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TECH. NOTE
G.W.89

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ROYAL AIRCRAFT ESTABLISHMENT

FARNBOROUGH, HANTS

TECHNICAL NOTE No: G.W.89

A THEORETICAL APPROACH TO THE MATCHING OF FUZE AND WARHEAD CHARACTERISTICS FOR ANTI-AIRCRAFT GUIDED WEAPONS

by

F.J.FAYERS, J.K.S.CLAYTON, B.A.,

R.G.KEATS, B.Sc. & W.R.B.HYND, M.A., B.Sc.

DEPARTMENT OF THE ARMY
UNITED STATES ARMY STANDARDIZATION GROUP, U.K.
BOX 65, F.P.O. NEW YORK, 09510

MASE-AM

11 July 1975

SUBJECT : Notification of Regrading Action

Commander
Picatinny Arsenal
ATTN: SARPA-TS-S # 59
Dover, New Jersey 07801

1. Reference : Your request for notification of regrading action, SARPA-TS-S, 7 Feb 75.

2. The following are the current security classifications of all reports listed in reference, with the exception of ARD TM 1/M/1947, which is unknown in RARDE, and the CARDE reports which are Canadian.

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ARD Expl 67/42	UNCLASSIFIED
RAE TN ARM 501	SECRET
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U.D.C. No.623.451-519:623.565.3:623.454.257

Technical Note No. G.W.89

November, 1950

ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

A theoretical approach to the matching of fuze and warhead characteristics for anti-aircraft guided weapons

by

F.J. Fayers,
J.K.S. Clayton, B.A.
R.G. Keats, B.Sc.
and
W.R.B. Hynd, M.A., B.Sc.

R.A.E. Ref: GW/S/100/10/171

SUMMARY

As a sequel to the publication of a preliminary study of warhead characteristics it was required to modify certain of the theoretical assumptions made therein to represent more precisely combat conditions. In this report, therefore, a theory is presented relating, generally, the warhead shape to the distributions of fragments and targets about the missile: it is shown that the problem of designing a warhead to satisfy a given fragment distribution is not capable of a unique solution. As an example of the use of the theory, and to indicate the nature of the results, it is applied to a simple probability model representative of the engagement of a target by missiles fitted with V.T. fuzes. Methods are suggested by which the general theory may be modified to meet more stringent demands.

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1 Introduction

An earlier report (Report of the Medium-Range Anti-Aircraft Guided Weapon Project Group - Appendix IV) has described an initial and general investigation into the desirable characteristics of a warhead for an anti-aircraft guided weapon - the necessary warhead weight, charge/case weight ratio and the manner in which these vary with the accuracy of guidance and the conditions of engagement. In carrying out this preliminary study it was convenient to make the following three assumptions (as well as others):-

(i) that the performance of the proximity fuze was highly idealized;

(ii) that the dimensions of the target were small compared with other distances, such as the guidance accuracy, involved in the problem; i.e. the target could be regarded as concentrated at a point;

and (iii) that the warhead could be regarded as cylindrical and yet the angular distribution of its fragments could be adjusted to match the fuze performance.

In the work to be described below it was desired to modify these assumptions so as to be more representative of actual conditions by

(i) showing how the actual burst pattern of any fuze could be incorporated into calculations, and how the fragment distribution could be matched to such fuze performance so as to give a maximum probability of destroying the target;

(ii) introducing some parameter into the problem which might be used to allow for the finite size of the target;

and (iii) determining, approximately, the shape of the warhead case which would give the desired fragment distribution in space - this last being evaluated with the object of estimating the general form of the warhead, so that its installation into the missile might be considered more realistically; and so that, the order of the curvature of the case being estimated, a study could be initiated as to the methods of controlling the fragment mass distribution of such cases.

The burst pattern of the actual fuze which will be used in the first designs of anti-aircraft guided weapons is unlikely to be known for some considerable time: this report must therefore be regarded as solely to establish a theoretical method of warhead-fuze matching, the calculations presented here being merely an example of how such a method may be applied, an example which will be superseded when the performance of the finalised fuze has been determined.

General equations are established determining the optimum fragment distribution, the total number of fragments and the probability of destroying the target, all in terms of the distribution of strikes on vulnerable targets about the point of burst, which is itself dependent on the fuze characteristics and the distribution of missiles about the target (or of targets about the missile, the two being equivalent). The equations are then examined for a particular missile distribution assuming a constant looking-angle fuze: the full analysis is shown in the degenerate case, corresponding to a looking-angle of 90°, but in the general case it has been necessary to resort to numerical methods, the resulting distributions being presented graphically together with specimen warhead contours.

2 Assumptions

In the general theory the target is assumed to have only one component which is singly vulnerable and spherically symmetrical, presenting a small area A: the distribution of possible positions of the vulnerable area about the point of burst is supposed to be known.

To reduce the general equations to a tractable form it has been necessary to construct a simple model in the particular case. It is assumed that a fuze with constant looking-angle γ is triggered when any point D of the target lies on the surface of the infinite cone of semi-angle γ about the missile axis (see Fig.2); further the distance from the detected point to the vulnerable area (here supposed concentrated at a point V) at the instant of strike is distributed according to a three dimensional Gaussian Error Law of modulus h, chosen to account both for the distance separating the detected and vulnerable points at the instant of burst and, also, for the motion of the vulnerable point before the instant of strike. The distribution of missile trajectories is a two dimensional Gaussian one of modulus k about a central trajectory through O.

3 Formal statistical approach

It can be shown that the probability of fragments ejected from a point striking an area A at range s is

$$1 - e^{-\rho A/s^2} \qquad (3.01)$$

where ρ is the fragment density in angular measure. Now suppose the vulnerable area, A, to be concentrated at a point V whose co-ordinates in a spherical polar system referred to the point of detonation, O, (Fig.1) and the axis of the missile are (s, ϕ, ψ). Obviously the probability distribution of V, being determined by the characteristics of the fuze and the performance of the guidance system will, in general, not be uniform and, if the probability of V lying within the element of volume dV be expressed by dP, then

$$dP_1 = G_1 \, dV \qquad (3.02)$$

where G_1 is a function of the position in space of V: expressed in polar co-ordinates this becomes

$$dP_1 = g(s, \phi, \psi) \, s^2 \sin\phi \, ds \, d\phi \, d\psi \qquad (3.03)$$

If the form of $g(s, \phi, \psi)$ is determinable it is obviously desirable that that warhead should be designed which will relate the number of fragments thrown in a given direction to the probability of V lying in that direction: this is equivalent to saying

$$\rho = f(\phi, \psi) \qquad (3.04)$$

where the form of $f(\phi, \psi)$ is to be optimised and depends on $g(s, \phi, \psi)$.

If the area A be singly vulnerable the total probability of destruction of the target, P_2 is given by the equation

$$P_2 = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\left(1 - e^{-f(\phi,\psi)\frac{A}{s^2}}\right) g(s,\phi,\psi)\, s^2 \sin\phi \, ds\, d\phi\, d\psi$$

$$= 1 - \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} e^{-f(\phi,\psi)\frac{A}{s^2}}\, g(s,\phi,\psi)\, s^2 \sin\phi \, ds\, d\phi\, d\psi \qquad (3.05)$$

The form of $f(\phi,\psi)$ may be determined by maximizing P_2 with respect to $f(\phi,\psi)$.

Clearly

$$\delta P_2 = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} e^{-f(\phi,\psi)\frac{A}{s^2}}\, A\delta f(\phi,\psi)\, g(s,\phi,\psi) \sin\phi \, ds\, d\phi\, d\psi \qquad (3.06)$$

to the first order of small quantities.

Moreover, the function $f(\phi,\psi)$ also determines the total number of fragments, n, for

$$n = \int_0^{2\pi}\int_0^{\pi} f(\phi,\psi) \sin\phi\, d\phi\, d\psi \qquad (3.07)$$

Hence, if $f(\phi,\psi)$ vary in such a manner that n remains constant, it follows that at a stationary value of P_2

$$\delta P_2 - \alpha\delta n = 0 \qquad (3.08)$$

where α is an arbitrary constant.

Therefore

$$\int_0^{2\pi}\int_0^{\pi}\left\{\int_0^{\infty} e^{-f(\phi,\psi)\frac{A}{s^2}}\, A\delta f(\phi,\psi)\, g(s,\phi,\psi) \sin\phi\, ds - \alpha\delta f(\phi,\psi) \sin\phi\right\} d\phi\, d\psi = 0$$

and since this equation must hold for any variation δf in f, it is necessary that

$$\int_0^{\infty} A e^{-f(\phi,\psi)\frac{A}{s^2}}\, g(s,\phi,\psi) ds = \alpha \qquad (3.09)$$

This is the condition to be satisfied by $f(\phi,\psi)$ for P_2 to be a maximum: it will be seen that n is dependent on α for α defines $f(\phi,\psi)$ which in turn, defines n: therefore, for a fixed n, α is also fixed. It follows that if the distribution of V about the warhead can be described by a function $g(s,\phi,\psi)$ (which may have to be determined empirically), then the form of $f(\phi,\psi)$ giving the maximum probability of destruction may be found.

The function so obtained defines the directions in space in which the fragments must travel: making allowance for the component of velocity imparted to the fragments by the motion of the missile a distribution function $f_{static}(\phi,\psi)$ is derived representing the directions of the fragment velocities relative to the missile.

It is supposed that the direction of flight of a fragment from any point on the missile surface is expressible in terms of certain parameters depending on the geometry of the warhead, the type of explosive and the position of the point of initiation. Conversely if the fragment distribution $f_{static}(\phi,\psi)$ is known a warhead contour may be determined for particular values of the other parameters: it will be shown, further, that such a contour is not unique.

3.1 A particular case: the constant looking-angle fuze

3.11 The determination of $g(s,\phi,\psi)$

Suppose that the warhead is fuzed to explode when any point D of the target, not necessarily a vulnerable point, is detected on the surface of a cone whose vertex is at O, axis coincident with that of the missile and semi-vertical angle γ. Let O be the origin of a system of cylindrical co-ordinates such that D is the point (z, r, θ) and V is (Z, R, Θ) as in Fig.2.

Then under the assumption that the distribution of V about O is Gaussian with modulus h,

$$\Pr.\ \{\ell < VD < \ell + d\ell\} = \frac{4h^3}{\sqrt{\pi}} \cdot e^{-h^2\ell^2}\, \ell^2 d\ell$$

and the probability of V lying in an element of volume $RdRdZd\Theta$ when D lies at a given point is

$$\frac{h^3 e^{-h^2\ell^2}}{\pi^{3/2}} \quad R\,dR\,dZ\,d\Theta \qquad (3.111)$$

Also under the assumption that the distribution of missile trajectories is Gaussian about a central trajectory through D

$$\Pr.\ \{r < PD < r + dr\} = 2k^2 e^{-k^2r^2}\, r\,dr$$

where k is the modulus of the distribution. Hence the probability of a detected point lying within the element of area $\frac{r\,d\theta dr}{\sin\gamma}$ of the surface of the cone is

$$\frac{k^2 e^{-k^2r^2}\, r}{\pi}\, dr\, d\theta \qquad (3.112)$$

It follows that, if dP_1 is the unconditional chance of V lying within the element of volume dV, then, in the notation of (3.02)

$$G_1 = \int_{\text{area of cone}} \cdot \frac{h^3 e^{-h^2\ell^2}}{\pi^{3/2}} \frac{k^2 e^{-k^2r^2} r}{\pi} \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\infty} \frac{h^3k^2}{\pi^{5/2}} e^{-(h^2\ell^2+k^2r^2)} r \, dr \, d\theta \qquad (3.113)$$

This function, expressed in polar co-ordinates as $g(s,\phi,\psi)$ will give $f(\phi,\psi)$ corresponding to a given α by (3.09).

Now

$$\ell^2 = (Z - z)^2 + (R \cos\Theta - r \cos \theta)^2 + (R \sin \Theta - r \sin \theta)^2$$

and

$$z = \frac{r}{t}$$

where

$$t = \tan \gamma$$

so that

$dP_1 =$

$$\left\{ \int_0^{2\pi} \int_0^{\infty} \frac{h^3k^2}{\pi^{5/2}} e^{-(\frac{h^2}{t^2}+h^2+k^2)r^2+2h^2r\,(\frac{Z}{t}+R\cos\theta\cos\Theta+R\sin\theta\sin\Theta)-h^2Z^2-h^2R^2} \cdot r \, d\theta \, dr \right\} R \, dR \, dZ \, d\Theta$$

$$= \left\{ \frac{h^3k^2}{\pi^{5/2}} \int_0^{\infty} e^{-(\frac{h^2}{t^2}+h^2+k^2)r^2+\frac{2rZh^2}{t}-h^2(R^2+Z^2)} \int_{\frac{\pi}{2}-\Theta}^{\frac{5\pi}{2}-\Theta} e^{-2h^2rR\sin u} \, r\,du\,dr \right\}$$

$$R \, dR \, dZ \, d\Theta$$

where $u = \frac{\pi}{2} + \theta - \Theta$. Hence, using the relationship

$$I_0(z) = \frac{1}{2\pi} \int_{-\pi+c}^{\pi+c} e^{-z \sin\theta} \, d\theta \; ,$$

dP_1 may be expressed in terms of a Bessel Function as

$$dP_1 = \left\{ \frac{h^3k^2}{\pi^{5/2}} \int_0^\infty e^{-\left(\frac{h^2}{t^2}+h^2+k^2\right)r^2 + \frac{2rZh^2}{t} - h^2(R^2+Z^2)} \, 2\pi I_0(2h^2rR) r \, dr \right\} dR \, dZ \, d\Theta$$

and, converting to polar co-ordinates where

$$s^2 = R^2 + Z^2$$

$$R = s \sin \phi$$

$$Z = s \cos \phi$$

$$dP_1 = \left\{ \frac{2h^3k^2}{\pi^{3/2}} \int_0^\infty e^{-\left(\frac{h^2}{t^2}+h^2+k^2\right)r^2 + \frac{2h^2rs\cos\phi}{t} - h^2s^2} I_0(2h^2rs \sin\phi) r \, dr \right\}$$

$$s^2 \sin\phi \, ds \, d\phi \, d\psi \qquad (3.114)$$

In this case, accordingly,

$$g(s,\phi,\psi) = \frac{2h^3k^2}{\pi^{3/2}} \int_0^\infty e^{-\left(\frac{h^2}{t^2}+h^2+k^2\right)r^2 + \frac{2h^2rs\cos\phi}{t} - h^2s^2} I_0(2h^2rs \sin\phi) \, r \, dr \qquad (3.115)$$

This function is, in fact, independent of ψ, indicating symmetry about the missile axis of the distribution of P_1 (this is to be expected owing to the nature of the fuze). The form of $f(\phi,\psi)$ follows directly from (3.09) and is found by solving the equation

$$\alpha = \frac{2h^3k^2A}{\pi^{3/2}} \int_0^\infty e^{-f(\phi,\psi)\frac{A}{s^2}}$$

$$\left\{ \int_0^\infty e^{-\left(\frac{h^2}{t^2}+h^2+k^2\right)r^2 + \frac{2h^2rs\cos\phi}{t} - h^2s^2} I_0(2h^2rs\sin\phi) r \, dr \right\} ds \qquad (3.116)$$

for given values of α .

It is easily shown that

$$V_F \sin(\phi_{static} - \phi) = V_M \sin\phi \qquad (3.117)$$

where V_M, V_F are, respectively, the velocities of the missile and of the fragments relative to the missile;

ϕ, ϕ_{static} are, respectively, the directions of the velocities of the fragments in space and relative to the missile.

Using this relation between ϕ_{static} and ϕ it is possible to derive $f_{static}(\phi,\psi)$ from $f(\phi,\psi)$: the property of symmetry is, of course, unaltered.

3.12 The solution when $\gamma = \frac{\pi}{2}$

The integrals in equations 3.114, 3.115 and 3.116 present some difficulty but in the case when $\gamma = \frac{\pi}{2}$, (that is, when the fuze cone degenerates to a plane) the solution is comparatively simple: (3.116) then becomes

$$\alpha = \frac{2h^3k^2}{\pi^{3/2}} \int_0^\infty Ae^{-f(\phi,\psi)\frac{A}{s^2}} \left\{ \int_0^\infty e^{-(h^2+k^2)r^2 - h^2s^2} I_0(2h^2rs\sin\phi)r\,dr \right\} ds \tag{3.1201}$$

Now it can be shown that

$$\int_0^\infty e^{-p^2t^2} I_0(at)t\,dt = \frac{1}{2p^2} e^{\frac{a^2}{4p^2}}$$

$$\therefore \int_0^\infty e^{-(h^2+k^2)r^2} I_0(2h^2rs\sin\phi)r\,dr = \frac{1}{2(h^2+k^2)} e^{\frac{4h^4s^2\sin^2\phi}{4(h^2+k^2)}}$$

$$\therefore \quad \alpha = \frac{h^3k^2A}{\pi^{3/2}(h^2+k^2)} \int_0^\infty e^{-f(\phi,\psi)\frac{A}{s^2} - h^2s^2 + \frac{h^4s^2\sin^2\phi}{h^2+k^2}} ds$$

Further it can be shown that

$$\int_0^\infty e^{-t^2 - \frac{a^2}{t^2}} dt = \frac{\sqrt{\pi}}{2} e^{-2|a|}$$

Write

$$p^2 = h^2\left(1 - \frac{h^2\sin^2\phi}{h^2+k^2}\right)$$

$$= \frac{h^2(h^2\cos^2\phi + k^2)}{h^2+k^2} \tag{3.1202}$$

$$q^2 = A\, f(\phi,\psi) \tag{3.1203}$$

Then

$$\alpha = \frac{h^3k^2\,A}{\pi^{3/2}(h^2+k^2)} \int_0^\infty e^{-p^2s^2 - \frac{q^2}{s^2}}\, ds$$

$$= \frac{h^3k^2A}{\pi^{3/2}(h^2+k^2)} \int_0^\infty \frac{e^{-y^2 - \frac{p^2q^2}{y^2}}}{p}\, dy$$

$$= \frac{h^2k^2A}{\pi^{3/2}\sqrt{h^2+k^2}\,\sqrt{h^2\cos^2\phi+k^2}} \cdot \frac{\sqrt{\pi}}{2}\, e^{-2h\sqrt{\frac{(h^2\cos^2\phi + k^2)A\, f(\phi,\psi)}{h^2+k^2}}}$$

$$\therefore \quad \alpha = \frac{h^2k^2A}{2\pi\sqrt{(h^2+k^2)(h^2\cos^2\phi+k^2)}}\, e^{-2h\sqrt{\frac{(h^2\cos^2\phi+k^2)A\, f(\phi,\psi)}{h^2+k^2}}}$$

(3.1204)

It is important to notice that a limit is imposed on the range of α by the modulus in the right hand side of (3.1204). Therefore

$$\alpha < \frac{h^2k^2A}{2\pi\sqrt{(h^2+k^2)(h^2\cos^2\phi+k^2)}} \tag{3.1205}$$

If

$$\alpha < \frac{h^2k^2A}{2\pi(h^2+k^2)}$$

this condition is satisfied for all values of ϕ but if

$$\frac{h^2k^2A}{2\pi(h^2+k^2)} < \alpha < \frac{h^2kA}{2\pi\sqrt{(h^2+k^2)(h^2\cos^2\phi+k^2)}} \tag{3.1206}$$

it is only satisfied for a certain range and any integrations with respect to ϕ must be over that range only, that is between the limits ϕ_0 and $\pi/2$ where

$$\cos\phi_0 = \frac{1}{h}\sqrt{\frac{h^4k^4A^2}{4\pi^2\alpha^2(h^2+k^2)} - k^2} \tag{3.1207}$$

The physical interpretation of this restriction is, simply, that if the number of fragments is small an optimum warhead would throw no fragments in those directions where a vulnerable area is rather unlikely to occur, but as the number of fragments increases so also does α and the zone of fragmentation steadily widens.

Rearranging, (3.1204) becomes

$$-4h\sqrt{\frac{Af(\phi,\psi)(h^2\cos^2\phi+k^2)}{h^2+k^2}} = \log_e\left(\frac{4\pi^2\alpha^2(h^2+k^2)(h^2\cos^2\phi+k^2)}{h^4k^4A^2}\right)$$

$$\therefore \quad f(\phi,\psi) = \frac{(h^2+k^2)}{16h^2A(h^2\cos^2\phi+k^2)}\left\{\log_e\left(\frac{4\pi^2\alpha^2(h^2+k^2)(h^2\cos^2\phi+k^2)}{h^4k^4A^2}\right)\right\}^2 \qquad (3.1208)$$

or, in dimensionless form,

$$f(\phi,\psi) = \frac{(1+\beta^2)\beta^2}{16a(\cos^2\phi+\beta^2)}\left\{\log_e\left(\frac{4\pi^2\alpha^2(1+\beta^2)(\cos^2\phi+\beta^2)}{a^2}\right)\right\}^2$$

where the dimensionless quantities β and a are defined by the equations

$$\beta = \frac{k}{h}\ ; \quad a = k^2A\ .$$

Again the function is independent of ψ and is expressed here in terms of ϕ and the parameters h, k, A and α.

It is now possible to evaluate P_2 from the equation

$$P_2 = 1 - \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} e^{-f(\phi,\psi)\frac{A}{s^2}} \left\{\frac{2h^3k^2}{\pi^{3/2}}\int_0^{\infty} e^{-(h^2+k^2)r^2-h^2s^2}\, I_0(2h^2rs\sin\phi)r\,dr\right\} s^2\sin\phi\,ds\,d\phi\,d\psi \qquad (3.1209)$$

Integrating with respect to r, as above,

$$P_2 = 1 - \frac{2h^3k^2}{\pi^{3/2}} \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} e^{-f(\phi,\psi)\frac{A}{s^2}} \frac{e^{\frac{h^4s^2\sin^2\phi}{h^2+k^2} - h^2s^2}}{2(h^2+k^2)} s^2\sin\phi \, ds\, d\phi\, d\psi$$

and since $f(\phi,\psi)$ is here independent of ψ

$$P_2 = 1 - \frac{2h^3k^2}{\pi^{1/2}(h^2+k^2)} \int_0^{\pi}\int_0^{\infty} e^{-f(\phi,\psi)\frac{A}{s^2} - \frac{h^2s^2(h^2\cos^2\phi+k^2)}{h^2+k^2}} s^2\sin\phi\, ds\, d\phi$$

(3.1210)

Now

$$\int_0^{\infty} e^{-p^2s^2 - \frac{q^2}{s^2}} s^2 ds = \frac{1}{2p^3}\int_0^{\infty} e^{-u - \frac{p^2q^2}{u}} u^{\frac{1}{2}} du \; : \; \text{also}$$

it can be shown that, for all values of ν,

$$K_\nu(z) = \tfrac{1}{2}\left(\tfrac{1}{2}z\right)^{\nu}\int_0^{\infty} \frac{e^{-w-\frac{z^2}{4w}}}{w^{\nu+1}} dw$$

$$\therefore \int_0^{\infty} e^{-p^2s^2 - \frac{q^2}{s^2}} s^2\, ds = \left(\frac{q}{p}\right)^{3/2} K_{-3/2}(2pq)$$

$$\therefore P_2 = 1 - \frac{2h^3k^2}{\pi^{1/2}(h^2+k^2)}\int_0^{\pi}\left(\frac{q}{p}\right)^{3/2} K_{-3/2}(2pq)\sin\phi\, d\phi$$

(3.1211)

Now

$$K_{-3/2}(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} e^{-z}\left(1+\frac{1}{z}\right)$$

$$\therefore \quad P_2 = 1 - \frac{2h^3k^2}{\pi^{\frac{1}{2}}(h^2+k^2)} \int_0^{\pi} \left(\frac{q}{p}\right)^{3/2} \frac{\pi^{\frac{1}{2}}}{(4pq)^{\frac{1}{2}}} e^{-2pq}\left(1 + \frac{1}{2pq}\right)\sin\phi \, d\phi$$

$$= 1 - \frac{h^3k^2}{(h^2+k^2)}\left\{\int_0^{\pi} \frac{q}{p^2} e^{-2pq} \sin\phi \, d\phi + \int_0^{\pi} \frac{1}{2p^3} e^{-2pq} \sin\phi \, d\phi\right\}$$

From (3.1204) we have the relation

$$\frac{h^3k^2}{h^2+k^2} e^{-2pq} = \frac{2\pi\alpha p}{A}$$

$$\therefore \qquad P_2 = 1 - \frac{2\pi\alpha}{A}\left\{\int_0^{\pi} \frac{q}{p}\sin\phi \, d\phi + \int_0^{\pi} \frac{1}{2p^2}\sin\phi \, d\phi\right\} \qquad (3.1212)$$

Now

$$\int_0^{\pi} \frac{q}{p}\sin\phi \, d\phi = \int_0^{\pi} \frac{A^{\frac{1}{2}}\sin\phi(h^2+k^2)^{\frac{1}{2}}}{h(h^2\cos^2\phi+k^2)^{\frac{1}{2}}} \cdot \frac{(h^2+k^2)^{\frac{1}{2}}}{4hA^{\frac{1}{2}}(h^2\cos^2\phi+k^2)^{\frac{1}{2}}}$$

$$\left\{\log_e\left(\frac{4\pi^2\alpha^2(h^2+k^2)(h^2\cos^2\phi+k^2)}{h^4k^4A^2}\right)\right\} d\phi$$

$$= \frac{(h^2+k^2)}{4h^2}\int_0^{\pi} \frac{\sin\phi}{(h^2\cos^2\phi+k^2)} \, 2\log_e\left(\frac{2\pi\alpha(h^2+k^2)^{\frac{1}{2}}(h^2\cos^2\phi+k^2)^{\frac{1}{2}}}{h^2k^2A}\right) d\phi$$

Let $\qquad h\cos\phi = k\tan\theta$

Then

$$\int_0^{\pi} \frac{q}{p}\sin\phi \, d\phi = \frac{(h^2+k^2)}{4h^2}\int_{\tan^{-1}\frac{h}{k}}^{\tan^{-1}-\frac{h}{k}} -\frac{2}{hk}\log_e\left(\frac{2\pi\alpha(h^2+k^2)^{\frac{1}{2}}\sec\theta}{h^2kA}\right) d\theta$$

$$= \frac{h^2+k^2}{h^3k}\left\{\log_e\left(\frac{2\pi\alpha(h^2+k^2)^{\frac{1}{2}}}{h^2kA}\right) . \tan^{-1}\frac{h}{k} + \int_0^{\tan^{-1}\frac{h}{k}} \log_e \sec\theta \, d\theta\right\}$$

Also

$$\int_0^\pi \frac{1}{2p^2}\sin\phi\, d\phi = \int_0^\pi \frac{(h^2+k^2)\sin\phi\, d\phi}{2h^2(h^2\cos^2\phi + k^2)}$$

$$= \frac{h^2+k^2}{h^3k}\tan^{-1}\frac{h}{k}$$

$\therefore$

$$P_2 = 1 - \frac{2\pi\alpha(h^2+k^2)}{Ah^3k}\left\{\tan^{-1}\frac{h}{k}\left[1+\log_e\frac{2\pi\alpha(h^2+k^2)^{\frac{1}{2}}}{h^2kA}\right] + \int_0^{\tan^{-1}\frac{h}{k}}\log_e\sec\theta\, d\theta\right\}$$

(3.1213)

This last integral must be evaluated numerically.

Again this expression for P_2 must be amended if

$$\frac{h^2k^2A}{2\pi(h^2+k^2)} < \alpha < \frac{h^2k^2A}{2\pi\sqrt{(h^2+k^2)(h^2\cos^2\phi+k^2)}}$$

for then (3.05) must be written

$$P_2 = \int_0^{2\pi}\int_{\phi_0}^{\pi-\phi_0}\int_0^\infty \left(1 - e^{-f(\psi,\phi)\frac{A}{s^2}}\right) g(s,\psi,\phi)\, s^2\sin\phi\, ds\, d\phi\, d\psi$$

where ϕ_0 is defined by (3.1207) and, finally,

$$P_2 = \frac{\sqrt{h^2+k^2}\,.\cos\phi_0}{\sqrt{h^2\cos^2\phi_0+k^2}}$$

$$- \frac{2\pi\alpha(h^2+k^2)}{Ah^3k}\left\{\tan^{-1}\frac{h\cos\phi_0}{k}\left[1+\log_e\frac{2\pi\alpha\sqrt{h^2+k^2}}{h^2kA}\right] + \int_0^{\tan^{-1}\frac{h\cos\phi_0}{k}}\log\sec\theta\, d\theta\right\}$$

(3.1215)

The function $\int_0^x \log\sec\theta\, d\theta$ has been tabulated.

3.13 The general solution

When γ is unrestricted the analytical solution of equations 3.05, 3.114, 3.115 and 3.116 presents considerable difficulty and no closed expressions for $g(s,\phi,\psi)$ $f(\phi,\psi)$ or P_2 have been obtained. Instead the integrals have been evaluated by numerical methods and the results are presented in the form of graphs of target and fragment densities about the point of burst together with a table of P_2 against α for a particular value of γ .

3.14 The barrel contours

In the special case when $f_{static}(\phi,\psi)$ is independent of ψ the warhead is a solid of revolutions, or barrel shaped. Using the notation of Fig.3, which illustrates a longitudinal section through a warhead initiated at I, it is possible to equate two expressions for the fragment density from an elementary segment of thickness dx, one due to the geometrical properties and one imposed by the fragment distribution function, namely

$$\frac{3}{4} \cdot \frac{2\pi y\, ds}{a_s} = 2\pi f_{static}(\phi,\psi) \sin\phi \, d\phi$$

i.e.
$$\frac{3y}{4a_s} \frac{ds}{d\sigma} = f_{static}(\phi,\psi) \sin\phi \frac{d\phi}{d\sigma} \qquad (3.141)$$

where a_s is the area of that face of a fragment which lies within the warhead surface and on the assumption (previously made in 'Report of the Medium-Range Anti-Aircraft Guided Weapon Project Group'- Appendix IV) that 75% of the weight of metal in the case is converted to controlled fragments: s is conventionally measured counter-clockwise round the contour.

From the figure

$$\phi = \sigma + \delta - \frac{\pi}{2} \qquad (3.142)$$

where σ is the angle between the positive x-axis and the tangent, measured in the positive direction of s, and δ is the angle between the normal to the surface and the direction of motion of the fragment, making the convention that δ is measured positively from the normal in a counter clockwise direction. Following the work of Shapiro (A Report on Analysis of the Distribution of Perforating Fragments for the 90 mm M71, Fuzed T74E6, Bursting Charge TNT, University of New Mexico UNM/T-234) the following equation has been adopted to define δ

$$\tan\delta = \frac{dr}{ds} \sec\sigma \cdot \frac{V}{2V_D} \qquad (3.143)$$

where V, V_D are the initial fragment velocity and the velocity of propagation of the explosion respectively.

To maintain the contour equation in a tractable form no attempt has been made here to allow for the dispersion of fragments about the direction ϕ although it is hoped to examine the effect of dispersion on the contour shape later.

Denoting $\frac{V}{2V_D}$ by j, which will be regarded as constant, equation (3.143) may be written

$$\tan\delta = j\frac{\cos\beta}{\cos\sigma} \qquad (3.144)$$

Now, from the figure,

$$\beta = \sigma - \theta \qquad (3.145)$$

where

$$\tan\theta = \frac{y}{(x - x_0)}, \quad \tan\sigma = \frac{dy}{dx} \qquad (3.146)$$

and the contour equation may be derived by the following method:- from (3.142)

$$\frac{d\phi}{d\sigma} = 1 + \frac{d\delta}{d\sigma}$$

where, by (3.144) and (3.145),

$$\sec^2\delta\,\frac{d\delta}{d\sigma} = j\sec^2\sigma\,(\sin\theta + \cos\sigma\sin\beta\,\frac{d\theta}{d\sigma})$$

also

$$\sec^2\theta\,\frac{d\theta}{d\sigma} = \sec^2\theta\,\frac{d\theta}{dx}\cdot\frac{dx}{d\sigma}$$

$$\therefore \frac{d\theta}{d\sigma} = \frac{(x-x_0)\frac{dy}{dx} - y}{(x-x_0)^2 + y^2}\cdot\frac{\sec^2\sigma}{\frac{d^2y}{dx^2}} \quad \text{using } (3.146)$$

$$= \frac{\sin\theta\;\sin\beta\;\sec^3\sigma}{y\,\frac{d^2y}{dx^2}} \quad \text{by } (3.145) \text{ and } (3.146)$$

using these results and (3.144) it follows that

$$\frac{d\delta}{d\sigma} = \frac{j}{(\cos^2\sigma + j^2\cos^2\beta)}\left\{\sin\theta + \frac{\sin\theta\;\sin^2\beta\;\sec^2\sigma}{y\,\frac{d^2y}{dx^2}}\right\}$$

from (3.141), therefore

$$\frac{3y}{4a_s} \cdot \frac{\sec^3\sigma}{\frac{d^2y}{dx^2}} = f_{static}(\phi,\psi)\sin\phi\left\{1+\frac{j\sin\theta}{(\cos^2\sigma+j^2\cos^2\beta)}\left(1+\frac{\sin^2\beta\sec^2\sigma}{y\frac{d^2y}{dx^2}}\right)\right\}$$

whence, finally,

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2\sigma(\cos^2\sigma+j^2\cos^2\beta+j\sin\theta)}\left\{\frac{3y(\cos^2\sigma+j^2\cos^2\beta)}{4a_s f_{static}(\phi,\psi)\sin\phi\cos\sigma} - \frac{j\sin\theta\sin^2\beta}{y}\right\}$$

(3.147)

It is important to realise that the fragment distribution does not define a warhead barrel uniquely: in fact since the differential equation (3.147) is of the second order it is formally possible to design a doubly infinite set of barrels each of which produces the same distribution of fragments, the selection of any one being determined solely by installation considerations. The computed contour will exist over that range of values of ϕ for which $f(\phi,\psi)$ exists and will not, in general, terminate on the axis: if it crosses the axis then no real contour exists satisfying the corresponding boundary conditions and if it stops short of the axis the ends of the barrel are supposed closed by non-fragmenting end plates. The boundary conditions, which are fixed arbitrarily, may be considered as the radius of the barrel and the direction of the tangent to its contour in a longitudinal section, both at the point of initiation.

4 The presentation of results

4.1 The distribution of vulnerable areas about the point of burst

It has been shown that the optimum fragment distribution and the corresponding warhead contour may be derived from the function $g(s,\phi,\psi)$ defining the distribution of vulnerable areas about the point of burst: accordingly the first result is a graphical representation of this function in the special case of a constant looking-angle fuze and a Gaussian distribution of vulnerable areas about the detected point evaluated numerically for the following values of the parameters (Fig.4):-

$$\gamma = 70^\circ \; ; \; a = 0.0004 \; ; \; \beta = 0.25$$

which would apply if, for example,

A = 4 sq.ft.

R.M.S. guidance accuracy = 100 ft.

R.M.S. value of ℓ = 30.6 ft.

The distribution being symmetrical about the missile axis, $g(s,\phi,\psi)$ is represented by contours in a plane containing the axis. Alternatively Fig.5 displays the density of vulnerable areas as a dot diagram in which the number of dots, spaced conventionally, in any given part is proportional to the probability of a vulnerable area lying within the volume described by the rotation of that part about the axis: the boundary within which 50% of the vulnerable areas lie is also shown. It will be observed that this diagram shows reasonable resemblance to the burst positions actually obtained from V.T. fuze trials.

4.2 The optimum fragment distribution and probabilities of destruction

In Fig.6 the function $f_{static}(\phi,\psi)$ is plotted against ϕ, the missile and mean fragment velocities being taken as 2000 ft/sec and 8000 ft/sec respectively. Corresponding to these values of α values of n and P_2 have been computed from equations (3.07) and (3.05) respectively by numerical integration:-

α an arbitrary parameter	n * the total number of 1/32 oz. fragments	P_2 the probabilities of destruction of the target
20×10^{-6}	13300	0.72
30×10^{-6}	8700	0.63
40×10^{-6}	5900	0.57

* No significance should be attached to the use of 1/32 oz. fragments in this example which is intended for purposes of illustration only: at the time the calculations were performed it was considered likely that the optimum fragment size was approximately 1/32 oz., but consideration is now being given to considerably larger fragments.

4.3 The barrel contours

Having regard to the complexity of equation (3.147) it was considered unreasonable to evaluate more than a few barrel contours. Accordingly it was decided to illustrate the variation in barrel shape as the arbitrary boundary conditions are changed by selecting a value of corresponding to a warhead of suitable size and then varying the radius of the barrel and the slope of its contour at the point of initiation: since no other parameters were varied, all the barrels studied should produce identically the same fragment distribution. Figs.7 and 8 represent longitudinal sections through two barrels which are approximately at the extremes of the possible range of variation. Both weigh about 150 lbs and break into 8700 fragments distributed according to curve (2) in Fig.6, but the point of initiation lies at opposite ends of the warhead: in each case the constant j was given the value $\sin 7^{\circ} = 0.122$ being consistent with a deflection of 7° from the equatorial plane, according to the earlier theory of G.I. Taylor for the detonation of a long cylindrical cased charge.

5 Some possible extensions

It is formally possible to modify the probability model in a number of ways in order to represent more closely combat conditions. Some of these are briefly indicated here and it is hoped that a more detailed discussion with examples will be published in the near future.

No attempt has been made in this report to consider variations in the effectiveness of fragments striking a vulnerable area A: to do so it is necessary, obviously, to know the striking velocity of the fragment which depends on a number of variables, including the weapon and target velocities and the angle between them, the direction of throw of the fragment, the polar co-ordinates of the target relative to the missile, and the height of attack. If all these are known the proportion of striking fragments which do lethal damage may be found, subject to some assumptions about the distribution of their presented areas at the moment of impact, and a new A defined as the total vulnerable area reduced by that ratio. The quantity A is then a function of the same variables.

The fuze burst pattern may be obtained more accurately by the following method. On each of a set of polar diagrams, representing planes through the target as pole, and for a given direction of missile attack in each plane, is superimposed a corresponding set of fuze burst points (experimental data) and contours to show the length of the flight path and necessary angle of throw for a fragment which strikes the target: the number of such diagrams must be sufficient to present a fair picture of the distribution of bursts in space and associated with each one there will be the probability of that direction of attack (dependent on the method of tactical use of the missile), and a particular value of the vulnerable area A. The ranges of flight and angles of throw, having been read from the contours, are all transferred to one new polar diagram, whose pole represents the position in space of the warhead; the points do not, of course, carry equal weight. The new diagram is interpreted as a density pattern such that the number of points falling within any elementary area is proportional to the probability that a target, at the instant a fragment reached the ring described by the rotation of the elementary area about the missile axis, would lie within that ring. It follows that the diagram also represents the distribution function of a vulnerable point about the warhead, denoted by $g(s,\phi,\psi)$ in the particular case of a barrel warhead, and that the fragment distribution function $f(\phi)$ may be derived from it.*

Whether or not it is practicable to treat the quantity A as a function of all the variables already mentioned, and yet to maintain the problem in a reasonably tractable form, is still to be determined: the use of the improved (experimental) fuze burst pattern, however, should create no great difficulty and permits precise evaluation of the manner in which $g(s,\phi,\psi)$ depends on the velocities of target, missile and fragment. To consider a number of subtargets having different vulnerable areas but all situated at the same point within the target aircraft is a simple extension; and it may prove possible to allow for the separation distances between subtargets.

More realistic barrel contours may be found by allowing for the dispersion of fragments about the predicted angles of throw: work now proceeding, assuming a Gaussian distribution, will be published shortly.

6 Acknowledgement

The authors wish to express their appreciation of the assistance rendered in the preparation of this Note by Miss D.M.C. Gilmore, who was responsible for the greater part of the numerical work, and other members in the Assessment Division of Guided Weapons Department.

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* Although it is unlikely that consideration will be given to the design of warhead not symmetrical about the axis for a number of years, the method may easily be adapted to the general case, in which f depends both on ϕ and ψ.

Tech. Note No. G.W.89

Attached: Addendum
Drg.Nos. GW/P/2091 to 2096; 2111 to 2113

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ADDENDUM

The comparative lethalities of shaped and cylindrical warheads

It was required to estimate the advantage to be gained by the use of a shaped warhead instead of a simple cylinder. In this addendum the lethality of a cylinder designed to contain the same quantity of fragmenting metal as any one of the family of barrel warheads, of which those illustrated in Figs.7 and 8 are typical, is calculated under the same conditions of engagement and to the same degree of accuracy.

Method of Assessment

The area of the curved surface of the cylinder being fixed by the requirement that both types of warhead should produce the same number of fragments a length and diameter were chosen broadly similar to those of the curved wall warheads (but such that the cylinder was slightly longer to improve the spread of fragments). The characteristics of the two types are compared below, certain inconsistencies being the result of a deliberate policy of favouring the cylinder. The warhead weights are of the same order.

	Barrel warhead	Cylindrical warhead
No. fragments (1/32 oz)	8700	8700
length	10 - 12 ins	17.9 ins
diameter	14.5 - 15.5 ins	12 ins
initial fragment veloc.	8000 ft/sec	8000 ft/sec
missile veloc.	2000 ft/sec	2900 ft/sec

This particular missile velocity, in the case of the cylindrical warhead, provides a dynamic fragment distribution peaked at $\phi = 70^\circ$ and, according to the relations between initial fragment velocity and charge/case weight ratio published in the Report of the Anti-Aircraft Guided Weapons Project Group, to which reference has been made, such a warhead would be annular.

Having regard to the indefinite nature of annular warhead theory at the present time it was decided to use Shapiro's Formula to predict the theoretical static fragment distributions, denoted by $f(\phi_s)$, assuming a solid charge initiated at the centre: in this instance we have, in the notation of Fig.9,

$$\cot\phi = \frac{jx}{(x^2 + R^2)^{\frac{1}{2}}} \qquad (1)$$

whence

$$\frac{dx}{d\phi} = -\frac{Rj \operatorname{cosec}^2\phi}{(j^2 - \cot^2\phi)^{3/2}}$$

Then, $f(\phi_s)$ being defined as the ratio of the number of effective fragments thrown from a 'slice' of thickness dx to the solid angle subtended at the point of explosion by the presented area, A, of the target,

$$f(\phi_s) = \frac{3j^2R^2 \operatorname{cosec}^2 \phi_s}{4A \sin \phi_s (j^2 - \cot^2 \phi_s)^{3/2}} \qquad (2)$$

The geometry of the case imposes certain limits on the range of ϕ_s, namely

$$|\cot \phi_s| \leqslant \frac{j\ell}{(\ell^2 + d^2)^{\frac{1}{2}}} \qquad (3)$$

and it follows that for a fixed value of the product $(\ell \times d)$ the fragment zone increases as ℓ increases. The function $f(\phi_s)$ is plotted in Fig.10.

In furtherance of the policy of favouring the cylinder it was decided moreover to assume that fragments were subject to a considerable Gaussian error about the predicted directions in a plane containing the missile axis (the symmetry of the warhead made it unnecessary to consider dispersion in a perpendicular plane); a standard deviation of 5° was chosen. The 'dispersed' static fragment distribution function $F(\phi_s)$, is compared with $f(\phi_s)$ and with the 'dispersed' dynamic fragment distribution function $F(\phi_d)$ in Figs.10,11 respectively.

It is easily shown that the direction of the fragment path in space is given by the equation

$$\phi_d = \tan^{-1}\left(\frac{\sin \phi_s}{0.364 + \cos \phi_s}\right) \qquad (4)$$

and, hence,

$$\frac{d\phi_s}{d\phi_d} = 1 + \frac{0.364 \cos \phi_d}{\cos (\phi_s - \phi_d)} \qquad (5)$$

Since, obviously, the number of fragments in corresponding regions of the static and dynamic distributions must be equal

$$F(\phi_s) \sin \phi_s \, d\phi_s = F(\phi_d) \sin \phi_d \, d\phi_d$$

and, accordingly, from equation (5)

$$F(\phi_d) = F(\phi_s) \frac{\sin \phi_s}{\sin \phi_d}\left(1 + \frac{0.364 \cos \phi_d}{\cos (\phi_s - \phi_d)}\right)$$

Finally, under the conditions of engagement assumed in the shaped warhead assessment, namely,

$$\gamma = 70^\circ; \quad a = 0.0004; \quad \beta = 0.25 \ ;$$

the probability of destroying the target represented by vulnerable area A is

Addendum to
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$$P_2 = 1 - \frac{4h^3k^2}{\pi^{\frac{1}{2}}} \int_0^{\pi} \int_0^{\infty} \left\{ \int_0^{\infty} e^{-\left(\frac{h^2}{t^2}+h^2+k^2\right)r^2 + \frac{2h^2 rs \cos\phi}{t} - h^2 s^2} I_0(2h^2 rs \sin\phi) r \, dr \right\} e^{-\frac{4F(\phi)}{s^2}} s^2 \sin\phi \, ds \, d\phi \qquad (7)$$

Here ϕ is written for ϕ_d for brevity.

Results

The fragment distribution functions are plotted in Figs. 10 and 11. Equation (7) was solved numerically to give

$$P_2 = 0.42$$

as compared with a lethality of 0.63 for the corresponding shaped warhead. It is recognised, however, that the 50% increase in lethality obtained in this particular case is not necessarily representative and that inspection of the effect of varying other combat, target and missile parameters, in particular the fragment mass, is desirable.

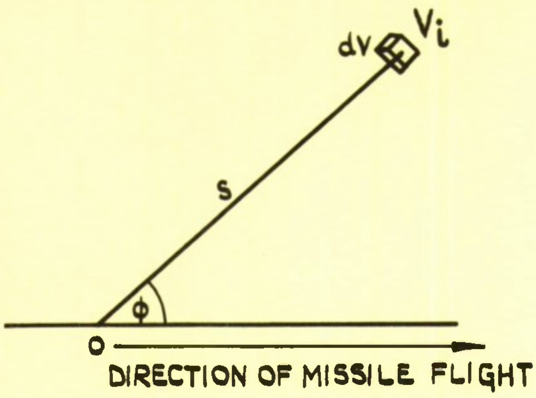


FIG. 1.

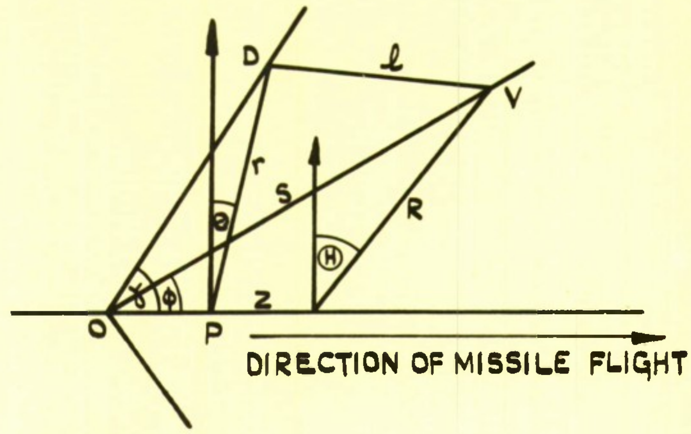
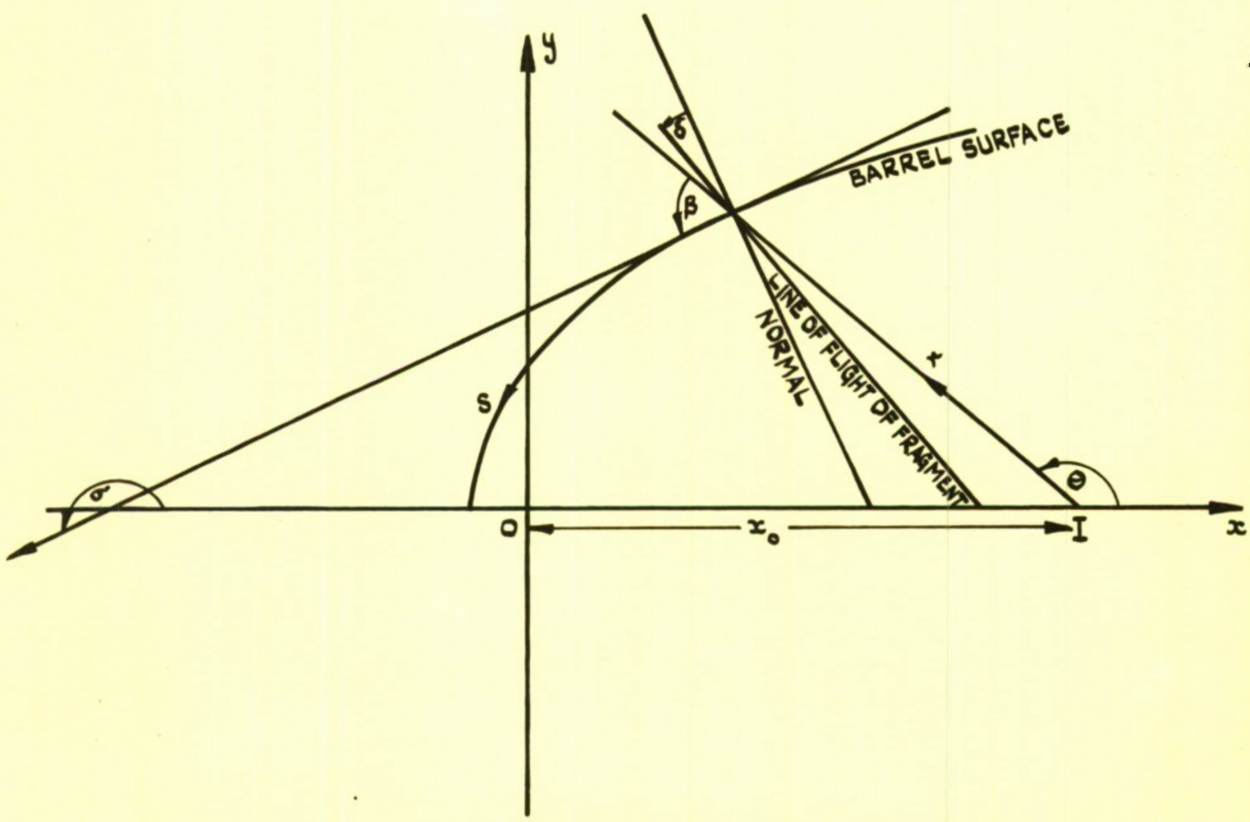
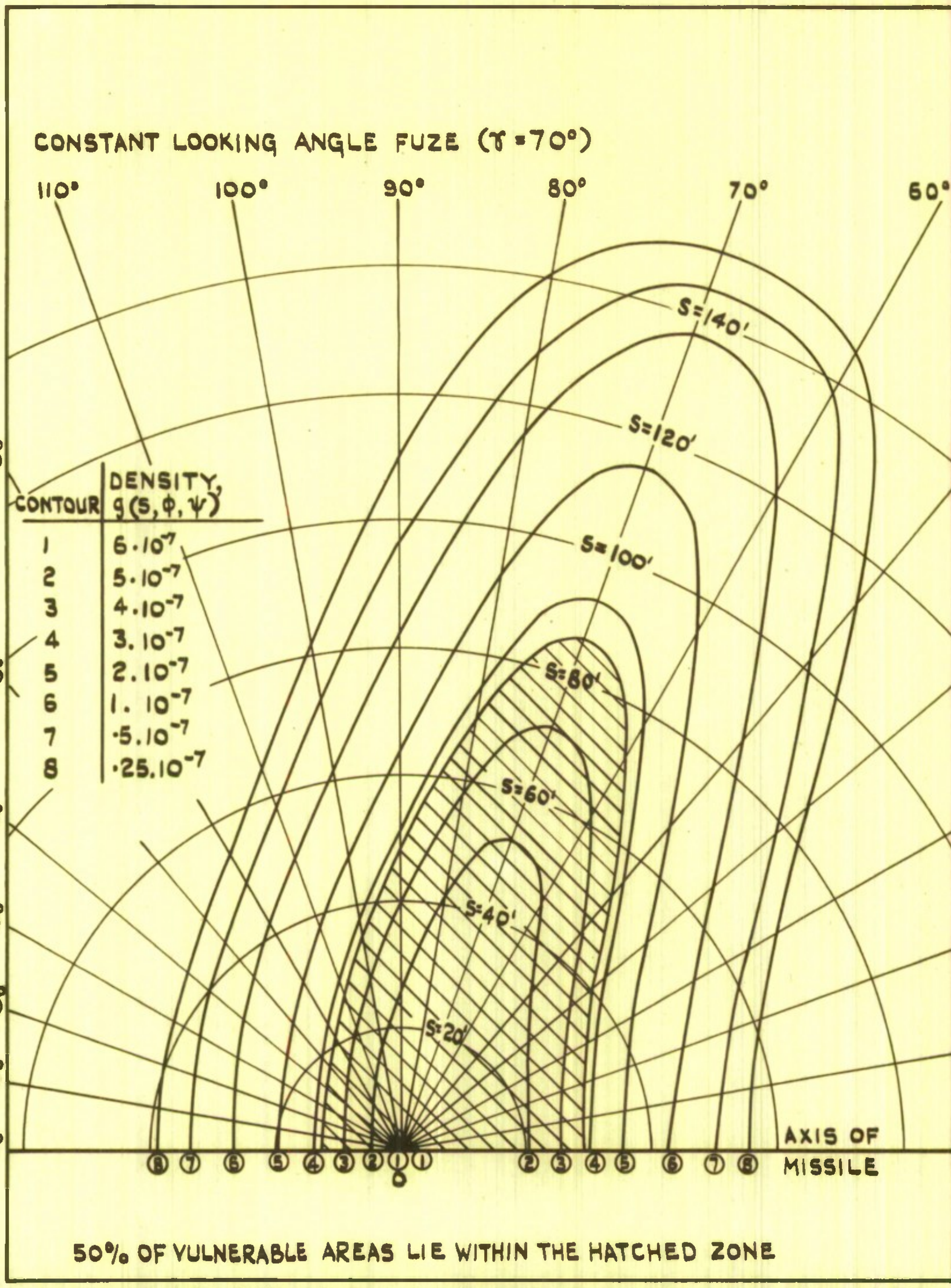


FIG. 2



THE ARROWS INDICATE POSITIVE DIRECTION OF MEASUREMENT.

FIG. 3.



THE FIGURE REPRESENTS THE GENERATOR OF A THREE DIMENSIONAL DISTRIBUTION, SYMMETRICAL ABOUT THE MISSILE AXIS AND, THEREFORE, INDEPENDENT OF ψ.

FIG. 4. CONTOUR DIAGRAM TO SHOW THE DISTRIBUTION OF VULNERABLE AREAS, $g(S, \phi)$ ABOUT THE BURST POINT, O, FOR A 'BARREL' WARHEAD.

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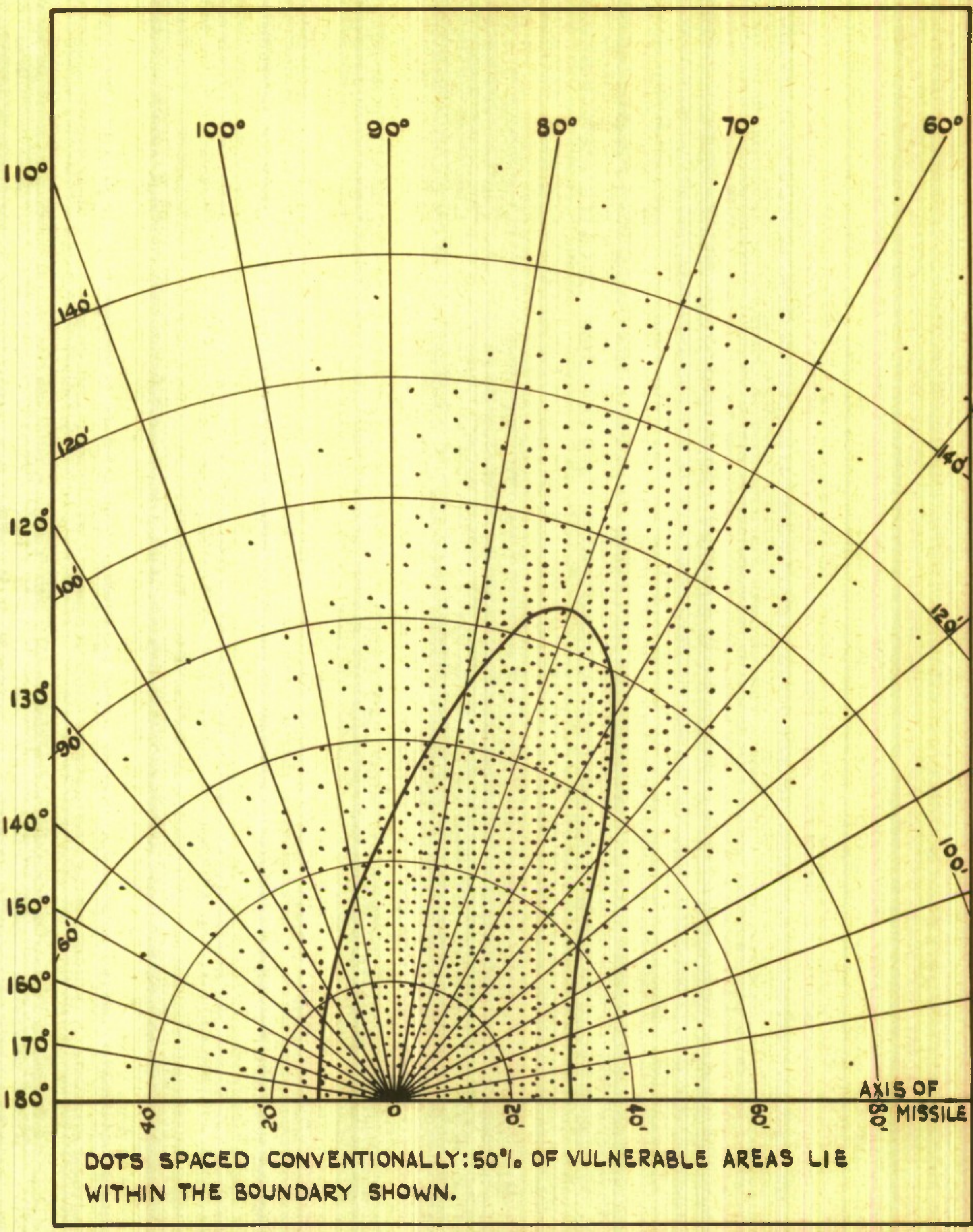
FIG.5.



THE NUMBER OF DOTS IN ANY ELEMENTARY AREA IS PROPORTIONAL TO THE FREQUENCY OF VULNERABLE AREAS IN THE SOLID OF REVOLUTION GENERATED BY THE ROTATION OF THAT AREA ABOUT THE MISSILE AXIS.

FIG.5. DOT DIAGRAM TO SHOW THE DISTRIBUTION OF VULNERABLE AREAS ABOUT THE BURST POINT, O, FOR A 'BARREL' WARHEAD.

FIG.6.

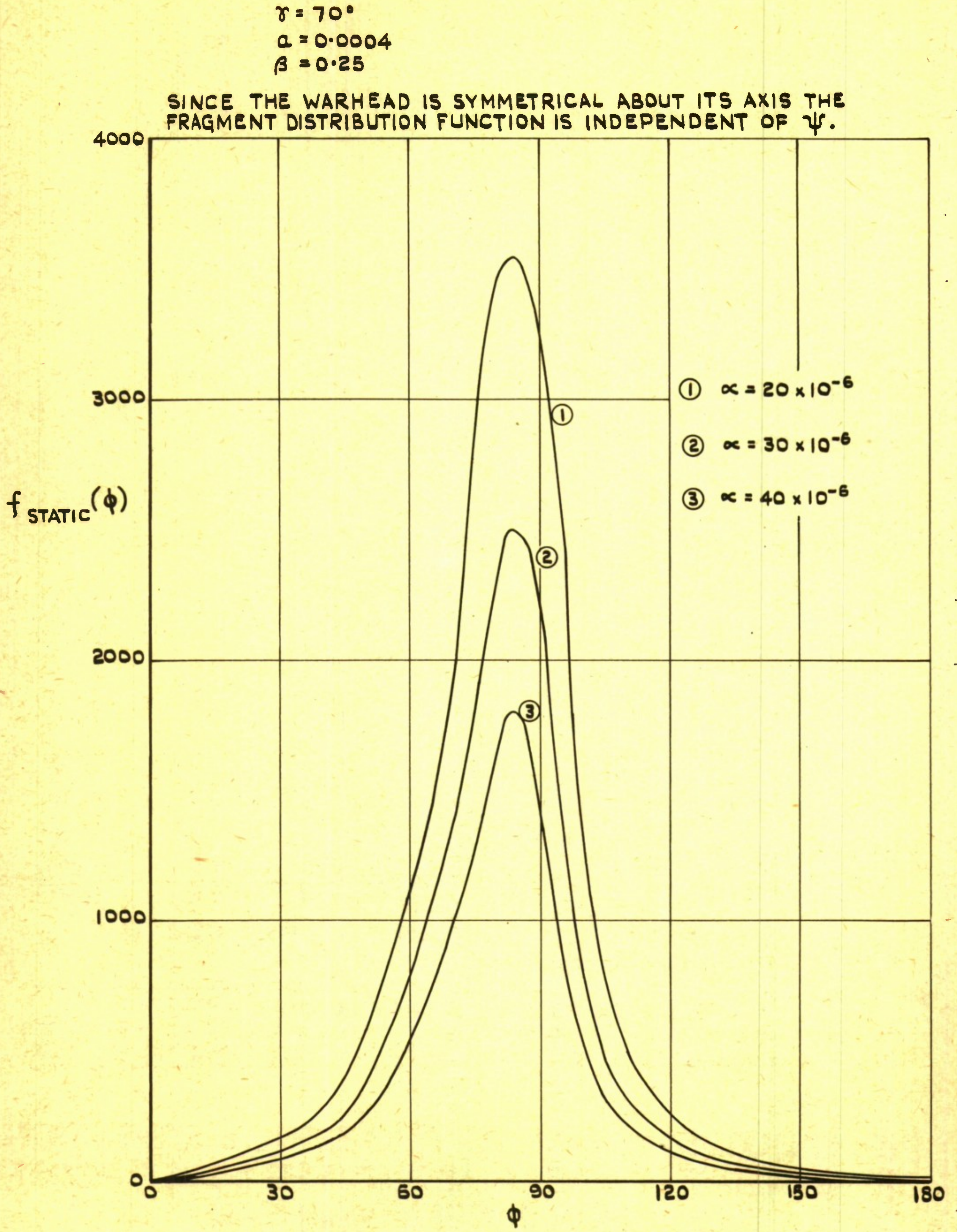


FIG.6. THE STATIC FRAGMENT DISTRIBUTION OF A 'BARREL' WARHEAD.

FIG.7.

$\gamma = 70°$

$a = 0.0004$

$\beta = 0.25$

$\alpha = 30 \times 10^{-6}$

⊗ POINT OF INITIATION.

WEIGHT (EXCLUDING END PLATES) = 147 LBS.

$\frac{\text{CHARGE}}{\text{CASE}}$ WT. RATIO = 5·7

FRAG. MASS = $\frac{1}{32}$ OZ.

No. FRAGMENTS = 8700.

THE BROKEN LINES REPRESENT NON-FRAGMENTING END PLATES.

SCALE 1CM.: 1 INCH.

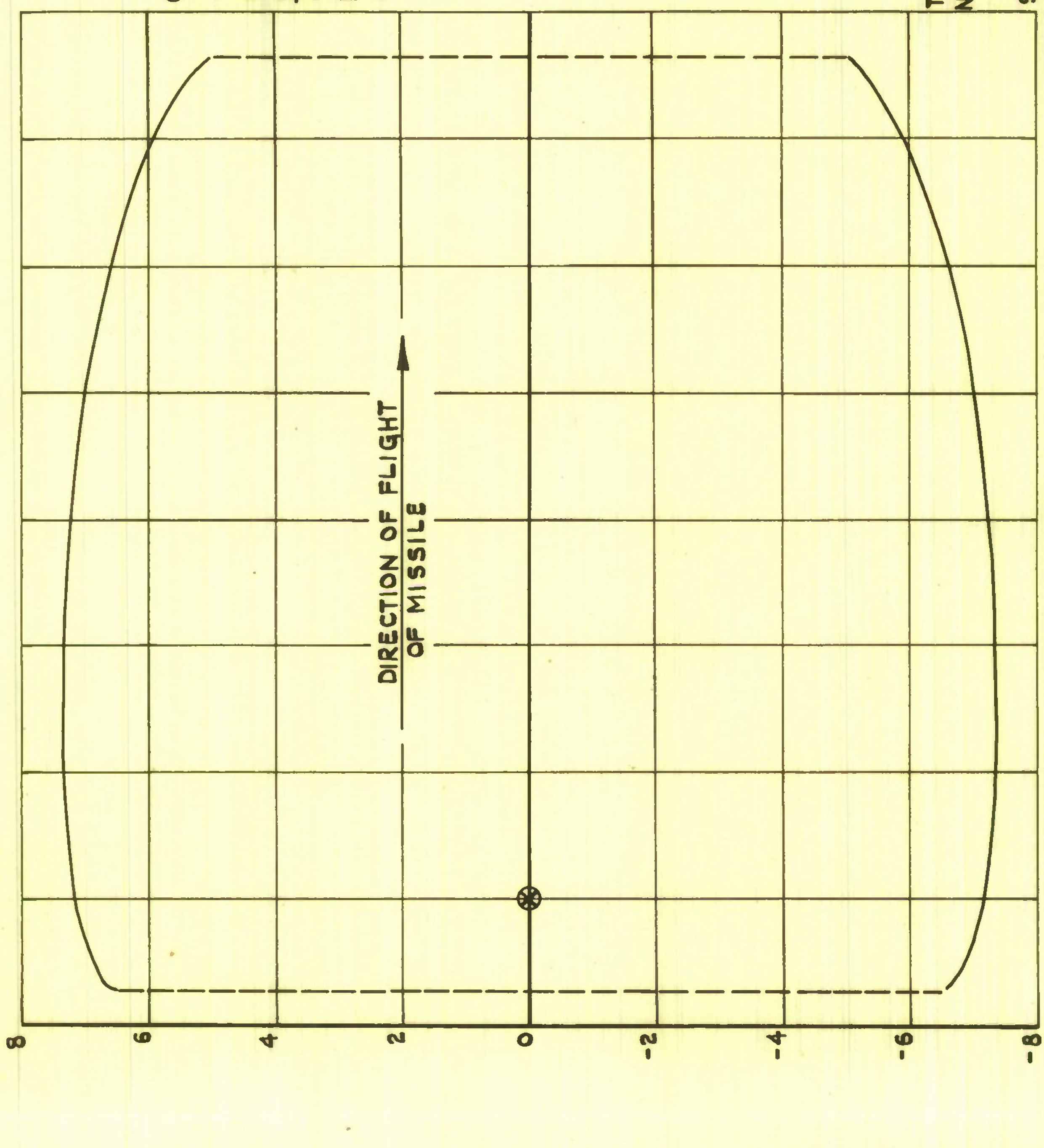
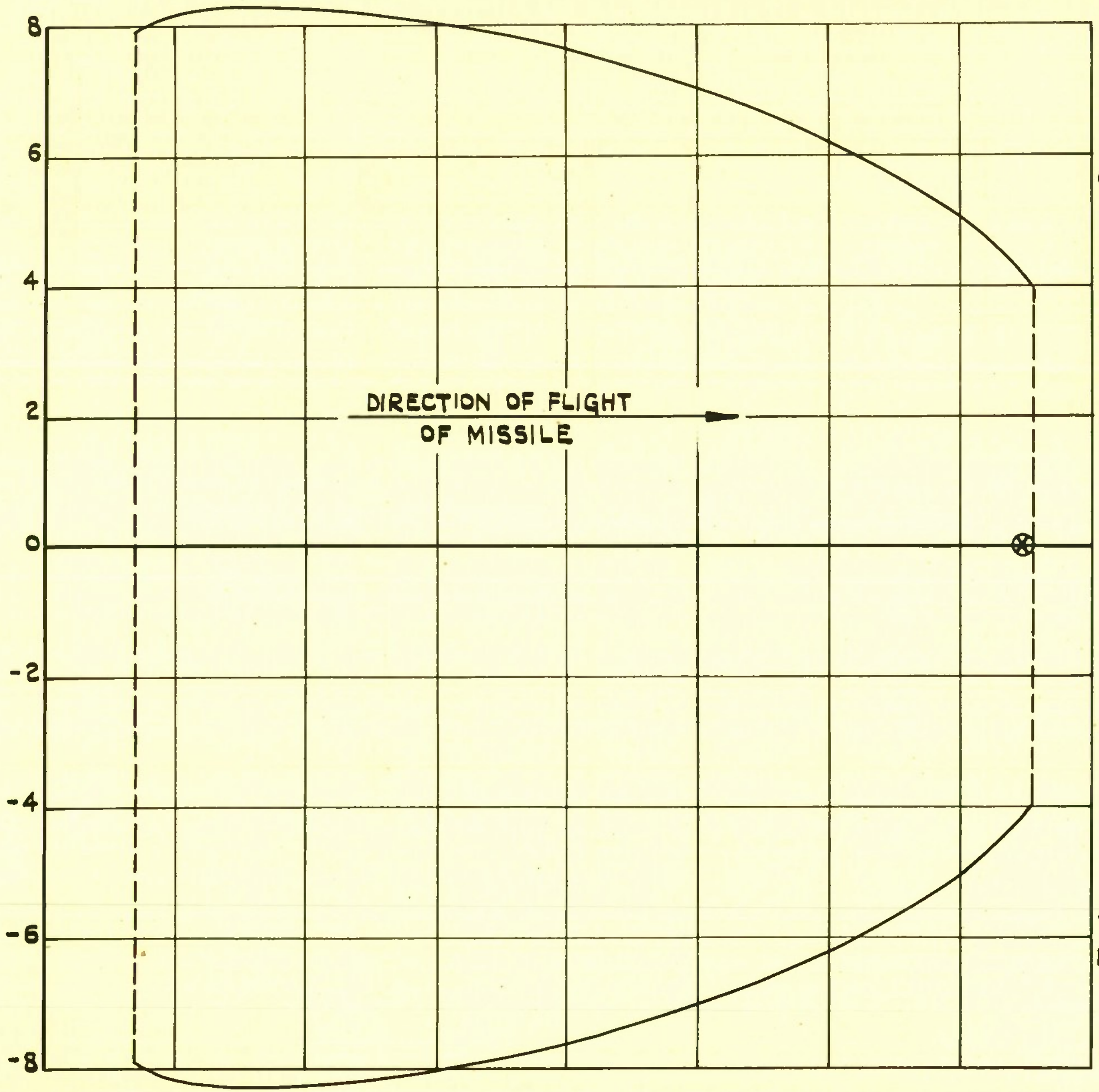


FIG.7. BARREL CONTOUR. POINT OF INITIATION AT REAR.

FIG.8.



$\gamma = 70^\circ$
$a = 0{\cdot}0004$
$\beta = 0{\cdot}25$
$\alpha = 30 \times 10^{-6}$

⊗ POINT OF INITIATION.

WEIGHT (EXCLUDING END PLATES) = 153 LBS.

$\frac{\text{CHARGE}}{\text{CASE}}$ WT. RATIO = 6·2.

FRAG. MASS = $\frac{1}{32}$ OZ.

No. FRAGMENTS = 8700.

THE BROKEN LINES REPRESENT NON-FRAGMENTING END PLATES.

SCALE 1cm.:1 INCH.

FIG.8. BARREL CONTOUR. POINT OF INITIATION IN NOSE.

SCALE 1:4.

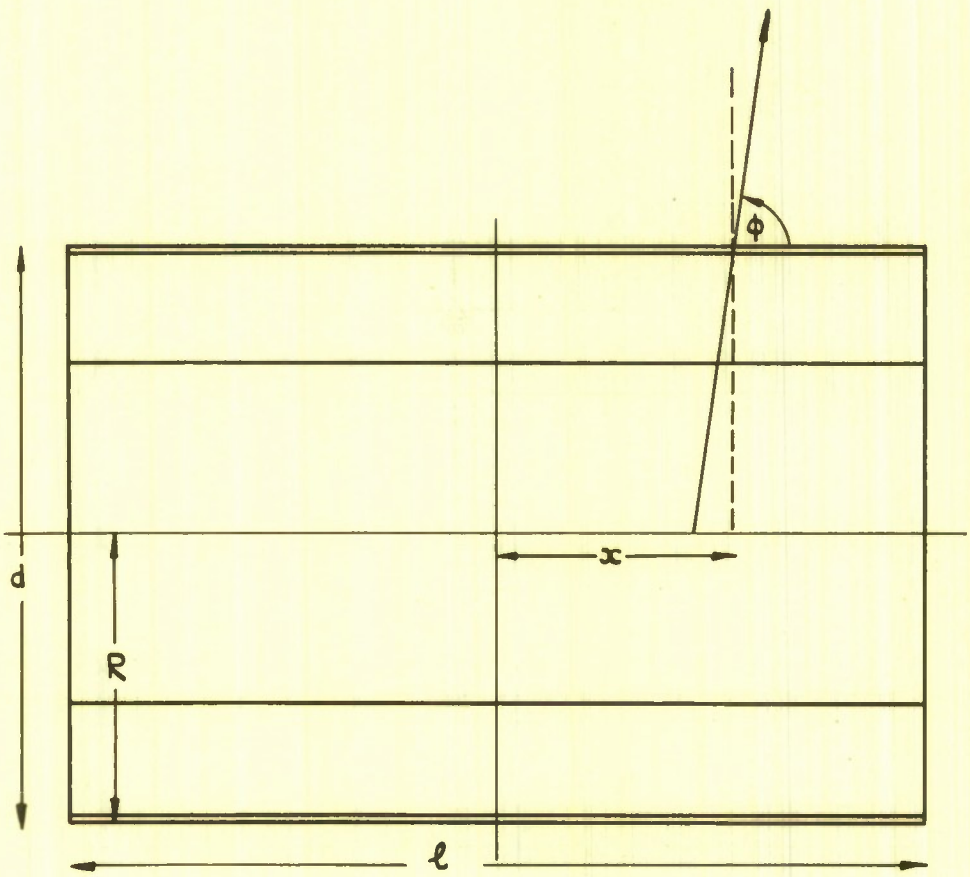


FIG. 9. THE CYLINDRICAL WARHEAD.

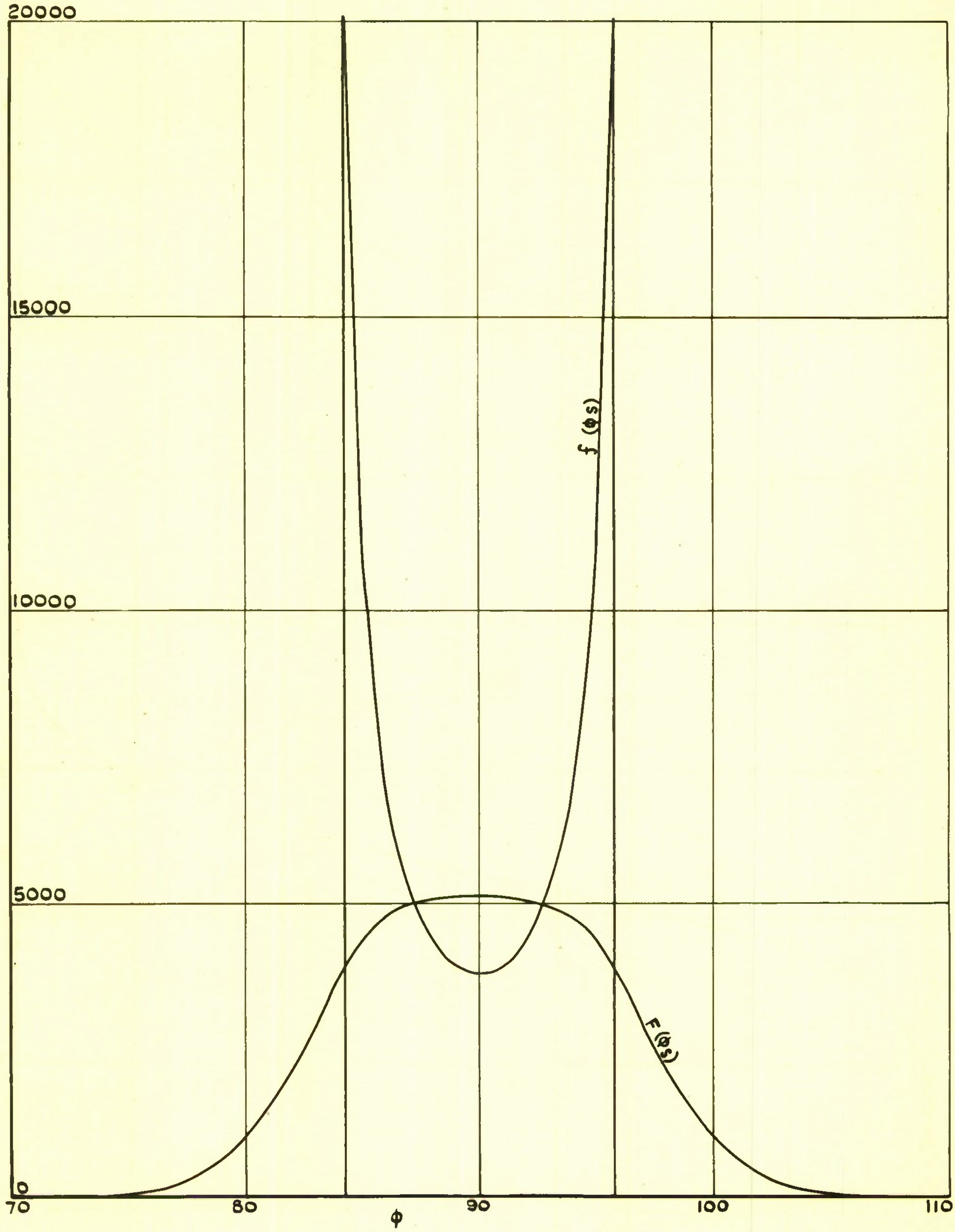


FIG.10. THE PREDICTED AND DISPERSED STATIC FRAGMENT DISTRIBUTION FUNCTIONS FOR THE CYLINDRICAL WARHEAD $f(\phi_s)$ AND $F(\phi_s)$

FIG.11.

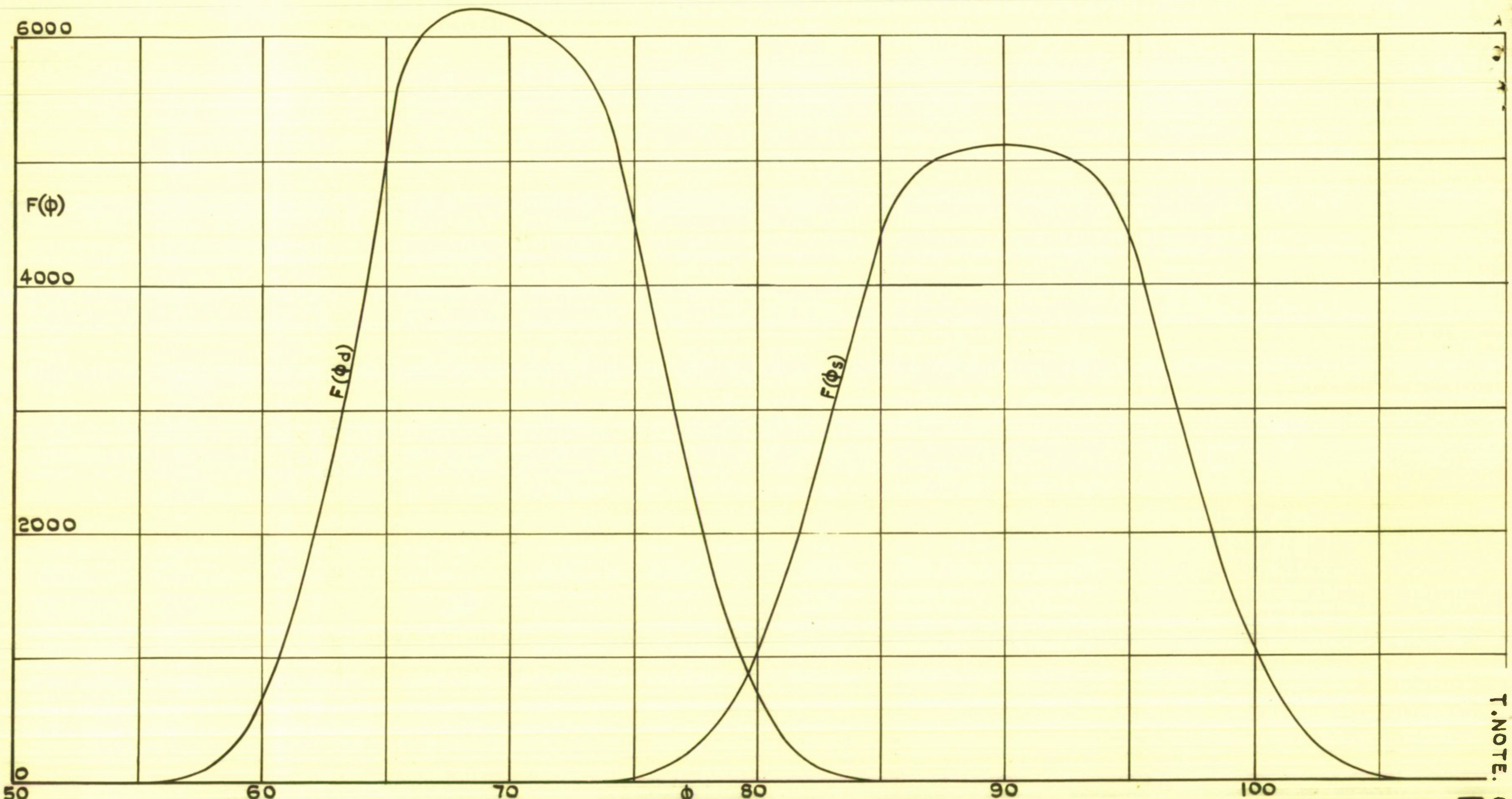


FIG.11. THE STATIC AND DYNAMIC DISPERSED FRAGMENT DISTRIBUTION FUNCTIONS FOR THE CYLINDRICAL WARHEAD, $F(\phi_s)$ AND $F(\phi_d)$.

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