

Modeling Uncertainties in the Prediction of the Acoustic Wavefield in a Shelfbreak Environment

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Abstract

The uncertainties in the predicted acoustic wavefield associated with the transmission of low-frequency sound from the continental slope, through the shelfbreak front, onto the continental shelf are examined. The locale and sensor geometry being investigated is that of the New England continental shelfbreak with a moored low-frequency sound source on the slope. Our method of investigation employs computational fluid mechanics coupled with computational acoustics. The coupled methodology for uncertainty estimation is that of Error Subspace Statistical Estimation. Specifically, based on observed oceanographic data during the 1996 Shelfbreak Primer Experiment, the Harvard University primitive-equation ocean model is initialized with many realizations of physical fields and then integrated to produce many realizations of a five-day regional forecast of the sound speed field. In doing so, the initial physical realizations are obtained by perturbing the physical initial conditions in statistical accord with a realistic error subspace. The different forecast realizations of the sound speed field are then fed into a Naval Postgraduate School coupled-mode sound propagation model to produce realizations of the predicted acoustic wavefield in a vertical plane across the shelfbreak frontal zone. The combined ocean and acoustic results from this Monte Carlo simulation study provide insights into the relations between the uncertainties in the ocean and acoustic estimates. The modeled uncertainties in the transmission loss estimate and their relations to the error statistics in the ocean estimate are discussed.

1. Introduction

Data-assimilative, high-resolution ocean physical models, when coupled to accurate acoustic propagation models, usually improve the prediction of the acoustic wavefield associated with the transmission of low-frequency underwater sound. Ideally, a comprehensive prediction should include a characterization of the reliability or uncertainty of forecast quantities. This allows the correct interpretation or processing of these quantities in a scientific or tactical application. Uncertainties in both the ocean and acoustic estimates arise from imperfect data, imperfect models and environmental variability not explicitly known. In a comprehensive coupled ocean physics and acoustic prediction system, the forecast of uncertainties involves the attribution of errors within each of the physical (oceanographic) and acoustical components, and the transfer of these error estimates or probabilities through the coupled system. In this paper, we outline an approach for carrying out such coupled predictions, focusing more on the transfer than on the attribution of errors. The approach is exemplified by a short hindcast of the large-mesoscale physics and transmission loss in a slope-to-shelf sound propagation experiment in the Middle Atlantic Bight (MAB) shelfbreak region, off the east coast of the United States, during the summer of 1996.

The basic concepts of the coupled physical-acoustical prediction approach presented here build on recent advances made in error estimation and data assimilation research in meteorology and oceanography (Ehrendorfer, 1997; Robinson *et al.*, 1998; Lermusiaux, 1999a), which in turn have their roots in classical estimation and control

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theory. To control and minimize errors in the prediction, dynamical models and data are optimally combined through data assimilation as a function of their respective error statistics or uncertainties. In numerical simulations, an issue that arises relates to the large dimension of the statistical properties of the coupled physical-acoustical fields. Efficient reduction and representation of uncertainties are necessary. To do so, the Error Subspace Statistical Estimation (ESSE) approach (Lermusiaux, 1999a,b; Lermusiaux and Robinson, 1999) is employed. This scheme is based on a reduction of the evolving error statistics to their dominant components or subspace. Presently, these statistics are measured by a variance or least-squares criterion (Tarantola, 1987; Robinson et al, 1998): a subspace is then characterized by the dominant components of the eigen-decomposition of a covariance matrix (subspace definitions that focus on higher-order moments can also be used). In our coupled prediction, a main task is thus to estimate and track the evolving uncertainty subspace of the coupled ocean physics and acoustic variables.

In what follows, Section 2 overviews the dynamics and sources of physical-acoustical variabilities in the MAB. Section 3 describes the data, models and methods employed. Section 4 consists of the results and Section 5 of the conclusions.

2. The Middle Atlantic Bight Variability and Uncertainty

The MAB shelfbreak marks a dramatic change, not only in water depth, but also in the dynamics of the waters that lie on either side (Beardsley and Boicourt, 1981; Colosi *et al.*, 2001). The shelf is about 100 km wide, extending from Cape Hatteras to Canada. The shelfbreak, which refers to the first rapid change in depth that occurs between the coastal and deep ocean, is near the 100-m isobath. The main oceanographic feature in the MAB is a mesoscale front of temperature, salinity and hence sound speed, separating the shelf and slope water masses. Located near the shelfbreak, this front is usually tilted in the opposite direction of the bottom slope. The shelf-water to the north is cold and fresh while the slope-water to the south is warm and salty. In the summer, the surface layers (top 10 to 30 m) are stratified (warmer and lighter) which has a tendency to reduce the influence of the atmosphere on the front below.

In the MAB region, atmospheric fluxes, buoyancy-induced pressure gradients, tides, river run-offs and shelf-slope water exchanges combine to generate a rich variety of physical phenomena. The shelf-water variability is often driven by wind forcings and tides. In the slope region, mesoscale variability is significant, especially that induced by Gulf Stream rings and meanders. Near the shelfbreak front, instabilities occur. Shelfbreak eddies, meanders, and internal waves, bores and solitons (SWARM Group, 1995; Colosi *et al.*, 2001) are frequently observed, leading to complex, energetic variability on multiple time and space scales.

Variability and uncertainty of an estimate are inherently related. The portion of the variability, oceanic and acoustic, that is estimated with errors contributes to uncertainty. For example, variability that is totally unresolved is pure uncertainty. Mathematically, uncertainty can be defined by the *probability density function (PDF) of the error in the estimate*. Errors here refer to differences between the true and estimated

fields. In a prediction, both errors in the initial data (conditions) and errors in the models and boundary conditions impact the accuracy of the forecast. Predicted uncertainties thus contain the integrated effects of the initial error and of the errors introduced continuously during the model integration.

3. Data, Models and Uncertainty Forecast Methodology

Data: During July and August of 1996, data were collected in the MAB south of New England, as part of the ONR Shelfbreak PRIMER Experiment (Lynch *et al.*, 1997). The main objective was to study the influence of oceanographic variability on the propagation of sound from the slope to the shelf. Intensive measurements were carried out in a 45 km by 30 km domain between the 85 m and 500 m isobaths. The measurements consisted of temperature, salinity, velocity, chlorophyll, bioluminescence and acoustic transmissions. Some additional wide-coverage data were obtained from outside that domain, including atmospheric fluxes and satellite surface temperature and height. To initialize physical fields and their uncertainties, these synoptic but also historical data are utilized as well as a feature model for the shelfbreak front (Lermusiaux, 1999a).

Ocean Physics Model: The physical variables are temperature, salinity, velocity and pressure. For this first coupled ocean physics-acoustic uncertainty study, only large mesoscales are considered (small mesoscales are not resolved by the 9 km grid resolution). Physical fields are initialized for August 1 in a domain (Fig. 1) of 320.26km by 355.29km centered on 39.86 N, 70.06 W. A simulated 5-day forecast is issued for August 5. The dynamical evolution is computed by the numerical ocean model of the Harvard Ocean Prediction System (HOPS) (e.g. Robinson, 1996 and 1999). Atmospheric fluxes based on buoy data are imposed at the surface. The model parameters and boundary condition schemes were calibrated based on data and sensitivity studies.

Acoustic Model: The acoustic model used in this study is that of Chiu, 1994, and Chiu *et al.*, 1995 and 1996. It is based on the physics of coupled normal modes. The basic formulation of the model involves decomposing the acoustic pressure into slowly-varying complex envelopes that modulate (mode by mode) analytic, rapidly-varying, adiabatic-mode solutions. Given sound speed, density, attenuation rate and bathymetry as a function of space, the acoustic solution is thus obtained by integrating a coupled set of differential equations governing these complex modal envelopes. Model output contains sound pressure, transmission loss, and travel time, phase and amplitude of the individual modes. One, several or all of these acoustic variables can be included in the joint ocean-acoustic state space for the coupled ocean-acoustic prediction. For simplicity, our discussion will focus on the prediction of the transmission loss and its uncertainty along an actual Shelfbreak-PRIMER acoustic path. Figure 1 shows the geometry of such a path and its relation to the HOPS model domain.

Coupled Uncertainty Forecasts: As discussed in Sec. 1, our approach is the ESSE methodology. For the MAB case study presented here, the principal components constituting the error subspace of the ocean physical state and their coefficients are first

initialized for August 1 combining data and dynamics, following Lermusiaux *et al.*, 2000. To account for nonlinearities, this initial uncertainty is forecast using an ensemble of Monte-Carlo prediction realizations obtained as follows. The initial physical oceanographic state is first perturbed using random combinations of the initial error principal components. For each of these perturbed initial conditions, the nonlinear ocean dynamical model is then integrated for 5 days.

These Monte-Carlo integrations are carried out in parallel until the size of the ensemble is large enough to describe most of the error variance in the forecast. This is assessed by a convergence criterion. For the scales considered, an ESSE estimate of the error covariance for the cross-slope sound-speed field along the MAB transmission path was obtained after 80 integrations. With these 80 forecast realizations of the sound speed field, 80 Monte-Carlo realizations of the acoustic wavefield are then computed to estimate the uncertainty in the predicted acoustic wavefield and its linkages to uncertainties in the ocean forecast. Note that larger ensembles of ocean forecast realizations (from 100 to 267 samples) have been computed for the region, with and without stochastic error forcings. The stochastic forcings aimed to account for the uncertainties associated with smaller-scale ocean processes including sub-mesoscale eddies and internal tides. In what follows, only forecasts of mesoscale uncertainties, i.e., corresponding to the case of no stochastic forcings, are presented.

4. Results

4.1 Physical Oceanography

The 5-day forecast of the large-mesoscale physical fields and their uncertainty estimates (based on 100 realizations) are illustrated by Fig 2. The focus is on the critical coupling variable for our study, sound speed (computed here using the UNESCO 1983 polynomial, based on the local pressure and forecast temperature and salinity). The surface sound speed map (Fig 2a) indicates the location the shelfbreak front. Note also three primary water masses: the Gulf of Maine water southeast of Cape Cod (lowest sound-speed), the shelf water to the north and slope water to the south. A meander develops in the western side of the domain. A slope-water eddy and a shelf-water eddy are starting to form upstream and downstream of this meander, respectively. The surface error standard deviation (Fig. 2b) relates to this mesoscale variability. It is largest along the front, especially downstream of the meander. Note also some streak patterns due to surface wind effects (to the southeast). The relatively large standard deviation at the inflow of the shelfbreak jet is due to uncertainties in the position and strength of the front,

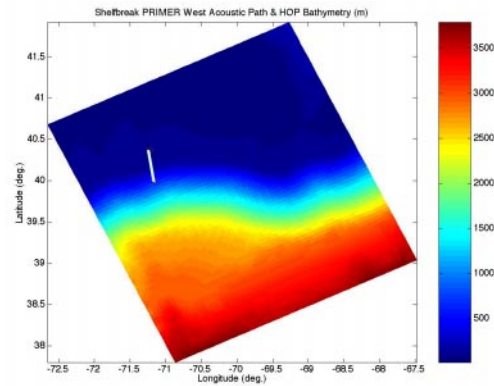


Figure 1. Geometry of the Shelfbreak PRIMER western acoustic path superposed on the HOPS model bathymetry.

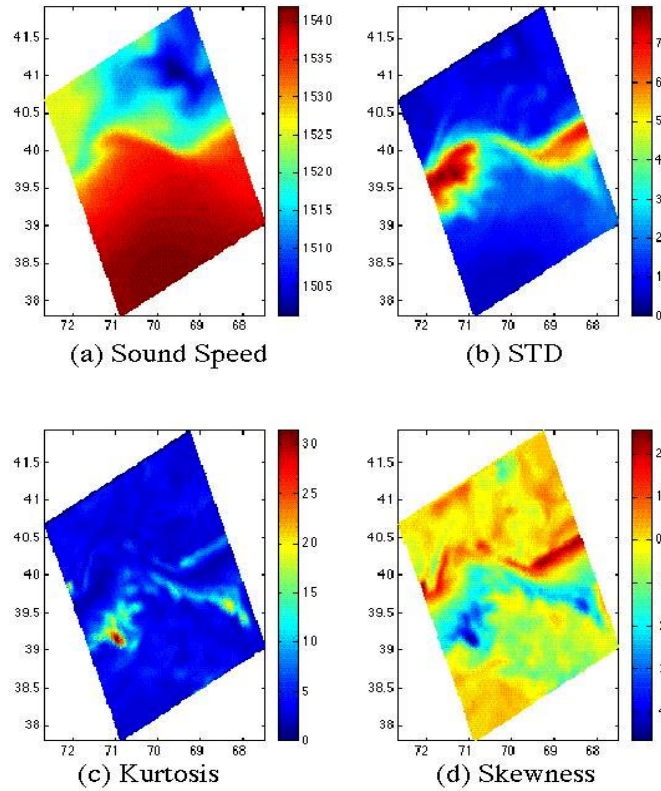


Figure 2. Surface values of the 5-day forecast sound speed and its error statistics: the ensemble mean, error standard deviation and higher-order statistics on the top model level are shown.

and also to uncertainties brought upon by the open-boundary condition. Higher-order error statistics (Fig. 2c-d) as well as error correlation functions (not shown) can also be computed by ESSE. If forecast errors were Gaussian, their skewness (3rd/2nd moment, Fig. 2d) would be null. An interesting result is that, on average, this skewness changes sign at the front. It is estimated to be positive on the shelf and negative on the slope. Its extrema corresponds to negative values on the slope because the day before the forecast time some westerly winds occurred and because of internal dynamics. If errors were Gaussian, the kurtosis (4th/2nd moment, Fig. 2c) would be 3. On average, it is here forecast to be maximum near the skewness extrema. The kurtosis extrema is near the region where a small mesoscale shelf-water eddy (mainly subsurface) is forming. Being able to compute such high-order statistics is important to characterize the statistical shapes of uncertainties. It has significant consequences in scientific, operational and also technical (e.g. data assimilation schemes) applications.

To illustrate the PDF of the errors in the physical estimates, a set of local sound speed error PDFs (based on 100 realizations) are plotted on Fig.3. Each of these histograms corresponds to a surface numerical grid point in the ocean physical dynamical model, going along a straight line across the shelfbreak from south (offshore) to north (along the US coast). At each of the 15 locations, the x-axis consists of 21 equally spaced bins in deviations (m/sec) with respect to the ensemble mean (the 0). The y-axis

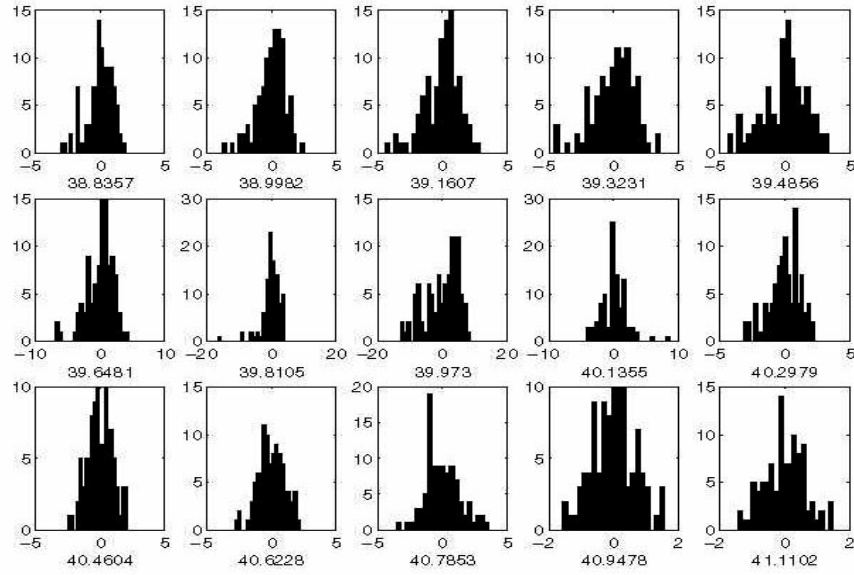


Figure 3. Forecast sound-speed error PDFs. Histograms of surface values (top model level) of the forecast sound speed error, at 15 locations in an across-shelf direction (top left is above slope near 39N, 69E; bottom right is at the shelf, near Nantucket).

consists of the number of ensemble members in each bin. The front location is close to the histogram number 8 (middle of the 15 plots, at 39.973N). The forecast results show a maximum variance near the front (see 20 m/sec extremes) and a lowest variance near Nantucket (see 2 m/sec extremes). The negative and positive skewness identified on Fig. 2 are again clearly visible just to the south and north of front, respectively. Note also that the error PDF of the sound speed is steepest on each side of the front and close to Gaussian away from the front except near the edges of strong eddies or meanders where high shear occur.

4.2 Acoustics

As discussed in the previous sections, based on observed oceanographic data during the 1996 Shelfbreak Primer Experiment, HOPS was initialized with perturbed physical oceanographic fields that are in statistical accord with a realistic error subspace and then integrated to produce 80 realizations of a regional forecast of the sound-speed field. One of these realizations of the sound-speed field along the transmission path is shown in Fig. 4. The different forecast realizations of the sound speed were then fed into a coupled-mode sound propagation model to produce realizations of the TL prediction for a low-frequency transmission from the slope, across the shelfbreak, onto the shelf. Specifically, the transmission frequency was 400 Hz and the sound source was located near the bottom at the 300-m isobath on the slope. Six of the 80 different realizations of the transmission loss (TL) prediction are shown in Fig. 5. They show that the structure in the spatial distribution of the acoustic energy is quite different from one realization to another, even in the shelf region where the ocean variance is minimal. The TL structure on the shelf is largely determined by what happens to the acoustic energy prior to

entering the shelf. With the large sound-speed variances over the slope, the initial distribution of the acoustic energy over the set of acoustic normal modes (i.e., modal excitation) near the source range, as well as the redistribution of modal energy along the slope due to mode coupling, are different for the different ocean realizations. This results in different TL structures on the shelf (Fig. 5).

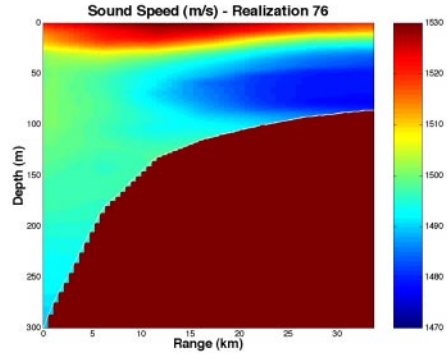


Figure 4. *A realization of the sound-speed forecast along the transmission path.*

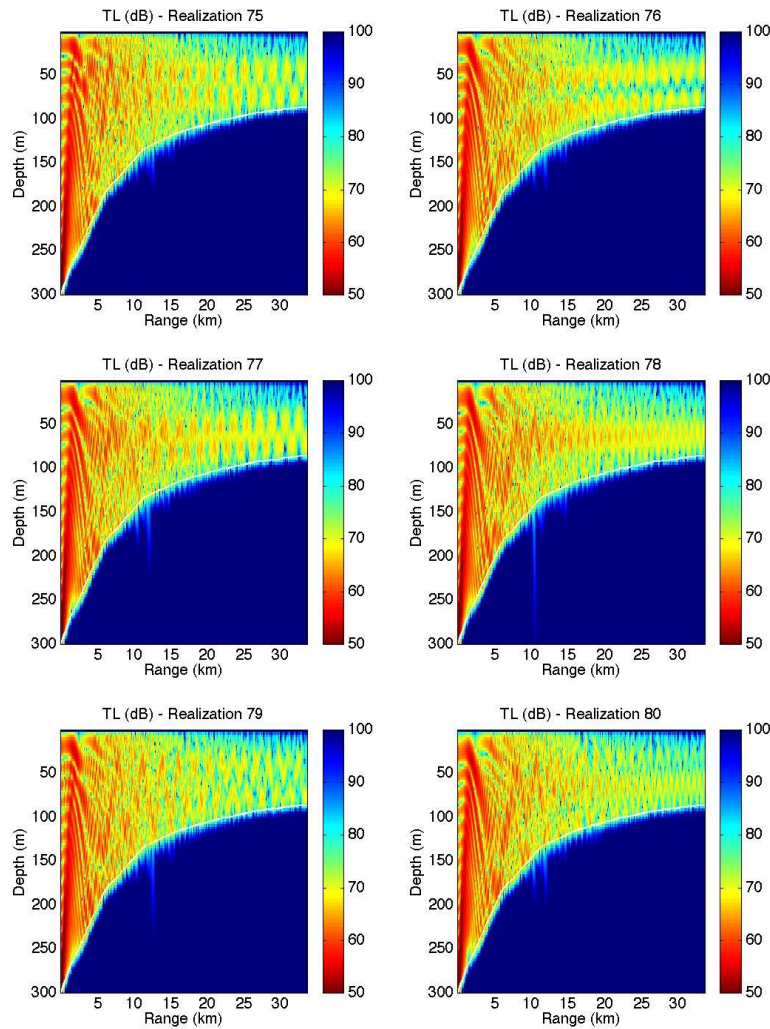


Figure 5. *Six different realizations of the TL prediction*

To summarize the forecast error statistics (uncertainty) for the TL prediction and its relation to the uncertainty in the ocean forecast, we show the error standard deviations of the sound-speed forecast in Fig. 6, the error standard deviations of the TL prediction in Fig. 7, and the corresponding histograms (error PDF estimates) for the sound speed and TL variables at two different locations, shelfbreak and shelf, in Fig. 8, respectively (all computed from 80 realizations). The error standard deviations of sound speed (Fig. 6) calculated from the 80 ocean realizations show that large uncertainties are confined in the top layers (from 0 to 50m depth, with a maximum around 30m depth) over the slope region at the frontal zone. Over the shelf, for the 5-day period considered, the HOPS model predicts only relatively small error variances at the mesoscale. Accordingly, the error variance in the TL prediction (Fig. 7) is small near the source below the top layers, but as the acoustic energy reaches these top layers where large sound speed error variances are confined, the error variances in TL increase. Note that the uncertainty in the TL does not grow in range over the shelf where sound speed uncertainties are relatively small. The complexity and inhomogeneity of the predicted error statistics in this slope-to-shelf transmission in the MAB are further revealed in the PDF estimates shown in Fig. 8. In particular, note the transformation of the PDF shape as uncertainties are transferred from the ocean (sound speed) estimate to the acoustic (TL) estimate. Because the sound pressure field, from which TL is computed, is composed of multiple acoustic modes, an in-depth understanding of the linkage between the error statistics of TL and sound speed behooves a careful analysis on the behavior of the errors in the amplitude and phase of each acoustic mode. This modal error analysis is being carried out at present.

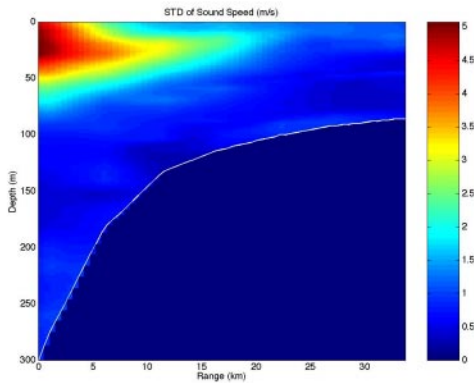


Figure 6. Error standard deviation estimate of sound-speed forecast.

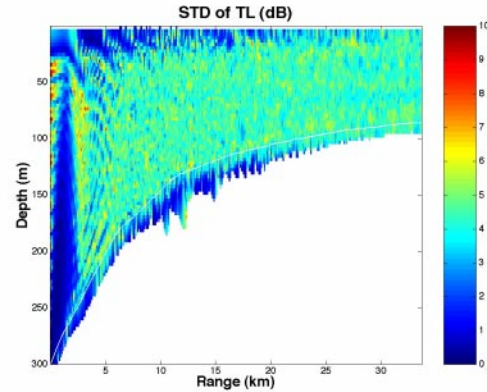


Figure 7. Error Standard deviation estimate of TL prediction.

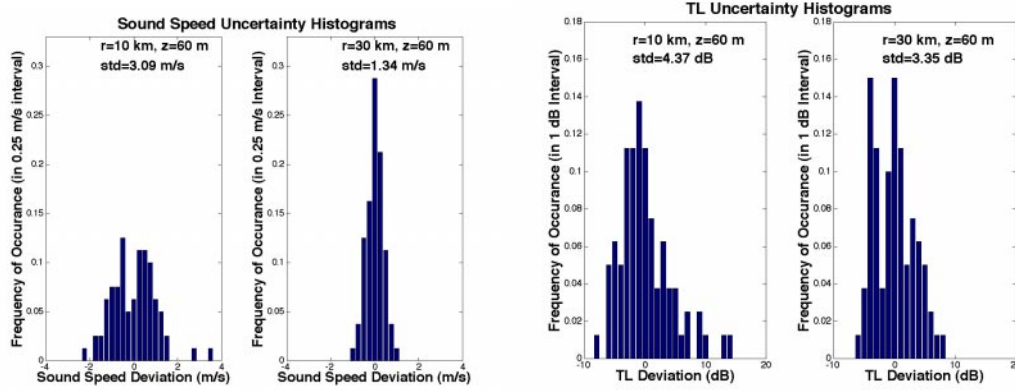


Figure 8. The histograms (PDF estimates) of the sound speed and TL uncertainties at two different locations (shelfbreak and shelf).

5. CONCLUSIONS

A methodology for the modeling and prediction of coupled ocean physics and acoustic uncertainties was outlined and exemplified. It is based on Error Subspace Statistical Estimation (ESSE). The focus was on the transfer of ocean physical uncertainties and their impact on the acoustical fields and uncertainties. The example considered consisted of the prediction of uncertainties for the large-mesoscale ocean physics and transmission loss in a slope-to-shelf sound propagation across the shelfbreak front in the Middle Atlantic Bight (MAB) region.

The results reveal the strong influence of the oceanic variability on the coupled uncertainties. They also illustrate possible modifications of the properties of error PDFs in their transfer from the ocean physics to the acoustic. Future challenges include the careful and comprehensive attribution of all error sources in such coupled predictions and their multiscale transfer through the coupled system. Scientific progress and error reduction should arise from the study and understanding of these error sources and transfers. Important feedbacks involve the coupled physical-acoustical data assimilation and the joint improvement of physical and acoustical data sampling schemes based on coupled error forecasts and quantitative adaptive sampling.

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