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Report Title

Group Theoretical Approach for Controlled Quantum Mechanical Systems

ABSTRACT

The aim of this research is the study of controllability of quantum mechanical systems and feedback control of de-coherence in order to gain an insight on the structure of control of quantum systems. During the reporting period, many important results have been obtained. Particularly, we have obtained results on (1) Controllability of quantum systems possesses infinite-dimensional system Lie-algebra, and (2) Feedback control of de-coherence in open quantum systems. Algebraic conditions for controllability and the control of de-coherences were obtained. To our knowledge no such conditions existed in the literature for quantum mechanical systems. The obtained results are very important. They will guide us on how to control a quantum system with scattering states (possesses continuous spectrum) and help us to design feedback control for quantum systems. Additionally we have published our results in high standard journals and presented our results at prestigious conferences. The PI was invited to give invited and plenary addresses at important control conferences.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

- (1) J. Zhang, C.W. Li, Re-Bing Wu, T.J.Tarn, and X.S. Liu, "Maximal suppression of decoherence in Markovian quantum systems," Journal of Physics A: Math. Gen., Vol.38, 2005, pp6587-6601.
- (2) Chunhua Lan, Tzyh Jong Tarn, Quo-Shin Cui, and John W. Clark, "Analytic Controllability of Time-Dependent Quantum Control Systems," Journal of Mathematical Physics, Vol.46, No.5, May, 2005, pp.052102-1 to 21.
- (3) Re-Bing Wu, T.J.Tarn, and C.W. Li, "Smooth controllability of infinite-dimensional quantum-mechanical systems," Physical Review A, Vol. 73, 2006, pp.012719-1 to 11.
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- (6) J. Zhang, R.B. Wu, C.W. Li, T.J. Tarn and J.W. Wu, "Asymptotically Noise Decoupling for Markovian Open Quantum Systems," Physical Review A, Vol. 75, 2007, pp.022324-1 to 11.
- (7) Narayan Ganesan and T.J. Tarn, "Decoherence Control in Open Quantum Systems via Classical Feedback," Physical Review A, Vol. 75, 2007, pp.032323-1 to 19.
- (8) M. Jiang, Z.K. Zhang, and T.J. Tarn, "Quantum Network Optimization Based on the Use of Relaxing Qubits," in Lecture Notes in Artificial Intelligence, Vol.4114, pp.838 to 843. Editors: D.S. Huang, K. Li, and G.W. Irwin, Springer-Verlag, 2006.
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- (10) Jiang Min, Zhang Zeng-Ke, Dong Dao-yi, Liu Bin, and T.J.Tarn, "The Improved Quantum Switching Mechanism Based on Contention," Chaos, Solitons & Fractals, Accepted for publication.
- (11) J. W. Wu, C. W. Li, T. J. Tarn, and J. Zhang, "Optimal Bang-Bang Control for $SU(1,1)$ Coherent States," Physical Review A, Accepted for publication.

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- (1) Chunhua Lan, Tzyh Jong Tarn, Quo-Shin Cui, and John W. Clark, "Strong Analytic Controllability for Hydrogen Control Systems," 43rd IEEE Conference on Decision and Control, December 14-17, 2004. Atlantis, Paradise Island, Bahamas.
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- (4) N. Ganesan and T.J. Tarn, "Feedback Control of Decoherence: Systems Theoretical Approach," Proceedings of the 25th Chinese Control Conference, pp.1 to 3, Harbin, China, August 2006.
- (5) T. J. Tarn and N. Ganesan, " Feedback Control of Decoherence in Quantum Mechanical Systems," Proceedings of the SICE-ICASE International Joint Conference, pp.1844 to 1847, Busan, Korea, October 2006.
- (6) Narayan Ganesan and T.J.Tarn, "Quantum Internal Model Principle and Enhanced Disturbance Decoupling," To appear in the Proceedings of the 46th IEEE Conference on Decision and Control, New Orleans, LA, Dec. 2007.
- (7) T. J. Tarn and Narayan Ganesan, "Quantum Computing and Information Acquisition," To appear in the Proceedings of the International Symposium on Micro-Nano Mechatronics and Human Science, Nagoya Japan, Nov. 2007.
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Number of Manuscripts: 0.00

Number of Inventions:

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Narayan Ganesan	1.00
FTE Equivalent:	1.00
Total Number:	1

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Narayan Ganesan	0.50
FTE Equivalent:	0.50
Total Number:	1

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Tzyh Jong Tarn	0.17	No
FTE Equivalent:	0.17	
Total Number:	1	

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

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Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

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The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

<u>NAME</u>
Narayan Ganesan
Total Number:

Names of personnel receiving PHDs

<u>NAME</u>
Narayan Ganesan
Total Number:

Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Inventions (DD882)

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Statement of the Problem Studied:

This research is an investigation of the use of geometric concepts utilized in the study of classical linear and nonlinear systems, which proves to be very useful in understanding the design problems such as disturbance decoupling and non-interacting control, in the quantum domain. It is recognized that successful implementation of quantum computation rests upon efficient control of de-coherence in the processing of quantum information. In this work the study is intended to control the de-coherence problem in quantum systems. Specifically, it is proposed to investigate the controllability and quantum non-demolition observations and subsequently using results obtained to gain an insight on the structure of quantum systems and determine whether the control of the de-coherence problem is solvable? We are looking for a controlled-invariant sub-manifold that contains the image of the interacting Hamiltonian connecting the environment and the controlled system. If this controlled-invariant sub-manifold is contained in the null space of the quantum non-demolition observable, then the problem is solvable; otherwise the problem does not have a solution. If the problem is solvable, then from the solvability condition one can derive algorithms for controlling the de-coherence problem.

During the reporting period we have concentrated our investigation on the question of controllability for a quantum control system in which the Hamiltonian operator which is not under the control of an external agent carry explicit time dependence. We consider the general situation in which the state moves in an infinite dimensional Hilbert space, a drift term is present, and the operators driving the state evolution may be unbounded. So getting the new results requires dealing with the unbounded operators that describe the dynamics. A relatively straightforward strategy allows the extension of Lie-algebraic conditions for strong analytic controllability derived by the PI earlier for the simpler, time-independent system in which the drift Hamiltonian and the interaction Hamiltonians

have no intrinsic time dependence. Enlarging the state space by one dimension corresponding to the time variable, we construct an augmented control system that can be treated as time-independent. Methods developed by Kunita can then be implemented to establish controllability conditions for the original time-dependent Schrodinger control problem. The end result gives a useful criterion that can be used to check complete controllability of concrete systems [2, 3, 4, 13.].

To apply the obtained result to a very important yet manageable system, the hydrogen atom, the realization and representation of Lie Algebra, $so(4,2)$, associated with the hydrogen atom Hamiltonian are derived. By choosing operators from the realization of $so(4,2)$ as interacting Hamiltonians, a hydrogen atom control system is constructed, and it is proved that this control system is strongly analytically controllable based on the time-dependent strong analytic controllability theorem obtained here. It should be noted that the result obtained by us on the time dependent generators of $so(4,2)$ associated with the hydrogen atom is new. We are not aware of any existing results [12].

De-coherence, which is caused due to the interaction of a quantum system with its environment plagues all quantum systems and leads to the loss of quantum properties that are vital for quantum computation and quantum information processing. In this work we address the problem of control of de-coherence in open quantum systems via a classical state feedback. While most of earlier work in literature deals with analyzing the behavior of the reduced density matrix of an open quantum system and designing controls so as to counteract the effects of environment on the density matrix of the system as one would plainly expect, we decided take a different approach to this problem. We plan to “decouple” the quantum information of interest from the effects of the interacting environment or bath. This leads to the discovery of “Quantum Internal Model Principle” which will play a very important role in the control design [1, 6, 7, 14, 15, 16, 17, 18.].

The study of controllability using Lie algebraic techniques is, by now, no longer new. This statement is true for systems with only finite states and thus finite-dimensional. In fact, if the quantum evolution equation is finite-dimensional, as appeared in many recent investigations in the literature, most system theoretic results were worked out decades ago including controllability. The proposed investigation is for the general situation where a quantum evolution equation with Hamiltonians which may possess discrete, continuous, and mixed spectrum. For such a quantum system, the Hamiltonian operator may be non-compact. We are not aware of any investigation of such a system from the system theoretic control and observation point of view, incorporating the true quantum characteristics, existing in the literature. Our investigation here is to attack this difficult problem. Since our investigation is dealing with an infinite dimensional system with unbounded/non-compact Hamiltonian operators. It is both conceptually and mathematically very difficult. Based on our proven records, we are confident that this research will produce important results.

To study feedback control problem for de-coherence of quantum mechanical systems, the challenges we faced are: (1) Lack of proper tools in the conventional literature to address feedback in quantum systems and to analyze the controllability effects of de-coherence

on the quantum system which we developed at the outset to apply in our work. (2) In order to understand the fundamental effects of de-coherence on controllability and decouplability, both finite, infinite dimensional systems are of equal importance and the theory of infinite dimensional systems is generally more involved and non-trivial as stated above.

We are still continuing our study in this important field. Currently we study:

(1) Studying the manipulation of hydrogen atom—the simplest atom—can help us understand the manipulation of other atomic systems. In this study, by picking up operators from the generators of the Lie algebra of hydrogen atom as interacting Hamiltonians, and choosing piecewise constant functions as control, we have constructed an ideal hydrogen atom control system which is strongly analytically controllable. The results obtained so far did not consider design schemes for control inputs. Based on this first step, we plan to investigate optimal transitions from bound states to bound states, scattering states to scattering states, bound states to scattering states and vice versa. This will facilitate the selection of desired control inputs to the system. We have started the control design problem for a simple sub-system of the hydrogen atom, i.e., the sub-algebra $\mathfrak{su}(1,1)$ of the dynamic algebra $\mathfrak{so}(4,2)$ of the hydrogen atom to gain more insight on the design problem using Cartan decomposition[10, 11].

(2) The processes that lead to dissociation or ionization by light absorption is well known. The inverses of these processes are two-body recombination processes. The simplest process of this kind consists of a direct radiative transition from a scattering state belonging to a continuous range to a lower discrete bound state (see attached Figure 1). The scattering state corresponds to a collision between two atoms or radicals, or between an electron and an ion and the collision time is very small (of the order of 10^{-13} sec) compared to the radiative lifetime ($\sim 10^{-8}$ sec), the intensity of such emission continua is very low. It is difficult to establish this mechanism for a given observed emission continuum. In this project, we ask the question on “how to control such processes?” This is the controllability problem. From our investigation we observed that to control scattering states, the controllability Lie algebra of the system under consideration must be infinite dimensional. We have obtained some initial results in this direction. It is a nontrivial extension of the existing results of controllability based on the analysis over finite dimensional manifolds to infinite dimensional manifolds. Currently we are prepared to use the obtained results to find desired inputs to the system.

(3) Quantum computation [8, 9, 10.] clearly possesses several advantages over classical methods of computing. However quantum information is highly prone to de-coherence and noise. Environmental interaction plays a destructive role and prevents coherent processing of quantum information (see attached Figure 2 and 3). This is the biggest hurdle against practical realization of quantum computers. Fortunately methods in control and systems theory provide answers to control and eliminate de-coherence from a quantum system. Quantum feedback is assuming increasingly important role in control quantum systems and in quantum information processing. Extraction and acquisition of quantum information is pivotal in employing such feedback methods to control of

quantum systems. To that end non-selective measurements, zeno effect, and non-demolition observation play an important role as data acquisition tools. In this work we propose to analyze strategies and conditions for de-coherence free quantum control and invariance of output equation of the system with respect to the environmental interaction (see attached Figure 4). We will proceed to develop a general theory that will make finite, infinite and time varying quantum systems amenable to the results.

Summary of the most important results:

(1) The advantage for a quantum computer over a conventional computer is our ability to use quantum effects to improve the computation with a quantum computer. Our realization of a practical quantum computer is far away from reality although physicists have constructed systems with a few Qubits. The difficulty arises from the fact that quantum effects usually occur on the atomic scale have to be controlled on the macroscopic level. Our study of the controllability problem is from system theoretical point of view. We are investigating controllability by studying the structure of a quantum system. This will provide a deeper insight for us to understand how to control atoms and molecules from macroscopic level and thus to control the quantum computation.

(2) At the simplest of all atoms, hydrogen atom plays a central role in quantum mechanical theory. It is the only atom for which the energy eigenvalue problem has been solved exactly. Wave functions of the hydrogen atom are the starting point for the description of all atoms and molecules, the theory of the whole periodic table of the elements and atomic spectra. Understanding on how to control the hydrogen atom is the starting point for understanding on how to control more complicated atoms and molecules. From the design of a control scheme for atoms and molecules via perturbation/approximation method the solution of the control problem of hydrogen atom furnishes the first step. From energy spectrum point of view, hydrogen atom possesses both discrete (bound state) and continuous (scattering states) spectra. Understanding the control problem of hydrogen atom will have no doubt help us to understand the controlled transition from bound states to scattering states and vice versa-a long outstanding control problem in the laboratory.

(3) We propose a novel strategy using techniques from systems theory to completely eliminate de-coherence and also provide conditions under which it can be done so. The strategy not only helps us get to the root of the problem but also helps us design the solution. The elegance of this approach yields interesting results on open loop decouplability. This approach unlike the others does not aim at mitigating or slowing down the effects of de-coherence rather aims at completely eliminating it by feedback control. We have now learnt a great deal about behavior of open quantum systems and have also stumbled on a few interesting results regarding the nature of control Hamiltonians and the Internal Model Principle analog for quantum systems that is first of its kind in the literature. The above results might prove important in their own right and in due course of time will influence the design of future quantum control systems.

In summary, since our research is basic and fundamental in nature, the research findings obtained from this research project will undoubtedly stimulate and enhance close related research projects, such as quantum computation/materials, supported by the Army Research Office.

Conclusions:

During the reporting period vigorous and effective research activities have been carried out in the area of controllability of time-dependent quantum mechanical systems and quantum systems with infinite dimensional control Lie algebra. As an application of the obtained result on controllability we studied the control problem of hydrogen atom, a simple but important atom for understanding the control problem of other atoms and molecules. Additionally, we have made significant progress in understanding feedback control of de-coherence of quantum systems. We have produced research results of both initial and fundamental nature. The obtained results may be very important in the realization of practical quantum computer and the control of atoms and molecules. Since our research is basic and fundamental it may stimulate further research and enhance close related research projects, such as quantum computation/materials, supported by the Army Research Office.

Our research aims at future technology. The problem formulations on the control of quantum mechanical systems and the proposed approaches are entirely new. Our methods of solutions are based on sound mathematical theory and physical principles, and these methods can be applied to a broad class of problems in the area of control of quantum mechanical systems. Our results place many of the “known” results in the quantum control literature on a firmer mathematical footing, while at the same time extending these results to problems for which no satisfactory solution has yet been reported, despite various attempts.

We have presented three papers at CDC and our paper submitted to the Journal of Mathematical Physics was accepted within three months from the date of submission and was rated as an important contribution by the reviewers. Several additional papers also appeared in high quality Journals. The PI has been invited to give seminars and plenary addresses all around the world.

Bibliography:

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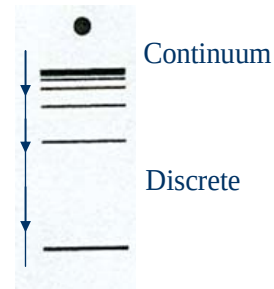
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- [18]T. J. Tarn and Narayan Ganesan, "Quantum Computing and Information Acquisition," To appear in the Proceedings of the International Symposium on Micro-Nano Mechatronics and Human Science, Nagoya Japan, Nov. 2007.

Recombination process

- Direct radiative transition from an upper state belonging to a continuous range to a lower discrete state.

➡ A continuous emission spectrum



- Upper state: Corresponds to a collision between two atoms or radicals or an electron and an ion (collision time 10^{-13} seconds radioactive lifetime 10^{-8} seconds).
- Currently very difficult to understand the mechanism of this transition due to difference in time scales. Hopefully based on our study a well controlled transition can be designed.

Figure 1

A Closed Quantum Control System

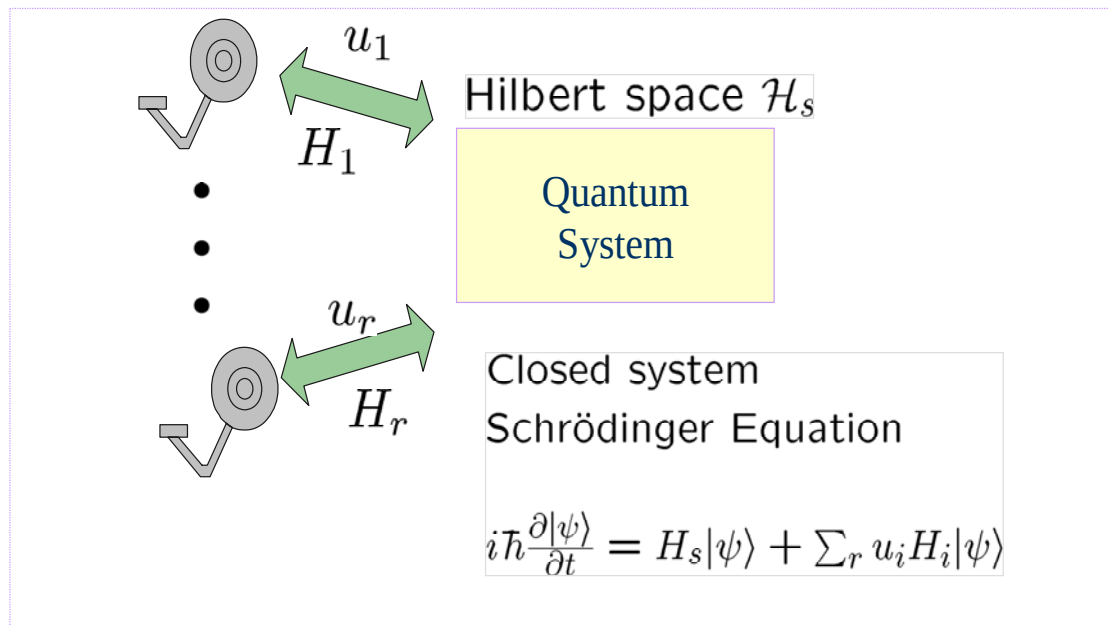
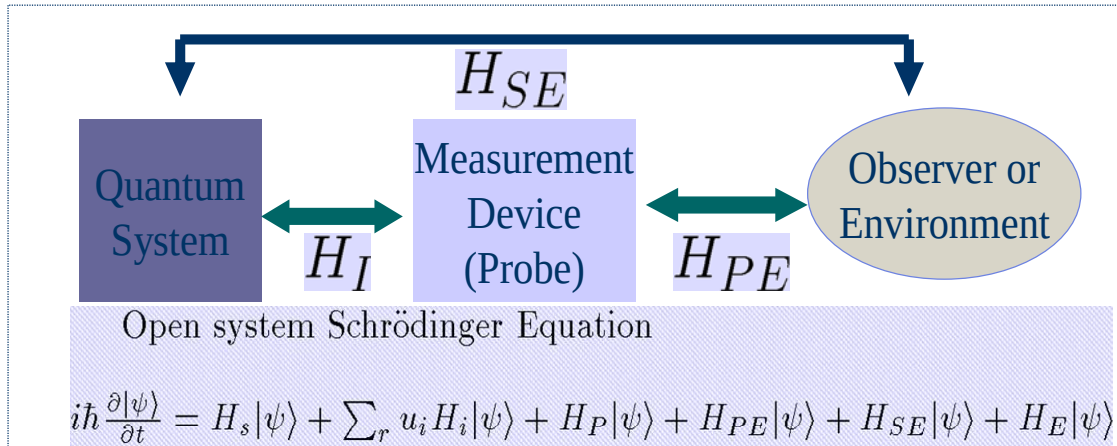


Figure 2

Open Quantum System Coupled to a Measurement device

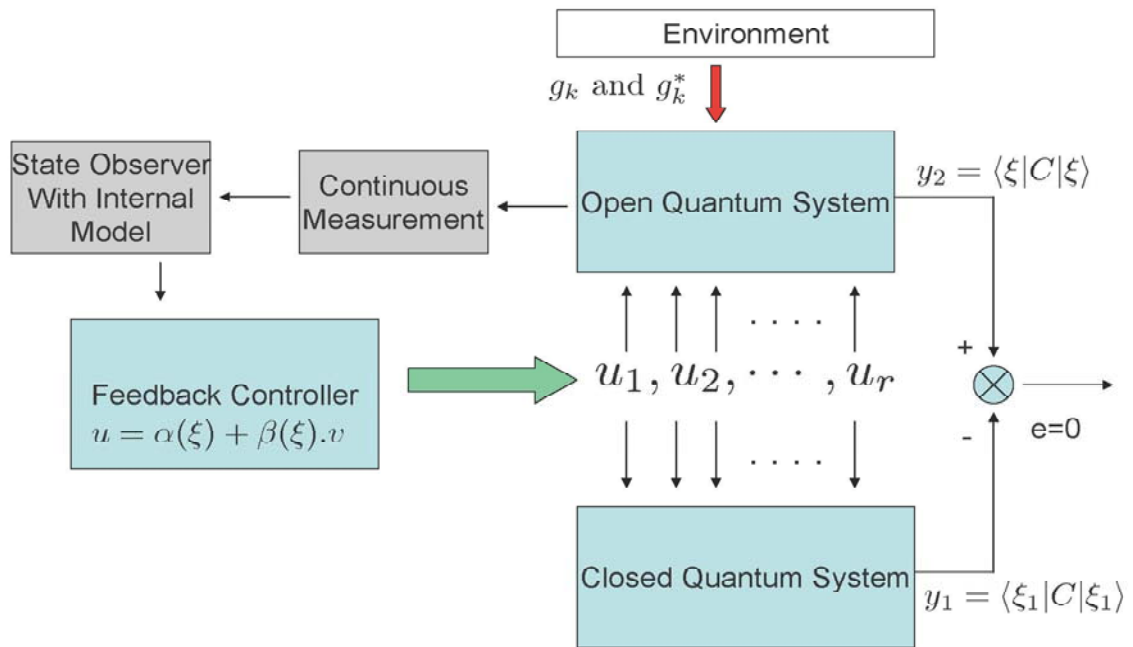


Evolves unitarily in System+Probe+Environment Hilbert Space

- Quantum Measurement can be explained at a non-selective level by the combined evolution of Probe+System in the presence of environmental interaction.

Figure 3

Internal Model Principle



In order to completely decouple we need to have a good estimate of the state of the system+environment as well as the model of the environment.

Figure 4