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CONTROL AND OPTIMIZATION OF COHERENCE OF A NANO-SIZED SPIN-TORQUE MICROWAVE OSCILLATOR FOR MILITARY NANO-ELECTRONICS

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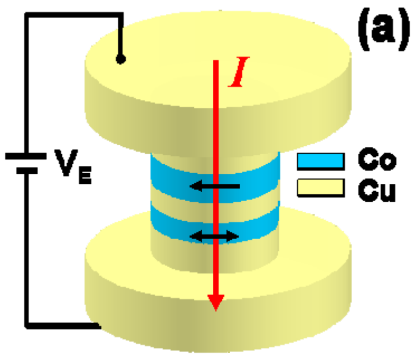
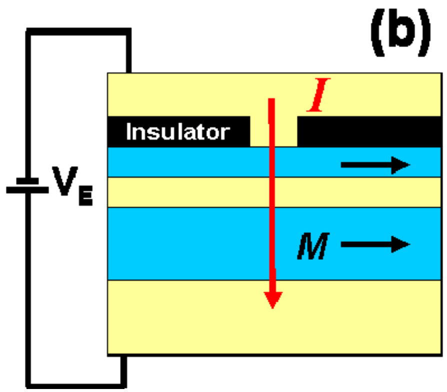
Spin-torque oscillator

Nano-pillar



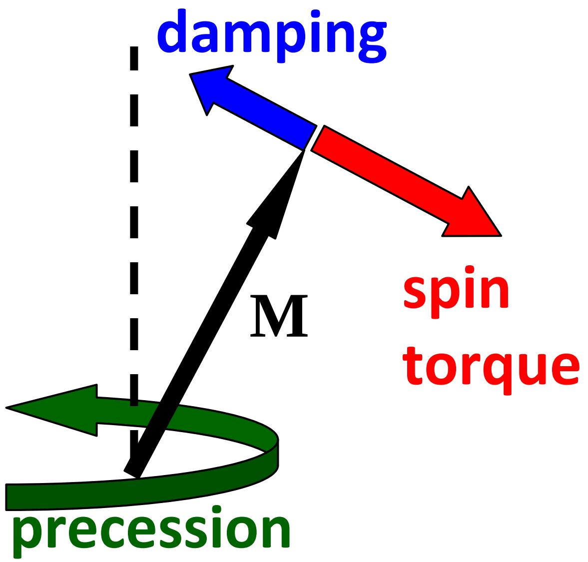
Nano-contact



Passage of an electric current through a multilayered magnetic nano-structure can lead to the excitation of a persistent magnetization precession in the thinner ("free") magnetic layer of the structure. The frequency of the excited auto-oscillations is close to the ferromagnetic resonance frequency and typically lies in the microwave frequency range. This effect can be used for the development of a novel class of fully metallic microwave nano-sized oscillators – **spin-torque oscillators (STO)**.

Equation of motion

Landau-Lifshits-Gilbert-Slonczewski equation:



$$\frac{d\mathbf{M}}{dt} = \gamma[\mathbf{H}_{\mathrm{eff}} \times \mathbf{M}] + \mathbf{T}_{\mathrm{G}} + \mathbf{T}_{\mathrm{S}}$$

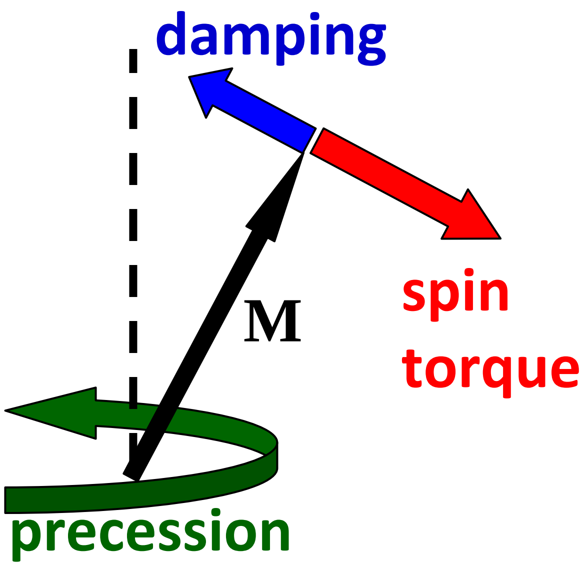
$\gamma[\mathbf{H}_{\mathrm{eff}} \times \mathbf{M}]$ – conservative torque (precession)

$\mathbf{T}_{\mathrm{G}} = -\frac{\alpha_{\mathrm{G}}\gamma}{M_0}[\mathbf{M} \times [\mathbf{M} \times \mathbf{H}_{\mathrm{eff}}]]$ – dissipative torque (**positive** damping)

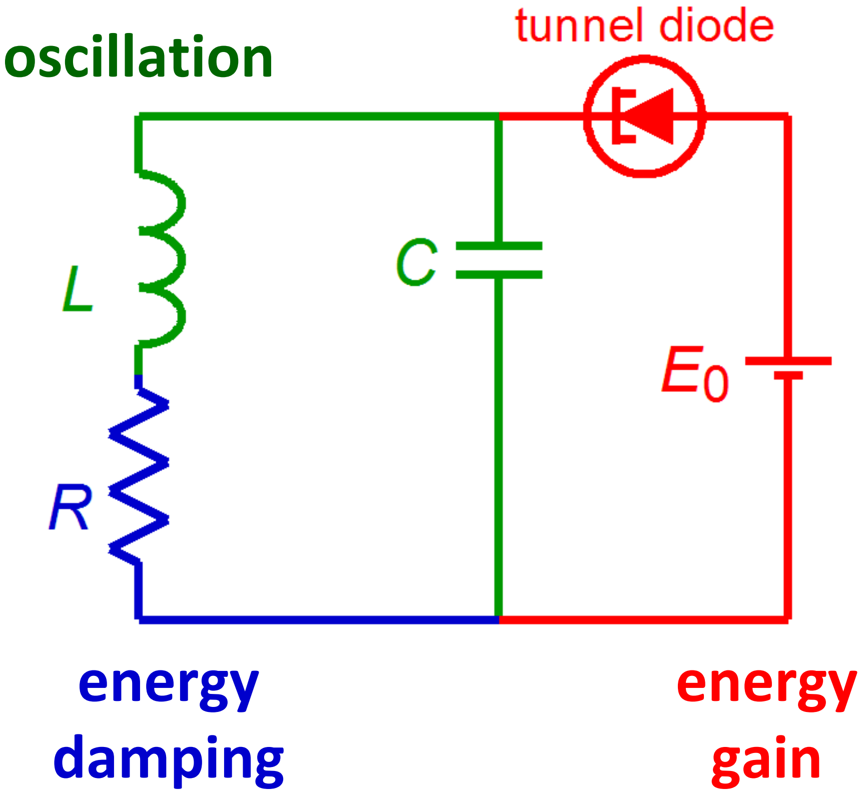
$\mathbf{T}_{\mathrm{S}} = +\frac{\sigma I}{M_0}[\mathbf{M} \times [\mathbf{M} \times \mathbf{p}]]$ – spin-transfer torque (**negative** damping)

STO and conventional oscillator

Spin-torque oscillator



Electrical oscillator (van der Pol)



Nonlinear oscillator model

oscillation (precession)

positive damping (Gilbert torque)

negative damping (spin-transfer torque)

$$\frac{dc}{dt} + i\omega(p)c + \Gamma_+(p)c - \Gamma_-(p)c = 0$$

$p = |c|^2$ – (dimensionless) oscillation **power**

$\phi = \arg(c)$ – oscillation **phase**

Nonlinear oscillator model

$$\frac{dc}{dt} + i\omega(p)c + \Gamma_{+}(p)c - \Gamma_{-}(p)c = 0$$

Conditions of applicability:

- Excitation of **only one** mode
- **Weakly** non-conservative system

Where is the magnetic field?

$$\frac{dc}{dt} + i\omega(p)c + \Gamma_+(p)c - \Gamma_-(p)c = 0$$

All physical parameters of spin-torque oscillators are hidden in the "material" functions

$$\omega(p) \qquad \Gamma_+(p) \qquad \Gamma_-(p)$$

Weakly-nonlinear expansion:

$$\omega(p) = \omega_0 + Np \qquad \Gamma_+(p) = \Gamma_G(1 + Qp) \qquad \Gamma_-(p) = \sigma I(1 - p)$$

- ω_0: FMR frequency
- N: nonlinear frequency shift
- Γ_G: FMR half-linewidth
- Q: nonlinear damping coefficient
- σ: spin-transfer efficiency of the current
- I: bias current

$$\frac{dc}{dt} + i\omega(p)c + \Gamma_+(p)c - \Gamma_-(p)c = 0$$

Stationary solution: $c(t) = \sqrt{p_0}\exp(-i\omega_g t + i\phi_0)$

Stationary power is determined from the condition of vanishing **total** damping:

$$\Gamma_+(p_0) = \Gamma_-(p_0)$$

Stationary frequency is determined by the oscillation power: $\omega_g = \omega(p_0)$

Weakly-nonlinear expansion:

$$\omega(p) = \omega_0 + Np \qquad \Gamma_+(p) = \Gamma_G(1+Qp) \qquad \Gamma_-(p) = \sigma I(1-p)$$

$$p_0 = \frac{\zeta - 1}{\zeta + Q} \qquad \omega_g = \omega_0 + N\frac{\zeta - 1}{\zeta + Q} \qquad \zeta = \frac{I}{I_{th}} = \frac{\sigma I}{\Gamma_G}$$

supercriticality

Stochastic Langevin equation:

$$\frac{dc}{dt} + i\omega(p)c + \Gamma_+(p)c - \Gamma_-(p)c = f_\mathrm{n}(t)$$

Random thermal noise:

$$\left\langle f_\mathrm{n}(t)\, f_\mathrm{n}^*(t') \right\rangle = 2D_\mathrm{n}\delta(t-t')$$

$$D_\mathrm{n}(p) = \Gamma_+(p)\eta(p) = \Gamma_+(p)\frac{k_\mathrm{B}T}{\lambda\omega(p)}$$

$$\eta = \frac{k_\mathrm{B}T}{\lambda\omega(p)}$$ – thermal equilibrium power of oscillations:

$$\left\langle |c|^2 \right\rangle_{\Gamma_-=0} = \left\langle p \right\rangle_{\Gamma_-=0} = \eta$$

Linearized power-phase equations

Stochastic Langevin equation: $\frac{dc}{dt} + i\omega(p)c + \Gamma_+(p)c - \Gamma_-(p)c = f_n(t)$

Power-phase ansatz: $c(t) = \sqrt{p_0 + \delta p(t)}\; e^{i\phi(t)}$ $\quad |\delta p| << p_0$

Stochastic power-phase equations:

$$\frac{d\delta p}{dt} + 2\Gamma_{\mathrm{eff}}\ \delta p = 2\sqrt{p_0}\ \mathrm{Re}\left[f_n(t)e^{-i\phi}\right]$$

$$\frac{d\phi}{dt} + \omega(p_0) = \frac{1}{\sqrt{p_0}}\mathrm{Im}\left[f_n(t)e^{-i\phi}\right] + N\ \delta p$$

Linear system of equations.
Can be solved in a general case.

Effective damping: $\Gamma_{\mathrm{eff}} = (G_+ - G_-)p_0$ $\quad G_\pm = d\Gamma_\pm(p)/dp$

Nonlinear frequency shift coefficient: $N = d\omega(p)/dp$

Nonlinear frequency shift creates additional source of the phase noise

Lorentzian lineshape with the full linewidth

$$2\Delta\omega = (1+\nu^2)\Gamma_+ \frac{k_B T}{E(p_0)} = (1+\nu^2)\, 2\Delta\omega_{\text{lin}}$$

[J.-V. Kim *et al.*, Phys. Rev. Lett. **100**, 017207 (2008)]

Frequency nonlinearity broadens linewidth by the factor

$$1+\nu^2 = 1+\left[\frac{N}{G_+ - G_-}\right]^2 = 1+\left[\frac{N}{\Gamma_G}\frac{1}{\zeta+Q}\right]^2$$

Region of validity $\quad k_B T \ll \left(\frac{\Gamma_{\text{eff}}}{\Gamma_+}\right)\frac{E(p_0)}{1+\nu^2} \sim 100\ \text{K}$

Linewidth of a spin-torque oscillator

Due to small nano-scale sizes, spin-torque oscillators are very vulnerable to the influence of thermal fluctuations. Thermal noise determines one of the main, from the practical point of view, characteristics of STO – **generation linewidth**.

The problem of stochastic dynamics of STO under the action of thermal noise was analyzed in [J.-V. Kim, V. Tiberkevich, and A.N. Slavin, Phys. Rev. Lett. **100**, 017207 (2008)], yielding the following expression for the generation linewidth:

$$2\Delta\omega = \Gamma_0 \frac{k_B T}{E_0}\left[1+\left(\frac{N}{G}\right)^2\right]$$

Here:

Γ_0 – linear Gilbert damping rate (half-linewidth of ferromagnetic resonance)

k_B – Boltzmann constant

T – absolute temperature

E_0 – energy of auto-oscillations

N – nonlinear frequency shift coefficient
(rate at which oscillator frequency changes with the oscillation power)

G – nonlinear damping coefficient
(rate at which effective STO damping changes with the oscillation power)

Linewidth of a spin-torque oscillator

The linear damping rate Γ_0 and nonlinear frequency shift coefficient N for spin-torque oscillator can be calculated using standard formulas for ferromegnetic resonance. The energy of auto-oscillations E_0 and nonlinear damping coefficient G are given by

$$E_0 = \frac{M_0 V_{\rm eff}}{\gamma} \frac{\zeta - 1}{\zeta + Q} \qquad G = (\zeta + Q)\Gamma_0$$

Here M_0 is the saturation magnetization of the “free” magnetic layer, $V_{\rm eff}$ is the effective volume of this layer, involved in the microwave precession, γ is the modulus of gyromagnetic ratio, Q is the phenomenological nonlinear damping parameter (dimensionless quantity of the order of unity), and ζ is the supercriticality of the bias current:

$$\zeta = \frac{I}{I_{\rm th}} = \frac{\sigma I}{\Gamma_0} \qquad \sigma = \frac{\varepsilon g \mu_{\rm B}}{2eM_0 V_{\rm eff}}$$

I is the bias current traversing the magnetic structure, $I_{\rm th} = \sigma/\Gamma_0$ is the threshold value of this current, at which self-sustained auto-oscillations starts, ε is the dimensionless spin-polarization efficiency, g is the spectroscopic Lande factor, $\mu_{\rm B}$ is the Bohr magneton, and e is the modulus of electron charge.

Nonlinear broadening of STO linewidth

In contrast with the majority of conventional auto-oscillators, STO is **strongly nonlinear** oscillator, in the sense that the precession frequency strongly depends on the precession magnitude. This strong dependence can be characterized by the inequality

$$|N| >> G$$

whereas as majority of conventional oscillators is characterized by the opposite inequality.

It is clear, that such strong nonlinearity of STO leads to a significant broadening of the generation linewidth. Therefore, for practical purposes it would be desirable to work in the region where the nonlinear frequency shift N is as small as possible.

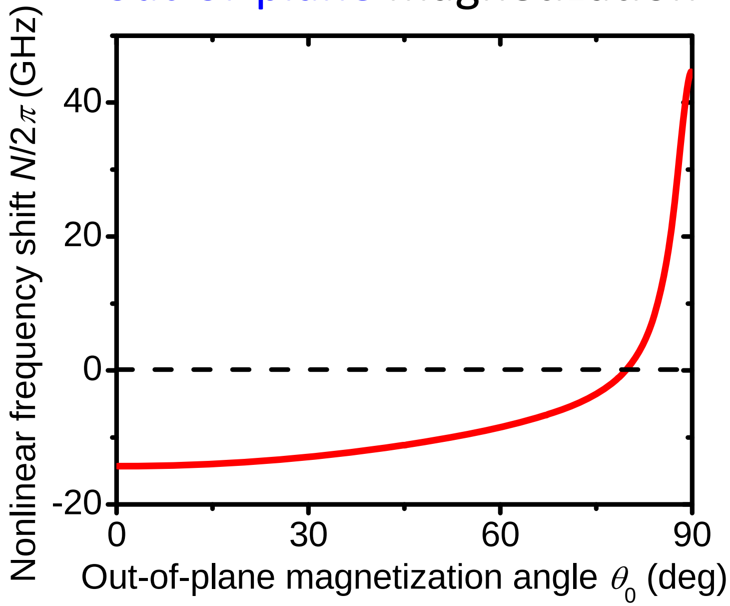
It is known that the nonlinear frequency shift N of magnetization precession strongly depends on the orientation of magnetization of the magnetic layer. This effect allows one to control the coherent properties of self-sustained magnetization precession and to choose optimal conditions for STO operation.

Angular dependence of the linewidth

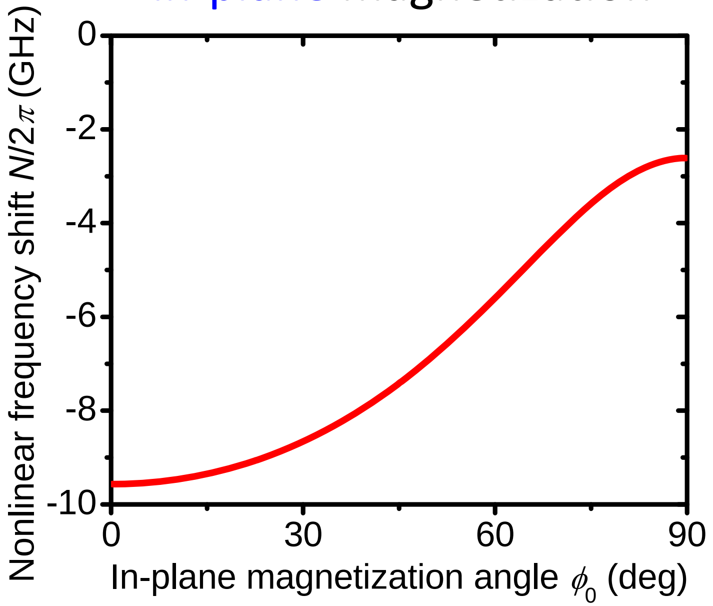
$$2\Delta\omega = (1+\nu^2)\Gamma_+ \frac{k_B T}{E(p_0)} = \left(1+\left(\frac{N}{G_+ - G_-}\right)^2\right)\Gamma_+ \frac{k_B T}{E(p_0)}$$

Nonlinear frequency shift coefficient N strongly depends on the orientation of the bias magnetic field

Isotropic film, out-of-plane magnetization



Anisotropic film, in-plane magnetization



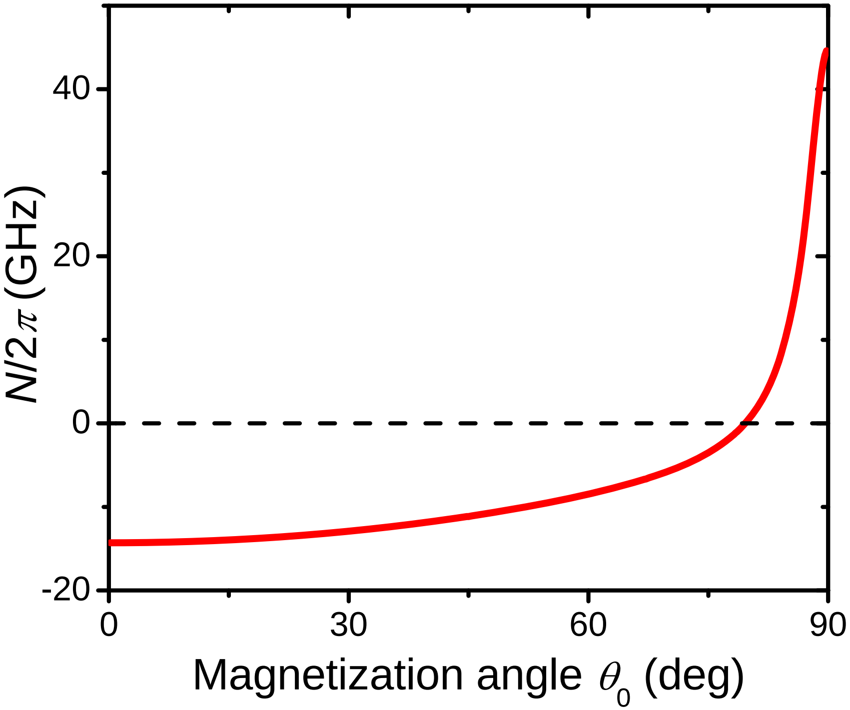
For an **isotropic** magnetic film nonlinear frequency shift coefficient N depends on the **out-of-plane** angle θ_0 that the external bias magnetic field H0 makes with the film plane. In two limiting cases of in-plane ($\theta_0 = 0$) and normal ($\theta_0 = \pi/2$) magnetizations expressions for N have the form

$$N(\theta_0 = 0) = -\frac{2\pi\gamma M_0\sqrt{H_0}}{\sqrt{4\pi M_0 + H_0}}\frac{4\pi M_0 + 4H_0}{4\pi M_0 + 2H_0}$$

$$N(\theta_0 = \pi/2) = 8\pi\gamma M_0$$

Here H_0 is the bias magnetic field.

For intermediate magnetization angles N can be found only numerically. The dependence of N on θ_0 is shown in the figure. It is clear that there is a magnetization angle $\theta_0 \approx 80$ deg, at which N vanishes and STO does not experience nonlinear linewidth broadening. At this point one expects to observe the smallest STO linewidth.



Dependence of the nonlinear frequency shift N of isotropic magnetic film on the **out-of-plane** magnetization angle θ_0. Saturation magnetization $4\pi M_0 = 8$ kG, magnitude of the bias magnetic field $H_0 = 10$ kOe.

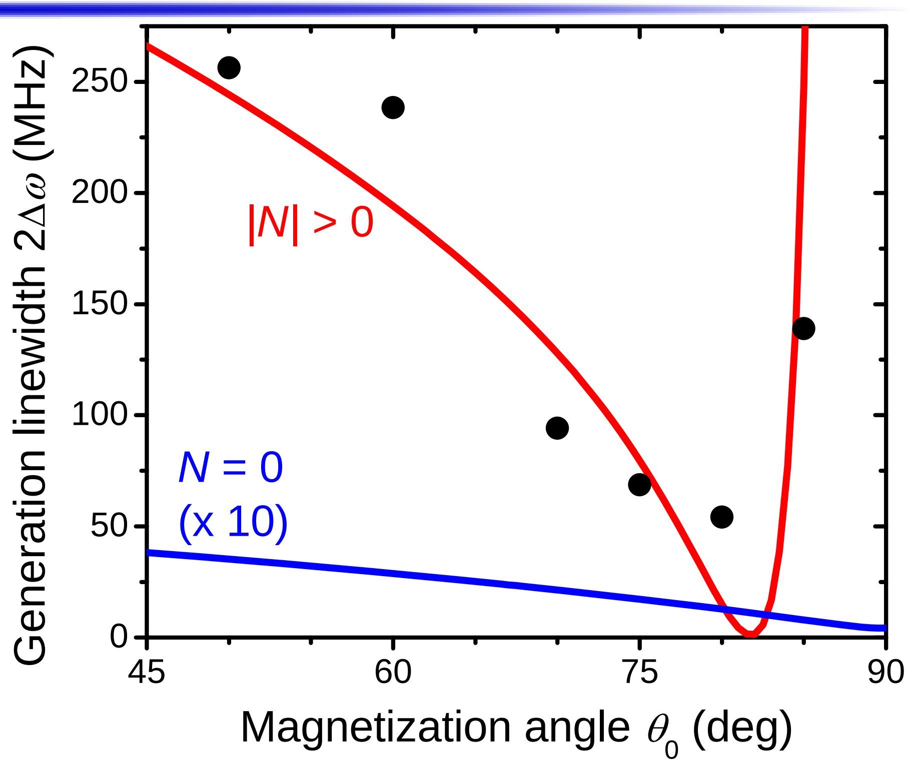
Note that N changes sign.

Such angular dependence of STO linewidth has been observed in the recent experiments [W.H. Rippard *et al.*, Phys. Rev. B **74**, 224409 (2006)] (see figure).

As it is clear from the figure, the nonlinear frequency shift coefficient N determines both the magnitude of the STO generation linewidth and its qualitative angular dependence.

The minimum linewidth is achieved at the magnetization angle $\theta_0 \approx 80$ deg, where the nonlinear frequency shift N vanishes.

However, almost normal magnetization ($\theta_0 \approx 80$ deg) requires use of large bias magnetic fields $H_0 \sim 4\pi M_0$ to saturate the free layer of STO. Thus, another way of reduction of influence of the nonlinear frequency shift N on the generation linewidth is desirable.



Dependence of the STO generation linewidth on the out-of-plane magnetization angle θ_0. Points – experiment [W.H. Rippard *et al.*, Phys. Rev. B **74**, 224409 (2006)], red line – theoretical dependence, blue line – theory for a linear oscillator without nonlinear frequency shift ($N = 0$), multiplied by 10.

STO based on anisotropic film

In this work, we consider generation linewidth of STO, based on **anisotropic in-plane magnetized** "free" magnetic layer. It is known, that even small anisotropy strongly modifies nonlinear properties of magnetization precession and can lead to a significant dependence of the nonlinear frequency shift coefficient N on the in-plane magnetization angle ϕ_0 that the in-plane bias magnetic field H_0 makes with the easy anisotropy axis of the layer.

At the same time, magnitude of the bias magnetic field H_0, necessary to saturate in-plane magnetized magnetic film, is substantially smaller than $4\pi M_0$ and is of the order of anisotropy magnetic field $H_A << 4\pi M_0$.

An additional advantage of the in-plane magnetized geometry is that, for the same value of the bias magnetic field H_0, the ferromagnetic resonance frequency ω_0 is substantially higher than for the out-of-plane magnetized case.

In our approach the effective anisotropy field H_A accounts both for the crystallographic anisotropy of the "free" layer and for the shape anisotropy caused, in the case of STO based on magnetic nano-pillar, by non-circular shape of the pillar.

Parameters of anisotropic film

To find the generation linewidth of STO, we need to obtain expressions for the damping rate Γ_0 and nonlinearity coefficient N of in-plane magnetized magnetic layer. We derived analytical expressions for these coefficients:

$$\Gamma_0 = \alpha_G A$$

$$N = -\frac{1}{\omega_0 A}\left[\left(2A^2 - 3AB + B^2\right)B + \omega_A\left(2A^2 + B^2\right)\cos(2\phi) + \frac{3\omega_A^2}{4\omega_0^2}(A+B)\left(2A^2 + 2AB + B^2\right)\sin^2(2\phi)\right]$$

α_G is the Gilbert damping constant, $\omega_0 = \sqrt{\left(\omega_H - \sin^2\phi\,\omega_A\right)\left(\omega_H + \omega_M\right)}$ is the FMR frequency,

$$A = \omega_H + \left(\omega_M - \sin^2\phi\,\omega_A\right)/2 \qquad B = \left(\omega_M + \sin^2\phi\,\omega_A\right)/2$$

$$\omega_H = \gamma H \qquad \omega_M = 4\pi\gamma M_0 \qquad \omega_A = \gamma H_A$$

and H and ϕ are the magnitude and in-plane angle of the *internal* magnetic field, which are connected with the *external* values H_0 and ϕ_0 by

$$H\cos\phi = H_0\cos\phi_0 + H_A\cos\phi \qquad H\sin\phi = H_0\sin\phi_0$$

Frequency and damping rate

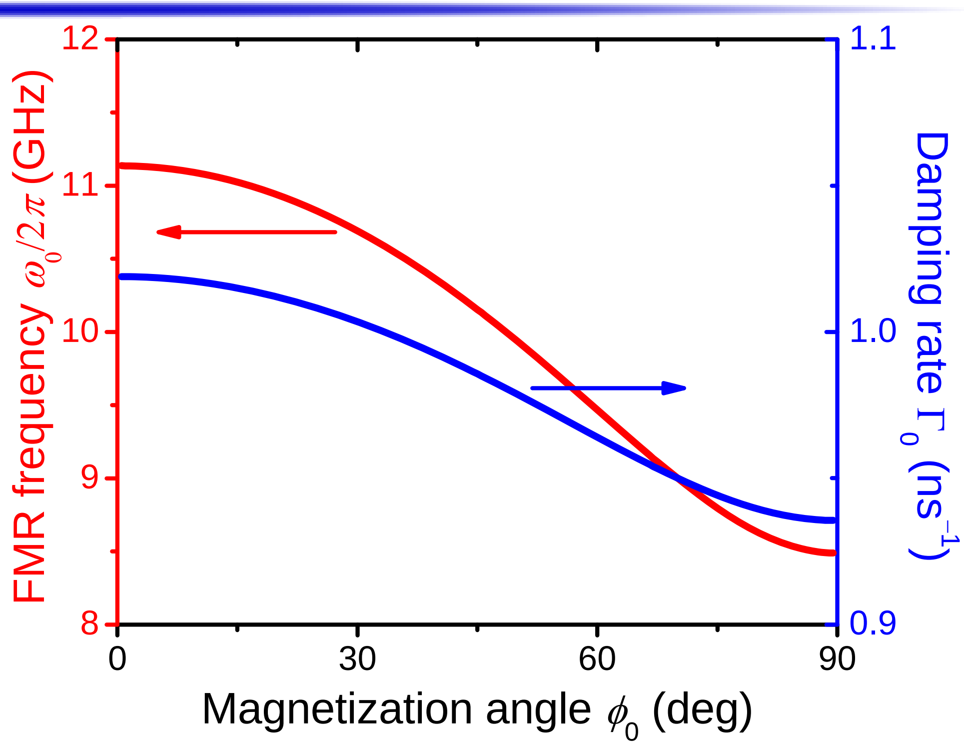
Analysis shows that both the FMR frequency ω_0 and Gilbert damping rate Γ_0 only weakly depend on the in-plane magnetization angle (see figure). Thus, in the most interesting case of small magnetic field $H_0 << 4\pi M_0$ expressions for the FMR frequency for magnetizations along the easy ($\phi_0 = 0$) and hard ($\phi_0 = \pi/2$) axes have the form

$$\omega_0^{\text{easy}} \approx \gamma\sqrt{4\pi M_0\left(H_0 + H_A\right)}$$

$$\omega_0^{\text{hard}} \approx \gamma\sqrt{4\pi M_0\left(H_0 - H_A\right)}$$

and differ by only 30 % for $H_0 = 4\,H_A$ (see figure).

Gilbert damping rate Γ_0, which determines both the threshold current I_{th} and range of variations of the generation linewidth $2\Delta\omega$, is almost independent of ϕ_0, due to a strong ellipticity of magnetization precession in in-plane magnetized film: $\Gamma_0 \approx \alpha_G\omega_M/2$.



Dependence of the FMR frequency ω_0 (red line, left axis) and Gilbert damping rate Γ_0 (blue line, right axis) on the in-plane magnetization angle ϕ_0. Bias magnetic field $H_0 = 1.2$ kOe, saturation magnetization $4\pi M_0 = 8$ kG, anisotropy field $H_A = 0.3$ kOe, Gilbert damping parameter $\alpha_G = 0.01$.

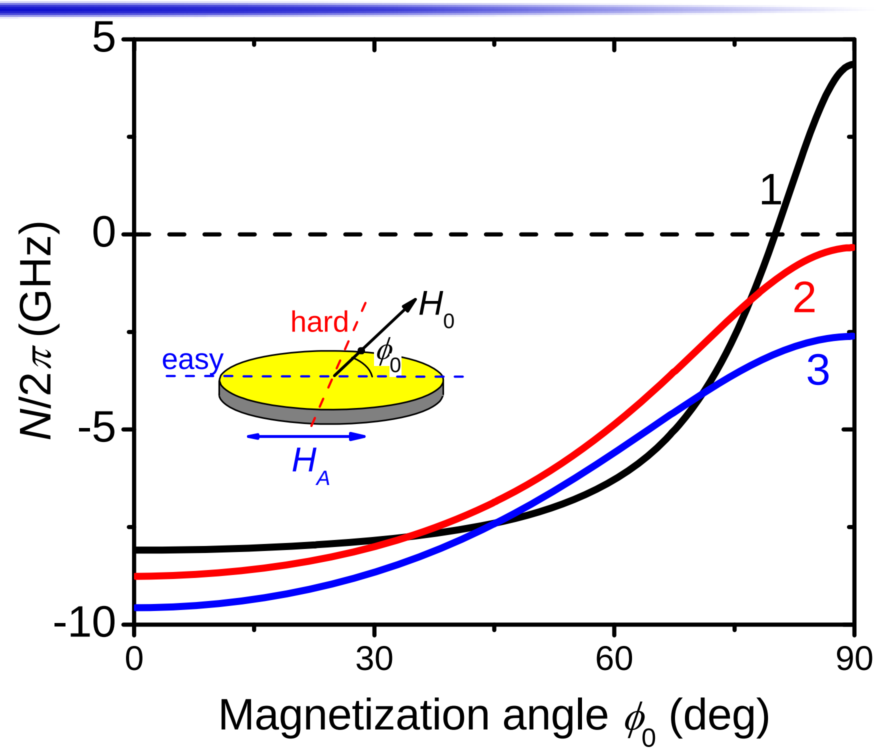
Nonlinear frequency shift

The nonlinear frequency shift coefficient N strongly depends on the magnetization angle ϕ_0 (see figure). In the limit $H_0 << 4\pi M_0$ one can obtain simple expressions for N for magnetizations along the easy ($\phi_0 = 0$) and hard ($\phi_0 = \pi/2$) axes:

$$N^{\text{easy}} \approx -\gamma\left(H_0 + 4H_A\right)\frac{\omega_M}{2\omega_0^{\text{easy}}}$$

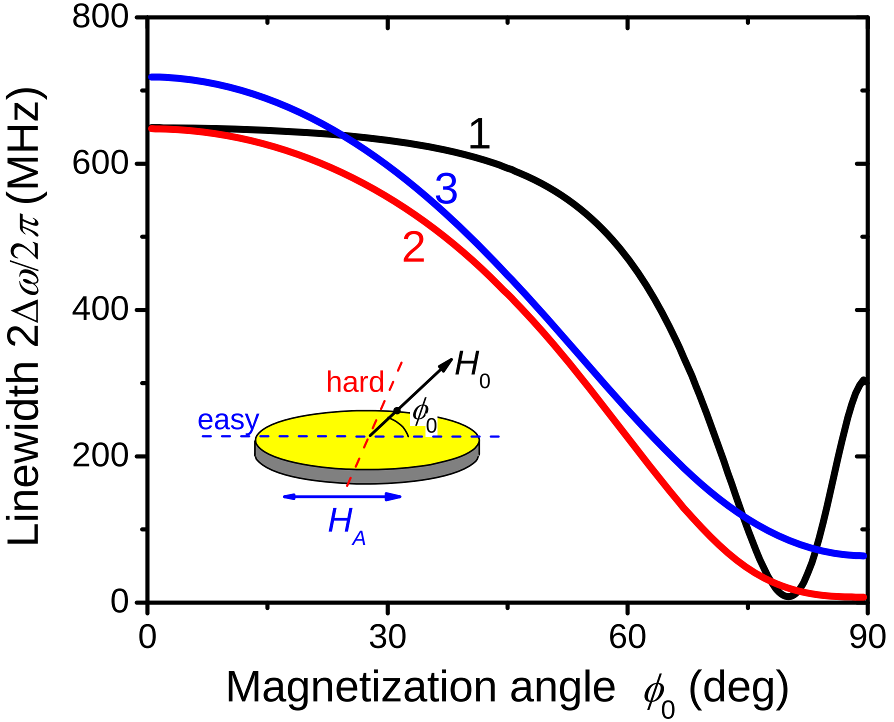
$$N^{\text{hard}} \approx -\gamma\left(H_0 - 4H_A\right)\frac{\omega_M}{2\omega_0^{\text{hard}}}$$

For the easy-axis case ($\phi_0 = 0$) N is always negative, whereas for $\phi_0 = \pi/2$ and magnetic field in the range $H_A < H_0 < 4H_A$ nonlinear frequency shift N is positive (see curve 1). In this field range N vanishes for a certain intermediate magnetization angle, where the minimum linewidth of STO should be observed. For larger fields $|N|$ monotonically decreases when ϕ_0 rotates from the easy- to hard-axis orientation (see curves 2 and 3) and STO generation linewidth should have minimum for the hard-axis magnetization (ϕ_0 = $\pi/2$).



Dependence of the nonlinear frequency shift N on the in-plane magnetization angle ϕ_0 for several values of the bias magnetic field H_0: 1 – 0.6 kOe, 2 – 1.2 kOe, 3 – 1.8 kOe. Other parameters of magnetic film: $4\pi M_0 = 8$ kG, anisotropy field $H_A = 0.3$ kOe.

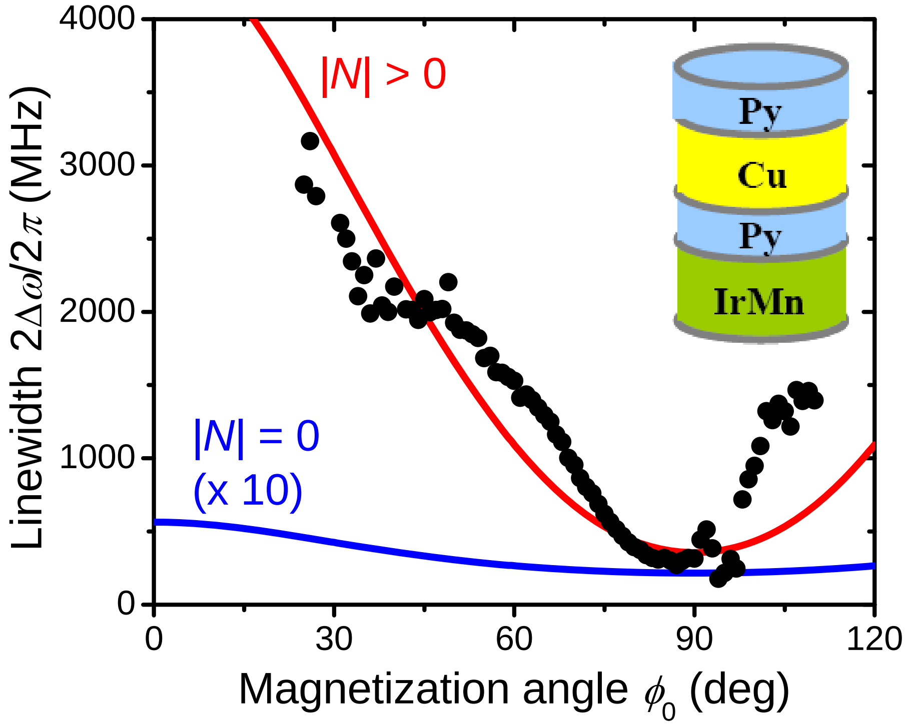
In this figure we showed the calculated dependence of the STO generation linewidth $2\Delta\omega$ on the in-plane magnetization angle for several values of the bias field H_0. One can see that this dependence virtually reproduces the angular dependence of N^2, since all other parameters of STO depends on ϕ_0 only weakly. In particular, for small bias field $H_0 < 4H_A$ (see curve 1) $2\Delta\omega$ has a strong minimum for a certain intermediate magnetization angle, whereas for large bias fields $H_0 > 4H_A$ (see curves 2 and 3) the generation linewidth $2\Delta\omega$ monotonically decreases with the increase of ϕ_0 and has a minimum for the hard-axis magnetization $\phi_0 = \pi/2$.



Dependence of the generation linewidth $2\Delta\omega$ of STO on the in-plane magnetization angle ϕ_0 for several values of the bias magnetic field H_0: 1 – 0.6 kOe, 2 – 1.2 kOe, 3 – 1.8 kOe. Other parameters of STO: $4\pi M_0$ = 8 kG, H_A = 0.3 kOe, α_G = 0.01, Q = 3, "free" layer thickness d = 3 nm, shape of the nano-pillar – circular with the radius R = 50 nm, spin-polarization efficiency ε = 0.2, bias current I = 3 mA, temperature T = 300 K.

Recently, experimental study of generation linewidth of in-plane magnetized anisotropic STO has been performed in [K. V. Thadani *et al.*, arXiv:0803.2871 (2008)]. In this figure we showed comparison of our theoretical results (red line) with experimental data (dots) for exchange-biased STO (see Fig. 2c in [K. V. Thadani *et al.*, arXiv:0803.2871 (2008)]). One can see good qualitative and quantitative agreement between the experiment and our nonlinear theory.

At the same time, the linear theory, that does not take into account nonlinear frequency shift N (see blue line in the figure), predicts much smaller quantitative values of the generation linewidth $2\Delta\omega$ and much smoother dependence of the linewidth on the magnetization angle ϕ_0.

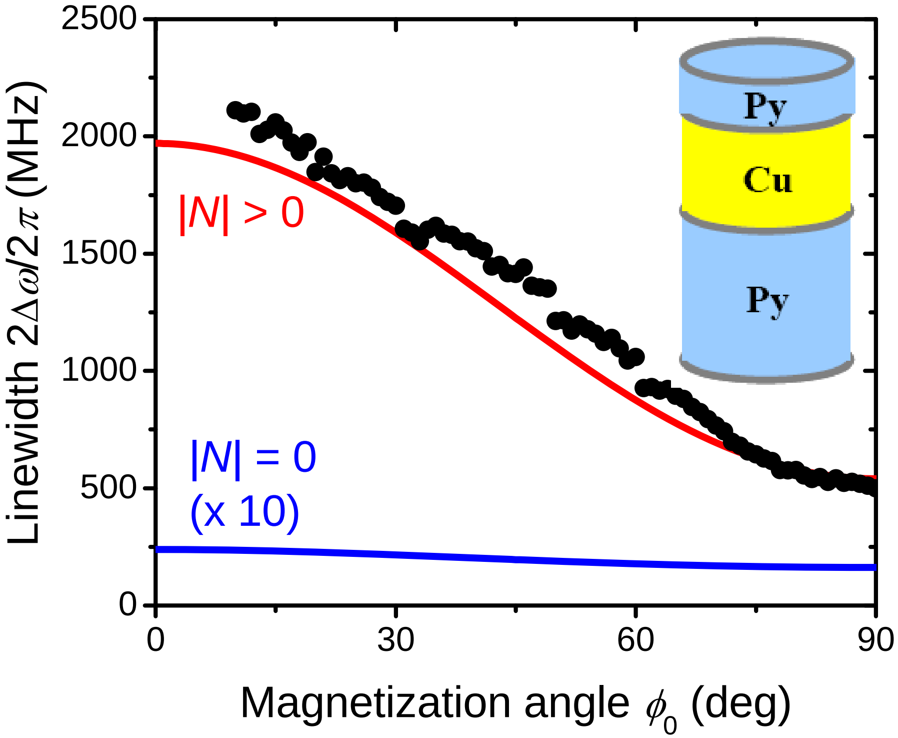


Comparison of experiment (dots) from Fig. 2c in [K. V. Thadani *et al.*, arXiv:0803.2871 (2008)] with our theory (red line). Parameters: $4\pi M_0 = 8$ kG, $H_0 = 1.08$ kOe, $H_A = 0.2$ kOe, $\alpha_G = 0.015$, $Q = 3$, "free" layer thickness $d = 4$ nm, shape of the nano-pillar – elliptical 150 nm x 50 nm, spin-polarization efficiency $\varepsilon = 0.32$, bias current $I = 5$ mA, temperature $T = 300$ K. Blue line shows theory for a linear oscillator ($N = 0$), multiplied by 10.

Comparison with experiment

In this figure we showed comparison of our theory (red line) with experimental data (dots) for thick-fixed-layer STO (see Fig. 2d in [K. V. Thadani *et al*., arXiv:0803.2871 (2008)]). Again, the agreement between the experiment and our nonlinear theory is rather good.

In both experimental cases the bias magnetic field H_0 was larger than then $4H_A$ and, according to our theory, generation linewidth of STO $2\Delta\omega$ monotonically decreases with the increase of the magnetization angle ϕ_0 and has a strong minimum for the hard-axis magnetization $\phi_0 = \pi/2$.



Comparison of experiment from Fig. 2d in [K. V. Thadani *et al*., arXiv:0803.2871 (2008)] with our theory (red line). Parameters: $4\pi M_0 = 8$ kG, $H_0 = 1.2$ kOe, $H_A = 0.1$ kOe, $\alpha_G = 0.015$, $Q = 3$, "free" layer thickness $d = 4$ nm, shape of the nano-pillar – elliptical 130 nm x 70 nm, spin-polarization efficiency $\varepsilon = 0.375$, bias current $I = 5$ mA, temperature $T = 300$ K. Blue line shows theory for a linear oscillator ($N = 0$), multiplied by 10.

Proposal for an optimum design of STO

Our theoretical results suggest an optimum design of STO, in which the influence of thermal fluctuations will be minimized and the generation linewidth $2\Delta\omega$ will have minimum possible value.

To achieve this goal, it is necessary to operate STO in a regime, when the nonlinear frequency shift N vanishes and the generation linewidth does not experience nonlinear broadening.

The simplest way to do this is to use **in-plane** magnetized along the **hard axis** nano-pillar with the bias magnetic field

$$H_0 = 4H_A$$

The generation frequency of STO in this case is equal approximately to

$$\omega_0 \approx \sqrt{3}\gamma\sqrt{4\pi M_0 H_A}$$

and can be tuned by changing the anisotropy field H_A (e.g., by changing the aspect ratio of ellipsoidal magnetic nano-pillar forming “free” layer of STO).

Summary

• We have developed theory of generation linewidth of spin-torque oscillator (STO) with an **in-plane magnetized anisotropic** "free" magnetic layer.

• The dependence of the generation linewidth of STO on the in-plane magnetization angle is determined mostly by the angular dependence of the nonlinear frequency shift coefficient.

• For relatively small bias magnetic fields ($H_A < H_0 < 4H_A$) the generation linewidth has a minimum for a certain intermediate magnetization angle, whereas for large bias fields ($H_0 > 4H_A$) the linewidth has a **minimum** value for the magnetization direction along the **hard in-plane axis**.

• Our theory provides qualitatively and quantitatively correct description of recent experimental studies of the angular dependence of the generation linewidth in in-plane magnetized anisotropic STO.

• Our results allows one to select an optimum design of STO, in which generation linewidth will have a minimum possible value.