TIMESCALE ALGORITHMS COMBINING CESIUM CLOCKS AND HYDROGEN MASERS

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Abstract

The USNO atomic timescale, formerly based on an ensemble of cesium clocks, is now produced by an ensemble of cesium clocks and hydrogen masers. In order to optimize stability and reliability, equal clock weighting has been replaced by a procedure reflecting the relative, time-varying noise characteristics of the two different types of clocks. Correction of frequency drift is required, and residual drift is avoided by the eventual complete deweighting of the masers.

INTRODUCTION

At timing laboratories, readings from an ensemble of clocks are combined mathematically by some sort of algorithm to produce a mean timescale in order to average down both random and systematic errors, thereby increasing overall stability and accuracy, respectively. Also, reliability is improved because individual clocks can be added as they become available or removed when they fail or need adjustment. In order to have a time signal continuously available, at least one "master clock" at USNO is steered in frequency so that its time approximates that of the mean "paper" timescale.

The optimum algorithm is not obvious and, indeed, depends on the needs of the user, which may favor stability over accuracy, for example. Algorithms can differ in their definition of mean timescale, in their clock weighting, in their use of filters to reduce measurement noise, and in their methods of predicting and steering clock frequencies. These aspects may depend on the type of clocks involved, since different clock types have different noise characteristics.

OLD AND NEW TIMESCALE ALGORITHMS

The atomic timescale at USNO has until recently been based entirely on an ensemble of commercial (nearly all Hewlett-Packard) cesium frequency standards whose frequencies have been averaged by a linear algorithm and equal clock weighting. Weighting by inverse Allan variances was not found to improve stability or accuracy significantly[1]. The algorithm employed was the following:

$$z_t = z_{t-T} + \sum_i W_t(i)[x_t(i) - x_{t-T}(i) + Tr_t(i)] \qquad (1)$$

where z_t is the difference between the readings of the Master Clock and the mean timescale, $x_t(i)$ the difference between the readings of the Master Clock and clock i, $W_t(i)$ is the weight of clock i,

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and $r_t(i)$ is the rate (frequency in time gained per time interval T) of clock i relative to the mean timescale, all at time t [2]. r_t was determined by least-squares using 5-day bins of hourly data, since over a span of 5 days oscillator noise can be well modelled by a combination of white FM noise and FM random walk [3].

The new algorithm mentioned in [2], based on ARIMA prediction modelling, was never implemented. Indeed, ARIMA modelling was found to yield a timescale no better than, and often significantly inferior to, that generated by Eq. (1), in spite of ancillary robust features, apparently because of model variations [1]. The apparent inferiority of the NIST algorithm on the short-term [1] may have been due to ignorance of an unpublished rate-change detector and upper weight limit [4].

Use of Eq. (1) assumes constant clock rates, aside from discrete changes that are monitored in real time and corrected for during postprocessing. This leads to a very occasional rejection of an otherwise satisfactory cesium clock because of a small frequency drift. More important, this algorithm would not be appropriate for an ensemble containing hydrogen masers in view of their significant, characteristic, and generally positive frequency drifts.

While Eq. (1) could be modified to incorporate a drift term, simultaneous solutions for rate and drift of comparable accuracy to previous solutions for rate alone would require data lengths greater than 5 days. Since solutions for rate alone are quite adequate for cesium clocks and because we have found (second-order) solutions for maser drifts to be insufficiently stable, we have chosen to retain first-order solutions and to derive drifts from long-term changes in the rates.

In the presence of a frequency drift $d_t(i)$, we have the following:

$$
\begin{aligned}
r_t(i) &= r_{t-T}(i) + T d_{t-T}(i) \\
x_t(i) &= x_{t-T}(i) + T r_{t-T} + 1/2\, T^2 d_{t-T}(i)
\end{aligned}
$$

(2)
(3)

and:

$$
z_t = z_{t-T} + \sum_i W_t(i)[x_t(i) - x_{t-T}(i) + T r_{t-T}(i) + 1/2\, T^2 d_{t-T}(i) \tag{4}
$$

Solving Eq. (2) for r_{t-T} and substituting in Eq. (4), we get:

$$
z_t = z_{t-T} + \sum_i W_t(i)[x_t(i) - x_{t-T}(i) + T r_t(i) - 1/2 T^2 d_{t-T}(i) \tag{5}
$$

which replaces Eq. (1) as the USNO timescale algorithm.

Another restriction of the old USNO algorithm is its assumption of a homogeneous ensemble with regard to clock types. While equal clock weighting may be satisfactory for an ensemble of cesium clocks, it would not be appropriate for an ensemble of both cesium clocks and hydrogen masers, due to the significantly greater short-term stability of the masers compared to the cesiums and the significantly greater long-term stability of the cesiums compared to the masers.

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STABILITY OF THE MEAN TIMESCALE

Over the past 21 months, the USNO ensemble has averaged 22 ±1 equally weighted cesiums. Since MJD 47842, between 3 and 7 masers have been added, at first with equal weight, each maser being retroactively unweighted 60 days in the past so as not to introduce a long-term frequency drift. Our six SAO masers have drifts relative to TAI of from +0.3 to +3.4 parts in 10^{15}/day and our four Sigma Tau masers have drifts of from +0.1 to +1.3 parts in 10^{15}/day [5]. Still, such a procedure does not minimize the noise because it overweights the cesiums relative to the masers in the short term.

One could construct a timescale based entirely on masers, determining and correcting their rates and drifts relative to another, pure cesium timescale. However, the small number of masers would make its operational reliability questionable, and the drifts would be difficult to determine relative to the noisier cesium timescale.

Optimal use of a given number of clocks in a mixed ensemble would be possible if the weight of a given maser, relative to a given cesium, were allowed to vary with time inversely as their relative Allan variances vary with sampling time. This would require that the entire mean timescale be retroactively recomputed every hourly time step, rather than a few times a week as before. Cesiums would be phased out with time (up to the present) while the masers would be phased in. All rates and drifts would be determined from a comparison with a combined timescale whose long-term trend would be determined by the cesiums and whose near-realtime stability would be determined by the masers.

In order to determine the weighting function, sigma-tau curves were constructed for our masers and an equal number of good cesiums. The intersection of a typical maser curve with a typical cesium curve, i.e. when their Allan deviations were equal, was found to occur at a sampling time of 7.5 days. Fitting second-order curves to the data, we obtained:

$$\log \sigma_{cs} = 0.219x^2 - 0.432x - 13.608 \tag{6}$$

$$\log \sigma_{HM} = 0.460x^2 + 0.049x - 13.917 \tag{7}$$

where σ_{CS} is the Allan deviation of a typical cesium, σ_{HM} is the Allan deviation of a typical maser, $x = \log t - 5.3$, and t is the time difference in seconds between a given hour's measurement and the most recent hour. At $t = 0$, x is arbitrarily set to x at $t = \frac{1}{3600}$. The weight of a maser relative to a cesium is taken to be the following:

$$w_{HM}/w_{CS} = \sigma_{CS}^2/\sigma_{HM}^2 \tag{8}$$

The relative weights change from 10:1 around the current hour to 1:10 around 25 days in the past (see Fig. 1). Accordingly, in practice, only the last 25 days of the timescale, rather than its entirety, are recomputed every hourly time step. Also, the masers are completely unweighted after 60 days to prevent the accumulation of any long-term drift.

Allan deviations were determined for a range of sampling times using 370 days of clock data and two of our masers as references. A three-cornered-hat analysis yielded the results in Fig. 2. The stability of the old algorithm was only slightly improved by the introduction of masers (at equal weight with the cesiums), but the new algorithm is significantly more stable for sampling times shorter than 11.5 days. The noise at shorter sampling times is mostly due to the old measurement

system. This noise will be reduced from 100 to 10 ps when transition is made to our new Erbtec measurement system currently under test[6]. The weighting Eqs. (6) and (7) will then have to be reevaluated.

A by-product of this analysis is the sigma-tau plot for the reference masers in Fig. 3. Similar results at long sampling times were obtained by Powers et al. [5] using Erbtec data.

REALTIME STABILITY AND ACCURACY

The next question is how well this increased stability in the mean timescale translates into realtime stability on the part of a Master Clock (MC). MC #1 is a maser that is steered daily by an internal frequency synthesizer toward a linear prediction based on a least-squares solution of the past 24 hours of data, but with a 10-day damping time for time offsets and a maximum frequency change of $300~ps/day^2$; both restrictions are due to user requirements for strict frequency stability and their exact values are currently under evaluation. A sigma-tau plot for MC #1 is given in Fig. 4, which shows a significant improvement using both the old and new mean timescales, when masers are incorporated, over use of the old mean timescale composed of cesiums alone. Only if the steering can be improved will full advantage be taken of the new algorithm; this possibility will be investigated.

While one measures and corrects for the relative rates and drifts of the masers and cesiums, some residual drift may affect the timescale, as will the rate and drift in common to all the clocks. It is of interest to compare the old and new means incorporating masers (never completely deweighting them) with the old, pure-cesium mean. Such a comparison is shown in Fig. 5. The old and new means incorporating masers drift about -11 ns and -35 ns, respectively, over 300 days. One would expect the new mean to drift more because of the greater weight of the masers. In practice, this drift never accumulates to this level in the USNO timescale because of the retroactive unweighting of the masers after 60 days. If this unweighting is not done, such a drift would presumably be a risk for any timescale that incorporates masers (e.g. TAI).

The USNO time signal actually derives from Master Clock #2, a maser that was formerly steered to the old mean. Recently it has been steered (by an internal frequency synthesizer) to TAI, or more specifically, to an extrapolation thereof based on the best performing USNO masers. This was done to synchronize UTC (USNO) with UTC (BIPM) within the limits required by NATO and other users. That being nearly accomplished, MC #2 will hereafter be steered toward the new mean, with occasional corrections to keep it within 200 ns (or perhaps less) of TAI (as is also done to MCs #1 and #3), with the same dampening factor (10 days) and limit on the steering ($300~ps/day^2$) that are being used and evaluated for MC #1. MC #3, a maser steered daily by a phase microstepper heretofore to MC #2, will also be steered similarly. These duplicative systems provide extra reliability. The maser of each MC is not weighted as a clock unless its unsteered signal is available (as it is if a microstepper is used) or its signal is corrected for steering.

Fig. 6 depicts the drifts of MCs #1 and #2 relative to TAI. MC #1 has some extra noise around a sampling time of 10 days as the result of its being steered toward our mean timescale; for the following sampling times, MCs #1 and #2 had the logarithmic Allan deviations listed below:

Tau (days)	MC #1	MC #2
10	-13.638	-13.867
20	-13.667	-13.700
30	-13.713	-13.543

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Still, steering MC #2 to the mean has the advantage of greater statistical independence from TAI, which in turn would dampen the influence of fluctuations in TAI.

Aside from checks for large time and rate deviations, our clock measurements are not filtered, in order to have a near-realtime measure of accuracy and environmental response. However, the performance obtained by this and the other procedures described above will be compared with that obtained by a Kalman-filter algorithm developed by Stein[7,8] as part of continuing effort to improve the accuracy and stability of UTC (USNO).

ACKNOWLEDGMENTS

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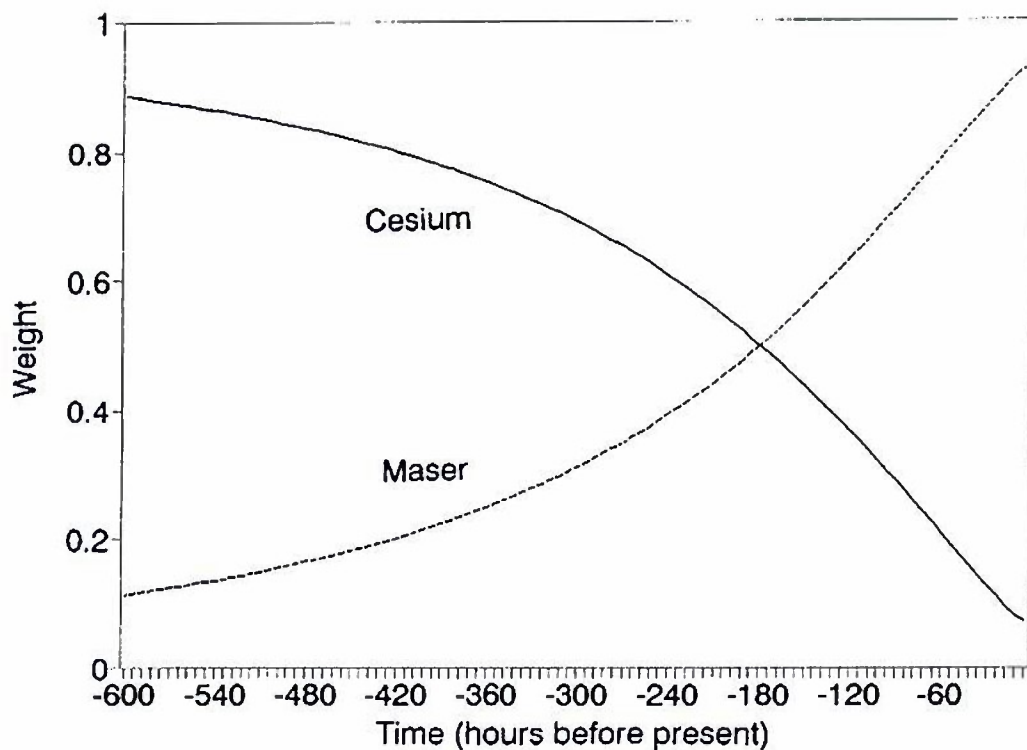
Fig. 1. The weight of a cesium clock and a hydrogen maser as functions of time in the new mean timescale algorithm.
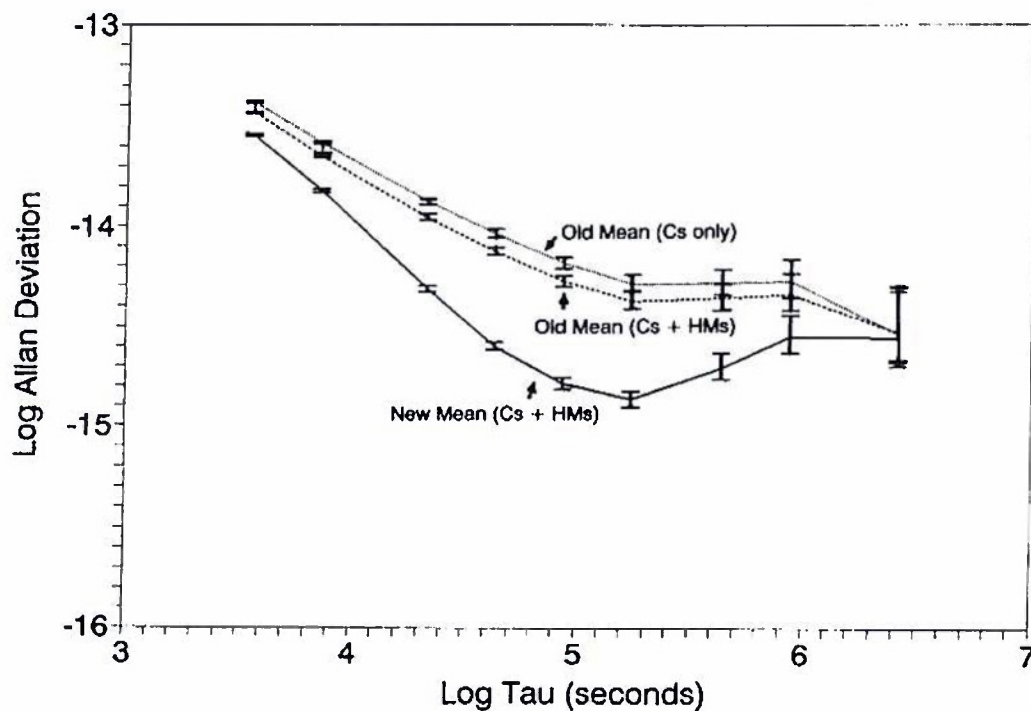

Fig. 2. The frequency stability of the old mean timescale (with and without masers) and the new mean timescale as a function of sampling time (tau). The error bars correspond to a confidence level of 90%.
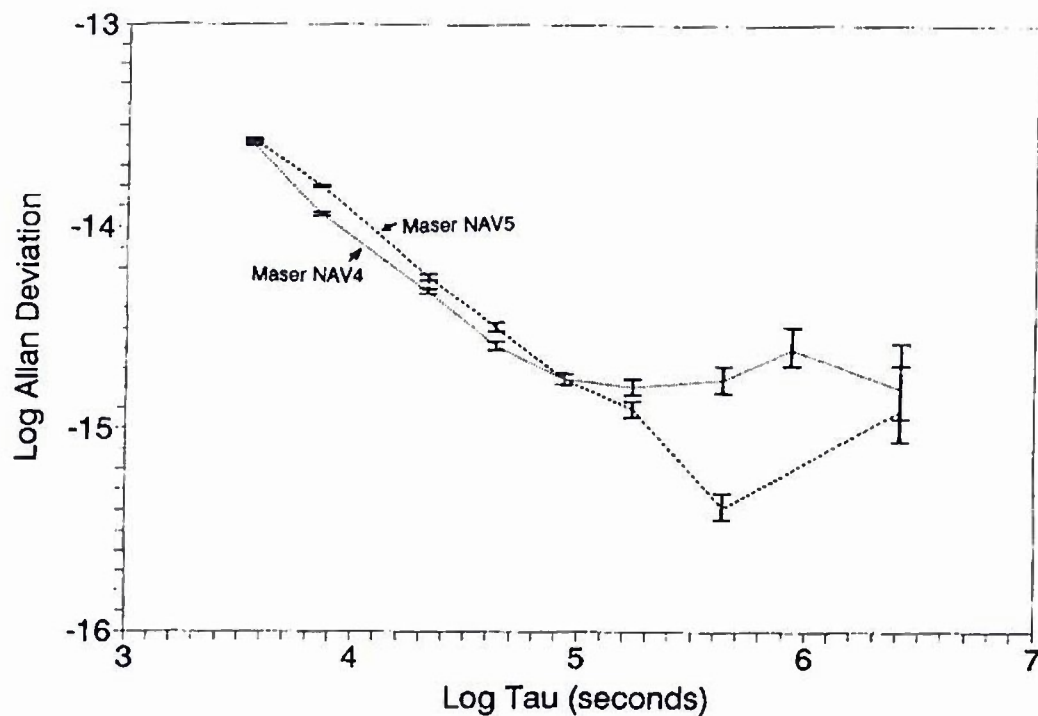
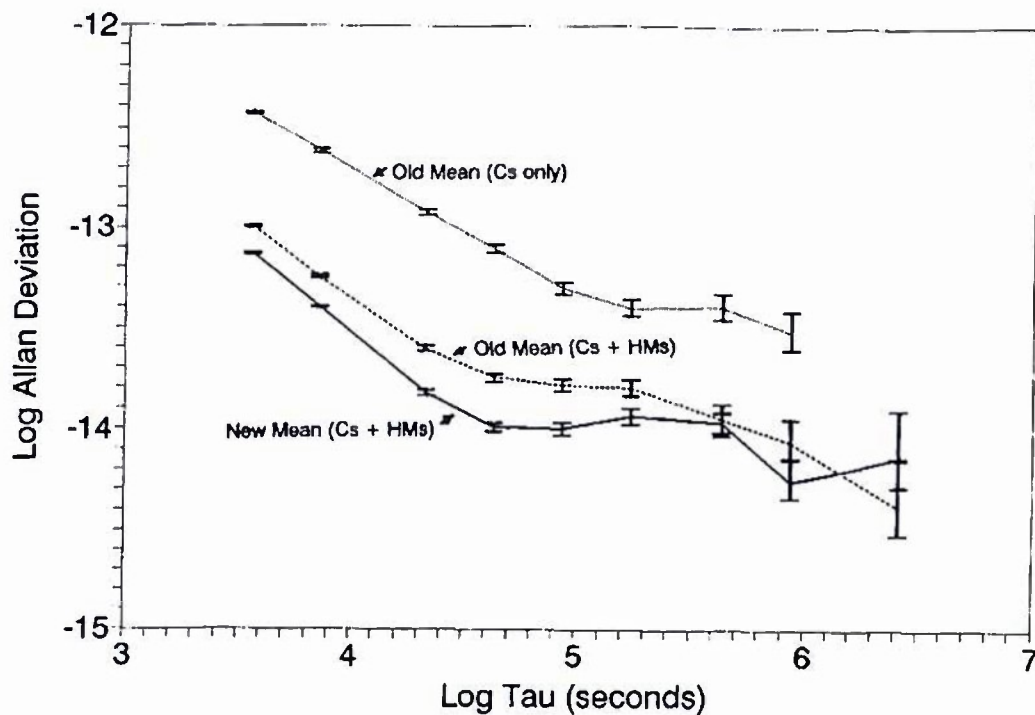Fig. 3. The frequency stability of the two reference masers.

Fig. 4. The frequency stability of Master Clock #1 while being steered toward the old mean timescale (with and without masers) and toward the new mean timescale.
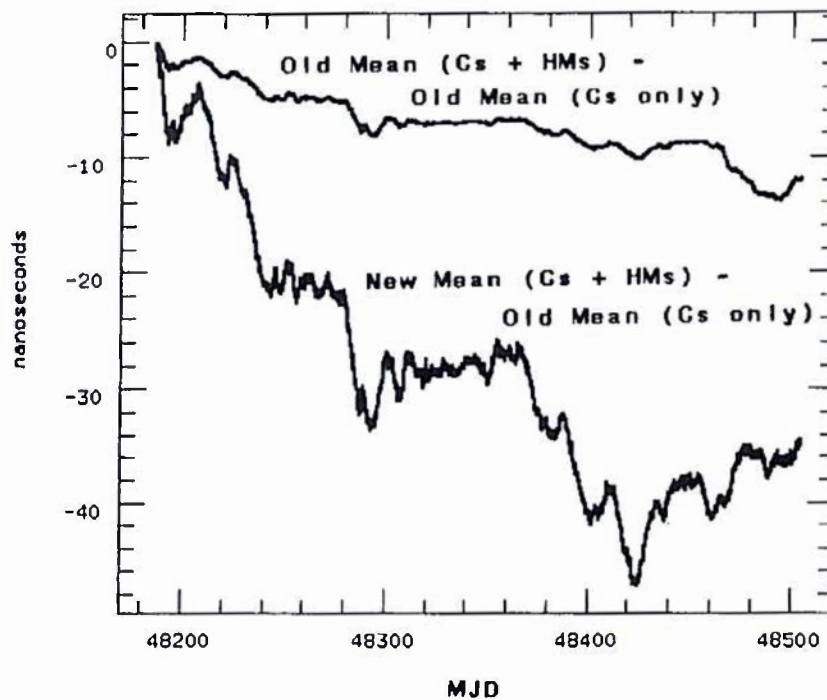
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Fig. 5. The time drift, starting from an arbitrary point of enforced synchronism, of the old and new mean timescales incorporating masers, relative to the old mean timescale based on cesiums only, when the masers are never completely deweighted.
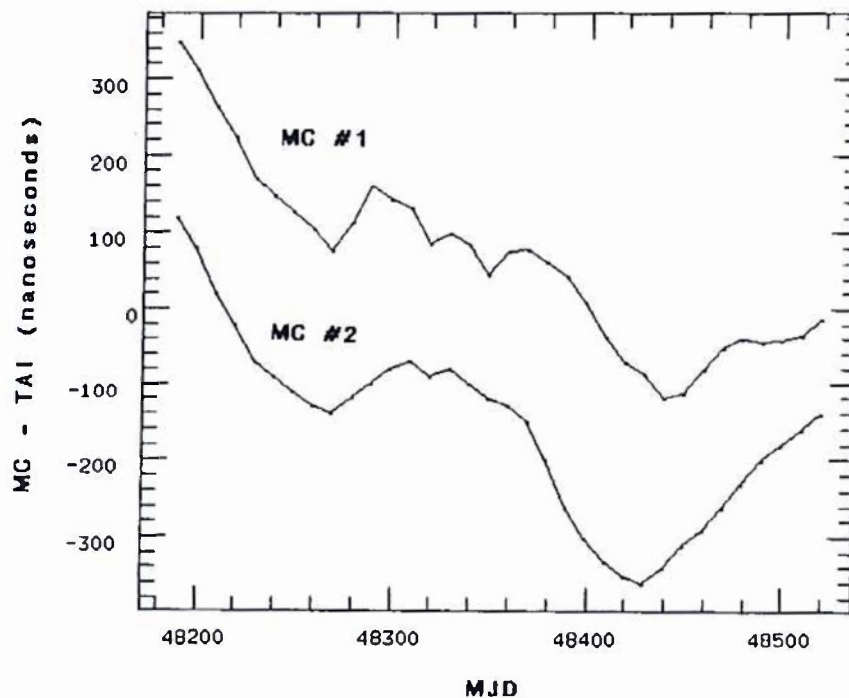

Fig. 6. The time drift of Master Clocks #1 and #2 relative to TAI.

CORRIGENDA TO PREVIOUS PAPER

The following typographical errors should be corrected in [1]:

[fixed]

A list of figure captions was not published. The figures, however, are self-explanatory, except for the fact that the filled squares in Fig. 1 represent clocks whose beam tubes were replaced before the start of the data set in 1986.