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ANALYSIS OF ANALYTIC MODELS FOR THE EFFECT OF INSURGENCY/COUNTERINSURGENCY OPERATIONS ON THE GENERAL POPULATION

by

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June 2008

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This thesis proposes and analyzes mathematical descriptive models of the effect of Insurgency/Counterinsurgency Operations on the population of a nation experiencing stability operations. The model is a system of differential equations representing insurgent activity, insurgent recruiting, insurgent removal by the coalition; the population's tolerance for insurgent violence; occurrence of actions by the coalition and insurgency the population perceives as beneficial and damaging, and the resulting change in the population's support for the government. The study focuses on a single population, attempting to identify and model the first order effects of stability force actions on the population. We represent and study the effect of possible strategies by local government and external stability forces to influence popular support toward the government. We find the greatest increase in popular support occurs when the coalition concentrates on performing actions perceived by the population as beneficial *and* mitigating the effects of its damaging actions. When the population does not perceive insurgent actions as damaging, we find the coalition has difficulty increasing popular support for the government. Coalition cooperation with local leaders in planning and executing beneficial actions may increase the perceived effect of coalition actions the population perceives as beneficial.

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ANALYSIS OF ANALYTIC MODELS FOR THE EFFECT OF INSURGENCY/COUNTERINSURGENCY OPERATIONS ON THE GENERAL POPULATION

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ABSTRACT

This thesis proposes and analyzes mathematical descriptive models of the effect of Insurgency/Counterinsurgency Operations on the population of a nation experiencing stability operations. The model is a system of differential equations representing insurgent activity, insurgent recruiting, insurgent removal by the coalition; the population's tolerance for insurgent violence; occurrence of actions by the coalition and insurgency the population perceives as beneficial and damaging, and the resulting change in the population's support for the government. The study focuses on a single population, attempting to identify and model the first order effects of stability force actions on the population. We represent and study the effect of possible strategies by local government and external stability forces to influence popular support toward the government. We find the greatest increase in popular support occurs when the coalition concentrates on performing actions perceived by the population as beneficial *and* mitigating the effects of its damaging actions. When the population does not perceive insurgent actions as damaging, we find the coalition has difficulty increasing popular support for the government. Coalition cooperation with local leaders in planning and executing beneficial actions may increase the perceived effect of coalition actions the population perceives as beneficial.

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EXECUTIVE SUMMARY

While traditional warfare focuses on military force-on-force engagements, the long-term goal of stability operations is often "to help develop indigenous capacity for securing essential services, a viable market economy…democratic institutions, and a robust civil society" (DoD Directive 1). Understanding the motivations of the indigenous population is crucial to achieving this goal, and it is useful to model the effects of the stability force actions on popular opinion.

We propose and analyze mathematical descriptive models of the effect of Insurgency/Counterinsurgency Operations on the general population using systems of differential equations. The system of differential equations represent insurgent activity, insurgent recruiting, and insurgent removal by the coalition; the population's tolerance for insurgent actions perceived as damaging; and the occurrence of actions by both the coalition and insurgent forces that are viewed by the population as being damaging or beneficial and their effects on changing the population's support for the government. Through analytical and numerical solutions, we study the model's implications of the effects of stability force actions on the support of the population for the government of a country experiencing stability operations. The primary goal is to provide coalition decision makers insight into how best to employ the forces and influence the factors under their control.

We first consider a case in which the support of the population for the government depends only on the actions of the coalition and focus our analysis on factors under a coalition decision maker's direct control. We find that the greatest positive effect comes when the decision maker concentrates on both performing actions deemed beneficial by the population *and* mitigating the effects of damaging coalition actions. Next we consider a case where both coalition and insurgent actions affect the population's support for the government; the insurgency recruits from outside of the population and there are no desertions from the insurgency. The results of this model illustrate a crucial point: the effectiveness of coalition actions on the population's support of the government depends on the population's perception of the actions of the insurgency. If the population tends to

view insurgency actions as predominantly beneficial, then even a coalition accomplishing actions seen as predominantly beneficial can do very little to influence the popular support for the government. In this case mitigation of the grievances that created the insurgency may increase the popular support for the government. If the population tends to view insurgency actions as predominantly damaging, then even a coalition accomplishing actions that are neither predominantly beneficial nor predominantly damaging has a positive significant influence on the popular support for the government. Finally, we consider a model with full interaction of coalition and insurgent actions, insurgent recruiting from the population, and parameter values informed by conflict data from Anbar province in Iraq. We consider three scenarios: pre-surge, post-surge and post-"Anbar Awakening." With our chosen parameters, we conclude the increase in government support in the post-surge scenario is a result of a sizeable decrease in population's tolerance for insurgent violence without a sizeable decrease in insurgent violence. The fact that government support increases faster in the post-surge scenario than the pre-surge scenario also contributes to the overall increased government support at 52 weeks in the post-surge scenario than the pre-surge scenario. The effect on the population of beneficial actions by the coalition may be increased by cooperation with local leaders in planning beneficial actions. Increased coalition neighborhood security operations may lower the population's tolerance for insurgent violence.

The effectiveness of the coalition stability operations depends on the population's perception of the actions of the insurgency and the population's tolerance for insurgent violence. The population is more likely to be influenced to support the government if the population has a low tolerance for insurgent violence, and the insurgency engages in actions that are perceived to be damaging by the population. Models of insurgency/ counterinsurgency operations must include: actions of coalition against the insurgency; the actions of the coalition and the insurgency and their effect on population support of the government; and a measure of the population tolerance for insurgent violence.

I. INTRODUCTION

> Everything is simple and neat – except, of course, the world.
>
> – Nigel Goldenfeld and Leo P. Kadanoff

Combat models are a widely accepted tool used to represent and analyze warfare processes and events. In 1916, W.F. Lanchester was among the first theorists to attempt to model warfare using a system of ordinary differential equations. Since the publication of "Lanchester's Laws" of combat, military analysts have turned to mathematical combat models, in many different forms, for insight into how opposing forces interact in battle. With the evolution of warfare from conventional (World War I and Lanchester's era) to an ever-increasing number of insurgent-based battles (Vietnam thru current times), mathematical combat models must also evolve.

An increased occurrence of asymmetric warfare and insurgent activities throughout the world necessitates an extension to existing combat models to better understand emerging asymmetric combat situations. As Gompert states,

> The relationship between insurgents and contested populations is such that knowing how, when, where, against whom, by whom, and …whether to use deadly force is both more difficult and more consequential (in irregular warfare) than in regular warfare. Appeals are increasing to 'fight smarter' against extremist insurgents, a response to frustration over the wars in Iraq and Afghanistan and the persistence of global terrorism (1).

To realize their full utility, the extended models must represent effects of popular opinion, culture, and social interactions. Lang argues that it is the link between the insurgent and the area population that allows "insurgents to attack coalition forces or to plant improvised explosive devises (IEDs) and then fade back into the population"(16).

Former Deputy Assistant Secretary of Defense for Stability Operations, Joseph Collins, loosely defines stability operations as "military operations outside of combat, which usually take place in a post-conflict situation" (Vinall 1). While traditional warfare missions focus on military force-on-force engagements, stability operations have, in the past, centered on "winning the hearts and minds" of the local government, militia, and civilian population thru actions other than war. The attendees of the

Counterinsurgency Symposium in 1962 called "the primary objective of counter guerrilla warfare…not merely the guerrilla's elimination, neutralization, and conversion, but winning over the apathetic majority of the people" (Hosmer 12).

In recent years, stability operations have become a focus activity of the United States military forces, and the Department of Defense has established a refined set of goals and objectives for military support for Stability, Security, Transition, and Reconstruction (SSTR) operations:

> Stability operations are conducted to help establish order that advances U.S. interests and values. The immediate goal often is to provide the local populace with security, restore essential services, and meet humanitarian needs. The long-term goal is to help develop indigenous capacity for securing essential services, a viable market economy, rule of law, democratic institutions, and a robust civil society (DoD Directive 1).

Often, insurgent groups emerge to take advantage of instability in post-conflict nations. The roots of insurgency are by no means clear-cut, but Jacobs, et al. offer some explanation:

> Typically, the presence of the insurgency and the existence of civil strife and terrorist actions are the result of grievances between a minority of the population and its government or between two or more subpopulations. The insurgency groups engage in violence to undermine the authority of the government. Successful elimination or suppression of the insurgency depends on the ability to address the grievances of the minority and thus reduce the popular support for the insurgency – a key factor in the ability of an insurgency to thrive. The military troops assigned to such a stability operation require ... knowledge of the culture and history of the region, including reasons for insurgency formation; language skills; and diplomatic skills...(to) include determining and implementing processes to mitigate the grievances of the minority. Mitigating the grievances of the minority often causes tensions with the host nation government (Jacobs "Effect" 1).

Clearly, understanding the underlying motivations of the indigenous population is crucial to effectively quelling an insurgent uprising and fulfilling the SSTR objectives. It is useful to model the effects of the stability force actions on popular opinion.

To this end, this thesis proposes and analyzes mathematical descriptive models of the effect of Insurgency/Counterinsurgency Operations on the general population. One such initial model is described in the working paper entitled "A Model for the Effect of Coalition Actions on a Population in a Stability Operation" authored by Professors Jacobs, Kress, and Gaver of the Naval Postgraduate School's Operations Research department. Other models are discussed in working papers by Jacobs, et al. entitled "Deterministic 'Fluid' Models for Consequences of Coalition Actions in a Stability Operation" and Gaver, et al. entitled "A Model for the Effect of Coalition Actions on a Population in a Stability Operation Resulting from Discussion of Gaver, Jacobs, and Kress on January 18, 2008" (Jacobs, et al. "Fluid"; Gaver, et al.).

The primary goal of our study is to identify and model the first order effects of stability force actions on the population and the trade-off decisions necessary for a stability force with limited resources. Lang proposes several recommended stability actions to include military actions such as border security systems and a permanent area security presence as well as non-military actions such as amnesty and reward programs (65-73). Although his suggestions for improvement are all valid, area commanders will likely not have enough resources to accomplish all missions simultaneously.

Through analytical and numerical solutions, our study investigates the mathematical models' operational implications for the counterinsurgency forces in the modeled region/population and investigates possible courses of action (COAs) by both main players in counterinsurgency operations — local government and external stability forces — based on varied initial states. In Jacobs, et al. and in this study, we present a dynamic model representing the social cultural and behavioral aspects of the interaction between the stability forces and the local population and study the influence of those aspects on stability operations.

The "indigenous population" for this study consists of 3 groups. The first, a majority group, is the collection of host nation individuals who are controlled at least nominally by a local government and can be thought of as supporting the government. The second, a minority group, is discriminated against by the government, has little political power, and its members are generally impoverished and experience frustration.

The second group is categorized as opposing the government. Finally, there is an insurgency consisting of members of the minority group who use violence against the government and may use coercive actions (kidnapping and assassinations, IEDs, etc.) against the majority and/or minority groups to induce the government to share resources and political power with the minority group or to intimidate members in the minority group as a means of recruitment (Jacobs, et al. "Effect" 2-3).

The "coalition force" in this study refers to a collection of multi-national allies the local government invites to assist in quelling the insurgency. Initially, the coalition forces are most likely lacking in host nation cultural knowledge and experience. The initial role of the coalition forces may center around military and police actions, and these actions may well be performed with little cultural sensitivity due to lack of cultural training. The coalition forces are likely garrisoned outside of population centers (Jacobs, et al. "Effect" 2-3).

The study focuses on a single population of influence and draws conclusions from analytical and numerical solutions that highlight the models' operational implications for counterinsurgency forces in the modeled region/population. With resource constraints in mind, this study represents and analyzes the effect of possible strategies by local government and external stability forces to influence popular support toward the government.

The models developed and conclusions put forth in this thesis are part of a project entitled "Representing Urban Cultural Geography in Urban Stability Operations", sponsored by the TRADOC Analysis Center-Monterey (TRAC-MRY). The model is intended to be descriptive, and the analysis done in this study may provide decision makers with insight into how best to employ the forces and influence the factors under their control.

Chapter II provides a brief review of some current sociological models as well as counterinsurgency operations, policies, and tactics. Chapter III develops the differential equations-based models, discusses their behavior, and presents analytical solutions. Finally, Chapter VI presents conclusions and recommendations for future research.

II. LITERATURE REVIEW

> When we try to pick out anything by itself, we find it hitched to everything else in the universe.
>
> – John Muir, Naturalist, Writer, and Conservationalist

A. VISCOSITY MODELS AND DIFFUSION OF INNOVATIONS

Social scientists and social network theorists have long studied analytic models of innovation diffusion throughout a population using "viscosity models." The term "innovation" has been applied to new ideas, new technologies, or new management practices that diffuse through an organization or population via agents within that social system/population (qtd. in Krackhardt 177). In his paper entitled "Viscosity Models and Diffusion of Controversial Innovations," David Krackhardt goes one step further to define "controversial innovations" as "an innovation whose value (and subsequent adoption) is socially determined and not rationally determined" (177). This extension of definition suggests an extension of the viscosity model itself to include "innovations" such as the validity of a rumor, the impact of an action, and the influence of fellow agents' opinions. In this context, it is useful to think of changes in popular opinion toward an action taken against that population as a diffusion of ideas and the viscosity model as a means of modeling the change in popular opinion over time.

Krackhardt's discrete time model can be found in its entirety in the paper cited in this study's references. A more complex extension of the model includes a matrix of attributes attached to each entity in the network, or agent, that may influence other agents, but the main points of the basic model remain the same and are summarized:

1. The model relies on a relationship of idea "adopters" and "non-adopters"
2. The model is based on agent migration between and among groups during each time period as each agent searches the system for like-minded individuals.
3. The model uses a probabilistic approach to determine if migrating agents will encounter like-minded agents and whether, having found none, the isolated agent will "convert" to the opposite view. The key driver for system change is this conversion rate of "adopters" and "non-adopters."

In a Naval Postgraduate School thesis, Lieutenant Commander Harrison Schramm provides a framework to study ideology propagation by adapting analytic models previously used to epidemics. Schramm employs infectious disease and rumor propagation models to investigate the spread of two opposing ideologies in a closed population. His study includes the case where "contrarians and supporters openly vie for a greater share of support from the public" (Schramm xv). With his model, Schramm finds "a relatively small number of contrarians are required to overcome a large increase in supporters" (27).

B. COMPARATIVE POLITICS: PROXIMITY vs. DIRECTIONAL MODELS

In their article "Issues and Party Competition in the Netherlands," Aarts, MacDonald, and Rabinowitz assert, "At the heart of any democratic political system is the question of how public opinion relates to government policy" (63). Comparing results from two models, the authors investigate the relationship between policy preferences of the Dutch voters and how the voter views each political party.

Aarts, MacDonald, and Rabinowitz first employ a directional model in which parties and individuals are placed on the directional continuum of political ideology. As shown in Figure 1 from Aarts, et al., the neutral center is assigned a value of zero, "left supporter" positions are assigned negative values, and "right supporter" positions are assigned positive values. Parties are positioned more distant from the neutral center based on how strongly they advocate positions of the left or the right.

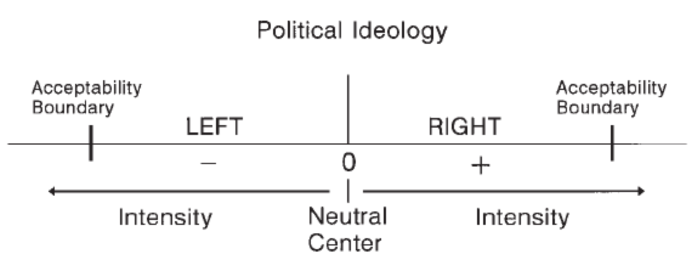

Figure 1. Directional Continuum of Political Ideology (From Aarts et al. 67)

Each voter is assumed to have a specific ideal point that represents the policy that he or she most prefers. The assumption is that voters are attracted to parties whose actions align with the voter's preferred position, and the effects of party cues and actions are mathematically modeled via a utility function (Aarts, et al. 67). Voters in the center are neutral. They have no issue preference and are assumed to support all parties about the same (Aarts, et al. 67). The "rightness" or "leftness" of a party or individual is determined by the model user based on how intensely the individual or party supports conventional political definitions. According to Wikipedia, left-wing parties and individuals generally "seek to reform social hierarchies and promote an equal distribution of wealth and privilege. The left advocates for a society where all people are on equal footing" (Wikipedia "Left"). On the other hand, right-wing politics are "generally defined as politics that seeks to uphold or a return to traditional authorities and/or the liberties of a civil society and the preservation of the domestic culture, usually in the face of external forces for change. In general, the right also advocates the preservation of personal wealth and emphasizes more self reliance" (Wikipedia "Right").

Aarts, et al. describe the placement of parties and individuals on the continuum:

> Parties that are consistent and vocal advocates of left policies would be positioned well to the left. Those that are more equivocal yet generally favorable toward the left would be located on the moderate left, and so on across the spectrum. Party position is not solely a function of the party's own efforts; it can be influenced by opposition strategy as well as by the media and ongoing events (66).

The party's success in capturing voter support is determined through a utility or affect function (Aarts, et al. 67-68):

$$A_{ij} = \sum_k s_k I_{ik} I_{jk} - P_{ij}$$

where:

s_k = intensity of the issue

I_{ik} = position of voter *i* on the directional continuum for issue *k*

I_{jk} = position of party *j* on the directional continuum for issue *k*

P_{ij} = penalty if the party becomes "too intense" compared to voter *i* preference

When party voters are on the same side of the issue, $I_{ik} * I_{jk}$ will be more positive, and product will higher the more intense either the voter or the party becomes (Aarts, et al. 68).

The authors next propose a Downsian spatial (proximity) model where each voter is assumed to have a specific ideal point that represents the policy that he or she most prefers. The origins of this model can be found in depth in Anthony Downs' book entitled "An Economic Theory of Democracy" (Downs). In this model, each party offers a set of policy alternatives that locate the party at a particular point in a multidimensional issue space. In contrast to a directional model where the center reflects neutrality, the center of a spatial model is a viable policy alternative. According to the spatial model, voters evaluate parties based on how close they are to their own position, and utility declines with distance (or, distance squared) (Aarts, et al. 68).

The party's success in capturing voter support is determined through a utility or affect function:

$$A_{ij} = -\sum_k s_k (\theta_{ik} - \theta_{jk})^2$$

where:

s_k = intensity of the issue

θ_{ik} = position of voter *i* on the proximity continuum for issue *k*

θ_{jk} = position of party *j* on the proximity continuum for issue *k*

(Aarts, et al. 68-69)

By varying individual initial positions and subsequent party actions, Aarts, MacDonald, and Rabinowitz plot the percentage of voters in 4 parties (center, moderate right, moderate left, and far right) versus the left-right continuum for each model. The authors compare the results and evaluate the relationship between the initial voter position and the party action. For directional theory model plot, the authors evaluate the slope of the plot for each party prediction--the more extreme the party, the steeper the slope will be (Aarts, et al. 69). For the proximity model plot, the authors look for monotonically increasing functions as predictors of an extreme party.

Lastly, Aarts, et al. compare both models with three historical case studies, the 1971, 1986, and 1994 Dutch National elections, using actual data and comparing known outcomes to verify the model findings. They conclude "... Dutch parties could be sharply differentiated in the minds of the voters. This occurred most dramatically in 1986, but there was some real differentiation between the parties in each of the three elections that we studied. What will happen in the future depends to a large extent on the actions of the parties" (Aarts et al. 96).

C. SOCIAL INFLUENCE EQUILIBRIUM MODELS

Friedkin and Johnsen use equilibrium models to investigate how networks of interpersonal influences affect an individual's opinion. They describe personal opinion formation as a process by which exogenous conditions such as group and individual characteristics (inputs) transform a member's or group's settled opinion (outputs) over time (Friedkin & Johnsen 193). Their models use other "peer" opinions (those which directly affect an individual) as the chief influential condition. Friedkin and Johnsen's deterministic approach assumes an individual's opinions are completely accounted for by a set of known causal variables. The process is decomposable into distinct predictive time periods, and the process of opinion formation continues until all changes that might have occurred play out. The highlighted conclusion of the analysis is to suggest that "social conflict and social conformity behaviors simultaneously exist in any group" (Friedkin & Johnsen 205).

D. COMPLEXITY ANALYSIS

As Dr Kathie Olsen of the National Science Foundation states, "The science of complexity seeks to develop tools and methods to study the emergence of collective properties in systems with large numbers of interacting parts" (1). Physical scientists have long studied complex systems to understand how these organisms behave under the standard laws of physics. In her discussion, Dr Olsen reminds the audience of the link between previous methods of studying physical systems and the emerging use of complexity analysis to study patterns of behavior. Among many applications, she cites two examples: the use of complexity analyses to analyze collective behaviors to predict the point at which a peaceful crowd may become a stampede; and the use of agent-based complexity models to better predict the patterns by which a disease is likely to spread into a global pandemic (Olsen).

Goldenfeld and Kandoff propose a key question in their article, "Simple Lessons from Complexity":

> The world contains complex 'ecologies' at all levels, from huge mountain ranges, to the delicate ridge on the surface of a sand dune, to the salt spray coming off a wave, to the interdependencies of financial markets, to the true ecologies formed by living things. Each situation is highly organized and distinctive, with biological systems forming a limiting case of exceptional complexity. So why, if the laws are so simple, is the world so complicated? (88)

Through their analysis, they reach a conclusion apropos to our study: "Apparently there are no general laws for complexity. Instead, one has to reach for 'lessons' which might, with insight and understanding, be learned in one system and applied to another" (Goldenfeld & Kandoff 89). In the context of this study, it is useful to view the indigenous population in a country experiencing stability operations as the system and model its behavior in light of prescribed "rules" and lessons learned from the fields of complexity analysis and chaos theory can be applied to the analysis of popular opinion.

One such modeling effort is currently being undertaken by Naval Postgraduate School thesis students Thorsten Seitz and Todd Ferris as part of the project entitled "Representing Urban Cultural Geography in Urban Stability Operations", sponsored by the TRADOC Analysis Center-Monterey (TRAC-MRY). As part of their analysis, Seitz and Ferris model stability operations in the agent-based modeling system Pythagoras. Seitz and Ferris define specific characteristics for each agent or entity in the model, to include whether the entity supports, opposes, or is neutral toward the government, and then introduce events which represent stability force actions. They examine the resulting interactions among entities and the changes in popular support, neutrality, or opposition of the government.

E. STABILITY, SECURITY, TRANSITION, AND RECONSTRUCTION (SSTR) OPERATIONS

Although counterinsurgency and stability operations, policies, and tactics have been relatively stagnant since the Vietnam Conflict, post- 9/11, military leaders have begun to redefine the goals and objectives of these operations. Department of Defense Directive 3000.05 on SSTR, released in 2005, states "stability operations are a core U.S. military mission but it also places much emphasis on the need for work closely with relevant U.S. Departments and Agencies, foreign governments and security forces, global and regional international organizations, even going as far as to suggest "many stability operations tasks are best performed by indigenous, foreign, or U.S. civilian professionals" (3). The directive suggests the DoD should provide opportunities for personnel to develop stability operations skills by: "…learning languages and studying foreign cultures, including long-term immersion in foreign societies" (DoD 7). This last suggestion clearly communicates a need for stability agents (forces engaging in stability operations) to understand the people and culture with which they will be interacting and opens the door for research and development such as the model proposed in this study.

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III. DYNAMIC MODEL

> Science, mathematics, and warfare have always been most closely linked; in fact, except for a certain portion of the nineteenth century, it may be fairly claimed that the majority of significant technical and scientific advances owe their origin directly to military or naval requirements.
>
> – British science historian, John Desmond Bernal, 1939.

A. NOTATION AND PRELIMINARIES

The mathematical notations and equations chosen for this study are identical to those in the working paper by Jacobs, et al. titled "A Model for the Effect of Coalition Actions on a Population in a Stability Operation" and are directly quoted in the following paragraphs. This model is one of many possible. Each member of the modeled population is either a member of the insurgency *(I)*, a supporter of the government *(G)* or a supporter of *I* with:

$S(t)$ = size of the population that support the government *(G)* at time t;

$O(t)$ = size of the population that support the insurgency (*I*) (opposition) at time t; the group is not active in actions against the government, but is inclined to assist the active insurgents;

$I(t)$ = size of the insurgency at time t;

The total population size at time *t* is:

$$\boxed{\Phi(t) = S(t) + O(t) + I(t)};$$

$C(t)$ = size of the coalition forces at time t.

We use the notation *S*, *O*, *I*, and *C* to denote the supporters, opposition, insurgency, and coalition forces, respectively. The set *S* is comprised mostly of people from the majority group but it may also contain people from the minority group who oppose the violence of the insurgents and seek a compromise and agreement. Similarly, the set *O* is comprised mostly of people from the minority group, but may also contain members of the majority group.

There are 2 Actors: the *Coalition Forces*, (*C*), and the *Insurgency* (*I*). The actions taken by each are interpreted by the population as being beneficial (B) or damaging (D) for a finite length of time, which may be affected by media coverage and public opinion manipulation by the actors. An action is called active if it is still influencing the population attitude and behavior. The labeling of an action as beneficial or damaging occurs at the time of the action and is determined by the nature of the action and the identity of the actor. The effect of beneficial (civilian affair) actions by *C* may be enhanced, and the effect of its damaging (military) actions lessened if the *C* forces are better trained in local culture and language. We assume that the labeling of an action does not depend on the (sub) population that is subject to this action.

State variables

Coalition

$C(t)$ = the size of the coalition force at time t

$B_C(t)$ = the expected number of actions by C that are active at time t and are perceived to be beneficial by the population.

$D_C(t)$ =the expected number of actions by C that are active at time t and are considered to be damaging by the population.

Insurgency

$I(t)$ = the size of the insurgency at time t

$B_I(t)$ = the expected number of actions by I that are active at time t and are perceived to be beneficial by the population.

$D_I(t)$ = the expected number of actions by I that are active at time t and are perceived to be damaging by the population.

Population

$S(t)$ = size of the population that support the government (*G*) at time t

$O(t)$ = size of the population that support the insurgency (*I*) (opposition) at time t

<u>Parameters</u>

1. γ_C = Removal rate of *I* by coalition forces. In Jacobs et al. this parameter is a linear function of the level of government support in the population.
2. α_C^+ = Arrival rate of actions per member of *C* that are perceived by the population as being beneficial. This parameter represents both the level of civil-affairs activity and its effectiveness, which depends on social, cultural and lingual training of the coalition forces, henceforth called *social training*. In the working paper, Jacobs, et al., further refines this parameter with a function that accounts for the mean time it takes to social-train the whole force and the impact of social training on the way military and civil-affair actions are perceived in the population. In this study, the impact of α_C^+ is investigated using sensitivity analysis by which the single-valued numerical input is assumed to incorporate the amount and impact of social training.
3. α_C^- = Arrival rate of damaging actions per member of *C*. This parameter represents collateral damage by military/police actions against the insurgency. This parameter increases as γ_C increases and decreases as social training of the force increases.
4. ν_C^+ = Rate at which beneficial actions by *C* cease to influence the attitude of the population towards the government. This parameter represents the effect of publicity of the beneficial actions and the type of actions that is perceived to be beneficial; e.g. actions that employ people from the population will exert influence for a longer time.
5. ν_C^- = Rate at which damaging actions by *C* cease to influence the population's attitude. This parameter represents the effectiveness of the coalition force to mitigate the effects of actions perceived as damaging by the population.

6. β_I^+ = Arrival rate of beneficial actions per member of *I*.

7. β_I^- = Arrival rate of actions per member of *I* perceived as damaging by the population. This parameter represents rate of coercive actions by I against the population.

8. μ_I^+ = Rate at which beneficial actions by *I* cease to influence the population's attitude.

9. μ_I^- = Rate at which damaging actions by *I* cease to influence the population's attitude.

10. ρ_I = Rate of recruitment from *O* to *I*.

11. δ_I = Desertion parameter from *I* into O.

12. φ_C^+ (respectively φ_I^+) = Rate at which active actions by C (respectively I) perceived as beneficial by the population influence popular support for the government. The parameter is influenced by the rate at which information about the beneficial actions travel through the population, the fraction of the population that perceive the action as beneficial, and the propensity of members of the population to be affected by beneficial actions

13. φ_C^- (respectively φ_I^-)= Rate at which active actions by C (respectively I) perceived to be damaging by the population influence popular support for the government. The parameter is influenced by the rate at which information about the damaging actions travel through the population, the fraction of the population that perceive the action as damaging, and the propensity of members of the population to be affected by damaging actions

14. D_I^* = Population tolerance for insurgent violence in units of the number of active damaging actions by I.

B. MODEL FORMULATION

The equations are those of Jacobs et al. from the working paper titled "A Model for the Effect of Coalition Actions on a Population in a Stability Operation."

$$\frac{dB_C(t)}{dt} = \underbrace{\alpha_C^+ C(t)}_{\substack{\text{Arrival rate}\\ \text{of beneficial}\\ \text{actions by C}}} - \underbrace{\nu_C^+ B_C(t)}_{\substack{\text{Beneficial actions}\\ \text{stop influencing}}} \quad (1)$$

$$\frac{dD_C(t)}{dt} = \underbrace{\alpha_C^- C(t)}_{\substack{\text{Arrival rate}\\ \text{of damaging}\\ \text{actions by C}}} - \underbrace{\nu_C^- D_C(t)}_{\substack{\text{Damaging actions}\\ \text{stop influencing}}} \quad (2)$$

$$\frac{dB_I(t)}{dt} = \underbrace{\beta_I^+ I(t)}_{\substack{\text{Arrival rate}\\ \text{of beneficial}\\ \text{actions by I}}} - \underbrace{\mu_I^+ B_I(t)}_{\substack{\text{Beneficial actions}\\ \text{stop influencing}}} \quad (3)$$

$$\frac{dD_I(t)}{dt} = \underbrace{\beta_I^- I(t)}_{\substack{\text{Arrival rate}\\ \text{of damaging}\\ \text{actions by I}}} - \underbrace{\mu_I^- D_I(t)}_{\substack{\text{Damaging actions}\\ \text{stop influencing}}} \quad (4)$$

$$\frac{dI(t)}{dt} = \underbrace{\rho_I O(t)}_{\substack{\text{Recruit}\\ \text{from O}\\ \text{to I}}} - \underbrace{\delta_I \frac{S(t)}{\Phi(t)} I(t)}_{\text{Desertion from I}} - \underbrace{\gamma_C C(t) I(t)}_{\substack{\text{Removal of I}\\ \text{by C actions}}} \quad (5)$$

The desertion parameter, δ_I, indicates desertion for "convenience" rather than "ideological" desertion. An example of "convenience" desertion may be those part-time insurgents who return to the fields at harvest time. Thus, in "convenience" desertion persons from *I* move to *O*. The desertion from *I* is determined by multiplying the desertion parameter (δ_I), the percentage of the population that supports the government at time t ($\frac{S(t)}{\Phi(t)}$), and the percentage of the population active in the insurgency at time t ($I(t)$).

$$\frac{dS(t)}{dt} = (\varphi_C^+ B_C(t) + \varphi_I^- (D_I(t) - D_I^*)^+) O(t) - (\varphi_C^- D_C(t) + \varphi_I^+ B_I(t) + \varphi_I^- (D_I^* - D_I(t))^+) S(t) \quad (6)$$

$$\frac{dO(t)}{dt} = \delta_I \frac{S(t)}{\Phi(t)} I(t) + (\varphi_C^- D_C(t) + \varphi_I^+ B_I(t) + \varphi_I^-(D_I^* - D_I(t))^+) S(t)$$
$$-(\varphi_C^+ B_C(t) + \varphi_I^-(D_I(t) - D_I^*)^+ + \rho_I) O(t) \qquad (7)$$

where $x^+ = \max(x, 0)$.

The insurgency may use D_I to coerce the population. The effect is favorable to *I* up to a certain level of violence D_I^* (population tolerance) beyond which the effect is reversed. The function $\varphi_I^-(D_I(t) - D_I^*)^+$ represents the effect of *I*'s coercive actions when they exceed the tolerance threshold of the population D_I^*, and it is monotone increasing. The function $\varphi_I^-(D_I^* - D_I(t))^+$ represents the effect of *I*'s coercive actions when the number of these actions is below the tolerance threshold D_I^* of the population.

IV. ANALYSIS

> Science, mathematics, and warfare have always been most closely linked; in fact, except for a certain portion of the nineteenth century, it may be fairly claimed that the majority of significant technical and scientific advances owe their origin directly to military or naval requirements.
>
> – British science historian, John Desmond Bernal, 1939.

A. INFLUENCING RELATIONSHIPS IN THE EQUATIONS

Because the purpose of this study is to provide insight to U.S. military decision makers, it is appropriate to focus the analysis on varying parameters under control of those decision makers. From equations (1) through (7) in the previous discussion, we can derive influencing factors.

Concentrating on the coalition actions perceived as beneficial by the population, we look at the steady-state, or limiting case, where the change in the state variables is zero. Setting the left hand side of Equation (1) equal to 0 results in:

$$B_C(\infty) = \frac{\alpha_C^+}{\nu_C^+} * C(\infty) \tag{8}$$

$B_C(\infty)$ represents the long run average number of beneficial actions influencing the population. Equation (8) suggests that decision makers may be able to increase the expected number of actions that are perceived to be beneficial by increasing the arrival rate of beneficial actions (the arrival rate of beneficial actions is proportional to the size of the coalition forces). Additionally, decision makers may choose to take actions that prolong the mean length of time the beneficial action is remembered, thereby *reducing* the parameter we call ν_C^+; an example may be programs that employ members of the population.

Alternatively, we can focus on reducing the expected number of coalition actions perceived as damaging by the population. Again we look at the steady-state (limiting) case where the change in the state variables is zero, and setting the right hand side of equation (2) equal to 0 results in:

$$D_C(\infty) = \frac{\alpha_C^-}{\nu_C^-} * C(\infty) \tag{9}$$

$D_C(\infty)$ represents the long run average number of damaging actions influencing the population. The equation suggests that, in the long run, decision makers may be able to decrease the expected number of actions perceived as damaging by decreasing the arrival rate of damaging actions (the rate at which damaging actions arrive is proportional to the size of the coalition force). In opposition to the previous case, decision makers may choose to focus on actions that shorten the mean length of time the damaging action is remembered, thereby *increasing* the parameter we call ν_C^-. This may be accomplished through training the coalition force in the social customs of the population and working in cooperation with local leaders.

B. GENERAL DISCUSSION OF MODEL IMPLEMENTATION

To approximate the solution to the systems of ordinary differential equations in this study, we implement the model using a Runge-Kutta fourth order iterative algorithm (RK4) executed with Matlab Student Version 7.1. More information on the Runge-Kutta algorithm can be found in Kendall Atkinson's book An Introduction to Numerical Analysis (see references).

In the Matlab model, we represent *S*, *O*, and *I*, as fractions of the total population, $\Phi(t)$. That is, $S(t)+O(t)+I(t)=\Phi(t)=1$. We represent *C* as a percentage *in relation to* the total population Φ(0). Therefore, the "initial population in the area", Φ(0) + *C*(0), will always be greater than 1. Parameters $\alpha_C{}^+$, α_C^-, ν_C^+, ν_C^-, φ_C^+, φ_C^-, φ_I^+, φ_I^-, β_I^+, β_I^-, μ_I^+, μ_I^-, ρ_I, δ_I, and γ_C are given fractional values to ensure all input magnitudes are consistent (except in cases of parameters informed by data). All data used are scaled to be consistent with 1 model time step = 1 week. With these assumptions, we use the algorithm to compare the numerical results to analytical solutions obtained for special cases of the equations. Specific input parameters and values chosen for each case are displayed in Tables 1, 2, and 3.

C. MODEL I—NO INSURGENCY INFLUENCE

When considering how decision makers can influence the outcome of the conflict in which they are engaged, it is natural to start with a simplified version of the equations, concentrating only on factors directly under the decision makers' control: α_C^{+}, α_C^{-}, ν_C^{+}, ν_C^{-}. We also consider the rates at which active actions by C influence popular support for or against the government (φ_C^{+} and φ_C^{-}). Of the many possible, we consider a model in which insurgent forces do not engage the coalition forces and have no impact on the scenario outcome. Numerical analysis of the full model can be found in section E of this thesis.

In our first model, all insurgency related parameters, β_I^{+}, β_I^{-}, μ_I^{+}, μ_I^{-}, φ_I^{+}, φ_I^{-}, ρ_I, δ_I, and D_I^{*} are set to zero. We assume *C*(t)=C and *S*(t)+*O*(t)=1. When γ_C, the attrition rate of *I* by coalition forces, is also set to zero the model reduces to four equations:

$$\frac{dB_C(t)}{dt} = \alpha_C^{+} C(t) - \nu_C^{+} B_C(t) \tag{10}$$

$$\frac{dD_C(t)}{dt} = \alpha_C^{-} C(t) - \nu_C^{-} D_C(t) \tag{11}$$

$$\frac{dS(t)}{dt} = (\varphi_C^{+} B_C(t) * O(t)) - (\varphi_C^{-} D_C(t) * S(t)) \tag{12}$$

$$\frac{dO(t)}{dt} = (\varphi_C^{-} D_C(t) * S(t)) - (\varphi_C^{+} B_C(t) * O(t)) \tag{13}$$

In the simplified model, the changes in *S* and *O* and the resulting percentage of the population in *S* and *O* at time *t* are complementary because the members of the population move only between these two groups. Although initial conditions may indicate a percentage of the population belonging to *I*, there is no movement into or out of *I*, and that percentage remains constant.

1. Analytical Limiting Solution

The left hand sides of Equations (10) through (13) are set equal to 0 and solved to find a limiting or steady-state solution. The steady-state (limiting) and analytic (finite time, t) solutions for this system are derived in detail in the Appendix. Algebraic manipulation yields the following steady-state *relationship* of the long run average number of government supporters to the long run average number of insurgent supporters as

$$S(\infty) = \frac{\varphi_C^+ * \dfrac{\alpha_C^+ * C(\infty)}{\nu_C^+}}{\varphi_C^- * \dfrac{\alpha_C^- * C(\infty)}{\nu_C^-}} * O(\infty) \tag{14}$$

The relationship in equation (14) suggests that if $\varphi_C^+ * \frac{\alpha_C^+}{\nu_C^+} > \varphi_C^- * \frac{\alpha_C^-}{\nu_C^-}$ support of the government will remain higher than support of the opposition. Equations (8) and (9) allow the following interpretation of this algebraic inequality: in order to ensure support for the government remains higher than support for the opposition, decision makers must ensure the effect (that is represented by the φ_C^+) of the long run average number of actions by C that are active and are perceived to be *beneficial* ($B_C(\infty)$) is greater than the effect of the long run average number of actions by C that are active and are perceived as *damaging* ($D_C(t)$). The values of $S(t)$ and $O(t)$ remain coupled as $t \rightarrow \infty$.

Thus in this simplified model the long run average number of people in the population that support the government is a function of the ratio of the effect of the number of coalition actions that are perceived to be beneficial to the effect of the number of coalition actions that are perceived to be damaging. With our assumption that C remains constant, equation (14) suggests that in this simplified model an increase or decrease in $C(\infty)$ has no effect on the end state outcome because the arrival rate of beneficial and damaging actions by C is proportional to the size of the coalition force.

2. Analytic Solution for Finite Time

If we examine the analytic solution at a finite time t, the equations suggest how coalition actions might more quickly influence a turning point in the conflict. Details of the finite time solution for this system are in the Appendix.

Assuming constant *C*,

$$B_C(t) = \frac{\alpha_C^+ C}{\nu_C^+} + \left(B_C(0) - \frac{\alpha_C^+ C}{\nu_C^+} \right) * e^{-(\nu_C^+)t} \text{ and } D_C(t) = \frac{\alpha_C^- C}{\nu_C^-} + \left(D_C(0) - \frac{\alpha_C^- C}{\nu_C^-} \right) * e^{-(\nu_C^-)t};$$

let

$$c_1(t) = \varphi_C^+ * B_C(t) \text{ and } c_2(t) = \varphi_C^- * D_C(t)$$

$$A(t) = \frac{c_2(t)(O(0)+S(0))}{c_1(t)+c_2(t)} \quad and \quad B(t) = \frac{c_1(t)*O(0)+c_2(t)*S(0)}{c_1(t)+c_2(t)}$$

then,

$$O(t) = A(t) - B(t) * e^{-(c_1(t)+c_2(t))t}$$
$$S(t) = A(t) * \frac{c_1(t)}{c_2(t)} - B(t) * e^{-(c_1(t)+c_2(t))t}$$

If $\frac{c_1(t)}{c_2(t)} > 1$, then $S(t) > O(t)$--the larger the ratio of $\frac{c_1(t)}{c_2(t)}$, the faster the government gains supporters. $c_1(t)$ is the rate at which actions by *C* which are viewed as beneficial by the population influence the population to support the government. If the members of the population are slow to be influenced by beneficial actions (small φ_C^+) then more beneficial actions may be needed. φ_C^+ may be influenced by publicity of actions and obtaining coalition/government support of local leaders.

3. Numerical Exploration of the Transient Solution and Analysis

In this section we numerically explore the transient solution of equations (10) through (13). The numerical results display the importance of the ratios $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+}$ and $\varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$ on the transient behavior of the equations. Figures 2 through 5 display numerical solutions comparing the effects of different ratios of $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+}$ to $\varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$ using the *non-zero* parameter values listed in Table 1. We see that a higher ratio results in a quicker time at which the initial minority of government supporters grows to equal the percentage of population that opposes the government.

Parameter Value	**Figure 2:** ratio=1	**Figure 3:** ratio=2	**Figure 4:** ratio=3	**Figure 5:** ratio=4
$O(0)$	0.6	0.6	0.6	0.6
$S(0)$	0.4	0.4	0.4	0.4
α_C^+,	0.2	0.5	0.75	0.8
α_C^-,	0.8	0.5	0.25	0.2
ν_C^+, φ_C^+	1.0	1.0	1.0	1.0
ν_C^-, φ_C^-	1.0	1.0	1.0	1.0
$\boldsymbol{S(t) \approx O(t)}$	**~52weeks**	**~10 weeks**	**~8 weeks**	**~6 weeks**
***S*(52)**	**0.50**	**0.65**	**0.72**	**0.87**
***O*(52)**	**0.50**	**0.35**	**0.28**	**0.13**

Table 1. Parameter Values for Model I Analysis

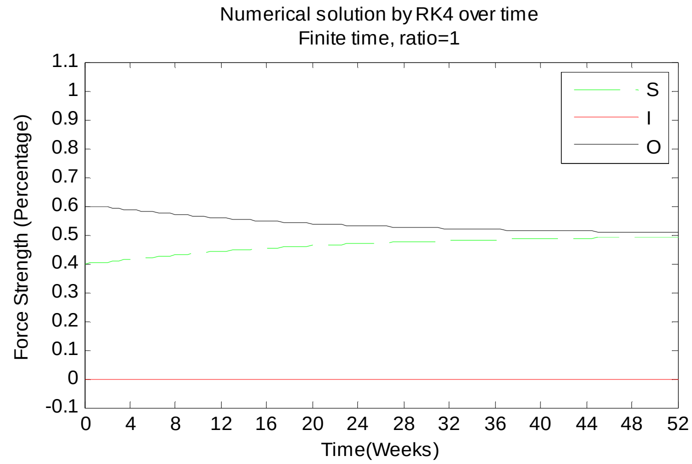

Figure 2. Numerical Approximation with ratio $\varphi_C^+ \dfrac{\alpha_C^+}{\nu_C^+}$ to $\varphi_C^- \dfrac{\alpha_C^-}{\nu_C^-} = 1$ (best viewed in color)

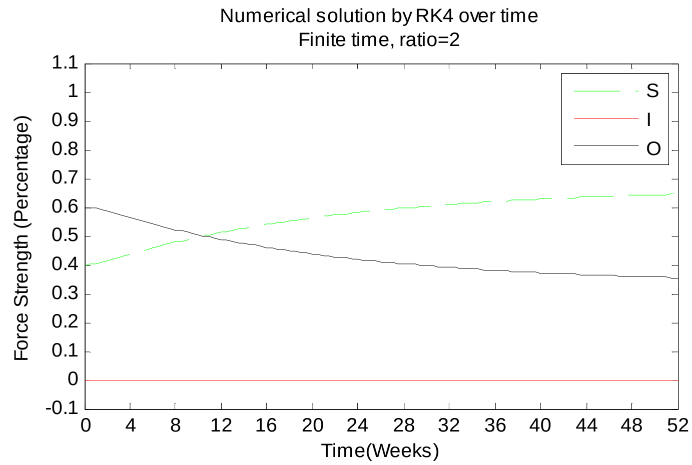

Figure 3. Numerical Approximation with ratio $\varphi_C^+ \dfrac{\alpha_C^+}{\nu_C^+}$ to $\varphi_C^- \dfrac{\alpha_C^-}{\nu_C^-} = 2$ (best viewed in color)

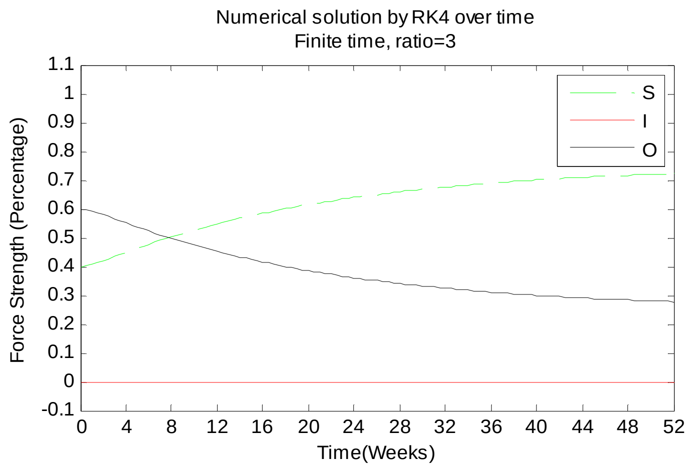

Figure 4. Numerical Approximation with ratio $\varphi_C^+ \dfrac{\alpha_C^+}{\nu_C^+}$ to $\varphi_C^- \dfrac{\alpha_C^-}{\nu_C^-} = 3$ (best viewed in color)

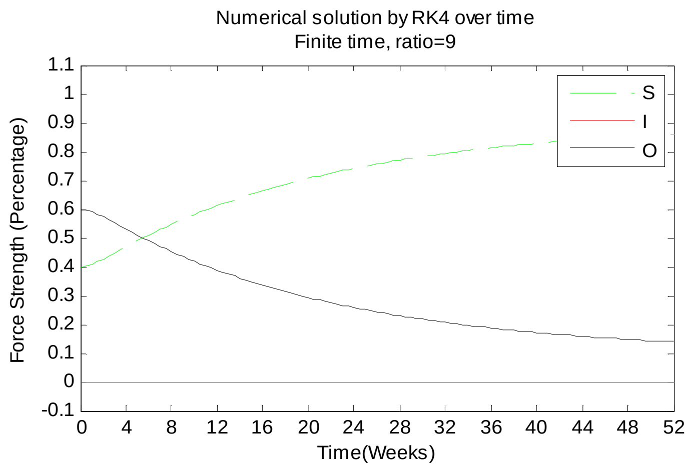

Figure 5. Numerical Approximation with ratio $\varphi_C^+ \dfrac{\alpha_C^+}{\nu_C^+}$ to $\varphi_C^- \dfrac{\alpha_C^-}{\nu_C^-} = 9$ (best viewed in color)

The results suggest the biggest "bang for the buck" occurs with a combination of effective publicity of beneficial coalition actions (decreasing ν_C^+; that is increasing the mean time a beneficial action remains active) and mitigation of damaging actions (decreasing $\frac{\alpha_C^-}{\nu_C^-}$). The mean time a beneficial coalition action remains active is also influenced by the type of beneficial action (e.g. employment of the local population). Our model results also suggest that factors that increase the rate of beneficial coalition actions (α_C^+) (respectively, decreasing the rate of damaging coalition actions), such as increased social training, have a strong impact.

D. MODEL II—INSURGENCY INFLUENCE

Expanding our analysis to include insurgency influence and incorporating all factors of the full model gives us a more complex representation of a typical scenario. The insurgents engage in beneficial and damaging actions which affect the population's support for the government. In the general model, the insurgents experience the removal of members by coalition action and desertion of existing members back to *O*. In other words, members of the population move $S \to O$, $O \to S$, $I \to O$, $O \to I$, or completely out of the population when members of *I* are removed by coalition action.

Unfortunately, when considering the full model there is no mathematical theory that leads to an analytical solution of the fully coupled, non-linear system with all parameters non-zero. However, the equations can be modified so that the size of the population, $\Phi(t)$, remains constant, and a limiting solution for the system can be found. An analysis of one of many possible models with constant $\Phi(t)$ is presented in the working paper by Jacobs, et al. entitled "Deterministic 'Fluid' Models for Consequences of Coalition Actions in a Stability Operation" (Jacobs, et. al, "Fluid" 15-17). The numerical analysis of our full model can be found in section E of this thesis.

Given that the full model is analytically intractable, our second model (one of many possible) is more complex than our first model with some insurgency interaction. In our second model the insurgents may recruit new members, but those members do not come from *O* or *S*. Insurgent members may be removed by the coalition, but the insurgency does not experience desertion (δ_I is set to zero). In our second model, insurgency and coalition actions influence the exchange of members between *O* and *S* as members of the population move *O*→*S* and/or *S*→*O*.

1. A Modified Model and its Analytical Limiting Solution

In our second model, all insurgency related parameters, $\beta_I^+, \beta_I^-, \mu_I^+, \mu_I^-$, φ_I^+, φ_I^-, ρ_I, and D_I^*, are non-zero except δ_I. The model becomes a system of seven ordinary, linear, differential equations:

$$\frac{dB_C(t)}{dt} = \underbrace{\alpha_C^+ C(t)}_{\substack{\text{Arrival rate}\\ \text{of beneficial}\\ \text{actions by C}}} - \underbrace{\nu_C^+ B_C(t)}_{\substack{\text{Beneficial actions}\\ \text{stop influencing}}} \tag{15}$$

$$\frac{dD_C(t)}{dt} = \underbrace{\alpha_C^- C(t)}_{\substack{\text{Arrival rate}\\ \text{of damaging}\\ \text{actions by C}}} - \underbrace{\nu_C^- D_C(t)}_{\substack{\text{Damaging action}\\ \text{stops influencing}}} \tag{16}$$

$$\frac{dB_I(t)}{dt} = \underbrace{\beta_I^+ I(t)}_{\substack{\text{Arrival rate}\\ \text{of beneficial}\\ \text{actions by I}}} - \underbrace{\mu_I^+ B_I(t)}_{\substack{\text{Beneficial actions}\\ \text{stop influencing}}} \tag{17}$$

$$\frac{dD_I(t)}{dt} = \underbrace{\beta_I^- I(t)}_{\substack{\text{Arrival rate}\\ \text{of damaging}\\ \text{actions by I}}} - \underbrace{\mu_I^- D_I(t)}_{\substack{\text{Damaging actions}\\ \text{stop influencing}}} \tag{18}$$

$$\frac{dI(t)}{dt} = \underbrace{\rho_I}_{\substack{\text{I recruits}\\ \text{from outside}\\ \text{the population}}} - \underbrace{\gamma_C C(t) I(t)}_{\substack{\text{Removal of I}\\ \text{by C actions}}} \tag{19}$$

$$\begin{aligned}\frac{dS(t)}{dt} &= (\varphi_C^+ B_C(t) + \varphi_I^- (D_I(t) - D_I^*)^+) O(t) \\ &\quad - (\varphi_C^- D_C(t) + \varphi_I^+ B_I(t) + \varphi_I^- (D_I^* - D_I(t))^+) S(t)\end{aligned} \tag{20}$$

$$\frac{dO(t)}{dt} = (\varphi_C^- D_C(t) + \varphi_I^+ B_I(t) + \varphi_I^- (D_I^* - D_I(t))^+) S(t)$$
$$- (\varphi_C^+ B_C(t) + \varphi_I^- (D_I(t) - D_I^*)^+) O(t) \qquad (21)$$

where $x^+ = \max(x, 0)$.

We assume the size of the coalition force is constant equal to *C*. Equations (15) through (21) can be solved analytically to find a limiting relationship when $t \to \infty$ and changes in the state variables *S*, *O*, *I* are zero. If we assume $\frac{dS(t)}{dt} = \frac{dO(t)}{dt} = 0$ as $t \to \infty$, equations (20) and (21) approach the following long run average solutions in equations (22) and (23) as $t \to \infty$. Equations (22) and (23) are reduced to the single equation (1.22) when *S*(t) and *O*(t) reach steady state equilibrium as $t \to \infty$. Because we assume $S(t) + O(t)$ =constant= $S(0) + O(0)$, the relationship $S(\infty) + O(\infty) = S(0) + O(0)$ is also satisfied.

$$S(\infty) = \frac{(\varphi_C^+ B_C(\infty) + \varphi_I^- (D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^- (D_I^* - D_I(\infty))^+)} * O(\infty) \qquad (22)$$

where:

$$O(\infty) = \frac{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^- (D_I^* - D_I(\infty))^+)}{(\varphi_C^+ B_C(\infty) + \varphi_I^- (D_I(\infty) - D_I^*)^+)} * S(\infty) \qquad (23)$$

and $$I(\infty) = \frac{\rho_I}{\gamma_C * C} \qquad (24)$$

and $$B_C(\infty) = \frac{\alpha_C^+}{\nu_C^+} * C \qquad (25)$$

and $$D_C(\infty) = \frac{\alpha_C^-}{\nu_C^-} * C \qquad (26)$$

and $$B_I(\infty) = \frac{\beta_I^+}{\mu_I^+} * I(\infty) \qquad (27)$$

and $$D_I(\infty) = \frac{\beta_I^-}{\mu_I^-} * I(\infty) \qquad (28)$$

with $x^{+} = \max\left(x, 0\right)$.

The relationships in equation (22) and (23) suggest that if $D_I(\infty) > D_I^*$ and $\varphi_C^+ B_C(\infty) > \varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty)$ members may move from $O \rightarrow S$; that is, if $\dfrac{(\varphi_C^+ B_C(\infty) + \varphi_I^- (D_I(\infty) - D_I^*)^+)}{\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty)} > 1$ (and $D_I(\infty) > D_I^*$), then more of the population will support the government than the insurgency. In other words, the government may gain supporters if the long run average number of actions by *I* that are active and are perceived to be damaging ($D_I\left(\infty\right)$) is greater than the population tolerance; and the long run expected number of beneficial coalition actions that are active is greater than the long run expected number of damaging coalition actions that are active plus the long run expected number of beneficial insurgent actions that are active. If $D_I(\infty) < D_I^*$ and φ_I^- is low (or, φ_C^+ is high), the population will tolerate the insurgency (and possibly move from $S \rightarrow O$) until the number of damaging actions by *I* becomes larger than the D_I^* threshold. Actions that influence the population to support the government are actions by C that are perceived to be beneficial and actions by I that are perceived to be damaging and are in excess of the tolerance of the population for damaging actions by *I*; actions that influence the population to support the insurgency are actions by C that are perceived to be damaging and beneficial actions by *I*. More of the population will support the government than support the insurgency if the effect of actions that influence to population to support the government is greater than the effect of actions that influence the population to support the insurgency.

Equations (22) through (28) suggest many possible operational strategies. Among those possible, the coalition could choose to focus efforts to increase γ_C, the removal rate of *I* by coalition forces, thereby decreasing the strength of insurgent forces in the limit (as a percentage of the total population). Reducing the strength of the insurgency would reduce $B_I\left(\infty\right)$ and, in turn, should increase $S\left(\infty\right)$; however, it would also reduce $D_I(\infty)$.

2. Numerical Exploration of the Transient Solution and Analysis

In this model, we numerically study the transient behavior of the model of equations (15)-(21). For each analysis case, the initial population is 50% government supporters, 50% opposition supporters. The coalition forces, $C(t)$, remain constant at 5% of the total population. In our second model, all insurgency related parameters, $\beta_I^+, \beta_I^-, \mu_I^+, \mu_I^-, \rho_I$ are non-zero except δ_I. For simplicity, each case assumes the rate of insurgency recruitment per member of the insurgency equals the rate of removal of the insurgents per member of the insurgency by the coalition; that is, $\rho_I = \gamma_C * C * I(0)$. In each case, φ_C^+ (respectively φ_C^-) takes on the same value as α_C^+ (respectively α_C^-). The values for the *non-zero* parameters for Figures 9 through 13 are displayed in Table 2.

Input Value	Fig 6	Fig 7	Fig 8	Fig 9	Fig 10
$S(0)$	0.5	0.5	0.5	0.5	0.5
$O(0)$	0.5	0.5	0.5	0.5	0.5
C	0.05	0.05	0.05	0.05	0.05
γ_C	1	1	1	1	1
α_C^+, φ_C^+	0.9	0.9	0.5	0.5	0.9
α_C^-, φ_C^-	0.1	0.1	0.5	0.5	0.1
ν_C^+	0.1	0.1	0.5	0.5	0.1
ν_C^-	0.9	0.9	0.5	0.5	0.9
$\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+}$	8.1	8.1	0.5	0.5	8.1
$\varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$	0.01	0.01	0.5	0.5	0.01
β_I^+, φ_I^+	0.1	0.1	0.1	0.9	0.9
β_I^-, φ_I^-	0.9	0.9	0.9	0.1	0.1
μ_I^+	0.9	0.9	0.9	0.1	0.1
μ_I^-	0.1	0.1	0.1	0.9	0.9

$\varphi_I^+ \dfrac{\beta_I^+}{\mu_I^+}$	0.01	0.01	0.01	8.1	8.1
$\varphi_I^- \dfrac{\beta_I^-}{\mu_I^-}$	8.1	8.1	8.1	0.01	0.01
$\dfrac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)}$	730	570	60	0.02	0.25
ρ_I	0.01	0.01	0.01	0.01	0.01
D_I^*	0	0.5	0	0	0
Government Support at 52 weeks*	**100%**	**100%**	**98%**	**2%**	**23%**
Government Opposition at 52 weeks*	**0%**	**0%**	**2%**	**98%**	**77%**

Table 2. Parameter Values for Model II

Figures 6 and 7 below both depict the situation when the arrival rate of beneficial coalition actions per coalition member is 0.9 and the arrival rate of damaging coalition actions per coalition member is 0.1. Figure 6 depicts the results when the population tolerance for insurgent violence is 0; whereas Figure 7 depicts the results when the population has a 0.5 tolerance for insurgent violence. Although both figures display the fraction of the population that support the government equal to 1 at 52 weeks, the population tolerance of insurgent damaging actions has a significant impact on the fraction of government supporters during the initial 20 weeks of the scenario. With a 0.5 population tolerance, Figure 7 shows a significant initial drop in government support (almost 30%) in the first month. The percentage of the population that supports the government does not equal the percentage of the population that opposes the government again until almost 12 weeks.

* Remainder is percentage of population active in the insurgency at 52 weeks.

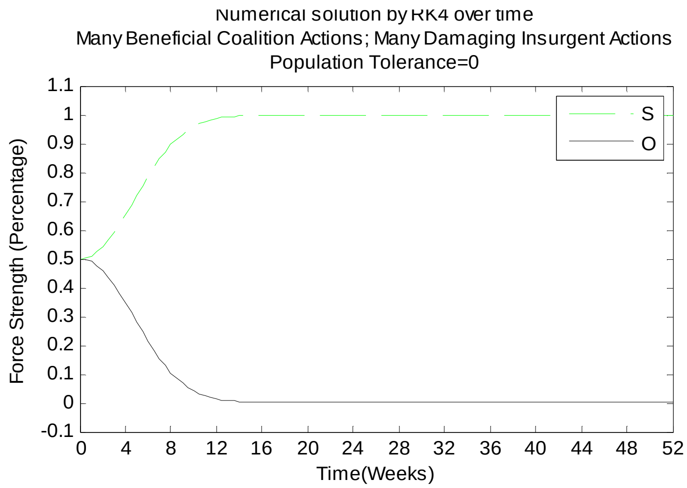

Figure 6. Many Beneficial Coalition Actions; Many Damaging Insurgent Actions; No Population Tolerance for Insurgent Violence=0 (best viewed in color)

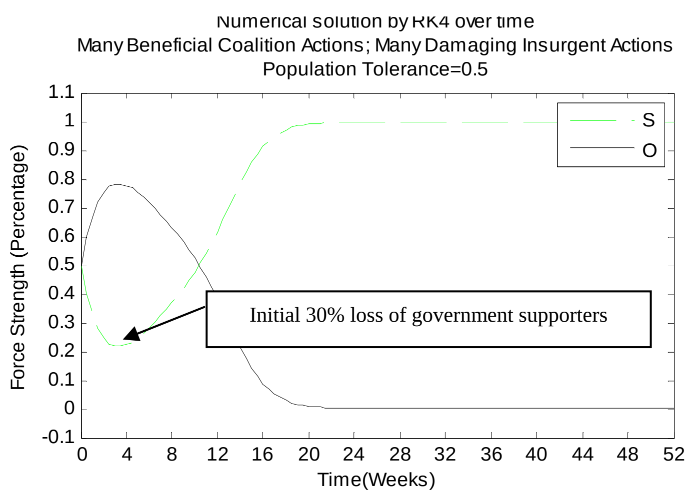

Figure 7. Many Beneficial Coalition Actions; Many Damaging Insurgent Actions; Population Tolerance for Insurgent Violence=0.5 (best viewed in color)

Figure 8 depicts the situation where the limiting effects of coalition actions perceived to be beneficial and damaging on the population support for the government are equal; that is $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+} = 0.5 = \varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$. We see that when the insurgency is engaged in many more damaging actions than beneficial, the coalition is still able to influence the population's support for the government. In Figure 6 (respectively Figure 8) $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+} = 8.2 > 0.01 = \varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$ (respectively $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+} = 0.5 = \varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$). Comparing Figures 6 and 8, we see the support for the government in Figure 6 is only slightly higher at 52 weeks than in Figure 8; in Figure 6 the ratio $\frac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)}$ =730 is greater than its value of 60 in Figure 8 and support for the government increases faster in Figure 6. Thus the effectiveness of coalition actions on increasing the support of the population for the government depends on the ratio of beneficial coalition actions to damaging coalition actions ($\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+}$ compared to $\varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$) and the rate of increase of support for the government (a faster rate of increase is indicated by a larger value of $\frac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)}$)

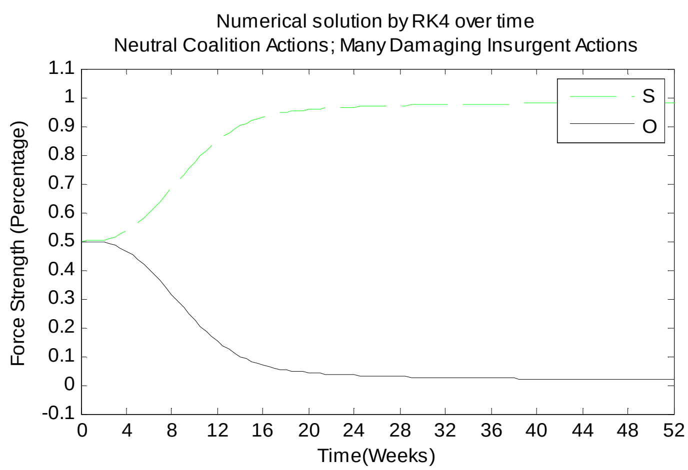

Figure 8. Coalition Actions Are Neither Predominantly Beneficial Nor Predominantly Damaging; Many Damaging Insurgent Actions (best viewed in color)

Finally, Figures 9 and 10 display the outcome of a situation in which many of the insurgent actions are perceived as beneficial by the population. In Figure 9 (respectively Figure 10) $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+} = 0.5 = \varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$ (respectively $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+} = 8.2 > 0.01 = \varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$), and $\frac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)} < 1$ in both figures. We see an initial surge of government support in Figure 10 where $\frac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)} = .025$; but at 52 weeks, supporters of the insurgency outnumber supporters of the government. These two figures illustrate a crucial point: if insurgent actions perceived as damaging by the population are very low or non-existent, there is very little the coalition can do to influence the population support or opposition of the government unless the reasons that lead to the insurgency are mitigated.

If we use our intuition based on our model with no insurgency involvement to compare Figures 6 and 10, we would expect to see the long run average number of government supporters to be high in both cases because the ratio of $\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+}$ to $\varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$ =810 in both cases. But, we see it is not enough simply to just "do more good things." Even with a high rate of beneficial coalition actions per member as in Figure 10, the support for the government in this scenario is only 23% at 52 weeks. We conclude that if the rate of damaging insurgent actions is low and the rate of beneficial insurgent actions is high, the government may continue to lose supporters regardless of coalition actions, and the coalition could be viewed as an occupying force rather than a positive influence. The effectiveness of coalition actions on the population's support of the government depends on the actions of the insurgency. If the population tends to view insurgency actions as predominantly beneficial, then even a coalition accomplishing actions seen as predominantly beneficial can do very little to influence the popular support for the government. In this case reasons for the insurgency need to be mitigated. If the population tends to view insurgency actions as predominantly damaging, then even a coalition accomplishing actions that are neither predominantly beneficial nor predominantly damaging has a significant positive influence on the popular support for the government.

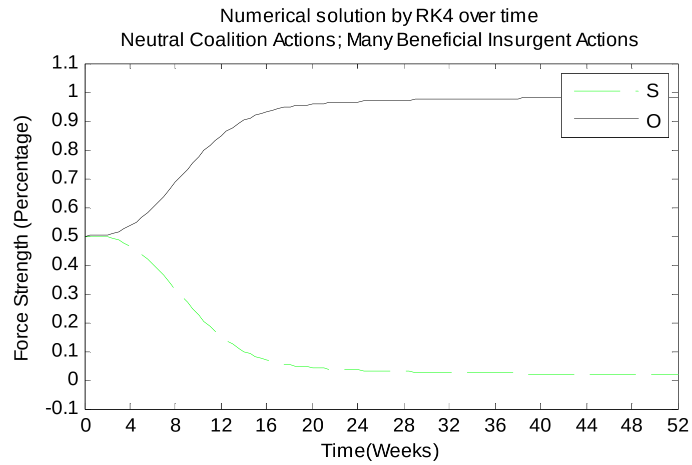

Figure 9. Coalition Actions Are Neither Predominantly Beneficial Nor Predominantly Damaging; Many Beneficial Insurgent Actions (best viewed in color)

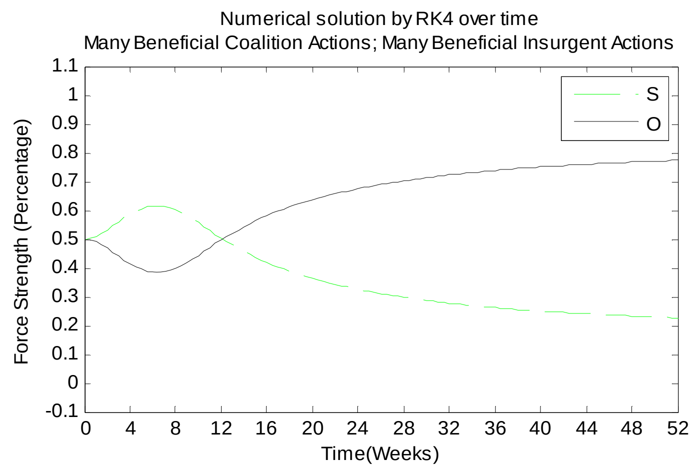

Figure 10. Many Beneficial Coalition and Insurgent Actions (best viewed in color)

E. FULL MODEL WITH PARAMETER VALUES INFORMED BY CONFLICT DATA

The full model, equations (1) through (7) with all parameters non-zero, gives a more complete relationship of how all factors interact to influence the scenario outcome. As previously stated, the full model cannot be solved analytically, but we can exercise the model numerically.

1. Parameter Value Selection for Pre- and Post-Surge

As with most numerical models, correct input parameter value selection is crucial to obtaining reasonable and useful results. When exercising our full model, we inform as many parameters as possible with available data. For parameters not closely related to available data, we use nominal values. Table 3 lists the parameter values, and the following paragraphs describe the method by which each parameter value was chosen. Our choices of parameter values serve as a suggestion for the model user. If the user has access to more accurate or applicable data the user may be able to better estimate values of parameters for the specific scenario he/she wishes to investigate.

Using the data as displayed in Table 3, we exercise the full model to investigate two scenarios—"pre-surge" and "post-surge." Pre-surge data are taken from sources dated June 2007 and earlier; post-surge data are taken from sources dated July 2007 to present. Both scenarios use data for Ar-Ramadi (aka: Ramadi, Ramadiyah) metro area of Al-Anbar province, Iraq. Based on 2002 United Nations census statistics, Global Security ranks Ramadi the 11th largest city (in population) in Iraq (Global Security). In her 2006 Washington Post article, Ann Tyson calls Ramadi "the deadliest city (in Iraq) for U.S. troops relative to its population" (Tyson 1).

While we recognize there exist vast differences in the regions of Iraq due to many factors such as the cultural disparity between urban and rural dwellers and the tribal nature of some regions versus others, many sources only publish statistics for "all of Iraq" or "all Iraqi Sunnis". We feel comfortable applying statistics given for "all Iraqi Sunnis" or "Mid-Euphrates" to the population of Ramadi; but it is with some hesitation we assume statistics given for "all of Iraq" apply to the population of Ramadi.

The March 2008 Central Intelligence Agency (CIA) factbook lists the Iraq total population as 27.5M (28,000,000) (CIA). Based on 2003 UN data, the Iraqi Coalition Provisional Authority (ICPA) lists Ramadi's population as 445K or, approximately 1.6% of the country's total population (ICPA). We scale each "all of Iraq" parameter to obtain a "percentage of Ramadi population" by multiplying by 0.02 (rounding the Ramadi population from 0.016 to 0.02).All parameters and scaling factors are rounded to two significant digits at each step of the scaling. Additionally, all "event data" are scaled to maintain 1 timestep=1 week consistency. For example, if 2000 prisoners were detained in one month in all of Iraq, then $\frac{2000*.02}{4}=10$ prisoners per week were detained in Ramadi in one week. A discussion of each parameter value selection method follows.

Although estimates of insurgent force strength can be suspect, we use the data for Fallujah from the Wikipedia article "Iraq Insurgency" to inform $I(0)$ as a percentage of the total population in Ramadi. Because of the proximity and similar demographics of the two cities, we are comfortable the ratio of insurgents to the general population in Fallujah is similar to the ratio of insurgents to the general population in Ramadi. The Wikipedia article lists the upper estimate of insurgents in Fallujah in mid-2004 as 5,000. Taken as a population ratio compared to the population of Fallujah $\approx 443,000$, our choice for $I(0)$ is $\frac{5,000}{443,000} \approx 0.01$. In the absence of data to inform a change in insurgent strength, we use $I\left(0\right)=0.01$ for both pre- and post-surge.

Polling data from ABC News is used to inform $S(0)$ and $O(0)$. In a March 2007 poll, 8% of Sunnis expressed confidence in the National Government of Iraq (NGI). In a February 2008 poll, 10% of Sunnis expressed confidence in the NGI (ABC News 27). We subtract $I(0)$ values discussed above from $O(0)$ to maintain $\Phi(t)=S(t)+O(t)+I(t)=1$. Based on the adjusted numbers, our parameters for $S(0)$ (respectively $O(0)$) for pre- and post-surge are 0.08 (respectively 0.91=1-0.08-0.01 support the insurgency) and 0.10 (respectively 0.89=1-0.10-0.01 support the insurgency). Our choice of low values for support of the government is supported by a similar question referenced in the ABC News report in which Iraqi Sunnis were asked "Do you approve or disapprove of the way Nouri Kame al-Maliki is handling his job as prime minister?" In March 2007, 96% of Iraqi Sunnis expressed disapproval, and in February 2008, 92% of Iraqi Sunnis expressed disapproval.

To inform our *C* parameter value, we again use data from the Wikipedia articles "Iraq Insurgency" and "The Battle of Ramadi (2004)." In "The Battle of Ramadi (2004)," Wikipedia lists the coalition troop strength at 1,500. In "Iraq Insurgency," Wikipedia estimates 3,500 more Marines were sent to Ramadi in 2006 to "reestablish control of the region" (Wikipedia "Iraq Insurgency"). We use the estimated ratio of coalition troops in Ramadi to the population of Ramadi to inform C. That is, $C = \frac{3,500+1,500}{445,000} = 0.01$ for both pre- and post-surge.

To inform γ_C, the removal rate of *I* by coalition forces, we use data from a USA Today article by Jim Michaels. Michaels' data on insurgents killed spans March 2003 to September 2007. The highest number of insurgents killed per month pre-surge is 1,623 in August 2004 and post-surge is approximately 600 in July 2007 (Michaels 1). The number of insurgents removed in Ramadi is calculated $R = 1600*0.02 = 32$ in pre-surge and $600*0.02 = 12$ in post-surge. An estimate of the number of coalition members in Ramadi is $C_{\mathrm{R}} = \textit{population of Ramadi} * \textit{coalition percentage} = 450000*0.01 = 4500$ for pre- and post-surge. An estimate of the number of insurgent members in Ramadi at time zero is $I(0)_{\mathrm{R}} = \textit{population of Ramadi} * \textit{insurgent percentage} = 450000*0.01 = 4500$ for pre-surge and $450000*0.01 = 4500$ for post-surge. For Ramadi, we calculate $\gamma_C = \frac{R}{I(0)_{\mathrm{R}} * C_{\mathrm{R}}} = \frac{32}{4500*4500} = 2\times10^{-6} \approx 0.00$ for pre- and post-surge (to two significant digits).

In the July 2007 *Iraq Index*, the estimates for the strength of the insurgency for each month between July 2004 and October 2006 are predominately 20,000 (26). In the absence of data, we assume the March 2007 insurgency strength of 70,000 holds true for each subsequent month. Based on O'Hanlon and Campbell's estimates, we assume the size of the insurgency changes very little month to month. Initially, we calculate

$\rho_I = \frac{\delta_I * S(0) * I(0) + \gamma_C * C * I(0)}{O(0)}$. Having determined γ_C =0.00 for pre- and post-surge, we choose δ_I =0.05 (in the absence of data to inform δ_I) and calculate pre-surge $\rho_I * O(0) \approx 0.00046$ and post-surge $\rho_I * O(0) \approx$ 0.00045. Rounded to two significant digits, both pre- and post-surge $\rho_I = 0.00$. The size of the insurgent force in our model changes very slowly by virtue of our parameter selection, but it is not constant.

We inform the parameter values used to model the population *perception* of beneficial and damaging coalition actions, α_C^+, φ_C^+, α_C^-, φ_C^-, ν_C^+, and ν_C^-, with data from the *Iraq Index*. To inform α_C^+, the arrival rate of beneficial actions per coalition member, we use the number of weapons caches found and cleared in Al Anbar (O'Hanlon and Campbell "May 2008" 13). Although the number of weapons caches cleared is not a direct measure of coalition actions perceived as beneficial by the population, we choose this data as an indication of the population perception of beneficial coalition actions relative to damaging coalition actions. There are other data available that could be used to inform α_C^+, such as the number of coalition patrols with the Iraqi army/police. O'Hanlon and Campbell list caches cleared in Al Anbar for all of 2006 as 941 and for all of 2007 as 3,155. We attribute $\frac{3{,}155}{2} \approx 1600$ to January-June 2007 and the same amount to July-December 2007. So, for the period January 2006-June 2007 (pre-surge), there was an average of $\frac{\textit{number caches cleared}}{\textit{number of weeks}} = \frac{940+1600}{18*4} \approx 35$ caches cleared per week in Al Anbar. From July-December 2007, there was an average of $\frac{1600}{24} \approx 67$ caches cleared per week in Al Anbar. $\alpha_C^+ = \frac{\textit{number of caches cleared by C}}{\textit{coalition as percentage of population}} = \frac{35}{0.01} = 3500$ for pre-surge and $\alpha_C^+ = \frac{67}{0.01} \approx 6700$ for post-surge.

Although the number of civilian deaths attributable to US forces is not a direct measure of coalition actions perceived as damaging by the population, we choose this data as an indication of the population perception of damaging coalition actions relative to beneficial coalition actions. In the absence of other data available that could be used to inform α_C^-, we estimate α_C^- using data from the Iraq Body Count (IBC) website. IBC lists a high estimate of 434 Iraqi civilian deaths attributable to US forces alone in all of Iraq in 2006 and a high estimate of 756 for 2007. The IBC also lists a high estimate of 623 Iraqi civilian bystanders killed in firefights and other attacks involving US-led coalition military forces in all of Iraq in 2006 and a high estimate of 1,326 for 2007. We attribute $\frac{760+1300}{2} \approx 1000$ to January-June 2007 and the same amount to July-December 2007. So, for the period January 2006-June 2007 (pre-surge), there was an average of $\frac{\textit{number of deaths} * \textit{Ramadi population scale}}{\textit{number of weeks}} = \frac{(430+1000)*0.02}{18*4} \approx 0.40$ civilian deaths due to coalition involvement per week in Ramadi. From July-December 2007 (post-surge), there was an average of $\frac{1000*0.02}{24} \approx 0.83$ civilian deaths due to coalition involvement per week in Ramadi. We calculate $\alpha_C^- = \frac{\textit{arrival rate of damaging actions by C}}{\textit{coalition as percentage of population}} = \frac{0.40}{0.01} \approx 40$ for pre-surge and $\alpha_C^- = \frac{0.83}{0.01} \approx 83$ for post-surge. We choose $\varphi_C^+ = \varphi_C^-$ =0.2 as the parameter of the effect of coalition actions on popular support for/opposition to the government.

For pre-surge, we choose ν_C^- =0.08 to reflect a population whose support for the government is influenced by damaging coalition actions for a mean time of 3 months. This is a notional parameter value, but our choice is based in an ABC News poll taken in August 2007. Although this poll is taken outside our pre-surge time period, questions reference events occurring within our timeframe. Of the 63% of the Iraqi Sunni population that answered "yes" to "tell me if unnecessary violence against citizens by US or coalition forces has occurred nearby here in the past 1-2 months, within 6 months, within the past year, or longer," almost half answered in the "1-2 months" or the "within 6 months" categories. We use the average of 3 months for insights into the population mean time coalition damaging actions continue to influence opinion (ABC News 36). In the absence of data, we choose $\nu_C^+ = \nu_C^-$ =0.08.

For post-surge, we choose ν_C^- =0.04 to reflect a population for which damaging coalition actions continue to influence opinion for a mean time of 6 months (12 weeks). This is again a notional parameter value based in an ABC News poll taken in February 2008. Of the 57% of the Iraqi Sunni population that answered "yes" to "tell me if unnecessary violence against citizens by US or coalition forces has occurred nearby here in the past 1-2 months, within 6 months, within the past year, or longer," almost half answered "within the past six months" (ABC News 36). In the absence of data, we choose $\nu_C^+ = \nu_C^-$ =0.04.

As with the coalition parameters, we inform the parameter values used to model the population *perception* of beneficial and damaging insurgent actions, β_I^+, φ_I^+, β_I^-, φ_I^-, μ_I^+, and μ_I^-, with data from the *Iraq Index*. To inform β_I^-, the arrival rate of damaging actions per insurgent, we use the number of multiple fatality bombings targeting civilians (Sunni) (O'Hanlon and Campbell "May 2008" 12). From January 2007-June 2007 (pre-surge), there was an average 5.5 multiple fatality bombings targeting Sunnis per month or 1.4 per week. From July 2007-May 2008, there was an average 3.5 multiple fatality bombings targeting Sunnis per month or 0.88 per week. We calculate $\beta_I^- = \dfrac{\textit{arrival rate of damaging actions by I}}{\textit{percentage of population active in the insurgency at time } 0} = \dfrac{1.4}{0.01} = 140$

for pre-surge and $\beta_I^- = \dfrac{0.88}{0.01} = 88$ for post-surge.

In the absence of data we assume the arrival rate of beneficial actions by *I* is lower than the arrival rate of damaging actions by *I* in both the pre- and post-surge environment. We choose pre-surge $\beta_I^+ = \dfrac{140}{3} = 47$ and post-surge $\beta_I^+ = \dfrac{88}{3} \approx 29$.

For pre-surge and post-surge, we choose μ_I^- =0.08 to reflect a population for which damaging insurgent actions affect public opinion for a mean time of 3 months (12 weeks). These are notional parameter values, but our choice is based in ABC News polls taken in August 2007 and February 2008. Although the August 2007 poll is taken outside our pre-surge time period, it references actions occurring within our timeframe. Of the roughly 50% of the Iraqi Sunni population that answered "yes" to "tell me if car bombs, suicide attacks or kidnappings for ransom have occurred nearby here in the past 1-2 months, within 6 months, within the past year, or longer," almost half answered in the "1-2 months" or the "within 6 months" categories (ABC News 34-35). We use the average of 3 months to gain insight into the population mean time to damaging insurgent actions continue to influence opinion. In the absence of data, we choose $\mu_I^+ = \mu_I^-$ =0.08.

We choose $\varphi_I^+ = \varphi_I^-$ =0.1 to represent the effects of insurgent actions on popular support for/opposition to the government. Our choice is strictly notional, but our choice for φ_I^- is consistent with our calculated D_I^*, indicating a large population tolerance for insurgent violence. We assume the large population tolerance indicates the population is not very responsive to damaging insurgent actions, thus our relatively low choice for φ_I^+ and φ_I^-.

Our choice for D_I^*, the population tolerance for insurgent violence, is informed by enemy initiated attacks against the coalition and its partners data from the May 2008 *Iraq Index* (O'Hanlon and Campbell 8). We assume that the number of attacks is an indication of the population tolerance for damaging actions by the insurgency. Attacks against the coalition might, in some cases, be viewed as beneficial by the population, but because of the likelihood of collateral damage, we have chosen to use this data to inform D_I^*. The average number of attacks per month in the pre-surge period in all of Iraq in September 2006-June 2007 is about 1200. The average number of attacks per month in the post-surge period in all of Iraq in July-October 2007 is about 900. We calculate $D_I^* = \frac{average\ \#attacks\ per\ month}{4} * 0.02$. Pre-surge $D_I^* = \frac{1200}{4} * 0.02 = 6$ and post-surge $D_I^* = \frac{900}{4} * 0.02 = 4.5$.

In our final example, we specify D_I^* informed by the effect of the Sahawah Al Anbar, or "Anbar Awakening," in which a 49-member council of provincial Sheikhs form a coalition to bring order to the Anbar region ("The Long War Journal"). We also choose notional values for φ_C^+, φ_C^-, μ_I^- based on scenario changes we assume are plausible in the post-Awakening environment.

We choose μ_I^-=0.04 to reflect a population for which damaging insurgent actions continue to influence opinion for a mean time of 6 months (24 weeks) versus 3 months in the post-surge analysis. Although our choice is notional, we assume the population is more likely to have damaging insurgent actions affect opinions in the Awakening environment; an environment where local leaders are discouraging insurgent violence.

We choose $\varphi_C^+ = \varphi_C^-$=0.3 for the parameters representing the effects of coalition actions on popular support for/opposition to the government during the Awakening period. Our choice is strictly notional, but we assume the population is more responsive to coalition actions given the coalition is working more closely with local leaders and is possibly more visible to the average citizen in this timeframe.

We choose $\varphi_I^+ = \varphi_I^-$=0.2 for the parameters representing of effects of insurgent actions on popular support for/opposition to the government. Our choice is strictly notional, but our choice for φ_I^- is consistent with our calculated decrease in D_I^* during the Awakening period, indicating a declining population tolerance for insurgent violence. We assume a declining population tolerance also indicates the population is becoming more responsive to damaging insurgent actions as the local leaders discourage insurgent violence in the post-Awakening environment.

As in the pre- and post-surge environments, our choice for D_I^*, the population tolerance for insurgent violence in the post-Awakening environment, is informed by enemy initiated attacks against the coalition and its partners data from the May 2008 *Iraq Index* (O'Hanlon and Campbell 8). We assume that the number of attacks that occur is an indication of the population tolerance. The average number of attacks per month in the pre-surge period in all of Iraq in January-June 2007 is about 1200. The average number of attacks per month in the post-Awakening period in all of Iraq from November 2007 to February 2008 is about 300. We calculate $D_I^* = \frac{average\ \#\, attacks\ per\ month}{4} * 0.02 = \frac{300}{4} * 0.02 = 1.5$.

Input Value	**Type**	**Fig 11 Pre-Surge**	**Fig 12 Pre-Surge**	**Fig 13 Post-Surge**	**Fig 14 Post-Surge**	**Fig 15 Post-Awakening**
$S(0)$	Informed	0.08	0.08	0.10	0.10	0.10
$O(0)$	Informed	0.91	0.91	0.89	0.89	0.89
$I(0)$	Informed	0.01	0.01	0.01	0.01	0.01
C	Informed	0.01	0.01	0.01	0.01	0.01
γ_C	Informed	0.00	0.00	0.00	0.00	0.00
α_C^+	Informed	3500	3500	6700	6700	6700
φ_C^+	Notional	0.20	0.20	0.20	0.20	0.30
α_C^-	Informed	40	40	83	83	83
φ_C^-	Notional	0.20	0.20	0.20	0.20	0.30
ν_C^+	Notional	0.08	0.08	0.04	0.04	0.04
ν_C^-	Notional	0.08	0.08	0.04	0.04	0.04
$\varphi_C^+ \frac{\alpha_C^+}{\nu_C^+}$	Calculated	8750	8750	33500	33500	50250
$\varphi_C^- \frac{\alpha_C^-}{\nu_C^-}$	Calculated	100	100	420	420	620
β_I^+	Informed	47	47	29	29	29
φ_I^+	Notional	0.10	0.10	0.10	0.10	0.20
β_I^-	Notional	140	140	88	88	88
φ_I^-	Notional	0.10	0.10	0.10	0.10	0.20
μ_I^+	Notional	0.08	0.08	0.08	0.08	0.08
μ_I^-	Notional	0.08	0.08	0.08	0.08	0.04
$\varphi_I^+ \frac{\beta_I^+}{\mu_I^+}$	Calculated	59	59	36	36	73

$\varphi_I^- \frac{\beta_I^-}{\mu_I^-}$	Calculated	180	180	110	110	440
D_I^*	Informed	6.0	0*	4.5	0*	1.5
ρ_I	Calculated	0.00	0.00	0.00	0.00	0.00
δ_I	Notional	0.05	0.05	0.05	0.05	0.05
***S*(52)**		**11%**	**92%**	**22%**	**95%**	**53%**
***O*(52)**		**88%**	**7%**	**77%**	**4%**	**46%**
***I*(52)**		**1%**	**1%**	**1%**	**1%**	**1%**

Table 3. Full Model Values Informed By Conflict Data

If we assume constant coalition size *C* and constant insurgent force size *I*, we investigate the long run average of number of beneficial (damaging) coalition (insurgent) actions still influencing the population and the fraction of active coalition (insurgent) actions perceived as beneficial (damaging) using the equations below:

$$\underbrace{B_C(\infty)}_{\substack{\text{Long run average number}\\ \text{of beneficial coaltion}\\ \text{actions still active}}} = \frac{\alpha_C^+ * C}{\nu_C^+} \tag{29}$$

$$\underbrace{D_C(\infty)}_{\substack{\text{Long run average number}\\ \text{of damaging coaltion}\\ \text{actions still active}}} = \frac{\alpha_C^- * C}{\nu_C^-} \tag{30}$$

$$\underbrace{B_I(\infty)}_{\substack{\text{Long run average number}\\ \text{of beneficial insurgent}\\ \text{actions still active}}} = \frac{\beta_I^+ * I}{\mu_I^+}. \tag{31}$$

$$\underbrace{D_I(\infty)}_{\substack{\text{Long run average number}\\ \text{of damaging insurgent}\\ \text{actions still active}}} = \frac{\beta_I^- * I}{\mu_I^-} \tag{32}$$

* Notional value for analysis.

$$\underbrace{\frac{B_C(\infty)}{B_C(\infty)+D_C(\infty)}}_{\substack{\text{Fraction of active}\\ \text{coalition actions}\\ \text{perceived as beneficial}}} = \frac{\varphi_C^+ * \dfrac{\alpha_C^+}{\nu_C^+}}{\varphi_C^+ * \dfrac{\alpha_C^+}{\nu_C^+} + \varphi_C^- * \dfrac{\alpha_C^-}{\nu_C^-}} \tag{33}$$

$$\underbrace{\frac{D_C(\infty)}{D_C(\infty)+B_C(\infty)}}_{\substack{\text{Fraction of active}\\ \text{coalition actions}\\ \text{perceived as damaging}}} = \frac{\varphi_C^- * \dfrac{\alpha_C^-}{\nu_C^-}}{\varphi_C^- * \dfrac{\alpha_C^-}{\nu_C^-} + \varphi_C^+ * \dfrac{\alpha_C^+}{\nu_C^+}}. \tag{34}$$

$$\underbrace{\frac{B_I(\infty)}{B_I(\infty)+D_I(\infty)}}_{\substack{\text{Fraction of active}\\ \text{insurgent actions}\\ \text{perceived as beneficial}}} = \frac{\varphi_I^+ * \dfrac{\beta_I^+}{\mu_I^+}}{\varphi_I^+ * \dfrac{\beta_I^+}{\mu_I^+} + \varphi_I^- * \dfrac{\beta_I^-}{\mu_I^-}} \tag{35}$$

$$\underbrace{\frac{D_I(\infty)}{D_I(\infty)+B_I(\infty)}}_{\substack{\text{Fraction of active}\\ \text{insurgent actions}\\ \text{perceived as damaging}}} = \frac{\varphi_I^- * \dfrac{\beta_I^-}{\mu_I^-}}{\varphi_I^- * \dfrac{\beta_I^-}{\mu_I^-} + \varphi_I^+ * \dfrac{\beta_I^+}{\mu_I^+}}. \tag{36}$$

$$\underbrace{\frac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)}}_{\text{Measure of increase of government support}} = \frac{\varphi_C^+ * \dfrac{\alpha_C^+ * C}{\nu_C^+} + \varphi_I^- * (D_I(\infty) - D_I^*)^+}{\varphi_C^- * \dfrac{\alpha_C^- * C}{\nu_C^-} + \varphi_I^+ * \dfrac{\beta_I^+ * I}{\mu_I^+} + \varphi_I^- * (D_I(\infty) - D_I^*)^+} \tag{37}$$

Table 4 gives the results of equations (29) through (37) to two significant digits based on our pre-surge, post-surge, and post-Awakening parameter values displayed in Table 3. We assume C=0.05 in pre-surge and 0.07 in post-surge and post-Awakening analysis. We assume I=0.01 in pre-surge and 0.03 in post-surge and post-Awakening analysis.

Approximate Long Run Number of Actions Active (assumes I is constant)	**Pre-Surge**	**Post-Surge**	**Post-Awakening**
$B_C(\infty)$	440	1700	1700
$D_C(\infty)$	5.0	21	21
$B_I(\infty)$	5.9	3.6	3.6
$D_I(\infty)$	18	11	22
Approximate Fraction of Active Actions			
$\frac{B_C(\infty)}{B_C(\infty)+D_C(\infty)}$	0.99	0.99	0.99
$\frac{D_C(\infty)}{D_C(\infty)+B_C(\infty)}$	0.01	0.01	0.01
$\frac{B_I(\infty)}{B_I(\infty)+D_I(\infty)}$	0.25	0.25	0.14
$\frac{D_I(\infty)}{D_I(\infty)+B_I(\infty)}$	0.75	0.75	0.86
Approximate Ratio of Effects of Increase to Government Support			
$\frac{(\varphi_C^+ B_C(\infty)+\varphi_I^-(D_I(\infty)-D_I^*)^+)}{(\varphi_C^- D_C(\infty)+\varphi_I^+ B_I(\infty)+\varphi_I^-(D_I^*-D_I(\infty))^+)}$	56	74	73

Table 4. Approximate Long Run Average Number of Active Actions and Fraction of Active Actions Influencing the Population

2. Numerical Exploration of the Transient Solution and Analysis

Figure 11 displays results for pre-surge operations using the parameter values listed in Table 3. In the pre-surge scenario, the percentage of the population that opposes the government after 52 weeks is 88% and the population that supports the government is 11%. Consistent with our parameter selection method for ρ_I, δ_I, and γ_C, the insurgency reduces slowly. At 52 weeks it is roughly its initial strength of 1% (to 2 significant digits). In the pre-surge scenario, we see the predominantly "beneficial" coalition is slow to influence popular support toward the government given a population with a very high tolerance for violence (D_I^* =6).

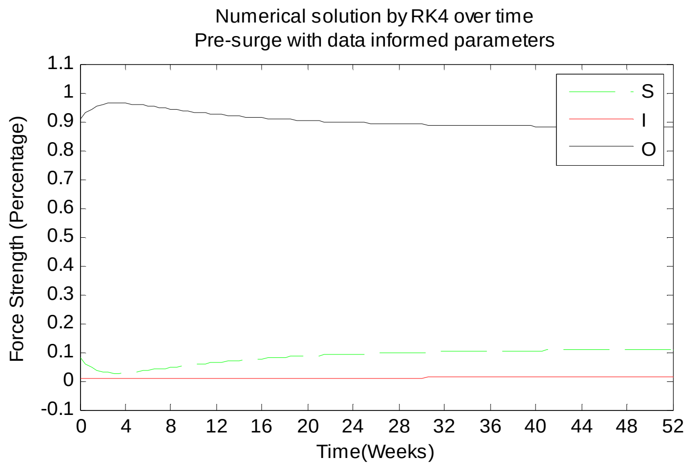

Figure 11. Pre-Surge Scenario with Informed Parameters (best viewed in color)

Figure 12 displays results for pre-surge operations with a value of 0 for D_I^*, population tolerance for insurgent violence. Comparing Figures 11 and 12 suggests the relative importance of lowering the population tolerance for insurgent violence. By focusing efforts to ensure the population has zero tolerance for insurgent violence, our analysis indicates support for the government is 81% higher at 52 weeks with a change in population tolerance from 6 to 0 attacks per week in Ramadi. Even an initial minority of government supporters is able to equal the percentage of the population that opposes the government at about 18 weeks when D_I^* = 0 in the pre-surge environment. We conclude the increase in government support in a scenario with zero population tolerance for insurgent violence is a result of a sizeable decrease in population's tolerance for insurgent violence without any decrease in insurgent violence.

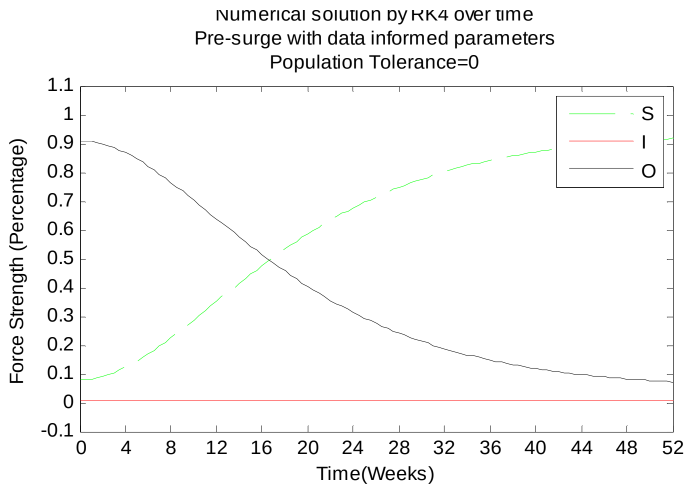

Figure 12. Pre-Surge; Population Tolerance for Insurgent Violence = 0 (best viewed in color)

Figure 13 displays the results for post-surge operations using the parameter values displayed in Table 3. In the post-surge scenario, the percentage of the population that opposes the government after 52 weeks is 77%, and the percentage of the population that supports the government is 22%. Consistent with our parameter selection method for ρ_I, δ_I, and γ_C, the insurgency reduces slowly but is still 1% at 52 weeks (to 2 significant digits). In the post-surge scenario, we see the coalition force with a predominant number of actions perceived by the population as beneficial begins to positively influence support for the government when D_I^* = 4.5. From the values in Table 3, we calculate the post-surge approximate (assumes *I* is constant) long run fraction of active insurgent actions perceived as damaging by the population as $\frac{D_I(\infty)}{D_I(\infty)+B_I(\infty)}$ = 0.74 to two significant digits. This long run fraction is only a slight (0.01) decrease from the pre-surge fraction; whereas the population's tolerance for violence decreased 25% (4.5/6) from the tolerance in the pre-surge scenario. From Table 3, we calculate the post-surge approximate ratio of effects of increase to government support as (post-surge) $\frac{(\varphi_C^+ B_C(\infty)+\varphi_I^-(D_I(\infty)-D_I^*)^+)}{(\varphi_C^- D_C(\infty)+\varphi_I^+ B_I(\infty)+\varphi_I^-(D_I^*-D_I(\infty))^+)} = 74 > 56$ (pre-surge). We conclude the increase in government support in the post-surge scenario is a result of a sizeable decrease in population's tolerance for insurgent violence without a sizeable decrease in insurgent violence. The fact that the percentage of the population that supports the government increases faster (as measured by a ratio of parameters specified in our approximate limiting solution discussion: $\frac{(\varphi_C^+ B_C(\infty) + \varphi_I^-(D_I(\infty) - D_I^*)^+)}{(\varphi_C^- D_C(\infty) + \varphi_I^+ B_I(\infty) + \varphi_I^-(D_I^* - D_I(\infty))^+)}$) in the post-surge scenario also contributes to the increase in government support from 11% in the pre-surge scenario to 22% in the post-surge scenario at 52 weeks.

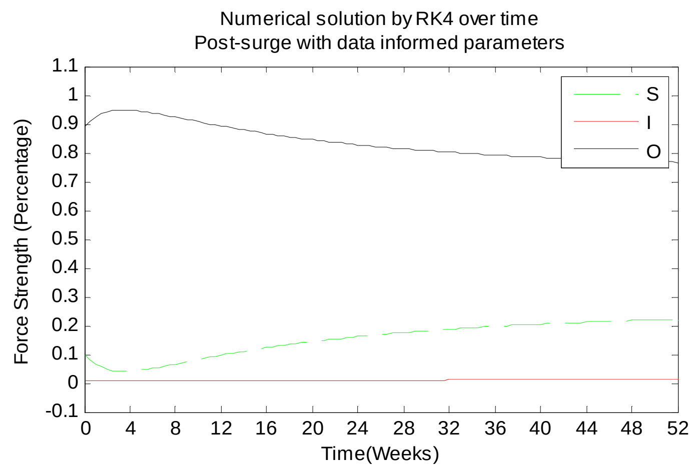

Figure 13. Post-Surge Scenario with Informed Parameters (best viewed in color)

Figure 14 displays results for post-surge operations with a value of 0 for D_I^*, population tolerance for insurgent violence. Comparing Figures 13 and 14 shows the relative importance of lowering the population tolerance for violence. Although the data indicate a one-quarter (4.5/6) drop in population tolerance from pre-surge to post-surge environment, we still see that focusing efforts to ensure the population has zero tolerance for insurgent violence may increase support for the government. Mitigating the reasons for the existence of the insurgency may decrease the population's tolerance of damaging actions. In our analysis of a population with zero tolerance for insurgent violence in the post-surge environment, support for the government is 73% higher (95% versus 22%) at 52 weeks with a change in tolerance from 4.5 attacks in Ramadi per week to 0. The time at which the initial minority of government supporters grows to equal the percentage of population that opposes the government is about 15 weeks, which is smaller than in the pre-surge scenario, displayed in Figure 12.

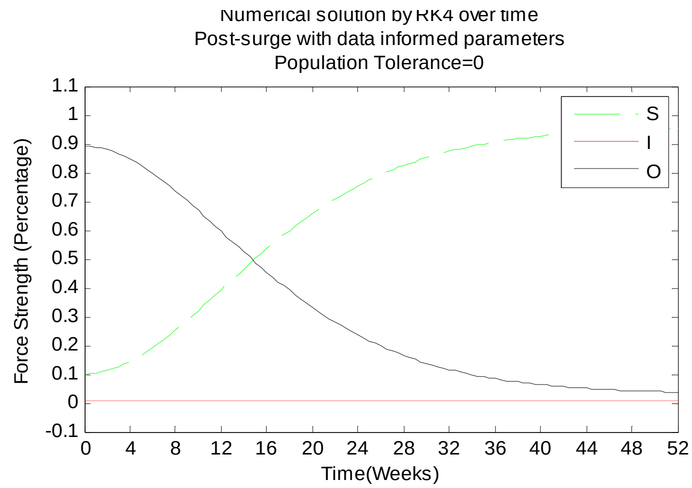

Figure 14. Post-Surge; Population Tolerance for Insurgent Violence = 0 (best viewed in color)

Figure 15 displays results using the D_I^* informed by post-Awakening data and notional values for φ_C^+, φ_C^-, μ_I^- based on scenario changes we assume are plausible in the post-Awakening environment.. We see the predominantly beneficial coalition influences popular support of the government enough to overcome the majority opposition at time 0 at about 40 weeks, and government support is at 53% at 52 weeks. The analysis indicates this is due to a lower population tolerance for violence (D_I^* = 1.5), an increased responsiveness to coalition beneficial and damaging actions ($\varphi_C^+ = \varphi_C^-$=0.30), an increased responsiveness to insurgent beneficial and damaging actions ($\varphi_I^+ = \varphi_I^-$=0.20), and an increase in the mean time damaging insurgent actions effect the popular opinion (μ_I^-=0.04) as compared to the post-surge scenario. If we make only the change to D_I^* from 4.5 in post-surge to 1.5 in this analysis (with other parameter values equal to those in post-surge analysis), we see similar results with 57% of the population supporting the government, 42% opposing the government, and 1% active in the insurgency at 52 weeks. The time at which the initial minority of government supporters grows to equal the percentage of population that opposes the government is approximately 32 weeks (displayed in Figure 16).

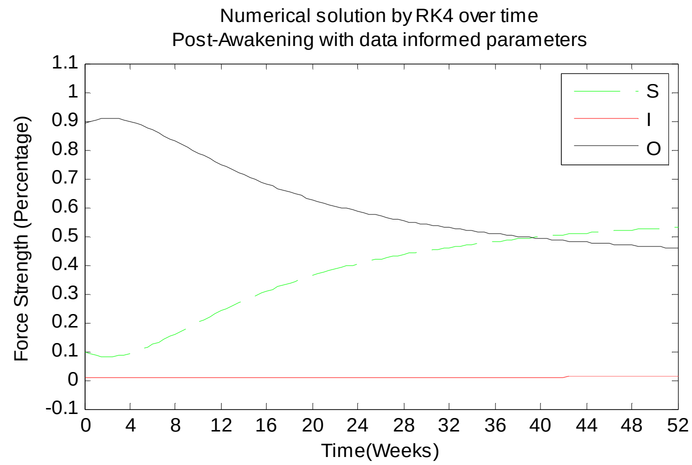

Figure 15. "Anbar Awakening" Informed Parameters (best viewed in color)

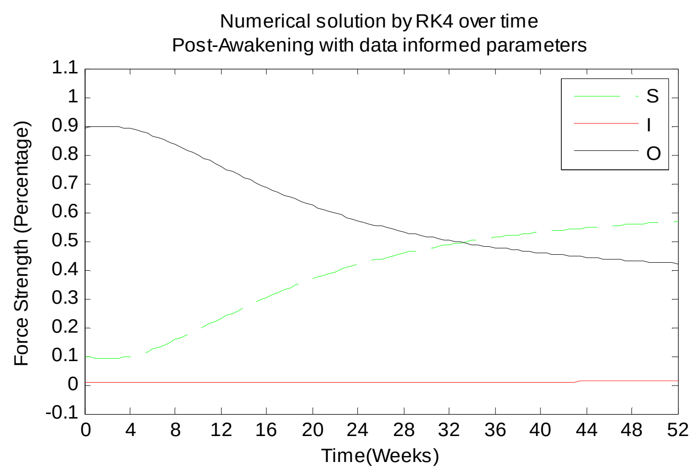

Figure 16. "Anbar Awakening" Informed D_I^* only (best viewed in color)

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V. CONCLUSION

A. RECOMMENDATIONS AND RESULTS

In this thesis we present and study a system of differential equations to represent the effect of coalition forces engaging in stability operations on the support of the local population for the government and insurgency. In Chapter III, we develop the baseline model that is exercised in the analysis of various special case scenarios in Chapter VI. Many other models are possible such as those discussed in the working papers by Jacobs et al. and Gaver et al., the theses by Seitz and Ferris, and suggested in this study's further research.

The primary goal is to provide decision makers insight into how best to employ the coalition forces and influence the factors under their control. Another goal is to provide insight into what a model for stability operations should represent. Using the results from our analysis in Chapter VI and the steady-state relationships resulting directly from the equations, we can distinguish the important factors that have the biggest positive impact on the scenario outcome for the models considered here.

First, we consider a case with no insurgent effect on the population where only the coalition actions influence the population’s support for the government. The analysis focuses only on factors under a coalition decision maker’s direct control and finds that intuition holds—the coalition decision maker must focus efforts to ensure the expected effect of the number of actions perceived as beneficial by the population outweigh the expected effect of number of actions perceived to be damaging. Overall, we find that the greatest positive effect comes when decision makers concentrate on both performing actions deemed beneficial by the population *and* mitigating the effects of coalition actions deemed damaging by the population.

Next we consider a case where the population is influenced by insurgent actions but there is no recruitment from the population to the insurgency or desertion from the insurgency to the population. The size of the population (excepting those active in the insurgency) is constant and the actions of both the coalition and insurgency can affect the support of the population for the government. In this scenario where all factors are not controlled by the decision maker, we find that the population's tolerance for actions by the insurgency perceived to be damaging (violence) influences the population's support for the government during the first month of the scenario for the cases studied. Although a higher population tolerance for violence does not influence the long run amount of support for the government for the cases studied, we see a significant loss of government support in the first 4 weeks. The coalition decision maker may focus efforts on ways to lower the population tolerance for violence. Perhaps, if the population is given a better sense of security and well-being they will be less tolerant of insurgent actions that would disrupt that sense of security. Perhaps if the population is made to feel more "in control" of their own land, they will be less willing to tolerate insurgent actions that usurp that control. A higher population tolerance for insurgent actions perceived as damaging may possibly be indicative of a population that is more likely to support the insurgency. The results of this model also illustrate a crucial point: if insurgent actions perceived as damaging by the population are very low or non-existent, there is very little the coalition can do to influence the population support or opposition of the government unless the causes that gave rise to the insurgency are addressed by the coalition or other appropriate agency (e.g. the state department). If the insurgency is participating in predominantly beneficial actions, the best the coalition can do is match the level of insurgent beneficial actions.

Finally, we consider a model with full interaction of coalition and insurgent actions and their effect on the population's support for the government. In our analysis of the full model, we proposed possible data points that could be used inform the parameter values. We hope the user would have access to more appropriate or more refined data to better inform parameter values for the specific scenario he/she wishes to investigate. We use the full model to investigate two scenarios—"pre-surge" and "post-surge." We then specify post-surge parameter values where data informs specific links to the effect of the Sahawah Al Anbar, or "Anbar Awakening," in which a 49-member council of provincial Sheikhs form a coalition to bring order to the Anbar region ("The Long War Journal"). We assume the coalition established an environment conducive to the formation of the

Awakening Councils, and attribute the positive effects of the Awakening to post-surge coalition parameters. With our chosen parameters, we conclude the increase in government support in the post-surge scenario is a result of a sizeable decrease in population's tolerance for insurgent violence without a sizeable decrease in insurgent violence. The fact that the percentage of the population that supports the government increases faster (as measured by a ratio of parameters specified in our approximate limiting solution discussion) in the post-surge scenario than in the pre-surge also contributes to an overall increase from the pre-surge to the post-surge scenario in the percentage of the population that supports the government at 52 weeks. When we examine the post-Awakening scenario, we find an even faster rate of increase in government support, and we conclude this faster rate of increase (as measured by a ratio of parameters specified in our approximate limiting solution discussion) is the reason our scenario in the post-Awakening environment is the only scenario informed by conflict data where the support for the government exceeds the opposition for the government at the end of the 52-week analysis period.

B. FURTHER RESEARCH

This study contains many possible follow on opportunities for both applied mathematicians and operations analysts. Additional relationships evident in the system of equations could be explored. Although we chose to analyze only three possible models, many extensions to the basic model proposed in this study exist. Natural extensions of our proposed model include, but are not limited to the following:

1. The expansion and or refinement of the state variables may be realistic and useful, but is avoided in this study to maintain simplicity. A progressively more complex sequence of models with additional actors and action classifications could be introduced and studied. State variables to account for the actions non-governmental organizations, the state department, and neutral members of the population may be of interest.

2. We assumed the labeling of actions as beneficial or damaging is strictly dependent on the (sub) population that is subject to this action. Future study could be devoted to the case including a relaxation of this assumption and could include a measurement of world opinion or media bias as a determining factor for whether an action is labeled beneficial or damaging, for example.

3. We assumed that ideological desertion from I was not possible. Subsequent studies could include both ideological desertion and the "convenience desertion" scenario described in this study. Inclusion of the possibility of ideological desertion would allow transition from $I \rightarrow S$.

4. We assumed only one subpopulation. Subsequent studies could investigate the effects of introducing more than one subpopulation.

5. The full model does not have a closed form analytic solution unless the total population, $\Phi(t)$, is held constant. Subsequent studies could examine the analytic solution of the full model with constant total population.

6. Our model does not represent the reintroduction of "removed" insurgents back into the population. Enhanced models could represent detention and release of people where the released detainees re-enter O.

APPENDIX: ANALYTICAL RESULTS FOR MODEL I

Model I equations from p.21 assuming C(t)=C and S(t)+O(t)=1:

$$\frac{dB_C(t)}{dt} = \alpha_C^+ C(t) - \nu_C^+ B_C(t) \tag{10}$$

$$\frac{dD_C(t)}{dt} = \alpha_C^- C(t) - \nu_C^- D_C(t) \tag{11}$$

$$\frac{dS(t)}{dt} = (\varphi_C^+ B_C(t) * O(t)) - (\varphi_C^- D_C(t) * S(t)) \tag{12}$$

$$\frac{dO(t)}{dt} = (\varphi_C^- D_C(t) * S(t)) - (\varphi_C^+ B_C(t) * O(t)) \tag{13}$$

A. STEADY STATE DERIVATION

Setting left-hand sides to zero yields:

$$B_C(\infty) = \frac{\alpha_C^+}{\nu_C^+} * C(\infty)$$

$$D_C(\infty) = \frac{\alpha_C^-}{\nu_C^-} * C(\infty)$$

$$S(\infty) = \frac{\varphi_C^+ * B_C}{\varphi_C^- * D_C} * O(\infty)$$

Specifically:

$$S(\infty) = \frac{\varphi_C^+ * \dfrac{\alpha_C^+ * C(\infty)}{\nu_C^+}}{\varphi_C^- * \dfrac{\alpha_C^- * C(\infty)}{\nu_C^-}} * O(\infty) \tag{14}$$

B. ANALYTIC SOLUTION FOR FINITE TIME

Assuming constant *C*, then

$$B_C(t)=\frac{\alpha_C^+ C}{\nu_C^+}+\left(B_C(0)-\frac{\alpha_C^+ C}{\nu_C^+}\right)*e^{-(\nu_C^+)t} \text{ and } D_C(t)=\frac{\alpha_C^- C}{\nu_C^-}+\left(D_C(0)-\frac{\alpha_C^- C}{\nu_C^-}\right)*e^{-(\nu_C^-)t}$$

Let

$$c_1(t)=\varphi_C^+ * B_C(t) \text{ and } c_2(t)=\varphi_C^- * D_C(t)$$

$$A(t)=\frac{c_2(t)(O(0)+S(0))}{c_1(t)+c_2(t)} \quad \textit{and} \quad B(t)=\frac{c_1(t)*O(0)+c_2(t)*S(0)}{c_1(t)+c_2(t)}$$

Then,

$$O(t)=A(t)-B(t)*e^{-(c_1(t)+c_2(t))t}$$
$$S(t)=A(t)*\frac{c_1(t)}{c_2(t)}-B(t)*e^{-(c_1(t)+c_2(t))t}$$

Confirming the limiting solution:

$$O(\infty)=A=\frac{c_2(\infty)*(O(0)+S(0))}{c_1(\infty)+c_2(\infty)}$$

$$S(\infty)=\frac{c_1(\infty)}{c_2(\infty)}*A=\frac{c_1(\infty)}{c_2(\infty)}*\frac{c_2(\infty)*(O(0)+S(0))}{c_1(\infty)+c_2(\infty)}=\frac{c_1(\infty)*(O(0)+S(0))}{c_1(\infty)+c_2(\infty)}$$

with $O(0)+S(0)=1$.

So,

$$O(\infty)=\frac{c_2(\infty)}{c_1(\infty)+c_2(\infty)}$$

$$S(\infty)=\frac{c_1(\infty)}{c_1(\infty)+c_2(\infty)}=\frac{c_1(\infty)}{c_2(\infty)}*O(\infty)=\frac{\varphi_C^+ * B_C}{\varphi_C^- * D_C}*O(\infty)=\frac{\varphi_C^+ * \frac{\alpha_C^+ * C(\infty)}{\nu_C^+}}{\varphi_C^- * \frac{\alpha_C^- * C(\infty)}{\nu_C^-}}*O(\infty) \qquad (14)$$

LIST OF REFERENCES

Aart, Kees, MacDonald, Stuart E, and Rabinowitz, George. "Issues and Party Competition in the Netherlands." Comparative Politics Study 25-2 (1994): 63-99.

ABC News "Security Gains Reverse Iraq's Spiral Though Serious Problems Remain." 17 March 2008. <http://www.abcnews.go.com/images/PollingUnit/1060a1IraqWhere ThingsStand.pdf> 29 May 2008.

Atkinson, Kendall E. An Introduction to Numerical Analysis. New York: Wiley, 1989. Accessed through Wikipedia: http://en.wikipedia.org/wiki/Runge-Kutta. 7 May 2008.

Blaufarb, Douglas S. The Counterinsurgency Era: U.S. Doctrine and Performance. New York: The Free Press, 1977.

Central Intelligence Agency. "The World Factbook." 2008. <https://www.cia.gov/library/publications/the-world-factbook/index.html> 7 May 2008.

Condit, D.M., et al. A Counterinsurgency Bibliography. Washington D.C.: The American University, 1963.

Corum, James S. Fighting the War on Terrorism. Boston: Zenith Press, 2007.

Downs, Anthony. An Economic Theory of Democracy. New York: Harper, 1957.

Friedkin, Noah E and Johnsen, Eugene C. "Social Influence and Opinions." Journal of Mathematical Sociology 15 (1990): 193-205.

Gaver, D.P., Jacobs, P.A., Kress, M., Glazebrook, K.D, and Szechtmann, R. "A Model for the Effect of Coalition Actions on a Population in a Stability Operation Resulting from Discussion of Gaver, Jacobs, and Kress on 18 January 2008." working paper. 18 January 2008.

Goldenfeld, Nigel and Kadanoff, Leo P. "Simple Lessons from Complexity." Science 284 (1999): 87-89.

Gompert, David C. Heads We Win: The Cognitive Side of Counterinsurgency (COIN). Arlington: RAND, 2007.

Global Security.Org. 4 October 2006: <http://www.globalsecurity.org/military/world/iraq/city.htm> 22 May 2008.

Gray, Colin S. Irregular Enemies and the Essence of Strategy: Can the American Way of War Adapt? Carlisle Barracks: U.S. Army War College, 2006.

Hosmer, Stephen T. The Army's Role in Counterinsurgency and Insurgency. Pittsburgh: RAND, 1990.

Hosmer, Stephen T. and Crane, Sibylle O. Counterinsurgency: A Symposium. 16-20 April 1962. New Orleans: RAND, 2006.

Iraqi Coalition Provisional Authority. "Al-Anbar Province Description." 2008 <http://www.iraqcoalition.org/regions/south-central/provinces/english/anbar.html> 22 May 2008.

Iraq Body Count. "Civilian Deaths from Violence in 2007" 1 Jan 08. < http://www.iraqbodycount.org/analysis> 28 February 2008.

Jacobs, P.A., Gaver, D.P., Kress, M. and Damalas, K. "Deterministic 'Fluid' Models for Consequences of Coalition Actions in a Stability Operation." working paper. 10 April 2008.

Jacobs, P.A., Gaver, D.P., Kress, M. and Damalas, K. "A Model for the Effect of Coalition Actions on a Population in a Stability Operation." working paper. 26 March 2008.

Krackhardt, David. "Viscosity Models and Diffusion of Controversial Innovations." Journal of Mathematical Sociology 22-2 (1997): 177-199.

Langer, Gary. Poll: "Iraqis Own Surge Assessment: Few see." Time: the Real Clear Politics Blog. 17 Mar 08. <http://time-blog.com/real_clear_politics/2008/03/poll_iraqis_see_progress.html> 29 May 2008.

Long, Austin. On "Other War": Lessons from Five Decades of RAND Counterinsurgency Research. Arlington: RAND, 2006.

Mathworks, Inc. Matlab and Simulink Student Version 7.1. Released 31 Aug 05. <http://www.mathworks.com/> 7 June 2008.

Metz, Steven. Counterinsurgency: Strategy and the Phoenix of American Capability. Carlisle Barracks: U.S. Army War College, 1995.

Michaels, Jim. "19,000 Insurgents Killed in Iraq Since '03." Sep 07. USA Today Online. <http://www.usatoday.com/news/world/iraq/2007-09-26-insurgents_N.htm> 26 May 2008.

Olsen, Kathie L. "Proposal for Convening a Workshop on Complex Systems and the Science of Unanticipated Consequences and Unrealized Opportunities." 2 Oct 07. <http://www.nsf.gov/news/speeches/olsen/07/ko071001_oecd.jsp> 18 February 2008.

O'Hanlon, Michael E. and Campbell, Jason H. Iraq Index: Tracking Variables of Reconstruction and Security in Post-Saddam Iraq. The Brookings Institution. 30 July 2006. <http://www.brookings.edu/saban/iraq-index.aspx> 28 February 2008.

O'Hanlon, Michael E. and Campbell, Jason H. Iraq Index: Tracking Variables of Reconstruction and Security in Post-Saddam Iraq. The Brookings Institution. 22 May 08. <http://www.brookings.edu/saban/iraq-index.aspx> 29 May 2008.

Pauker, Guy J. Notes on Non-military Measures in Control of Insurgency. Arlington: RAND, 1962.

Rabasa, Angel, et al. Money in the Bank: Lessons Learned from Past Counterinsurgency (COIN) Operations. Arlington: RAND, 2007.

Radha Iyengar and Jonathan Monten. "Is There an "Emboldenment" Effect? Evidence from the Insurgency in Iraq." National Bureau of Economic Research Working Paper No. 13839, March 2008. <http://people.rwj.harvard.edu/~riyengar/insurgency.pdf> 1 June 2008.

Roggio, Bill. "The Long War Journal." 11 May 2007. <http://www.longwarjournal.org/archives/2007/05/anbar_rising.php> 22 May 2008.

Tyson, Ann Scott. "An Iraqi City Where Tasks Often Turn Into Combat." 22 July 2006. Washington Post Online. <http://www.washingtonpost.com/wpdyn/content/article/2006/07/21/AR2006072101667_pf.html> 26 May 2008.

U.S. Department of the Army and U.S. Marine Corps. Counterinsurgency: FM 3-24/MCWP. Washington, D.C.: Department of the Army Headquarters, 2006.

U.S. Department of Defense. DoD Directive 3000.05: "Military Support for Stability, Security, Transition, and Reconstruction (SSTR) Operations." 28 November 2005. <http://www.dtic.mil/whs/directives/corres/pdf/300005p.pdf> 18 February 2008.

U.S. Department of Defense. Measuring Stability and Security in Iraq. Report to Congress, 7 Mar 2008. <http://www.defenselink.mil/pubs/pdfs/ Master%20%20Mar08%20-%20final%20signed.pdf> 29 May 2008.

U.S. Department of Defense. Measuring Stability and Security in Iraq. Report to Congress, 29 August 2006. < http://www.defenselink.mil/pubs/pdfs/Security-Stabilty-ReportAug29r1.pdf> 29 May 2008.

Vinall, Casie. "Profile of Deputy Assistant Secretary of Defense for Stability Operations: Joseph Collins." Defense America. 2003. <http://www.defendamerica.mil/profiles/june2003/pr062403a.html> 21 April 2008.

Wikipedia.org. "Battle of Ramadi (2204)." < http://en.wikipedia.org/wiki/Battle_of_Ramadi_2004>. 16 June 2008.

Wikipedia.org. "Iraq Insurgency." < http://en.wikipedia.org/wiki/Iraqi_insurgency>. 16 June 2008.

Wikipedia.org. "Left-wing Politics." <http://en.wikipedia.org/wiki/Left-wing_politics>. 23 May 2008.

Wikipedia.org. "Right-wing Politics.". <http://en.wikipedia.org/wiki/Right-wing_politics>. 23 May 2008.

Wolf, Charles Jr. Insurgency and Counterinsurgency: New Myths and Old Realities. RAND, 1965. <http://www.rand.org/pubs/papers/P3132-1> 28 November 2007.

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