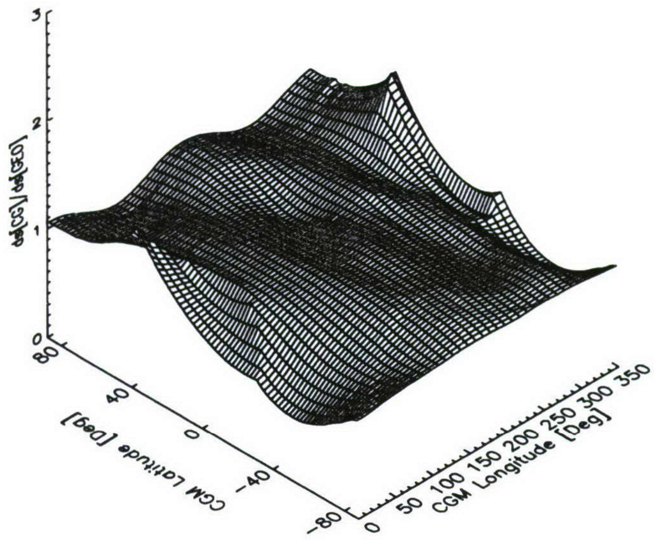

Figure 4. Arc length derivative curves from Figure 3 after smoothing and regridding the original coordinate distribution. Note the loss of the variations near the equator and South Atlantic Anomaly regions, but not the total loss of the anomaly.

For each of the other 19 non-zero altitude values, the field lines for the IGRF 2005 magnetic field model from altitude to ground were computed using a precise field line trace routine. CGLALO 2005 was then used to compute the CGM tables for the respective altitudes using the altitude adjustment algorithm. As before, this resulted in a uniform width to the altitude-dependent gap, and improved the spherical harmonic coefficient fits, which had the effect of eliminating "clumping" in the vicinity of the South Atlantic Anomaly.

The resulting 20 tables were used to generate the spherical harmonic coefficient fits for the selected altitudes. For each spherical harmonic coefficient, a fourth order polynomial fit was computed using altitude normalized by 7200 km as the independent variable, as was done in the past. In the fitting process, weighting of the various altitude terms was performed, in such a way that only small deviations were permitted in the 0 km altitude terms. The same set of weights was used for fitting both the forward and inverse coefficients.

3.2. CGM to Geographic Transformation

For the inverse transformation, an inverse altitude dependent routine was written, based upon the CGLALO 1995 inverse routine known as CGLLINV 1995. The original CGLLINV 1995 inverse routine used the CGLALO 1995 routine together with a Newton-Ralphson algorithm for computing the inverse. The CGLLINV 2005 routine duplicates the algorithm of a revised CGLLINV 1995 routine which uses the altitude dependent fourth order polynomial fit of the direct spherical harmonic fit. For each of the 20 altitudes, the desired tables are computed for a standard grid of CGM coordinates using CGLLINV 2005.

These resulting inverse tables were used to generate the required spherical harmonic expansion coefficients in the same manner as the forward coefficients, and the inverse tables were used to compute the required inverse fourth order altitude dependent polynomial spherical harmonic fits.

4. RESULTS

By replacing the spline fitting routine from Hein and Bhavnani's 1995 code with the arc length derivative algorithm, we attain additional improvements in the representation of the South Atlantic Anomaly and the equatorial region. Figures 5, 6, and 7 are graphs of CGM coordinates at 0, 800, and 7200 km altitude, respectively, as obtained from codes updated with the IGRF 2005 model coefficients and the arc length derivative routine. In Figures 6 and 7 there is a marked bending of the constant geographic longitude curves at the edge of the equatorial gap for certain longitudes. As explained in Hein and Bhavnani [1996], this bending is not an artifact, but reflects the CGM longitude variation of the lines of constant geographic longitude in the vicinity of the dip equator.

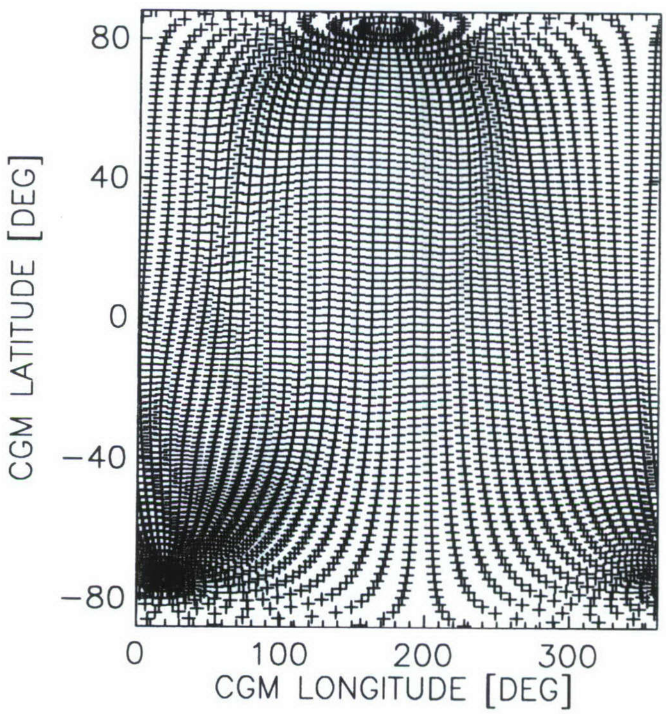

Figure 5. Grid for new routine, at 0 km altitude. The points have been smoothed using the arc length derivative approach.

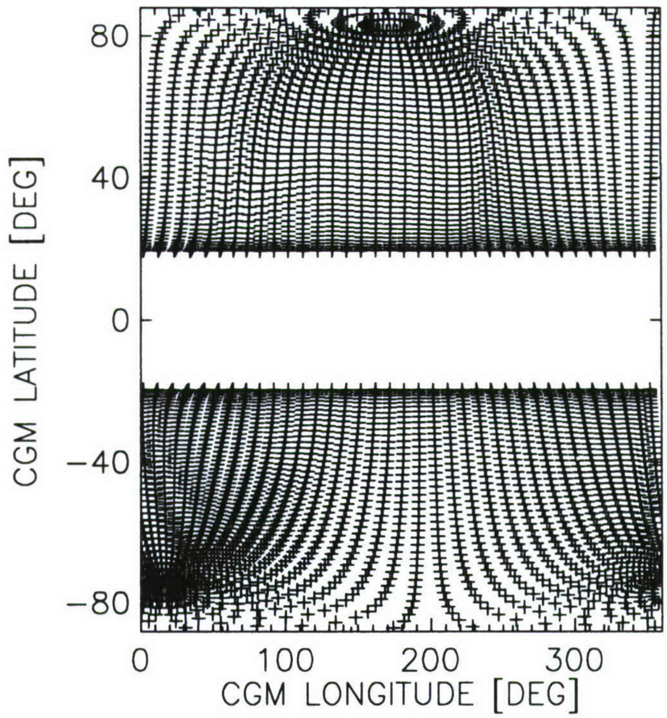

Figure 6. Same grid for new routine at 800 km altitude.

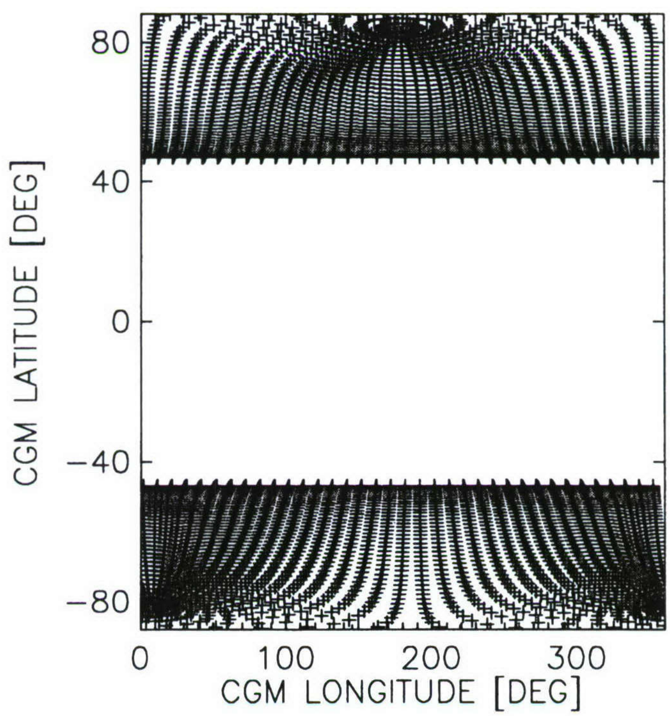

Figure 7. Same grid for new routine at 7200 km altitude.

Ideally, the output of the Geographic to CGM calculations, fed into the inverse computation, should reproduce the original coordinate grid. This test of the consistency of the direct and inverse transformations is illustrated in Figures 8, 9, and 10 for 0, 800, and 7200 km altitude, respectively. These graphs exhibit deviations from the uniform spacing of the original grid, particularly in the vicinity of the poles, and in the vicinity of the South Atlantic Anomaly.

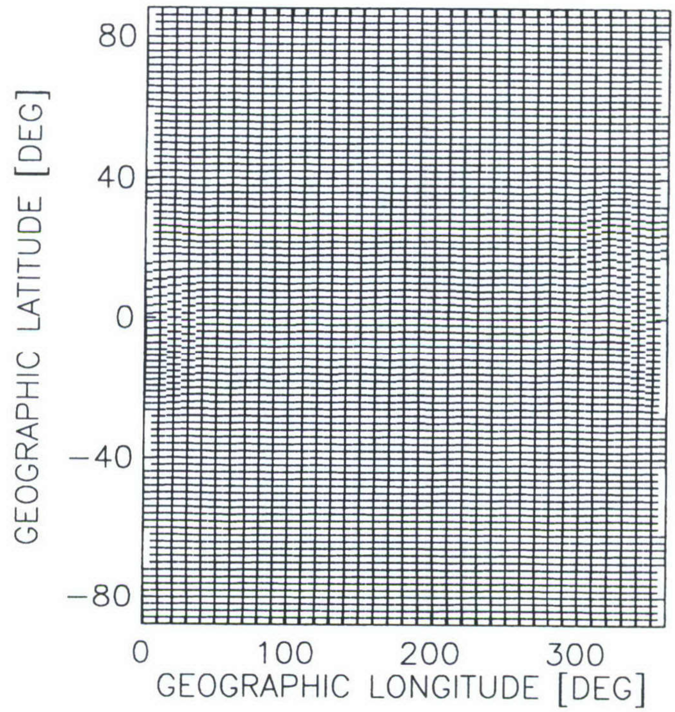

Figure 8. Same grid, showing consistency of direct and inverse conversion at 0 km altitude.

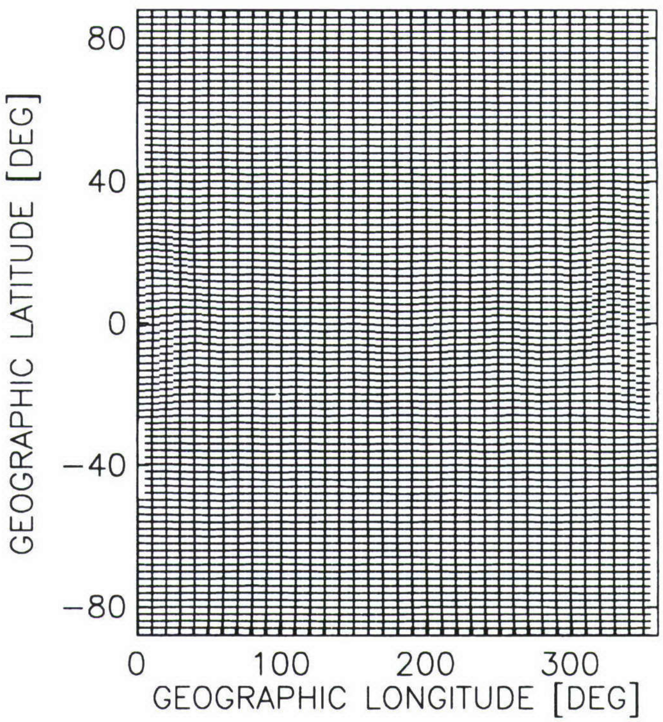

Figure 9. Grid showing consistency of direct and inverse conversion at 800 km altitude.

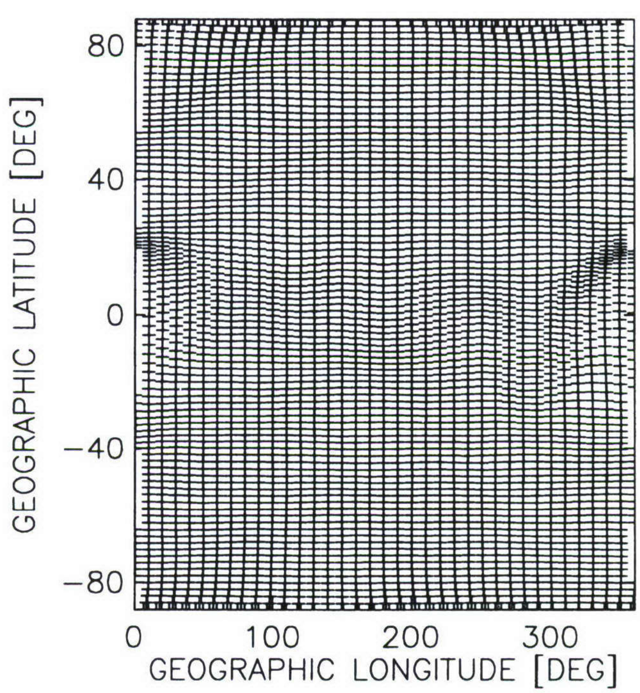

Figure 10. Grid showing consistency of direct and inverse conversion at 7200 km.

Since longitude differences at the poles are less relevant than at the equator, a more accurate measure of the differences between the original coordinates, and those obtained from the consistency calculation is the great circle arc between the coordinate pairs. Tables 1 and 2 provide the fraction of values which lie in the following error intervals (in degrees) for the Geographic to CGM to Geographic coordinate transformation and (where they exist) the CGM to Geographic to CGM coordinate transformation, respectively:

0 – 0.1, 0.1 – 0.2, 0.2 – 0.5, 0.5 – 1.0, 1.0 – 2.0 and > 2.0

Except for the standard problems near the South Atlantic Anomaly and near the "forbidden" band at altitude, the updated algorithm performs well throughout the 0 – 7200 km altitude regime. There is some slight distortion seen in the region of the South Atlantic Anomaly at 7200 km, and the reason for this is not immediately known. In a comparison of the distribution of errors at 7200 km altitude between Tables 1 and 2 it can be seen that errors are introduced when making the CGM to geographic transformation. In the future it may be necessary to try to restrict the deviations of the spherical harmonic coefficients at 7200 km as well as at 0 km when computing polynomial fits.

We did not attempt to extend the algorithm to process altitudes above 7200 km, as one could argue that above that altitude external field effects (i.e., ring currents) must be considered. These altitudes are not modeled by any code used here and are outside the scope of this work.

Table 1. Inversion error analysis. Geographic → CGM → Geographic. Fraction of errors in range (degrees).

Alt (km)	0.0 - 0.1	0.1 – 0.2	0.2 – 0.5	0.5 - 1.0	1.0 - 2.0	> 2.0
0	0.692572	0.191323	0.078964	0.021224	0.015918	0
100	0.715044	0.185705	0.067104	0.020287	0.01186	0
200	0.734082	0.167603	0.071785	0.020599	0.00593	0
300	0.736267	0.155743	0.085518	0.019039	0.003433	0
400	0.732522	0.147004	0.095194	0.023096	0.002185	0
500	0.730025	0.135144	0.105493	0.027778	0.001561	0
600	0.731586	0.125468	0.109863	0.027154	0.00593	0
800	0.733458	0.108926	0.107366	0.035893	0.014357	0
1000	0.729401	0.105805	0.104869	0.036517	0.019663	0.003745
1200	0.72191	0.108926	0.098627	0.041823	0.020287	0.008427
1600	0.705056	0.112984	0.095194	0.045256	0.02372	0.01779
2000	0.687266	0.117665	0.094881	0.047441	0.028714	0.024033
2500	0.668539	0.125156	0.092697	0.053683	0.029963	0.029963
3000	0.654182	0.125156	0.09613	0.057428	0.034332	0.032772
3500	0.628277	0.142634	0.098003	0.059301	0.035893	0.035893
4000	0.598002	0.167603	0.098003	0.060549	0.036829	0.039014
4500	0.580212	0.176342	0.10362	0.06211	0.035893	0.041823
5000	0.573346	0.177591	0.10799	0.058989	0.039014	0.043071
6000	0.560549	0.1804	0.112984	0.059613	0.041199	0.045256
7200	0.544007	0.188826	0.121411	0.057116	0.041823	0.046817

Table 2. Inversion error analysis. CGM → Geographic → CGM. Fraction of errors in range (degrees). Note: points for which the CGM input values were not valid were excluded.

Alt (km)	0.0 – 0.1	0.1 – 0.2	0.2 - 0.5	0.5 - 1.0	1.0 - 2.0	> 2.0
0	0.71654	0.175189	0.072917	0.022096	0.013258	0
100	0.777439	0.1521	0.049797	0.017615	0.003049	0
200	0.81802	0.122507	0.043447	0.016026	0	0
300	0.835892	0.109284	0.04386	0.010965	0	0
400	0.853228	0.095721	0.046171	0.00488	0	0
500	0.849099	0.100976	0.047673	0.002252	0	0
600	0.86034	0.092593	0.044753	0.002315	0	0
800	0.868254	0.085714	0.043651	0.002381	0	0
1000	0.875	0.082108	0.038807	0.004085	0	0
1200	0.875421	0.08165	0.037879	0.005051	0	0
1600	0.882616	0.080645	0.02957	0.007168	0	0
2000	0.870833	0.089352	0.031019	0.008796	0	0
2500	0.881944	0.085318	0.024802	0.007937	0	0
3000	0.875514	0.08642	0.028292	0.009774	0	0
3500	0.871795	0.089209	0.029915	0.009081	0	0
4000	0.865556	0.095556	0.031111	0.007778	0	0
4500	0.851273	0.116898	0.024884	0.006944	0	0
5000	0.829282	0.128472	0.030093	0.012153	0	0
6000	0.828914	0.143308	0.019571	0.008207	0	0
7200	0.68452	0.17791	0.06151	0.06548	0.01058	0

5. FIELD LINE TRACING, REVISITED

Field line tracing calculations are implicitly incorporated in the algorithm which generated altitude dependent spherical harmonic coefficients for the direct (Geographic to CGM) transformations. Thus, although the algorithm was not developed for use as a field line tracing routine, it is possible to use it for that purpose, since by definition the CGM coordinates along a field line (in either the Northern or the Southern CGM hemisphere) are constant. Such use is limited by the accuracy of both the Geographic to CGM and inverse computations, and the consistency of both these transformations.

The improvements in the consistency of the direct and inverse computations gained by the arc length derivative method over the spline fitting method can be seen in Table 3. Each column shows the percentage of field line traces over all points at all twenty altitudes that fit within the specified error ranges for each method. The arc length derivative method led to a 7 percent increase in the overall number of field line traces with errors fewer than 0.1 degrees. This can be explained by decreases in the percentages over all other error ranges, with decreases in some error ranges being quite substantial. Therefore, the revised algorithm exhibits a significant improvement for the purposes in field line tracing over the previous versions of the algorithm.

Table 3. Error distribution of the inversion error analysis data between the original spline fitting algorithm of Hein and Bhavnani [1996] and the arc length derivative method, in percent.

	0.0 – 0.1	0.1 – 0.2	0.2 – 0.5	0.5 – 1.0	1.0 – 2.0	> 2.0
Spline	58.0275	20.3745	11.6183	4.8408	2.372	2.7669
Arc length derivative	65.0578	20.0031	10.3995	3.2772	1.0004	0.2622

The use of the algorithm for field line tracing would proceed as follows:

1. Compute the CGM coordinates for the initial point (altitude, geographic latitude and longitude referenced to a spherical Earth).

2. Use the computed CGM latitude and longitude, and the desired altitude for the end point, as input to the inverse calculation. Note that if the end point altitude is greater than that for the initial point, it is possible that the field line trace never reaches the end point altitude. The error flags must be checked to exclude this possibility. If the error flag is zero, then the computed Geographic latitude and longitude is, to the accuracy limits of the algorithm, the desired end point of the field line trace.

We evaluate the performance of the field line tracing algorithm by using a precise field line tracing routine to compute the geographic coordinates and latitudes along a field line, and then using the algorithm to compute the corresponding CGM coordinates for each point. The latter should be the same at each point on the field line, therefore, the maximum angular deviation along a great circle arc should be a suitable measure of the accuracy of the algorithm.

A test of this type was performed for a uniform grid, for line traces from 7200 km to the ground, and for 800 km to the ground. The maximum angular deviation (great circle arc) along each field line traces were output at intervals of ~10 km in altitude. Maps which exhibit the regions in which the maximum deviation along a field line of greater than 0.2 degrees were also generated for the two cases described above. Different colors are used to indicate different "bins" for the maximum deviation along field line traces. Figures 11 and 12 are the maps for the 800 km – 0 km and 7200 km – 0 km cases, respectively. The increased region of errors as shown in Figure 12 for 7200 km as compared to the region of errors at 800 km as shown in Figure 11 correspond to the errors of the same altitudes as listed in Table 2.

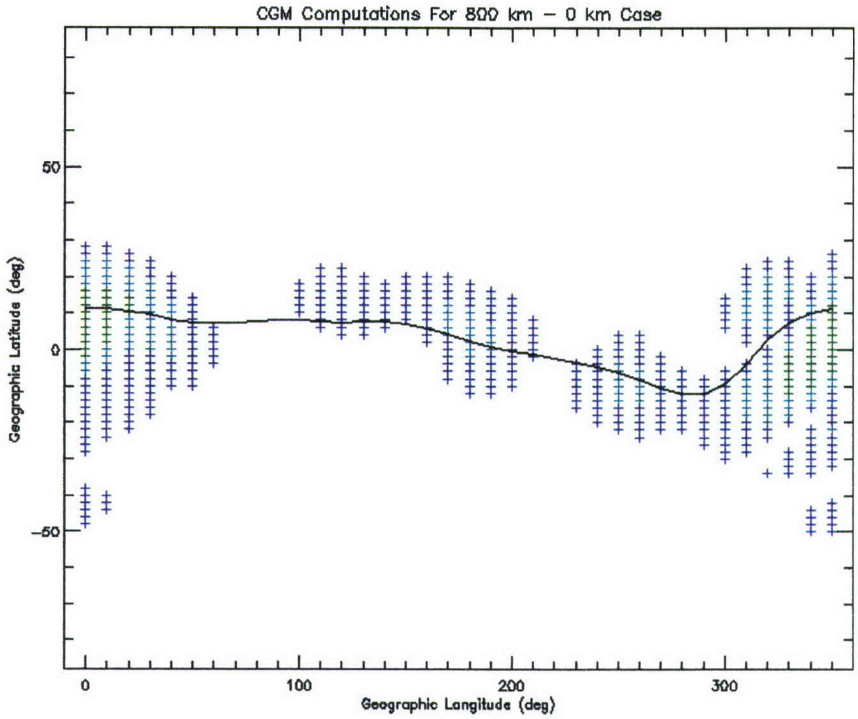

Figure 11. Map (standard grid, Geographic Coordinates) indicating grid points for which line traces from 800 km to ground exhibit a maximum deviation (great circle arc) greater than 0.2 degrees. Blue symbols: 0.2 – 0.5 deg; Light blue symbols: 0.5 – 1.0 deg; Green symbols: 1.0 – 2.0 deg; Red symbols: > 2.0 deg. The geomagnetic equator is shown as a solid black line.

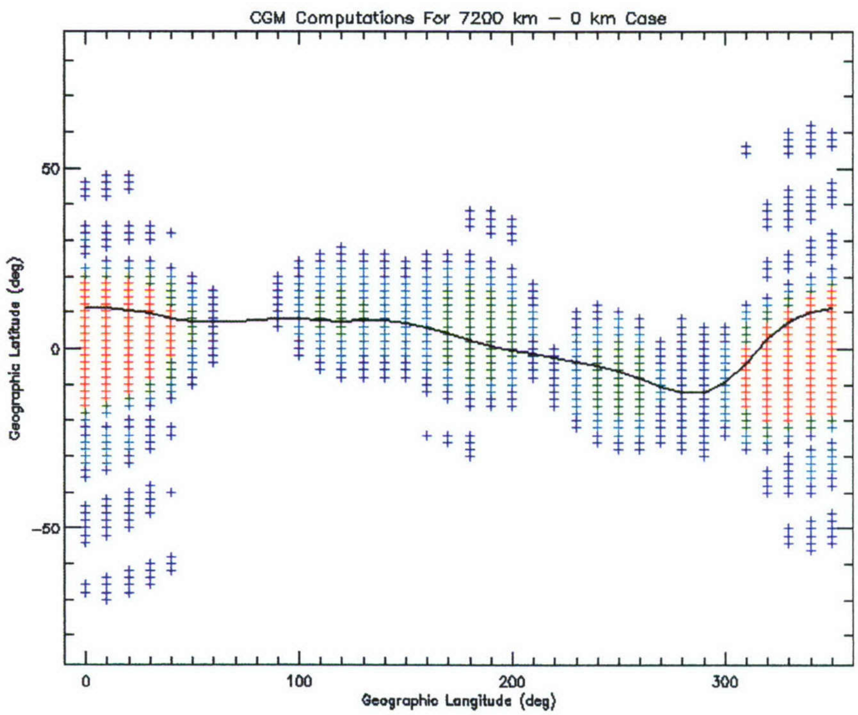

Figure 12. Same as for Figure 11, for field line traces from 7200 km altitude to the ground. The geomagnetic equator is shown as a solid black line.

The lack of a symbol at a grid point indicates that the maximum deviation along a field line beginning at the specified geographic coordinates and altitude and ending at the ground is less than 0.2 degrees. Otherwise, different colors are used to indicate the range of maximum deviation. As expected, the worst behavior for both cases is exhibited near the geomagnetic equator and in the vicinity of the South Atlantic Anomaly.

6. CONCLUSIONS

The IGRF 1995 version of the codes developed by Bhavnani and Hein [1996] have been updated by replacing the pre-existing set of spherical harmonic coefficients with an updated set based on the IGRF 2005 model. The revised version retains the Cartesian spherical harmonic approach of Baker and Wing [1989] for geographic to corrected geomagnetic coordinate conversion and the inverse and continues to make use of auxiliary coordinates (dipole coordinates at altitude) that are derived by applying a simple altitude adjustment algorithm to the CGM latitudes. In this auxiliary coordinate system the magnetic equator discontinuity described above is eliminated, permitting accurate fitting to tenth order spherical harmonic expansions.

The revised version differs from previous versions by performing a fourth order polynomial fit to the spherical harmonic expansions generated at 20 fixed altitudes for both the direct and inverse transformations. The new 2005 version also differs from the 1995 version by using a new technique to minimize or eliminate discontinuities near the South Atlantic Anomaly and the geomagnetic equator while performing the geographic to geomagnetic coordinate transformation.

An "arc length derivative" approach was introduced which is intended to replace Hein and Bhavnani's use of a simple spline fitting routine to solve the problem of the discontinuities. The spline fitting routine itself was found to cause irregularities which produced problems in the subsequent mapping of horizontal grids to spherical harmonic coefficients.

The arc length derivative approach calls for computing the great circle arc length between two CGM points on a constant CGM longitude line, normalizing the CGM arc length with the great circle arc length between the two corresponding geographic points, and smoothing the resulting derivative curves. The derivative curves are then used to redistribute the actual CGM points along the constant CGM longitude line, taking care to not alter the position of the geomagnetic equator.

The tables of spherical harmonic coefficients resulting from the use of the arc length derivative algorithm with the rest of Hein and Bhavnani's procedures were tested using the same methods as listed in Hein and Bhavnani [1996]. The errors generated by performing direct and inverse transforms using the new tables were found to be equal to, or in most cases less than, those that would be generated by using Hein and Bhavnani's tables of spherical harmonic coefficients as generated by their original methodology.

The update to the new CGM computation, beyond the inclusion of the arc length derivative algorithm, was accomplished by simply replacing the existing sets of spherical harmonic expansion coefficients with new ones that correspond to the IGRF 2005 magnetic field model.

The procedure for computing the new sets of coefficients was described in Section 3. In the FORTRAN code implementation of the new algorithm, this was accomplished by replacing the existing BLOCKDATA section of the source code containing the spherical harmonic expansion coefficients table with a new table. The CGLALO lookup table was also updated using the latest set of CGM coordinates, produced in the process of generating the updated set of expansion coefficients.

7. REFERENCES

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APPENDIX: GLOSSARY AND NOTES

The Earth's magnetic field arises from contributions both within and external to it. For most near Earth applications (typically one Earth radius), the external field may be ignored. The internal field is described in terms of its geomagnetic potential, and is available in mathematical form as spherical harmonic coefficients and their secular variations. This model is the responsibility of the International Association of Geomagnetism and Aeronomy (IAGA), and is published periodically as revisions to the International Geomagnetic Reference Field (IGRF). Those more interested in the history and development of this subject are referred to the classic book Geomagnetism by Chapman and Bartels which in turn describes the original work of Gauss, Schmidt, and many others.

Ionospheric phenomena near Earth are intimately controlled by the Earth fixed geomagnetic field and, because of the substantial and unnecessarily repetitive calculations involved in field line tracing, simplified models and procedures become essential. Although the dipole and offset dipole models can be determined directly from the first and second order terms of the IGRF spherical harmonics and useful for conceptual purposes, field line traces cannot be inferred with adequate accuracy from these models. Fortunately, the internal geomagnetic field is Earth fixed, and extensive a priori computations can be carried out to provide tables which relate geographic locations to their corresponding field line trace environment. This approach was used by Hultqvist to define and introduce Corrected Geomagnetic Coordinates, and subsequently revisions were made by Hakura and Gustafsson. Their work defines these coordinates with tables at the surface of Earth only. Later work leading to our present effort is described in the main text of this report.

Below we describe many of the terms covered or related to the present work. The asymmetrical nature of the geomagnetic field has given rise to the need for dipole, eccentric (or offset) dipole, corrected geomagnetic, and dip-pole representations, all of which are distinct in some manner. Thus, for instance, geomagnetic field lines are not truly perpendicular to Earth's surface at the corrected geomagnetic poles, but rather at the dip-poles. The reader should also be aware that the field undergoes a secular variation, and the assorted magnetic poles migrate one to a few kilometers per year.

Altitude Dependent Corrected Geomagnetic Coordinates

Extension of Corrected Geomagnetic Coordinates to altitudes above the Earth's surface. Defined so that all points along a field line in the Northern or Southern hemisphere (in this paper, defined as the dip equator) possess the same coordinates. Not part of Hultqvist's original definition of CGM coordinates.

Corrected Geomagnetic (CGM) Coordinates

Earth fixed magnetic latitude and longitude. Altitude is undefined. Prescribed by

Hultqvist, Hakura, and Gustafsson and used in this report. Entails tracing along field lines to the dipole equator, and then determining the geomagnetic coordinates corresponding to this point on the dipole equator as if it had been reached by tracing along a pure dipole. Zero corrected geomagnetic longitude is the meridian which passes through the geographic South Pole, with East positive.

Corrected Geomagnetic (CGM) Coordinate Poles

Locations in the polar regions from where internal geomagnetic field line traces effectively intercept the dipole equatorial plane at an infinite distance. For Epoch 2005, the north and south corrected geomagnetic poles are at 82.05 N latitude and 277.1 E longitude, and at 74.21 S latitude and 126.11 E longitude, respectively. Inversely, the corrected geomagnetic coordinates of the geographic north and south poles are at 83.11 N latitude and 170.38 E longitude, and 74.06 S latitude and 18.88 E longitude, respectively.

Dip Equator

The plane at low latitudes where Earth's field becomes horizontal, so that the magnetic dip angle is zero. This resolves the problem with the Hultqvist procedure of tracing to the dipole equator, which results in imaginary latitudes when the field lines terminate inside 1 Earth radius. Field lines undulate and the geographic latitude corresponding to zero CGM latitude sometimes had to be estimated by curve fitting.

Dip Poles

North and south polar locations where the geomagnetic field at Earth's surface is vertical. For Epoch 2005, these are reported as being 83.21 N latitude and 118.32 W longitude, and at 64.53 S latitude and 137.86 E longitude, respectively.

Dipole

Simple first order Earth-centered representation of the geomagnetic field. Calculated by using the first three Gauss coefficients. For Epoch 2005, the first order X, Y, Z dipole moments are 1671.8, -5080, 29556.8 nT respectively, which places the dipole north magnetic pole roughly at 79.74 N latitude and 71.78 W longitude, and the south magnetic pole at 79.74 S latitude and 108.22 E longitude. The plane through the center of Earth normal to this axis determines the dipole equator.

Dipole or Eccentric Dipole Equator

Since the eccentric offset is roughly the plane of the pure Dipole equator, the same equatorial plane through Earth's center, normal to the axis of the poles, applies to Dipole or Eccentric Dipole. See Dipole and Eccentric Dipole.

Dipole Poles

North and south intercepts of dipole axis with Earth's surface. See Dipole.

Eccentric (Offset) Dipole

Field lines do not trace out normal to Earth's surface at the dipole poles, but considerably removed particularly for the South geomagnetic pole, which implies an eccentric rather than centered dipole. The first eight Gauss coefficients found in the representation of Earth's field in

IGRF 2005 suggest an offset dipole axis centered approximately at 522 km, 22.2 N latitude and 141.6 E longitude.

Eccentric Dipole Poles

In 2001, the North and South intercepts of eccentric dipole axis with the Earth's surface, was measured to be at 82.03 N latitude and 93.3 W longitude, and at 75.34 S latitude and 118.66 E longitude, respectively. The IGRF 2005 model predicts the North intercept as 82.7 N latitude and 114.4 W longitude. See Eccentric Dipole.

Geocentric

Earth centered. Since latitude is defined by the angle between the vector to the location and the equatorial plane, geocentric coordinates imply a spherical earth model. Magnetic field models, such as the International Geomagnetic Reference Field (IGRF), use 6371.2 km as the mean Earth radius to normalize radial distance and, for convenience, geocentric altitude refers to this radius when describing particle locations and the geomagnetic environment.

Geodetic

Refers to oblate Earth and the local horizontal plane. Not used or implied in this report.

Geographic Coordinates

Earth fixed latitude, longitude, and altitude. Although commonly loosely applied to geodetic or geocentric, the spherical 6371.2 km reference radius applies throughout this report, with all latitudes, altitudes, and field line trace terminations determined by this model. Geodetic and geocentric longitudes are identical, with 0 degrees passing through the Greenwich meridian, and East positive.

Geomagnetic Coordinates

Earth fixed magnetic dipole latitude and longitude. Altitude is undefined. The pure dipole axis is tilted with respect to Earth's axis and the poles approach the magnetic poles. Zero magnetic longitude is the geomagnetic meridian which passes through the geographic South Pole, with East positive.

Inverse Coordinate Conversion

Obtains geographic coordinates, given corrected geomagnetic coordinates. Reverse of the geographic to corrected geomagnetic coordinate conversion. Since the conversions are altitude dependent, the altitude at which the geographic coordinates are desired must be specified. Except for the fitting approximations arising from analytical modeling, inversion following a coordinate conversion should return to the original latitude and longitude.

South Atlantic Anomaly

A region over the South Atlantic Ocean, off the coast of Brazil, where the shielding effect of the magnetosphere is not spherical and exhibits a dip or "pothole." This is a result of the eccentric placement of the center of the magnetic field from the center of the Earth as well as the displacement between the magnetic and geographic poles.