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**BLOCK-REPETITIVE ITERATED PROCESSING FOR  
SOFTWARE GPS RECEIVER: DICHOTOMIZED SEARCH  
OF CORRELATION PEAK (PREPRINT)**

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**Sigtem Technology, Inc.**

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An advantage of this dichotomized search is its reduced sensitivity to multipath. This can be understood as if it implements a sequential software narrow correlator vs. parallel hardware narrow correlators. Since it adapts to data with a variable spacing, it is computationally more efficient than a software implementation that uses a fixed number of correlators with the same coverage. Simulation results are presented to show the multipath-desensitized delay estimation as compared to the theoretical multipath error envelope and to the thermal noise errors under various SNR conditions for different multipath parameters.

# Block-Repetitive Iterated Processing For Software GPS Receiver: Dichotomized Search of Correlation Peak

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## ABSTRACT

The software GPS receiver offers a block-repetitive iterated processing capability that is difficult, if not impossible, to realize with a hardware correlator-based GPS receiver. Applications of such block-repetitive iterated processing include iterative estimation of GPS signal parameters under high dynamics and nonlinearities, cross-correlation cancellation of multi-access interference, and multipath mitigation.

In this paper, we first analyze the block-repetitive iterated processing capability offered by a software GPS receiver in both the acquisition and tracking modes as compared to conventional hardware correlator-based GPS receivers. Then we present a dichotomized search of the true correlation peak for delay estimation as an illustrating example of block iterative processing. This is in contrast to a conventional delay-locked loop (DLL) that moves several correlators of fixed spacing so as to drive the delay error discriminator to zero (an early-minus-late type). An advantage of this dichotomized search is its reduced sensitivity to multipath. This can be understood as if it implements a sequential software narrow correlator vs. parallel hardware narrow correlators. Since it adapts to data with a variable spacing, it is computationally more efficient than a software implementation that uses a fixed number of correlators with the same coverage.

Simulation results are presented to show the multipath-desensitized delay estimation as compared to the theoretical multipath error envelope and to the thermal noise errors under various SNR conditions for different multipath parameters.

## INTRODUCTION

Correlation is a critical operation step in any GPS receiver. When an incoming signal can successfully correlate with a locally generated code reference, it reveals which satellite the signal comes from. The

correlation process also removes the pseudo random number (PRN) code on the signal, accumulates the signal energy to go above the thermal noise through the despreading integration, and provides a timing measurement of the incoming code.

GPS signal processing is block-oriented in the sense that all meaningful operations are based on the results of despreading integration. This despreading integration accumulates on a block (or a segment) of incoming signal samples. More generally, this is true for all direct sequence spread-spectrum (DS/SS) receivers.

In conventional GPS receivers, a delay-locked loop (DLL) is used to track the incoming code phase, which consists of a delay error discriminator, a loop filter, a code numerical controlled oscillator (NCO), and a code generator [Parkinson and Spilker, 1996; Kaplan, 1996; Tsui, 2000; Misra and Enge, 2001]. The most widely used delay error discriminator is the early-minus-late correlation type, which attempts to equalize the correlation power on the early and late correlators by driving the code delay error discriminator to zero. At this point, the prompt correlator outputs its placement as the timing estimate.

According to [Moon and Stirling, 2000], iterative algorithms are those for which a new solution is obtained as a refinement of a previous solution (it is implied that the operation is done over the same data). Recursive systems provide an update of a solution as new data become available (the operation is sequential in nature). The DLL is a recursive estimator of the incoming code phase. If viewed as an iterative estimator, the iteration is done over time for a hardware correlator-based tracking channel, each with a new block of signal samples.

With a software GPS receiver, since each block of signal samples are held in memory (while another block is being filled in), this block can be accessed, if necessary, as many times as the computing power permits. As a result, the repetitive processing can be applied to each and every block so as to obtain the best estimate for each block, rather than iterating from one block to the next and waiting for the process to converge in time. In the latter case, the steady-state may never be reached because the incoming signal is under constant change.

This block-repetitive iterated processing capability is one of the features unique to the software GPS receiver that is difficult, if not impossible, to realize with a hardware correlator-based GPS receiver. Applications of block iterative processing include iterative estimation of signal parameters under high dynamics and nonlinearities, cross-correlation cancellation of multi-access noise (de-masking a weak signal), and multipath mitigation.

In this paper, we show an example of block-repetitive iterated processing by presenting the dichotomized search for the true correlation peak in delay estimation vs.

driving the delay error discriminator to zero. One advantage of this dichotomized search is that is less sensitive to multipath than a conventional delay error discriminator. This can be understood as if it actually implements a sequential software narrow correlator vs. parallel hardware narrow correlators. This offers a computational benefit for software implementation.

The rest of the paper is organized as follows. We first analyze the block-repetitive iterated processing capability offered by a software GPS receiver in both the acquisition and tracking modes in comparison to conventional hardware correlator-based GPS receivers. We then discuss the relationship between the conventional delay error discriminator's zero crossing and the direct signal's true timing as well as the true correlation peak of a composite signal (i.e., the direct signal plus multipath). We next describe the dichotomized search algorithm as applied to delay estimation. Finally, simulation results are presented to show the multipath-desensitized delay estimation performance as compared to the theoretical multipath error envelope and to the thermal noise errors under various SNR conditions for different multipath parameters.

## **BLOCK-REPETITIVE ITERATED PROCESSING**

Despreading integration acts as the boundary between hardware and software in a conventional GPS receiver while performing data compression or data rate reduction. When there are a large number of possible values of a signal parameter to test, a conventional receiver typically operates in one of three ways. In the first way, it uses a large number of hardware correlators in parallel, each assigned to a possible parameter value, to process the same block of signal samples simultaneously. This method is shown as "parallel processing" in Figure 1(a).

In the second way, only one set of correlators is used. It operates on a block of signal samples for a possible parameter value and goes through all possible parameter values sequentially on an equal number of blocks, one at a time. This method is shown as "sequential processing" in Figure 1(b). It is implied for this method that the signal parameters do not change too much before the cycle of sequential processing is completed.

In between is the third method, which is called "sequential parallel processing" in Figure 1(c). It uses a small number of parallel hardware correlators to process several parameter values per block and then goes through the remaining parameter values on a reduced number of blocks as compared to the second method.

In contrast, a software receiver offers the fourth method of "block-repetitive processing" as shown in Figure 1(d). The same block of data is processed repeatedly until all signal parameters are exhausted. If the memory is large enough and the computer is fast enough so that the processing can be completed within the length of a data

block, the block-repetitive processing can be sustained in real time. In average, only half of the uncertainty parameters need to be searched in the acquisition mode. In the tracking mode, only a small number of parameters are needed to construct signal parameter error discriminators.

One example of the block-repetitive processing is the use of the fast Fourier transform (FFT) to implement correlation, which searches all code phases for correlation per data block. Another example is the shifting of the signal spectrum up and down for Doppler removal. Another word that is frequently used to refer to what we call the “block-repetitive processing” in this paper is “parallel search” in software for time and/or frequency. In our opinion, the term parallel search in software is less accurate than block-repetitive processing. Furthermore, block-oriented processing can be generalized from one satellite search as described above to multiple satellite searches.

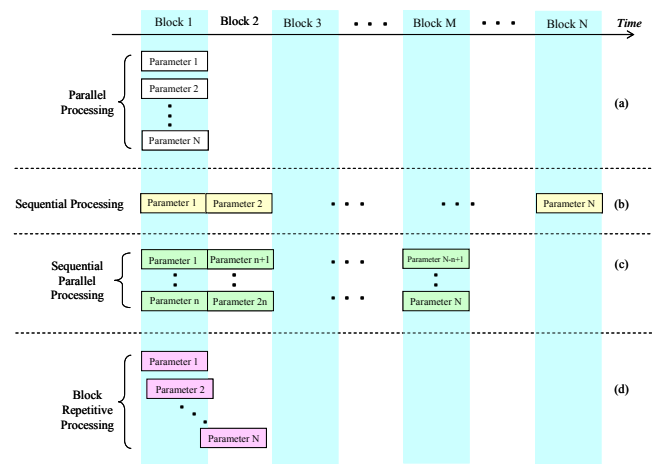


Fig. 1 Block-Oriented Processing in the Acquisition Mode

The above analysis of block-oriented processing was focused on signal acquisition in time and frequency over a large number of search cells. In the acquisition mode, the methods that a conventional hardware receiver can use include (1) parallel processing, (2) sequential processing, and (3) sequential parallel processing. A software receiver can either emulate (or mimic) the hardware receiver using the sequential processing or the block-repetitive processing. The latter is unique to software receivers and advantageous in that it can achieve the performance of parallel processing without massive parallel hardware.

Signal tracking is also block-oriented because of the need for despreading integration. It is used to catch up with the variations in the incoming signal, to reduce the initial errors in the signal parameter estimates, and to average out the effects of noise. It is typically implemented in a closed loop fashion and consists of an error discriminator, a loop filter, and a code/carrier numerical controlled

oscillator (NCO), and a code/carrier generator. Any perturbation to the loop undergoes transition before correction and/or convergence to a new value. Therefore, the closed-loop signal tracking can be viewed as an iterated estimation process where the iteration takes place per data block. In other words, a recursive formula is applied over time where an estimate at  $t-1$  is used to obtain the estimate at time  $t$ . In addition to the initial estimation error, signal variation, and noise, another reason for iteration is nonlinearities involved in the error discriminator and code/carrier generator. This sequential iterated processing is illustrated in Figure 3-2(a). Examples include delay-locked loop (DLL), phase-locked loop (PLL), and frequency-locked loop (FLL) in a GPS receiver.

With a software GPS receiver, the block-repetitive processing can be used to conduct the iterative estimation. This leads to “block-repetitive iterated processing” as shown in Figure 3-2(b) where a block of data is processed repeatedly for many times, one iteration making use of the results of the previous iteration so as to improve speed and performance.

In some applications, a bias may result from the block-repetitive iterated processing as illustrated in Figure 3-2(b) due to error terms and correlation between iterations. In conventional sequential iterated (recursive) processing, the error terms in measurements can be considered random and their effect can be averaged out if enough independent measurements (blocks of data) are used. However, with the block-repetitive iterated processing, only one block of data is used where error terms in measurements act as deterministic biases. To remove such biases, the operation needs to be carried out over several blocks.

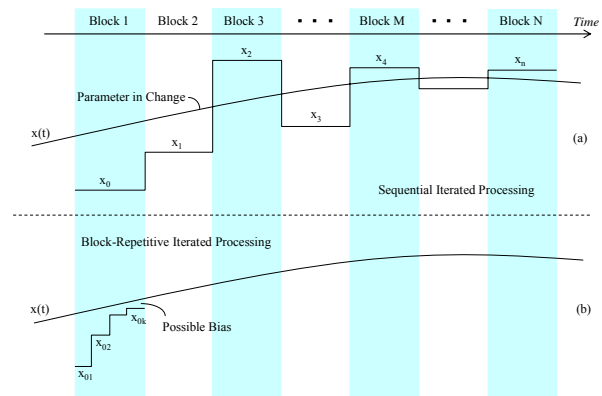


Fig. 2 Block-Oriented Processing in the Tracking Mode

The block-repetitive iterated processing as described in this paper can be applied to the following areas for a software GPS receiver [Yang, 2006]:

- **Multipath mitigation.** Narrowly spaced correlators are known to produce better multipath performance than the conventional  $\frac{1}{2}$ -chip spacing. In theory, a software GPS receiver can adjust its spacing to any small value as it desires. However, this is a tradeoff in coverage vs. spacing. The use of a large number of narrowly spaced correlators to obtain the desired coverage, though being possible, is computationally expensive. An alternative is to adjust the spacing iteratively. One example is the dichotomized search for peak timing as detailed in this paper.
- **Near-far interference.** A block of data is processed, in the first iterations, for all strong signals that can be acquired and tracked, starting from the strongest downwards. The acquired/tracked signals are then reconstructed and removed “coherently” from the block of data. The remaining weak signal is then processed in iterations for acquisition and tracking without the masking effects from the strong signals. Similar to this iterative cancellation of near-far interference in the signal domain, iterative cancellation can also be formulated in the correlation domain to further take advantage of software GPS receivers.
- **Nonlinearity.** Small signal error models are used in the design of conventional tracking loops. In ultra-tightly coupled GPS/INS integration, the correlation values are expressed in terms of signal parameters via linearization. The linear model parameters and linearization partials are evaluated at the previous estimates. As a result, the linearization process can be iterated several times because better estimates provide better linear approximation, which leads in turn to even better estimate.
- **High dynamics.** When the tracking loop bandwidth is small compared to the signal variation dynamics, it will attenuate the high-frequency components of the signal, thus lagging behind in changes. The net effect is an immediate increase in signal tracking errors and even leads to instability if the tracking loop cannot catch up with the signal in subsequent data blocks. One popular technique to handle high dynamics is to design upon a nominal trajectory in such a way that the residual dynamics falls within the loop bandwidth. With repetitive processing, a software GPS receiver can apply repeated tracking to blocks of data, an iteration using the estimates of the previous to remove known dynamics while narrowing the loop bandwidth accordingly.

### CORRELATION PEAK UNDER MULTIPATH

The code multipath errors as experienced by most GPS receivers in use today are due to the way in which the code phase is measured in the receivers. To illustrate, consider the simple case with one reflected signal on top of the direct signal. The code tracking loop correlates the

incoming signal with three locally generated code replicas called “early,” “prompt,” and “late.” This is equivalent to sampling the correlation function at the three lags. The spacing between two adjacent correlation lags (or the sampling interval) is assumed to be  $S$ .

The correlation between the incoming signal and the “prompt” replica is approximated with an equilateral triangle with its base length of two chips,  $2T$  where  $T$  is the chip duration, ignoring the effect of finite bandwidth on the shape, which otherwise rounds the correlation around the peak. The tracking loop adjusts the timing of its code replicas so as to equalize the “early” and “late” correlations. When this is achieved, the “prompt” replica indicates the timing of the incoming signal.

As shown in Figure 3, when the time delay  $p$  of the multipath signal relative to the direct signal varies, the placement of the “prompt” correlation moves around the direct signal. This results in the multipath-induced timing error (or code range error) denoted by  $q$ . Figure 4 illustrates the multipath-induced timing error envelope where the corner points and the slopes for all segments are identified using the technique presented in [Byun, Hajj, and Young, 2002].

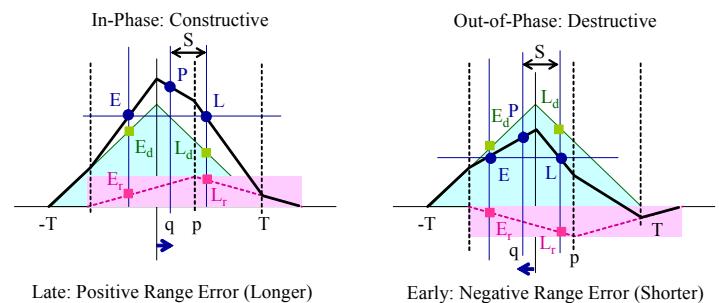


Figure 3 – Correlation Function with One Multipath

The upper region of Figure 4 corresponds to constructive multipath, which adds to the direct signal. As shown in the left plot of Figure 3, the composite correlation function is elevated and the “early” and “late” pair is moved right. As a result, the “prompt” develops a positive timing error. The time of arrival thus measured is larger (later) than the actual one, producing a longer range.

The lower region of Figure 4 corresponds to destructive multipath, which subtracts from the direct signal. As shown in the right plot of Figure 3, the composite correlation function is pushed down and the “early” and “late” pair is moved left. As a result, the “prompt” develops a negative timing error. The time of arrival thus measured is smaller (earlier) than the actual, producing a shorter range.

Mathematically, the biased timing error ( $q$  in Figures 3 and 4)  $\Delta\tau_q$  for  $S < T/2$  (narrow sampling interval) [Byun, Hajj, and Young, 2002] is given by:

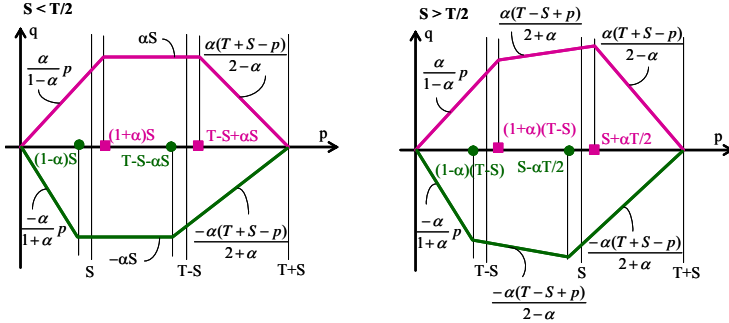


Figure 4 – Multipath-Induced Timing Error Envelope

$$\Delta\tau_q = \begin{cases} \frac{\Delta\tau_1\alpha\cos(\omega\Delta\tau_1 + \phi_1)}{1 + \alpha\cos(\omega\Delta\tau_1 + \phi_1)} & \Delta\tau_1 < S + \Delta\tau_p \\ \frac{S\alpha\cos(\omega\Delta\tau_1 + \phi_1)}{(T + S - \Delta\tau_1)\alpha\cos(\omega\Delta\tau_1 + \phi_1)} & S + \Delta\tau_p < \Delta\tau_1 < T - S + \Delta\tau_p \\ \frac{2 - \alpha\cos(\omega\Delta\tau_1 + \phi_1)}{0} & T - S + \Delta\tau_p < \Delta\tau_1 < T + S \\ 0 & \Delta\tau_1 > T + S \end{cases} \quad (1)$$

where  $\Delta\tau_1$  is the delay of the multipath component relative to the direct signal ( $p$  in Figures 3 and 4),  $\phi_1$  is the relative phase,  $\alpha$  is the relative strength, and  $\omega$  is the carrier frequency.

Similarly, the timing error for  $S > T/2$  (wide sampling interval) is given by:

$$\Delta\tau_q = \begin{cases} \frac{\Delta\tau_1\alpha\cos(\omega\Delta\tau_1 + \phi_1)}{1 + \alpha\cos(\omega\Delta\tau_1 + \phi_1)} & \Delta\tau_1 < T - S + \Delta\tau_p \\ \frac{(T - S + \Delta\tau_1)\alpha\cos(\omega\Delta\tau_1 + \phi_1)}{2 + \alpha\cos(\omega\Delta\tau_1 + \phi_1)} & T - S + \Delta\tau_p < \Delta\tau_1 < S + \Delta\tau_p \\ \frac{(T + S - \Delta\tau_1)\alpha\cos(\omega\Delta\tau_1 + \phi_1)}{2 - \alpha\cos(\omega\Delta\tau_1 + \phi_1)} & S + \Delta\tau_p < \Delta\tau_1 < T + S + \Delta\tau_p \\ 0 & \Delta\tau_1 > T + S + \Delta\tau_p \end{cases} \quad (2)$$

It is clear from Figure 4 that the smaller  $S$  and  $T$ , the smaller are the multipath code errors. That is why the P(Y)-code has better multipath performance than C/A-code and so does the narrow correlator receiver. It is also clear from Figure 3 that no matter how the slopes on either side are distorted (or the zero crossing is offset), the true correlation peak still corresponds to the true direct signal timing. This is the basis for the dichotomized search method as an example of the block-repetitive iterative processing presented in the next section.

## DICHOTOMIZED SEARCH FOR THE TRUE CORRELATION PEAK

Since the multipath signals are always later than the direct signal, it affects more the late side of the correlation function than the early side, thus changing the shape of the correlation function asymmetrically as shown in Figure 3 and moving the zero crossing of the early-minus-late correlation away from the true signal as shown in

Figure 4. As a result, this null-locking scheme is vulnerable to multipath.

The concept of multipath-desensitized delay estimation is based on the fact that the direct signal arrives earlier and is stronger than the multipath components (except for the rare cases where the direct signal is masked but not the multipath). By consequence, the correlation of the direct signal has a larger peak than the multipath counterpart, whose peak is located in the late side. Since both the correlation functions have the same support ( $\pm 1$  code chip), the direct signal's correlation has a larger slope. The composite correlation, as the sum of the direct signal and multipath correlations, still has its global peak at the original direct signal peak location corresponding to the true timing even though the shape of the composite correlation has been distorted on both sides.

Instead of null locking as in the conventional DLL, the multipath-desensitized delay estimation technique performs the true peak seeking. In general, finding a null is a better practice than finding a peak simply because the null is well defined while the peak is not, particularly for unknown signals. Besides, the early minus late delay error discriminator has a slope which is twice larger at zero than at the peak, thus having better noise performance. But the null locking is more vulnerable to multipath than the peak seeking as analyzed above.

In principle, the time-domain multipath-desensitized approach requires the evaluation of the entire main lobe of correlation function in order to find the peak to a great accuracy. This can be done by running a large number of closely spaced correlators in parallel. Alternatively, the prompt channel of a conventional DLL may be converted into a sweeping correlator, which does the peak search sequentially, while the early and late correlators keep locking onto the incoming signal (even at an erroneous timing). Once the peak is found, a multipath correction term can be derived so as to compensate for it in pseudorange measurements. A third technique is to implement a tau-dither correlator, which, instead of equalizing the power on either side, seeks the maximum power.

With a software GPS receiver, it is rather easy to use its block repetitive processing capability to conduct a dichotomized search iteratively on the same block of data rather than sequentially with different data blocks from one iteration to another. Figure 5 is the block diagram of a possible implementation of the time-domain multipath-desensitized delay estimation technique [Yang, 2005]. Starting with the initial correlation spacing of  $T_s = 1/2$  chip, the timing error after the  $n^{\text{th}}$  search step is  $\Delta t = T_s 2^{-(n+1)}$ ,  $n \geq 0$ . After three iterations, the timing error is reduced to 1/16 of a chip.

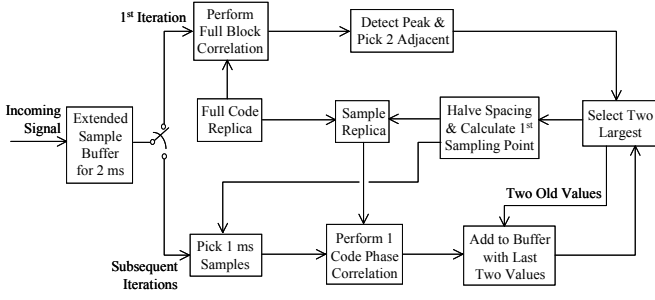


Figure 5 – Time-Domain MDDE with Iterative Dichotomized Search

Figure 6 illustrates the dichotomized search process. The initial correlator spacing is denoted by  $d_1$ . The three large correlations values are denoted by  $P_1$ ,  $P_2$ , and  $P_3$  where  $P_2$  is the maximum value. The correlations are calculated with the replica sampled with the first sample right on the rising edge of ms boundary. Since  $P_3$  is larger than  $P_1$ , it is reasonable to believe that the true peak lies somewhere between  $P_2$  and  $P_3$ .

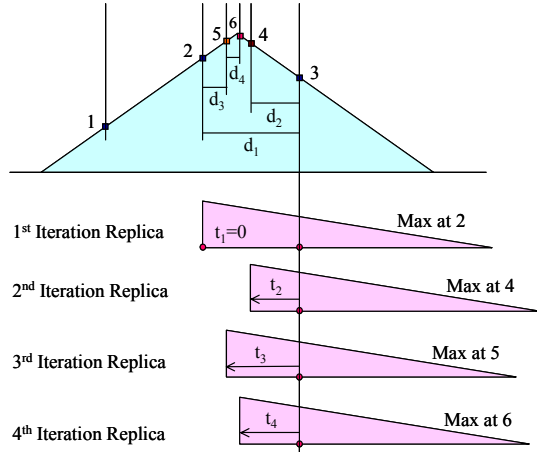


Figure 6 – Dichotomized Search for Peak Timing

The 2<sup>nd</sup> iteration starts with the halved correlator spacing denoted by  $d_2 = d_1/2$ . The correlation value  $P_4$  is calculated by preparing a replica with its first sample at  $t_2$  from the ms boundary. For the particular case of Figure 6, since  $P_2$  is larger than  $P_3$ , the true peak is most likely to lie somewhere between  $P_2$  and  $P_4$ .

Similarly, the 3<sup>rd</sup> iteration starts again with the halved correlator spacing denoted by  $d_3 = d_2/2$ . The correlation value  $P_5$  is calculated by preparing a replica with its first sample at  $t_3$  from the ms boundary. Since  $P_4$  is larger than  $P_2$ , the true peak is most likely to lie somewhere between  $P_4$  and  $P_5$ .

Finally, the 4<sup>th</sup> iteration starts with halving the correlator spacing as  $d_4 = d_3/2$ . The correlation value  $P_6$  is calculated by preparing a replica with its first sample at  $t_4$  from the ms boundary. Since  $P_5$  is larger than  $P_4$ , the true peak is most likely to lie somewhere between  $P_5$  and  $P_6$ . The process continues until the number of iterations reaches a preset threshold or the successive correlation values are smaller than a preset threshold.

The search performance is affected by such factors as the prefiltering bandwidth, which tends to flatten out the peak. The flattened shape at the peak makes it more vulnerable to noise, thus being biased to a wrong location. It may affect the speed of convergence. However, if the correlation peak location for a given data block corresponds to the true timing, the dichotomized search is able to find it. It is believed to perform no worse than narrow correlators. When the aggregated bandwidth of RF and IF filtering is 5 to 10 times larger than that of the GPS signal code of interest, the correlation function can preserve the correlation shape with a quite sharp peak. Nevertheless, a calibration curve may be built, which relates the apparent peak to the true signal timing as a function of  $\tau$  and  $\alpha$  for the given pre-filtering bandwidth.

## SIMULATION RESULTS AND ANALYSIS

The time-domain multipath-desensitized delay estimation technique is compared, via simulation, with a conventional delay error estimator that fits a quadratic curve to the three largest correlation values and interpolates for the analytic peak and its location. Such a quadratic fitting produces a timing error estimate comparable to the early minus late delay error discriminator used in most conventional DLL.

Denote the maximum correlation and its two adjacent values by  $C_{m-1}$ ,  $C_m$ , and  $C_{m+1}$  in the ascending order of their index. The complex correlation can be written as  $C_m = |C_m|e^{j\phi_m}$  where  $|C_m|$  is the magnitude and  $\phi_m$  is the phase. Fitting a quadratic form to these values produces the interpolated peak and the peak offset as:

$$|C|_{\text{peak}} = |C_m| + \frac{|C_{m+1}| - |C_{m-1}|}{4} \delta \quad (3a)$$

$$\delta = \frac{1}{2} \frac{|C_{m-1}| - |C_{m+1}|}{|C_{m-1}| + |C_{m+1}| - 2|C_m|} \quad (3b)$$

where  $\delta$  is the offset in units of fractional sampling interval (or the correlator spacing) away from the maximum correlation index  $m$ .

The corresponding delay and amplitude are obtained as:

$$\hat{\tau} = (m + \delta)T_s \quad (4a)$$

$$\hat{\alpha} = \frac{|C|_{\text{peak}}}{N} e^{j\phi_m} \quad (4b)$$

where  $N$  is the number of data samples included in the correlation.

Two peak seeking methods are implemented in the simulations: one with massive closely spaced parallel correlators (similar to narrow correlators) and the other conducting the dichotomized search.

The initial phase is drawn uniformly from  $[0, 2\pi)$  without Doppler frequency error for simplicity. A complex white Gaussian noise of unit variance is added to the signal samples and the signal amplitude is adjusted to simulate the desired SNR level. The signal amplitude is calculated as:

$$A = (T_i 10^{\frac{C/N_0}{10}})^{\frac{1}{2}} \quad (5)$$

where  $T_i = 0.001$  s and  $C/N_0$  is varied to simulate different SNR levels.

In the first simulation, we set the signal strength as  $C/N_0 = 30$  dB-Hz and choose one multipath with a delay of  $\tau$  chips and a complex amplitude of  $\alpha = 0.4 + 0.3i$  (a strong multipath with only 6 dB attenuation relative to the direct signal). For a 2 ms worth of signal samples, the true ms boundary of the incoming signal is at  $1002 - 0.45 = 1001.55$  samples. In other words, the ms boundary is between two samples, thus making it more difficult for discrete search.

We start with the multipath delay  $\tau = 0.05$  chips, then vary it from 0.1 to 1.5 chips by 0.2 chips per step. The results of three algorithms are shown in Figure 7. They are (1) the quadratic fitting algorithm of Eq. (4) applied to the equivalent narrow correlators of 0.2 chip spacing (the green curve), (2) the use of parallel correlators with 0.02 chip spacing (i.e., 1/10 of the sampling interval) (the red curve), and (3) the iterative dichotomized search (the cyan curve). Also shown in the same figure for easy comparison are (4) the timing estimate from the signal and noise only without multipath using the quadratic fitting (the blue curve) and (5) the theoretical multipath error calculated with spacing of 0.2 chips and the single multipath strength  $\alpha = 0.5$  (the magenta curve).

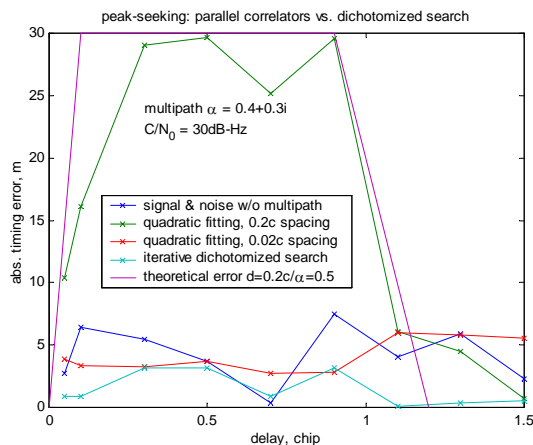


Figure 7 – Timing Errors vs. Multipath Delay

The multipath-induced errors as measured by the quadratic fitting (the green curve) match quite close to the theoretical prediction (the magenta curve). See Figure 4 where the multipath-induced error is  $\alpha S = 0.5 \times 0.2 = 0.01$  chips or 30 m for the GPS C/A-code. The former is a little bit lower than the latter. This may be explained by the fact that the multipath strength used in the simulation ( $\alpha = 0.4 + 0.3i$ , complex) differs from the value used in theoretical prediction ( $\alpha = 0.5$ , real). Besides, it is only a sample behavior where random noise contributes to variations from sample to sample.

Both the parallel correlators with 0.02 chip spacing (the red curve) and the iterative dichotomized search (the cyan curve) are close to the timing errors estimated from the signal and noise only without multipath using the quadratic fitting (the blue curve), indicating the performance of multipath insensitivity of the peak-seeking concept. The number of iterations used in the dichotomized search is 10, which is equivalent to the correlator spacing of  $2^{-10} = 1/1024$  of a sampling interval.

We now take a closer look at the iterative dichotomized search algorithm. Figures 8 and 9 show the estimated correlation peak value and timing (the location of ms boundary relative to the first sample) as a function of iteration for the case where there is no multipath ( $\alpha = 0$ ). After six or seven iterations, the algorithm converges to the desired values.

Figures 10, 11, and 12 show the results for the case where the multipath delay is  $\tau = 0.05$  chips. In Figure 10, the correlation of signal and noise without multipath (the blue curve) is compared with that with multipath (the green curve). The magenta curve indicates the true timing. Since the multipath is very close to the direct signal, the two signals “add” up. That is why the correlation with multipath (the green curve) is almost double of that without multipath (the blue one). Figures 11 and 12 show the estimated timing and correlation peak value as a function of iteration, respectively, for this case with  $\alpha = 4+3i$  and  $\tau = 0.05$ . After about seven iterations, the algorithm converges to the desired values.

Now consider the case with two multipath components. The first multipath component is set with  $\tau_1 = 0.2$  chips and  $\alpha_1 = 4+3i$  and the second multipath component is with  $\tau_2 = 0.4$  chips and  $\alpha_2 = 0.2$ . As can be seen from Figure 7, the conventional algorithm starts to experience the worst multipath error around this delay for the correlation spacing of  $\tau_1 = 0.2$  chips.

Figure 13 shows the correlation of signal and noise without multipath (the blue curve) as compared to that with multipath (the green curve). The two correlation functions are slightly distorted in shape as compared to those in Figure 10.

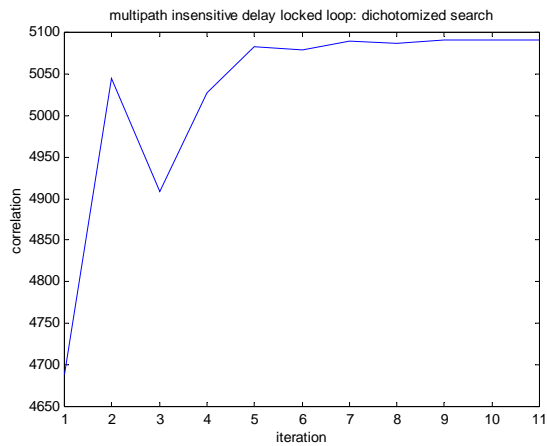


Figure 8 – Correlation Magnitude vs. Iteration

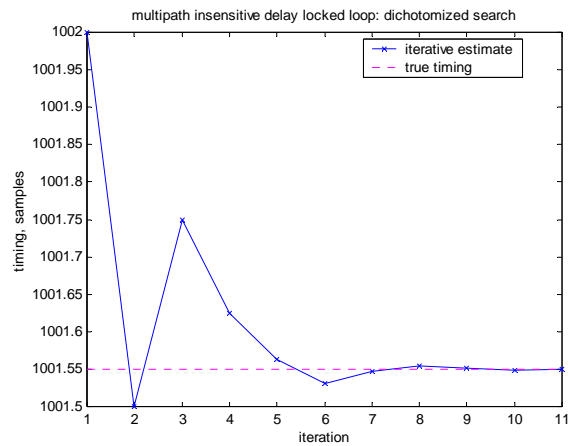


Figure 9 – Timing Estimate vs. Iteration

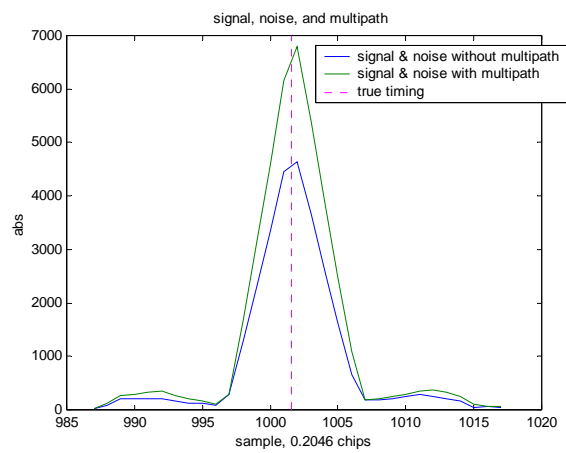


Figure 10 – Correlations with and without Multipath

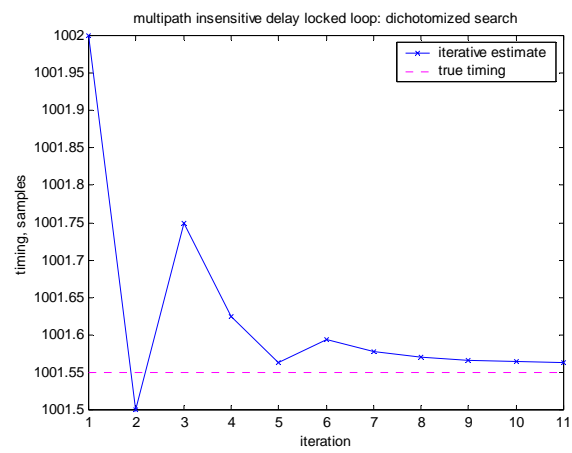


Figure 11 – Timing Estimate vs. Iteration

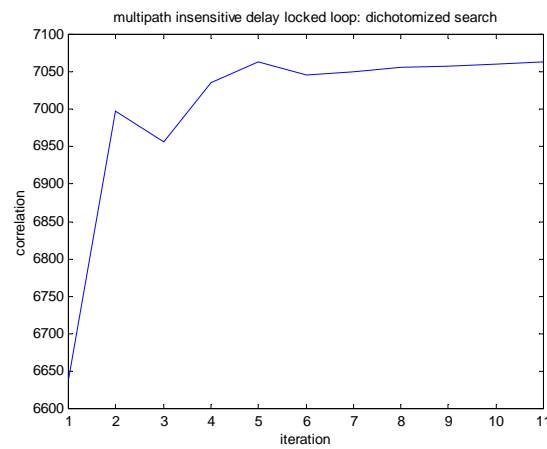


Figure 12 – Correlation Magnitude vs. Iteration

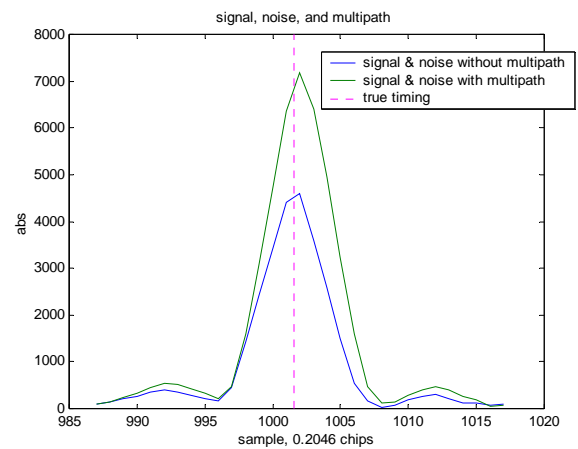


Figure 13 – Correlations with and without Multipath

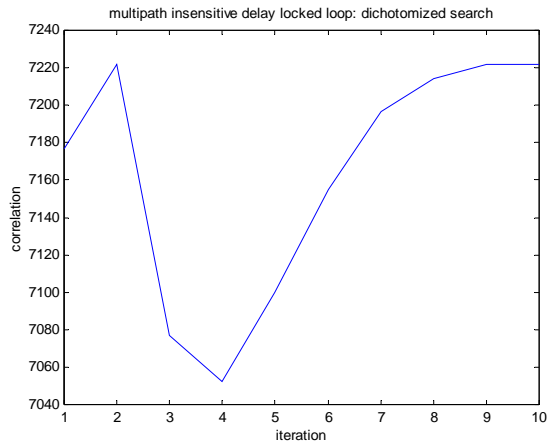


Figure 14 – Correlation Magnitude vs. Iteration

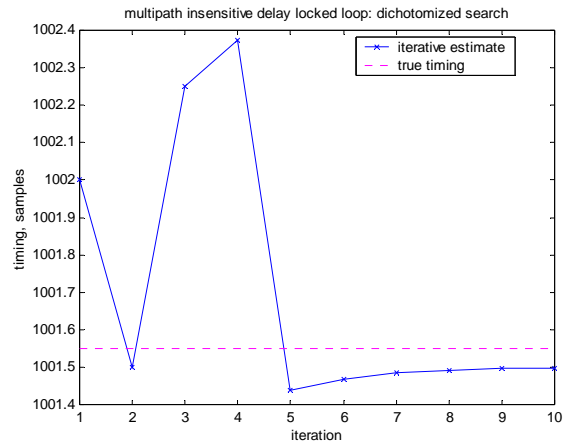


Figure 15 – Timing Estimate vs. Iteration

Figures 14 and 15 show the estimated correlation peak value and timing as a function of iteration, respectively, for this case with two multipath components. Compared to previous cases with a single multipath, there are large swings in the estimated values during the search process. Nevertheless, the algorithm converges to the desired values after six iterations.

## CONCLUSIONS

In this paper, we presented the dichotomized search of the true correlation peak for delay estimation as an example of the block-repetitive iterated processing capability unique to software GPS receivers. It was shown by simulations that this dichotomized search was less sensitive to multipath than a conventional delay error discriminator. This is based on the fact that the composite correlation peak still corresponds to the true timing of the direct signal. By implementing a sequential (adaptive) narrow correlator, the dichotomized search approach offered computational efficiency as compared to software implementation but using fixed-spacing correlators, albeit narrowly spaced. In other words, for the same amount of correlations over a given coverage, the dichotomized search approach can reach a much finer spacing.

In addition to multipath mitigation, the block-repetitive iterated processing can be used for estimation of GPS signal parameters under high dynamics and nonlinearities and cross-correlation cancellation of multi-access interference among others. Initial simulation results presented in this paper show the potential of the block-repetitive iterated processing capability for software receivers in general and the dichotomized search method in particular warrants further studies.

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