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Hurry up and Wait ... Optimizing the Initial Skills Training Scheduling Process

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A NETWORK FLOW APPROACH TO THE INITIAL SKILLS TRAINING SCHEDULING PROBLEM

Lt. Tony Illig

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The views expressed here are those of the authors and do not represent the official policy of AFIT, AU, or the USAF.

Overview

- Problem Description
- Motivation
- Network Flow Approach
- Why Network Flow Works
- Results
- Future Research

Problem Description

- Initial Skills Training Scheduling Problem (ISTSP):
  - After commissioning, new Air Force officers typically require training to be certified to perform their duties.
  - Commissioning sources: USAFA, ROTC, OTS
  - All officers must attend ASBC. (Mandated by CSAF)
  - Following ASBC, many officers continue training to be certified
    - Pilots
    - Combat Systems Officers (CSOs)
    - Air Battle Managers
    - Intel Officers
    - Space and Missile Officers
    - Air Field Operations Officers

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Problem Description

- Initial Skills Training Scheduling Problem (ISTSP):
  - Course sequences are rigid and vary by AFSC
  - Class sizes and intervals vary
  - Course length is fixed
  - Class blends must be met
    - AETC desires that certain blends are met according to commissioning source and AFSC (in ASBC).

Motivation

- Average 2nd Lt Pay:
  - $124 / day (2007 dollars)
- Number of Annually Commissioned 2nd Lts:
  - ~ 4000
- Estimated savings of 1 day:
  - $496,000
- Current average number of down days:
  - ~ 180
- Huge Potential for Savings!!!!

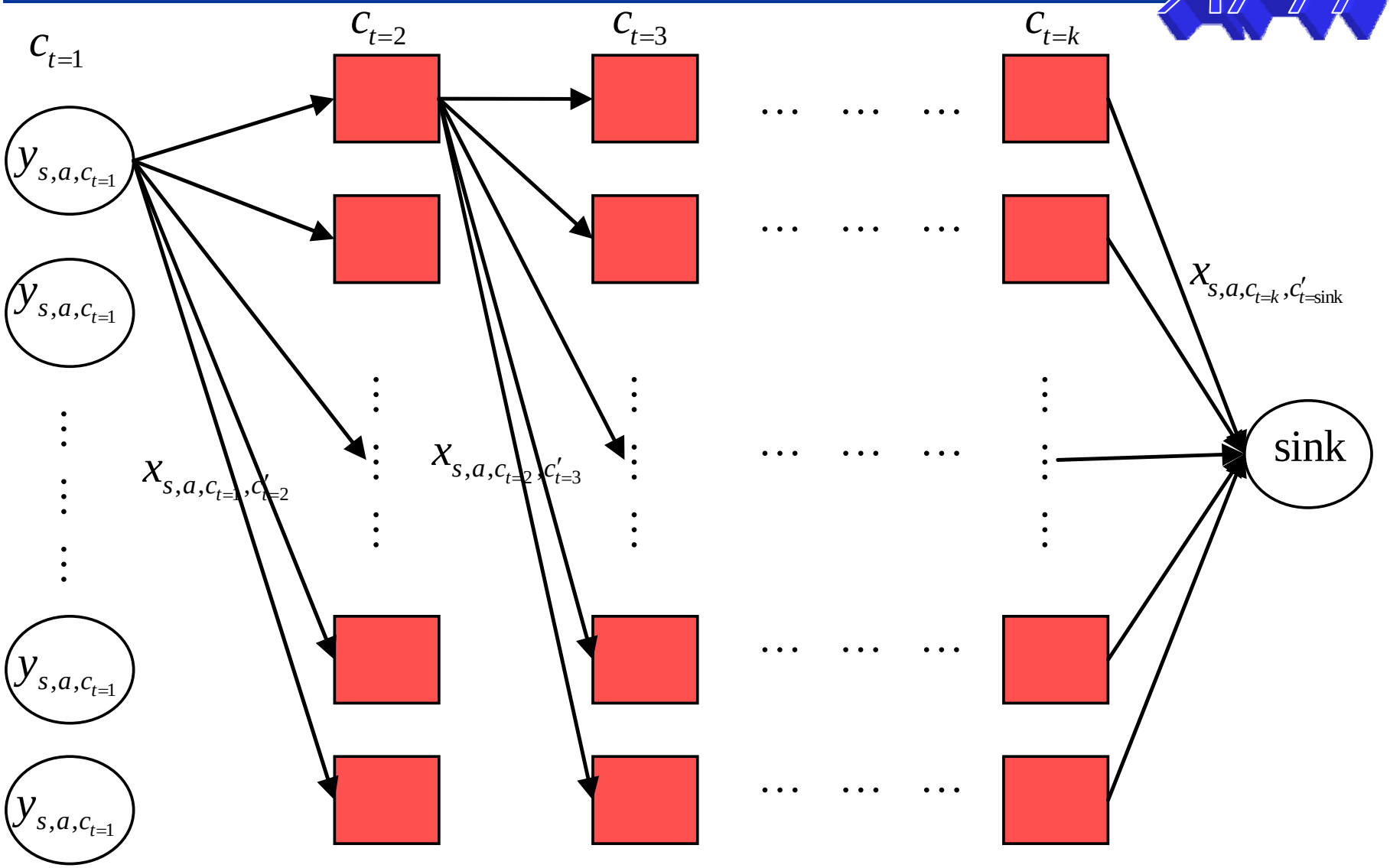
Network Flow Approach

- General personnel scheduling techniques are computational intensive
- ISTSP has structure which can be exploited
- Developed network flow approach (NFISTSP)
  - How it works
    - Commissioning sources are source nodes.
    - Create nodes to represent each class.
    - Connect the nodes with arcs if it is possible to go from the starting node's class to the ending nodes class.
    - The cost on the arc is the number of days between the classes.
    - Use minimum cost flow to solve.

NFISTSP Approach

- What it accomplishes
  - Enforces blends:
    - Source of commissioning (SOC) blends enforced by creating 3 separate nodes at each class (one for each SOC) then capacitating arcs accordingly.
    - AFSC blends are done in a similar way at ASBC.
  - Does not allow skipping.

NFISTSP Graph



NFISTSP Graph

Sum all nodes for class UB

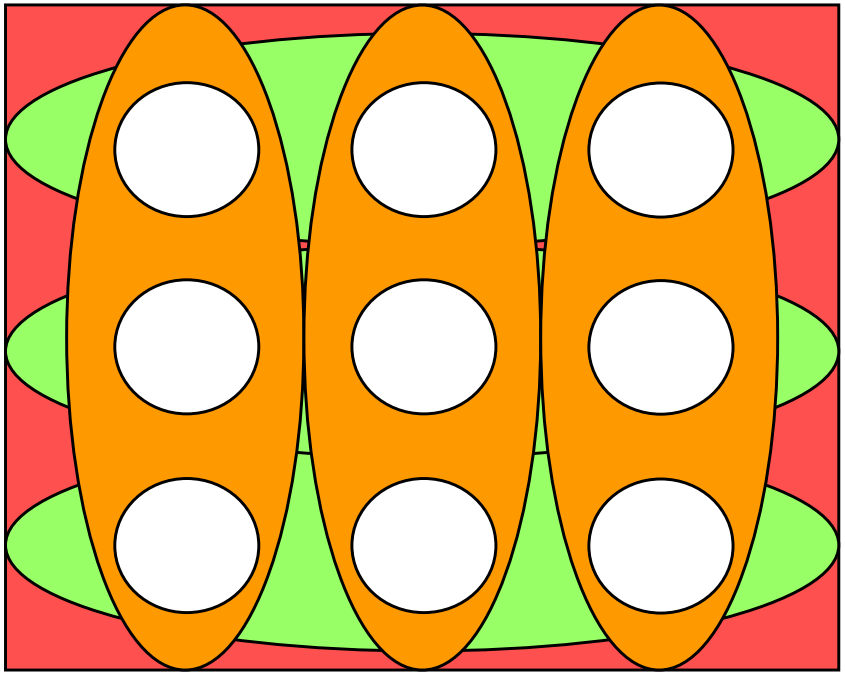
$$\sum_{\forall s \in \mathbf{S}} \sum_{a \in \mathbf{A}'} \sum_{\forall c \in \mathbf{C}_t} x_{s,a,c,c'} \leq UB_{c'}$$

Sum all nodes for AFSC UB

$$\sum_{\forall s \in \mathbf{S}} \sum_{\forall c \in \mathbf{C}_t} x_{s,a,c,c'} \leq UB_{a,c'}$$

Sum all nodes for SOC UB

$$\sum_{\forall a \in \mathbf{A}'} \sum_{\forall c \in \mathbf{C}_t} x_{s,a,c,c'} \leq UB_{s,c'}$$



Why does it solve quickly?

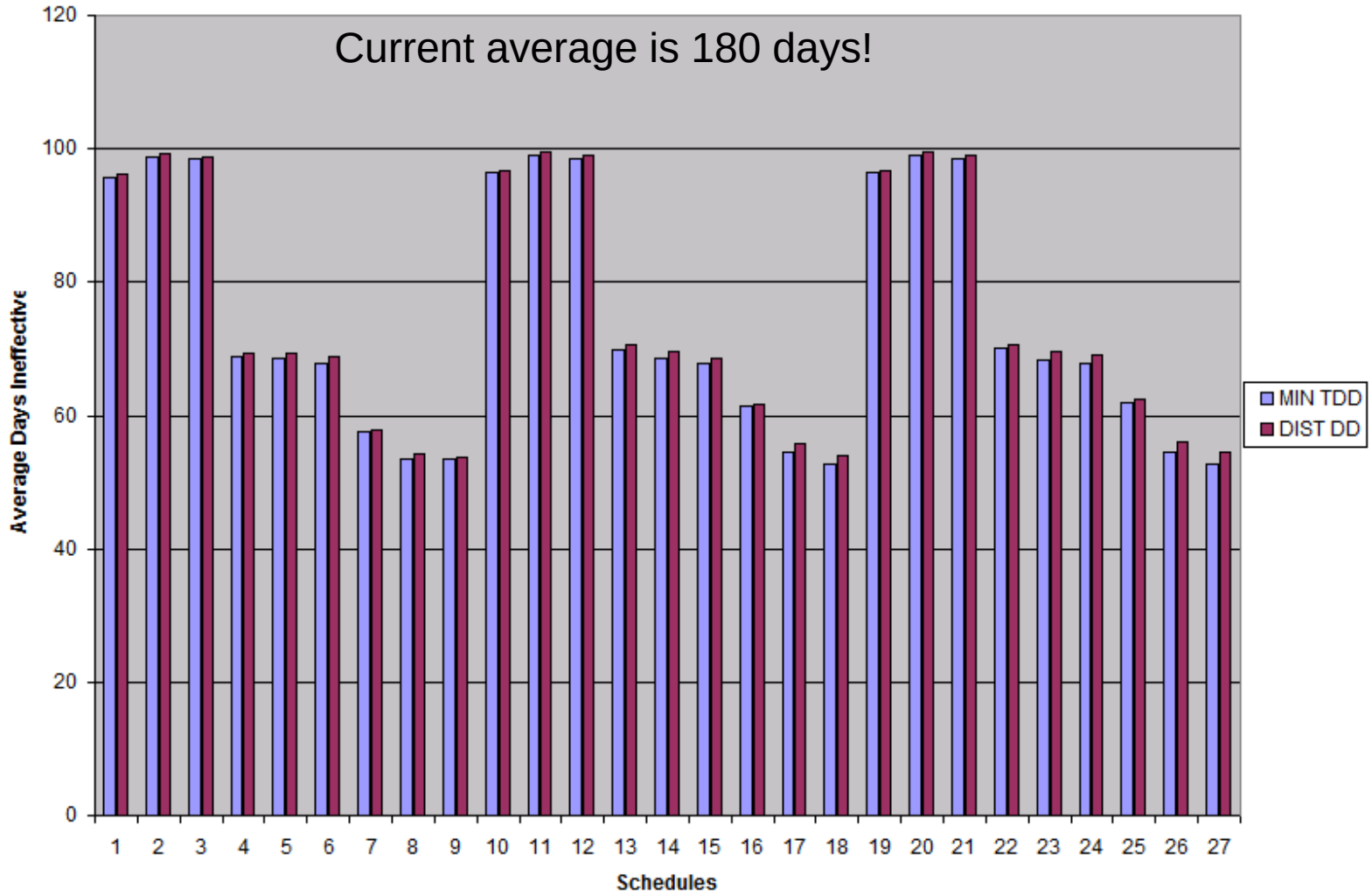
- Using this formulation allows large personnel scheduling problems to solve in seconds
  - Why?
- Total Unimodularity (TU)
  - Definition: ***A*** is totally unimodular iff every square sub matrix in A has a determinant of -1, 0 or 1.
  - Implications: Suppose ***A*** is the constraint matrix for a integer programming (IP) problem. If ***A*** is TU and the RHS is all integer then the LP relaxation of the IP will yield integer results.

Test Cases/Scenarios

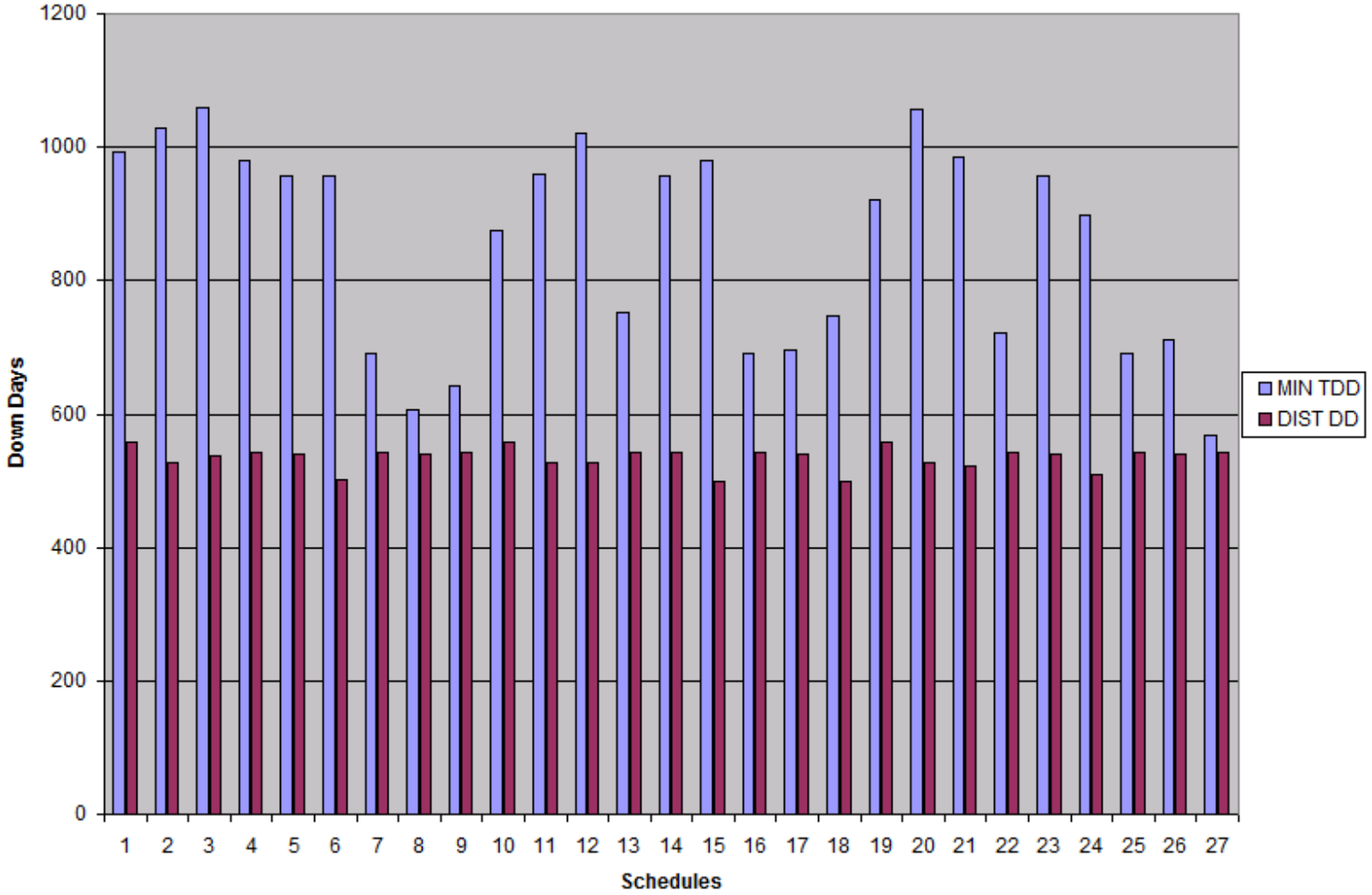
- Min USAFA Leave
  - 0, 30, 60 Days — 3 Levels
- Max ROTC Delay
  - 0, 180, 365 Days — 3 Levels
- Class Blend Levels
  - 50%, 75%, No Forced Blend — 3 Levels
- Arc Weighting Schemes (OF)
  - Minimize TDD, Distribute DD — 2 Levels

Total Scenarios: 54

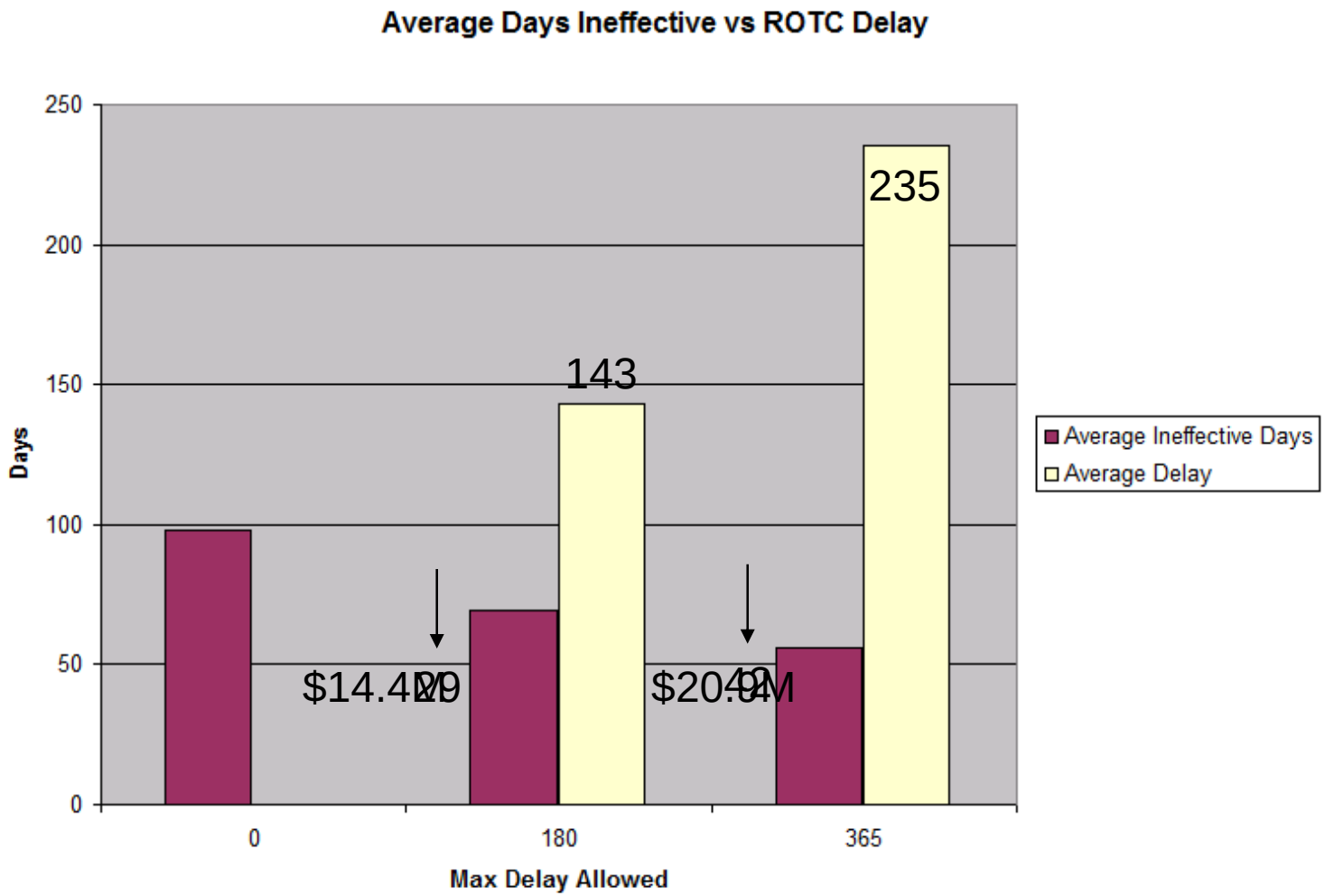
Minimize TDD vs Distribute DD



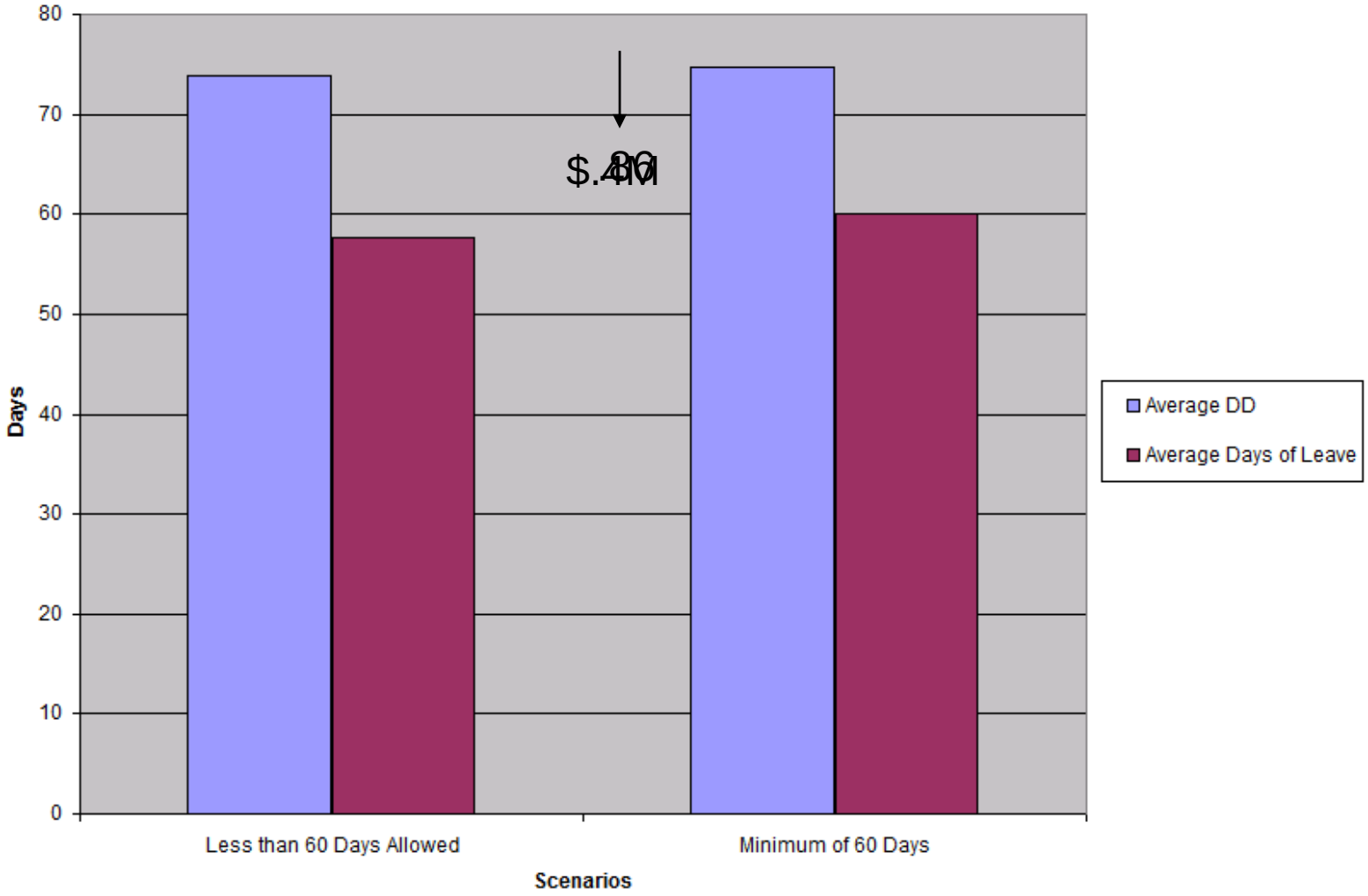
Minimize TDD vs Distribute DD



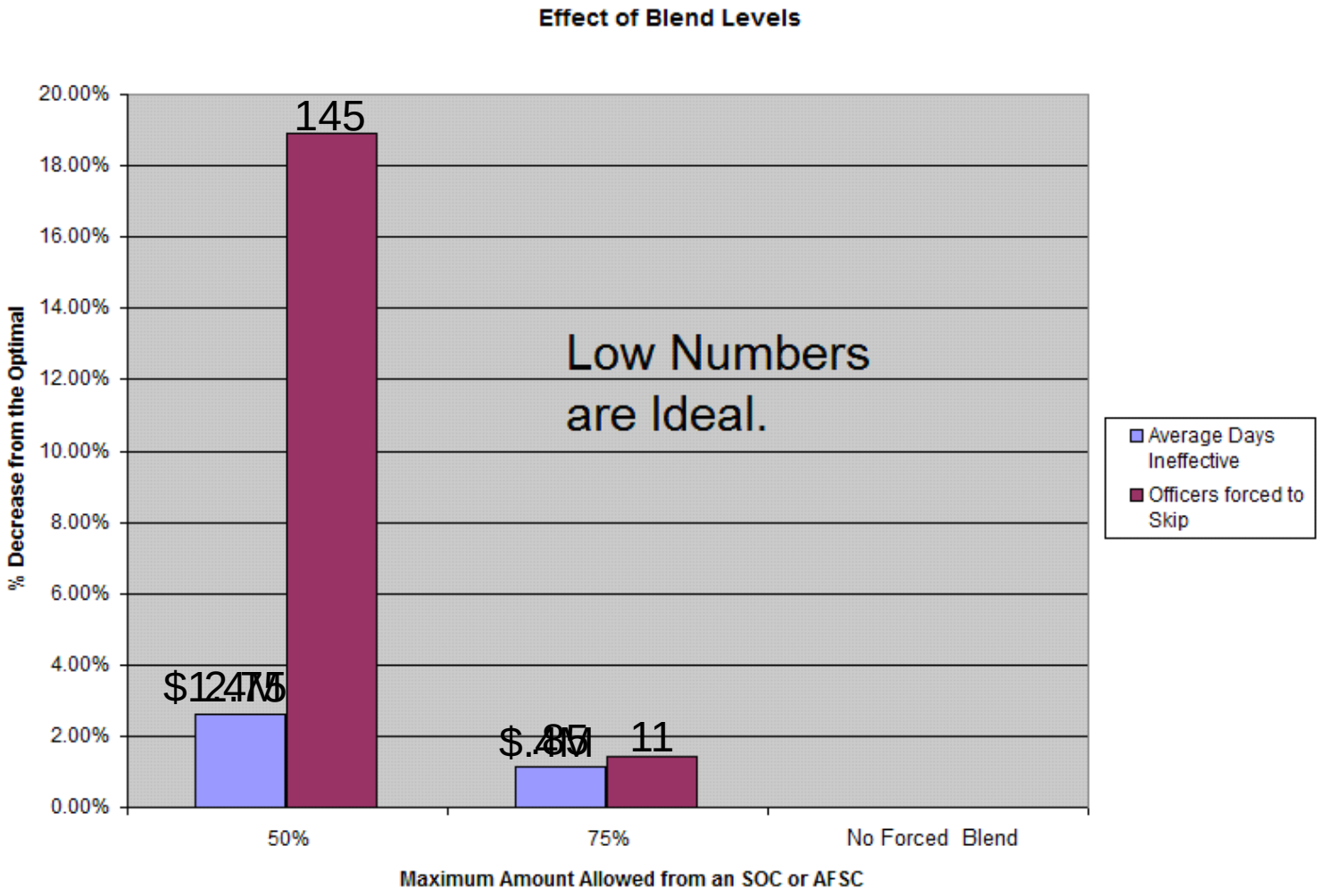
Impact of ROTC Delay



Impact of USAFA Leave



Impact of Blend Levels



Computational Conclusions

- The savings (dollars or ineffective days) from reducing the minimum USAFA Leave is not significant.
- ROTC delay extension results in a sizable effect.
- Restricting blends significantly impacts total number of down days; partial blends occur naturally.

Robustness of Formulation

- Theoretically, any timeline can be scheduled.
- This formulation handles heterogeneous workforces well.
  - In this example we had 2 heterogeneous levels (AFSC and SOC).
  - Can handle any number of types and levels efficiently.

Future Research

- Use of Decision Analysis
- Use new optimization package in SAS/OR
- Use model to schedule optimal class sizes and class start dates
- Use probabilities and stochastic analysis to include drop out rates

Questions?

Thanks for your time …

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NFISTSP Mathematical Formulation

- Sets

$\mathbf{S}$ = set of all commissioning sources = $\{s_1, s_2, ..., s_n\}$

$\mathbf{A}$ = set of all AFSCs = $\{a_1, a_2, ..., a_m\}$

$\mathbf{T}$ = set of all training courses = $\{t_1, t_2, ..., t_k\}$

$\mathbf{C}_t$ = set of all classes of training course t = $\{c_{t_1}, c_{t_2}, ..., c_{t_l}\}$

- Variables

x_{s,a,c_t,c'_t} = number of officers from commissioning source s in AFSC a going from class c_t to class c'_t

NFISTSP Mathematical Formulation

- Parameters

$d_{c_t,c_t'}$ = number of days bettween class c_t and class c_t'

y_{s,a,c_t} = number of officers from commissioning source s in AFSC a graduating from class c_t

- Objective Function

Minimize $$Z = \sum_{\forall t \in T} \sum_{\forall c_t' \in \mathbf{C}_{t+1}} \sum_{\forall c_t \in \mathbf{C}_t} d_{c_t,c_t'} \sum_{\forall a \in \mathbf{A}} \sum_{\forall s \in \mathbf{S}} x_{s,a,c_t,c_t'}$$

NFISTSP Mathematical Formulation

- Subject to

$$y_{s,a,c_1} = \sum_{\forall c' \in \mathbf{C}_2} x_{s,a,c_1,c'_2} \qquad \forall s \in \mathbf{S}, a \in \mathbf{A}, c_1 \in \mathbf{C}_1$$

$$\sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'_{t+1}} = \sum_{\forall c'_{t+2} \in \mathbf{C}_{t+2}} x_{s,a,c_{t+1},c'_{+2}} \; \forall s \in \mathbf{S}, a \in \mathbf{A}, c'_{t+1} = c_{t+1} \in \mathbf{C}_{t+1}, t = 1, 2, \ldots, n-1$$

$$\sum_{\forall s \in \mathbf{S}} \sum_{\forall a \in \mathbf{A}} \sum_{\forall c_k \in \mathbf{C}_k} x_{s,a,c_k,c'} = \sum_{\forall s \in \mathbf{S}} \sum_{\forall a \in \mathbf{A}} \sum_{\forall c_t \in \mathbf{C}_1} y_{s,a,c_1} \qquad c' = \text{"sink"}$$

$$\sum_{\forall s \in \mathbf{S}} \sum_{a \in \mathbf{A}'} \sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'_{t+1}} \le UB_{c'_{t+1}} \qquad \forall c'_{t+1} \in \mathbf{C}_{t+1}, t = 1, 2, \ldots, n-1$$

$$\sum_{\forall s \in \mathbf{S}} \sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'} \le UB_{a,c'_2} \qquad \forall a \in \mathbf{A}, c'_2 \in \mathbf{C}_2$$

$$\sum_{\forall a \in \mathbf{A}'} \sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'_{t+1}} \le UB_{s,c'_{t+1}} \qquad \forall s \in \mathbf{S}, c' \in \mathbf{C}_{t+1}, t = 1, 2, \ldots, n-1$$

Properties of Total Unimodularity

- Properties of TU that will be useful:
  - If A is TU then A^T is TU
  - If A is TU then (A, I) is TU
  - Row and Column swaps in A do not affect TU.
  - The Node-Arc incidence matrix of a Network Flow problem is always TU
  - If A is TU and B is obtained by removing a row or column of A, then B is TU

- The first four are from Bazaraa, Jarvis and Sherali

Proving Properties of Total Unimodularity

- If A is TU and B is obtained by removing a row or column of A, then B is TU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2j} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3j} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{4j} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

Total Unimodularity

- A look at the constraint matrix for a network (the node incidence matrix)

Arcs ↓

Nodes →

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\ 1 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

Total Unimodularity

- Another sufficient condition for TU is:
  - For every J ⊆ N={1,…,*n*}, there exists a partition J_1, J_2 of J such that

$$\left| \sum_{j \in J_1} a_{ij} - \sum_{j \in J_2} a_{ij} \right| \leq 1 \qquad \forall i = 1, \ldots, m$$

Total Unimodularity

- The side constraints for this problem are of this form:

$$C = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2j} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3j} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{4j} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

$$C \in \{0,1\}^{mxn}$$

Total Unimodularity

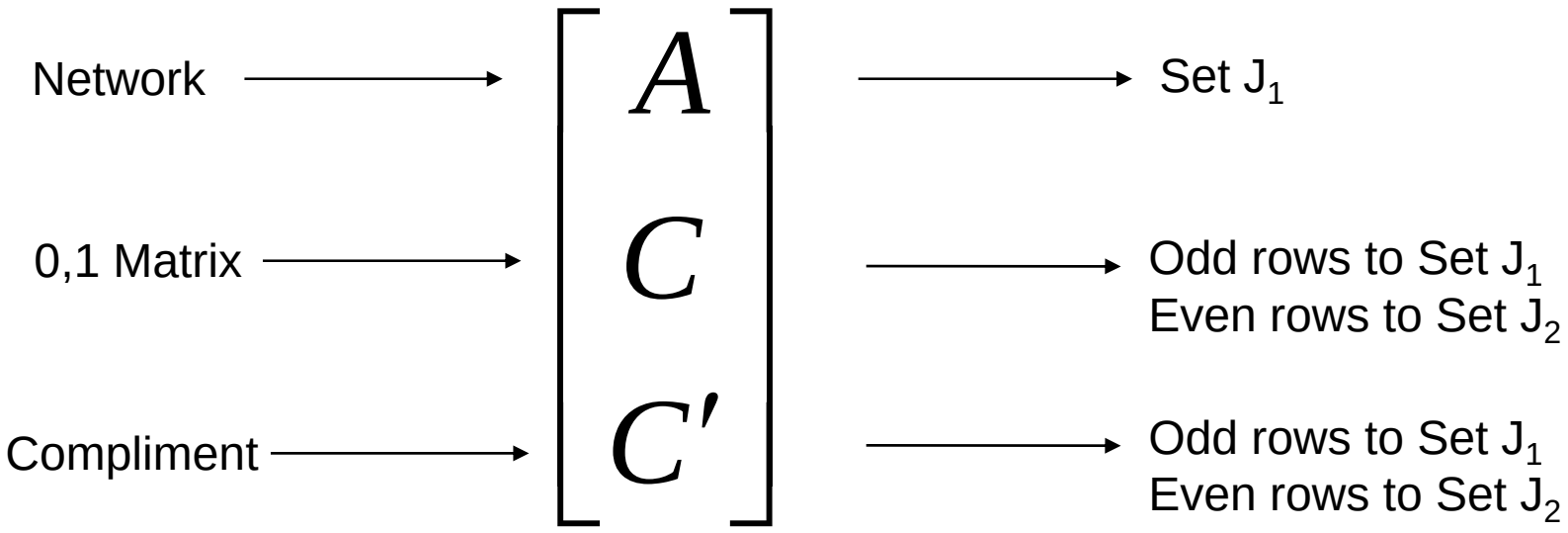
- The side constraints for this problem are of this form:

$$C' = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2j} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3j} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{4j} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

$$C' \in \{1, 0\}^{mxn}$$

Total Unimodularity

- Consider the Matrix:



Total Unimodularity

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  - For every J $\subseteq$ N={1,…,*n*}, there exists a partition J_1, J_2 of J such that

$$\left| \sum_{j \in J_1} a_{ij} - \sum_{j \in J_2} a_{ij} \right| \leq 1 \qquad \forall i = 1, \ldots, m$$

n/2 or (*n*+1)/2

n/2 or (*n*-1)/2

Total Unimodularity

- Therefore, this matrix is TU

$$\begin{bmatrix} A \\ C \\ C' \end{bmatrix}$$