



# On the Use of Path Diversity with Bursty Channels

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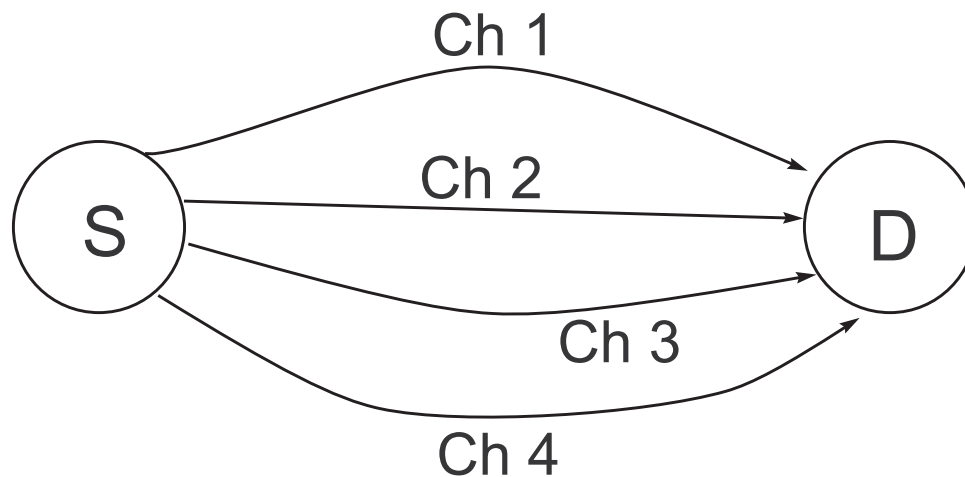
# Motivation

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- Avoid retransmissions by using some coding.
- Increase the Probability of Success of the transmission.
- Given a minimum desired probability of success, increase the Effective Rate of the system.
- “Robust Networking”

# Connectivity Model

- Multiple available paths with different long-term characteristics.
- We consider the case where **two** of the available paths are used.





# Setting

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- Open-loop environment
  - No real-time (feedback) information on the state of the channels.
  - Only long term statistics are known.
- Applicable in wireless & wired settings
  - More than one channel may be available
  - Long term channel characteristics can be estimated (measured).



# Coding

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- Use an  $(N, k)$  code
  - A block of  $N$  symbols can be reconstructed if  $k$  or more symbols are received correctly.
  - Introduced under different names in various settings, e.g.,
    - Information Dispersal (Rabin, 1989)
    - Diversity Coding (Ayanoğlu *et al.*, 1990)



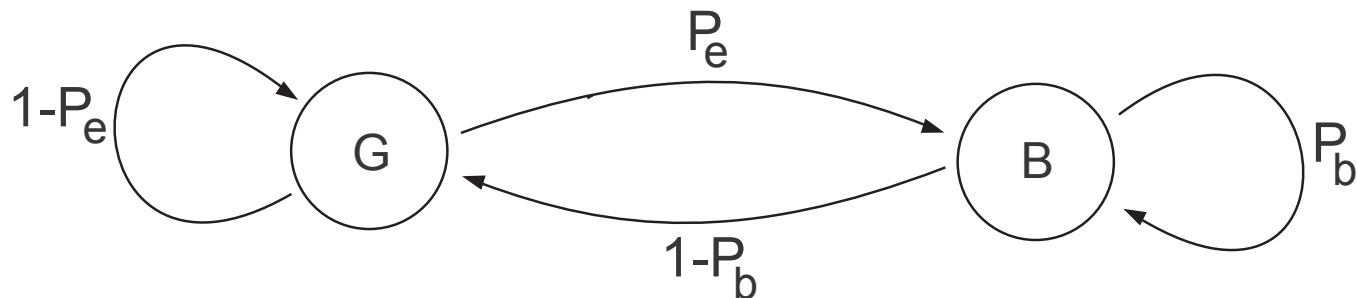
# Transmission Policy

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- How many packets on which channel?
- Focus on simple random policy
- Policy is specified by  $p \in [0,1]$ 
  - Each of the  $N$  packets is sent over channel 1 with probability  $p$  and over channel 2 with probability  $(1-p)$ .
- Objective: Maximize performance over all possible policies (values of  $p$ ).

# Channel Model

- Use a simplified Gilbert-Elliott model for each individual channel.

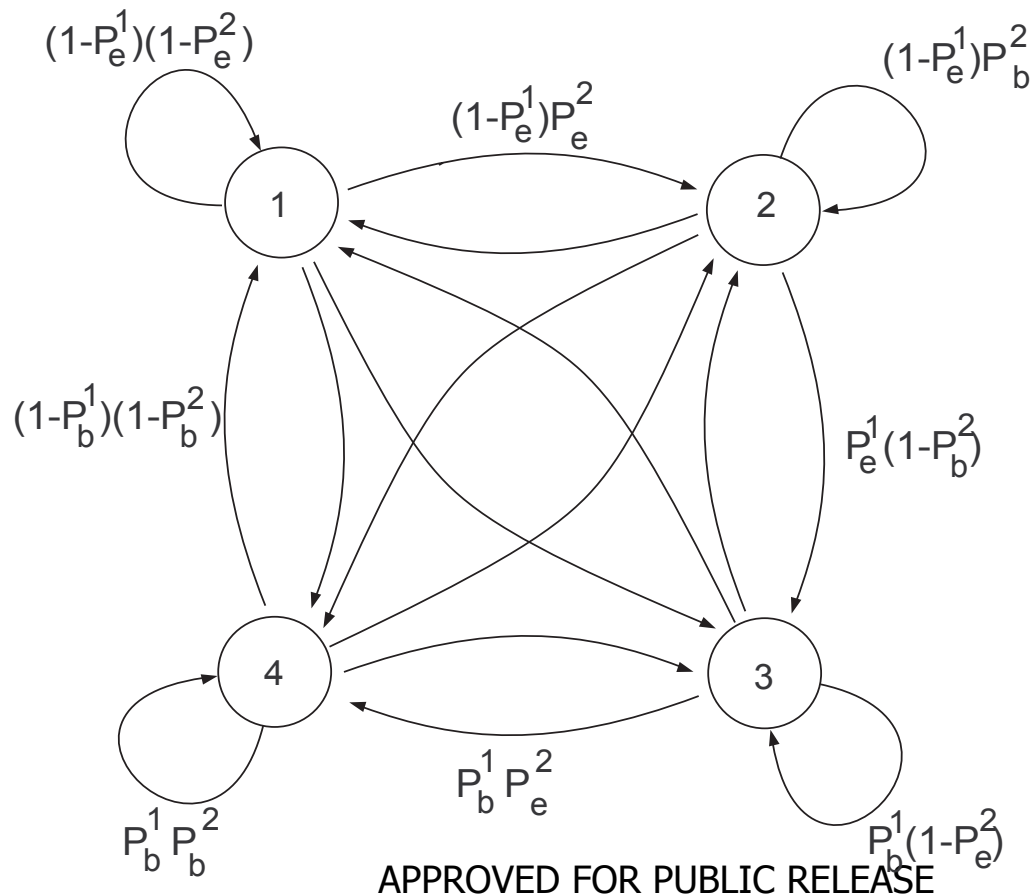


- $P_e = P\{\text{error} \mid \text{previous symbol was correctly received}\}$
- $P_b = P\{\text{error} \mid \text{previous symbol was in error}\}$

# System Model

## Two bursty channels

- 4-state Markov Chain



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# Recursive Expressions

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- The probability of success of an  $(N, k)$  code can be written as

$$P_{succ}(N, k) = \sum_{m=0}^{N-k} \sum_{i=1}^4 \sum_{j=1}^4 \pi_i P_{ij}(m, N)$$

where  $P_{ij}(m, n)$  corresponds to the probability of exactly  $m$  errors in  $n$  symbols, starting from state  $i$  and ending in state  $j$ . It can be recursively calculated via

$$P_{ij}(m, n) = \sum_{k=1}^4 P_{ik}(m, n-1) \cdot P_{kj} \cdot P\{\text{no error} \mid \text{state is } j\} \\ + \sum_{k=1}^4 P_{ik}(m-1, n-1) \cdot P_{kj} \cdot P\{\text{error} \mid \text{state is } j\}$$



# Performance Metrics

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- For a given  $(N, k)$  code, by how much can the **probability of success** be increased?
- Assuming a minimum desired probability of success  $P_{\min}$ , by how much can the **Effective Rate** of the system be increased using the *smallest* possible code?
- By how much can the **Effective Rate** be increased when we use the *best* possible code (given a **maximum allowable code length**)?



# Channel Configurations

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- We consider two sets of channels
  - Low error rate channels (1-9%)
  - High error rate channels (25-40%)
- The low error rate channels range from random, to bursty (expected length of burst  $E[B] = 10$ ).
- The high error rate channels range from random, to bursty (expected length of burst  $E[B] = 20$ ).

# Results

## Increasing the Probability of Success I

- Consider a (20,12) code and the high long term error rate channels
  - Using both channels can lead to an increase in  $P_{\text{succ}}$  of up to 70%.
  - The average increase in  $P_{\text{succ}}$  is about 16.9%.
  - The average  $P_{\text{succ}}$  using both channels is about 80.2%.
- Average increase in  $P_{\text{succ}}$  for the (20,18) code is about 387.4%!!
  - The average  $P_{\text{succ}}$  using both channels is about 28.7%.



# Results

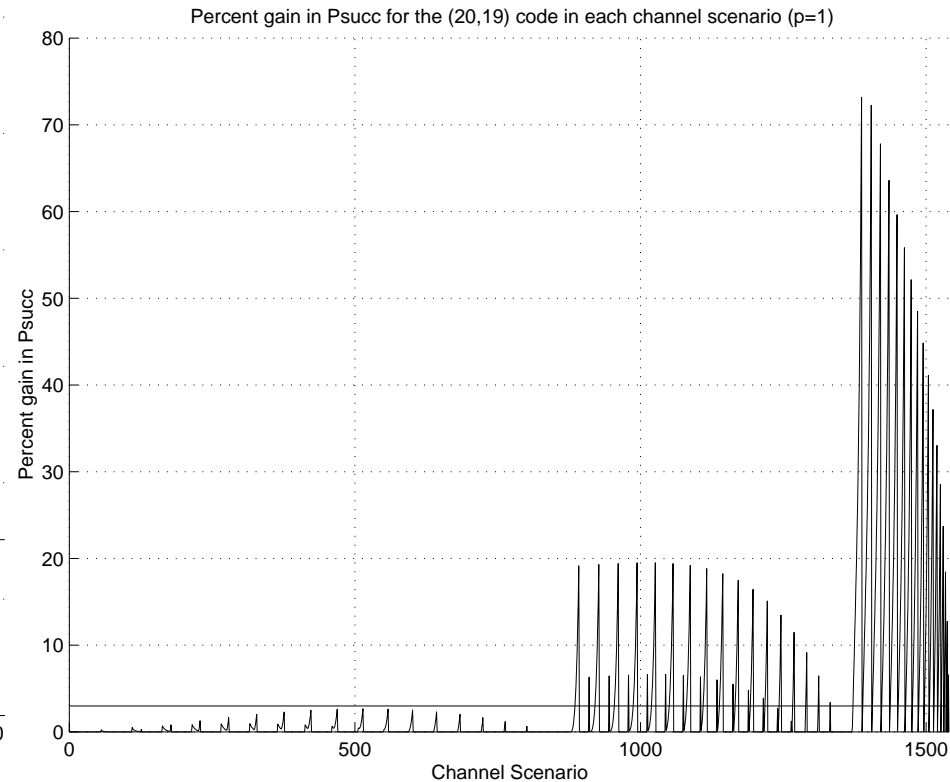
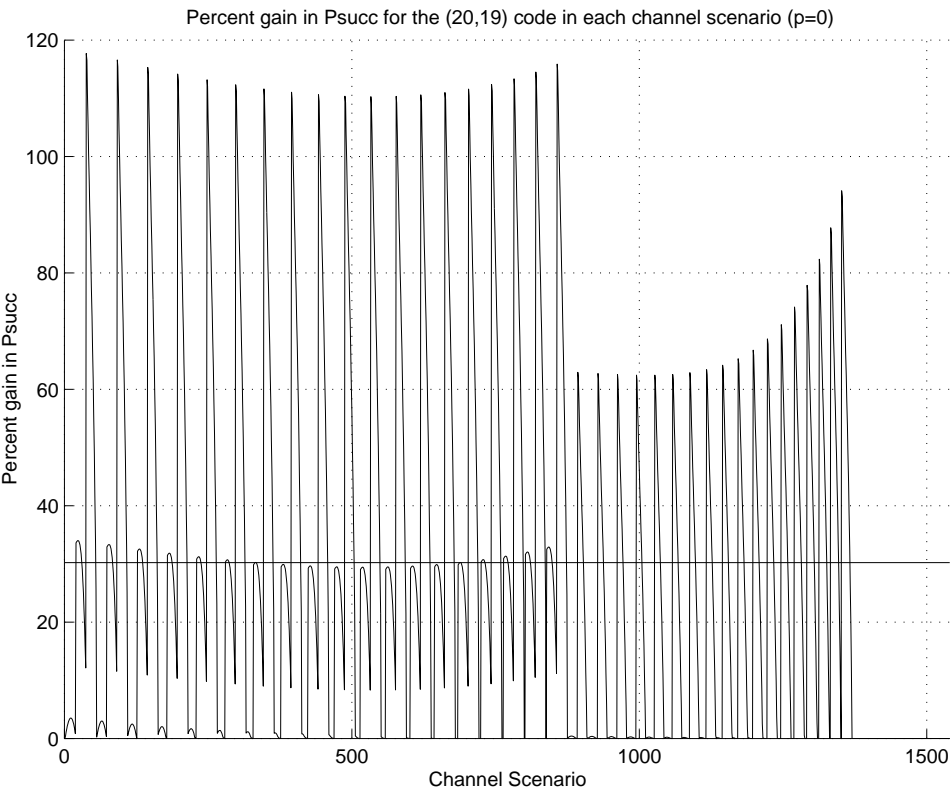
## Increasing the Probability of Success II

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- Consider a  $(20,19)$  code and the low long term error rate channels
  - Using both channels can lead to an increase in  $P_{\text{succ}}$  of more than 115%.
  - The average increase in  $P_{\text{succ}}$  is about 17%.
  - The average  $P_{\text{succ}}$  using both channels is about 86.41%.

# Results

## Increasing the Probability of Success II





# Results

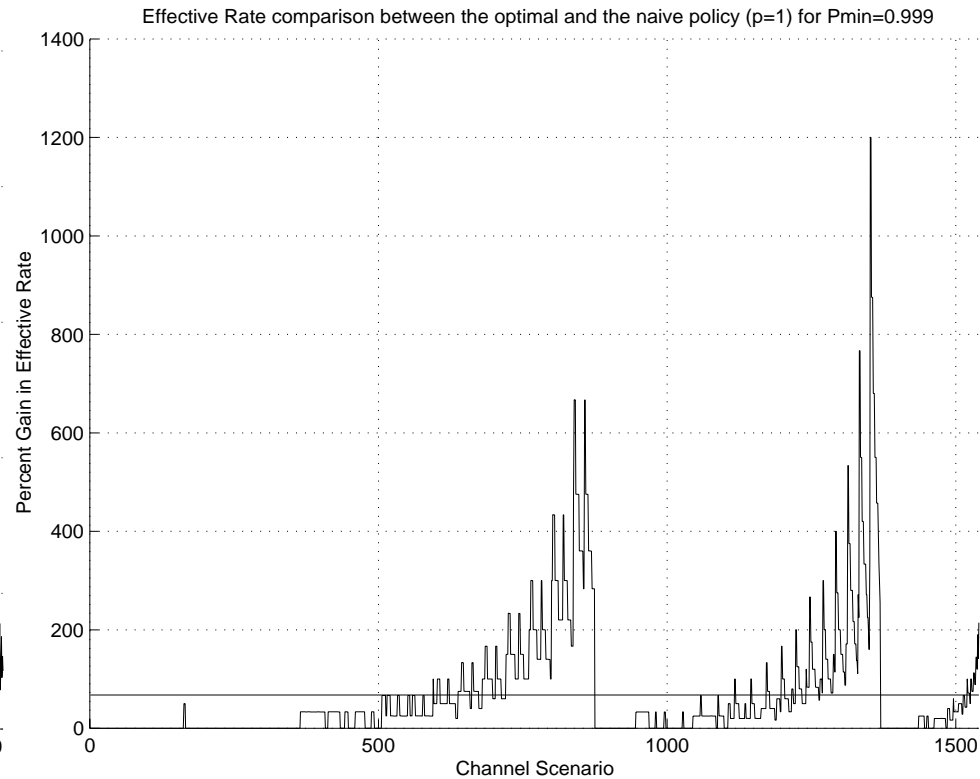
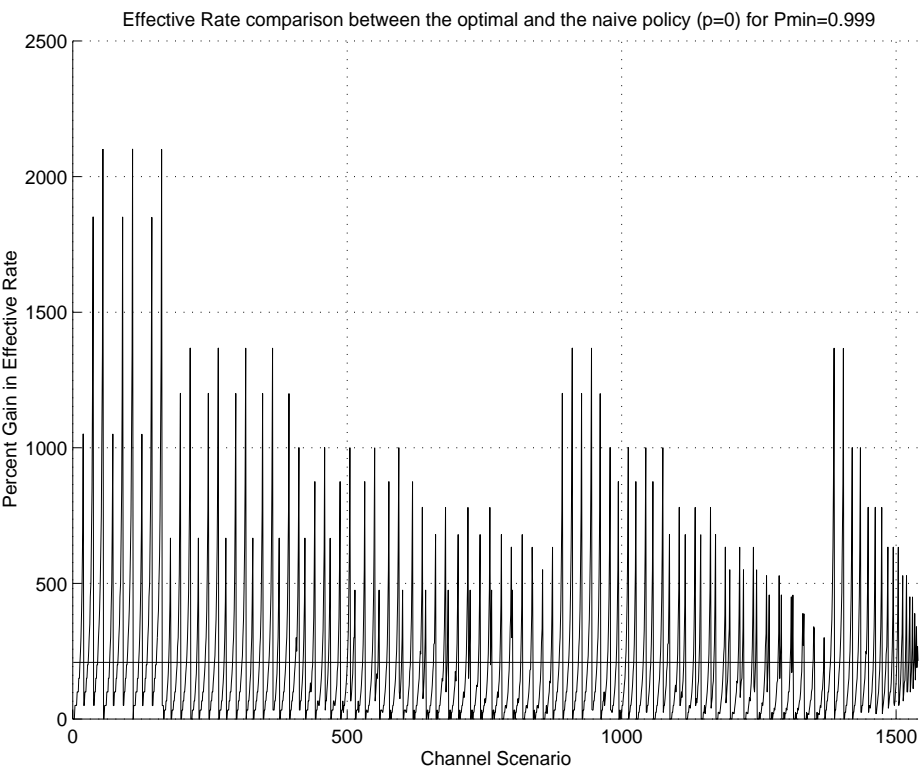
## Increasing the Effective Rate

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- Consider the low error rate channels.
  - Let the minimum desired probability of success ( $P_{\min}$ ) be 0.999.
  - Use the smallest  $(N, k)$  code that achieves  $P_{\min}$ .
- The increase in Effective Rate can be more than 2000%.
- The average increase in Effective Rate is about 138%.
- Gains are even higher when we consider the high error rate channels.

# Results

## Increasing the Effective Rate





# Results

## Increasing the Effective Rate for Large Codes

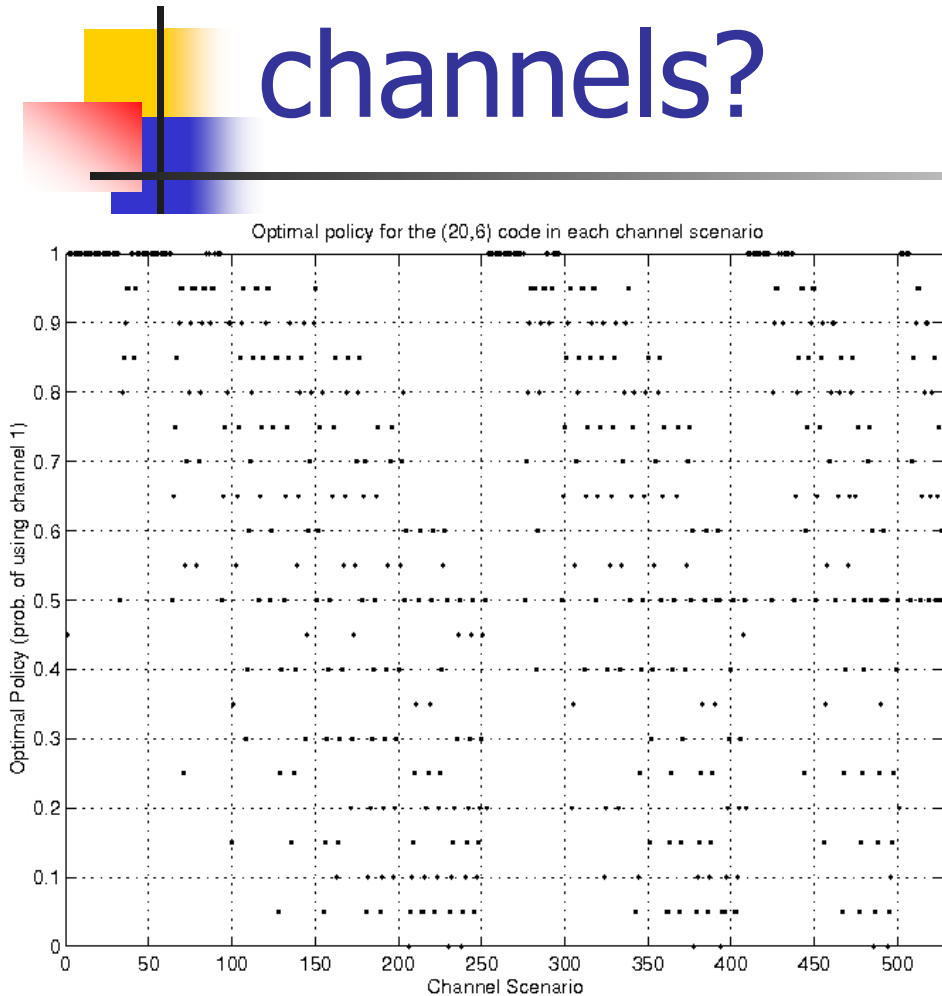
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- Consider the case of two identical GSM channels
  - Both channels are bursty ( $E[B] = 6.67$ ).
  - Long term error rate is 5%.
- When the length of the code is restricted to 50, the gain is about 33%.
- When the length of the code is restricted to 100, the gain is about 17%.
- When the length of the code is restricted to 500, the gain is about 4.2%.

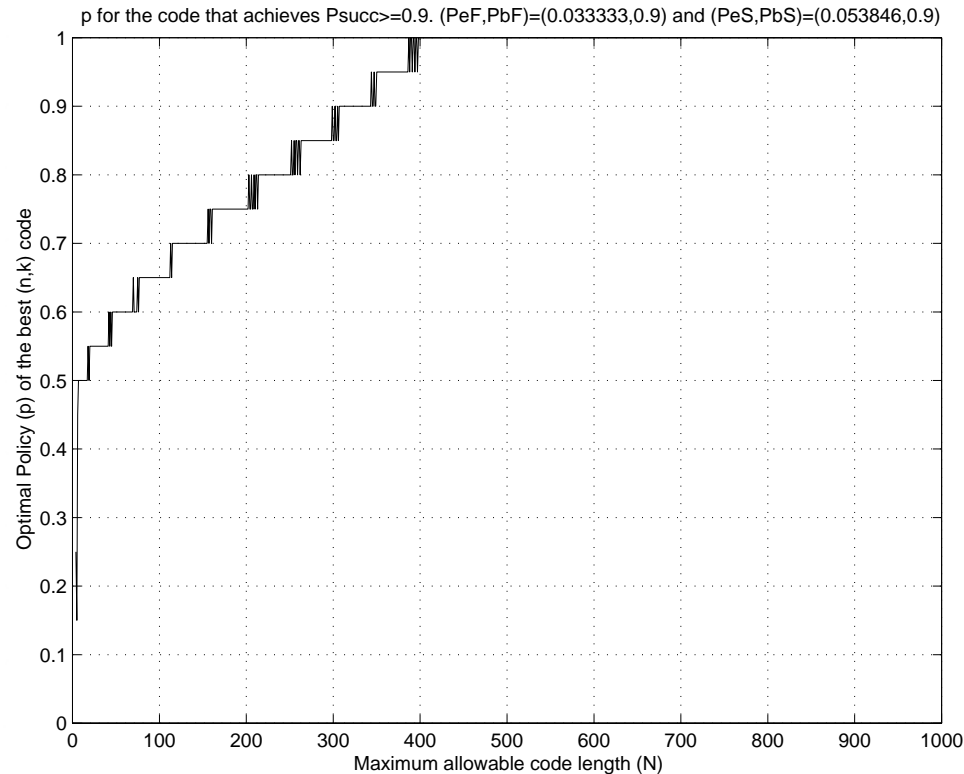
# When does it pay to use *both* channels?

- The optimal policy depends on
  - the code length,
  - the relative degrees of “burstiness” of the two channels, and
  - the long term error rates of the channels.
- Identical channels: Optimal policy need **not** be  $p=0.5$ .
- When both channels have the same long term error rate (but different expected burst lengths), using both channels can improve performance.
- When one channel has a better long term error rate
  - For a large enough code length, the optimal policy is to always choose the better channel.
  - However, if we restrict the length of the code, it can be still be beneficial to use both channels.

# When does it pay to use *both* channels?



Optimal Policy of the (20,6) code for each channel scenario of the high error rate channels.



Optimal Policy as a function of the maximum allowable code length. Both channels are bursty but the second one has a worse long term error rate. High error rate channels.



# Conclusion

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- Intelligent use of two channels can improve performance significantly with minimum cost.
  - Only limited channel knowledge is required.
  - Simple (random) transmission policy can be used.
- The optimal transmission policy, i.e.,  $p$ , can be computed efficiently based on
  - the channel characteristics,
  - the code being used (or maximum allowed code length), and
  - the desired target loss probability,  $P_{\min}$ .



# Reference

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- Full paper available at <http://einstein.seas.upenn.edu/mnlab/publications.html>
- E. Vergetis, R. Guerin, S. Sarkar, "[Benefiting from Path Diversity with Bursty Losses.](#)"