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Sinusoidal vertical motion of a sonobuoy suspension: experimental data and a theoretical model

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Also published in: The Technical Cooperation Program, Subgroup G (Undersea Warfare) Technical Panel GTP-10 25th Sonobuoy Working Party, Warminster, PA, 3-5 December 1990.

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Abstract

To isolate its acoustic sensor from the large vertical motions of the buoy at the ocean surface, a typical sonobuoy employs a low-pass mechanical filter comprising a bungee cord (spring) and a damper disk (hydrodynamic added mass and damping). If the system were to behave as a driven simple harmonic oscillator, the transfer function of vertical motion would roll off at 40 dB/decade above resonance; however, experimental systems of this sort exhibit a rolloff of approximately 30 dB/decade. Experiments have shown that the inertia coefficient C_I and the drag coefficient C_D of a circular disk in sinusoidal motion are in fact not constants, but depend on the dimensionless ratio of amplitude of motion to disk diameter, A/d. Incorporating this concept, the simple harmonic oscillator model has been enhanced to produce an iterative model for the transfer function of the bungee/disk system that agrees well with experimental data. Well above resonance, the model simplifies to an expression giving a 27 dB/decade rolloff.

Résumé

Pour que les grands mouvements verticaux qu’elles font à la surface de l’océan ne nuisent pas à la performance de leurs capteurs acoustiques, les bouées acoustiques sont généralement munies d’un filtre mécanique passe-bas composé d’un extenseur (ressort) et d’un disque d’amortissement (masse ajoutée et amortissement hydrodynamiques). La pente de diminution du transfert du mouvement vertical du filtre mécanique des oscillateurs harmoniques simples commandés est de 40 dB par décade. La pente du filtre expérimental est cependant de 30 dB par décade. Selon des expériences, le coefficient d’inertie C_i et le coefficient de frottement C_f d’un disque circulaire en mouvement sinusoïdal ne sont pas constants et dépendent du rapport adimensionnel entre l’amplitude du mouvement et le diamètre du disque, soit un rapport A/d. Grâce à l’intégration de ce concept, nous avons été en mesure d’améliorer le modèle de l’oscillateur harmonique simple et ainsi de produire un modèle itératif dont la fonction de transfert du filtre mécanique correspond aux données expérimentales. Bien au-delà de la fréquence de résonance escomptée, le modèle amélioré a une pente de diminution de 27 dB par décade.

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Executive summary

Sinusoidal vertical motion of a sonobuoy suspension: experimental data and a theoretical model

David M. F. Chapman and Dusko B. A. Kezele; DRDC Atlantic TM 2008-121; Defence R&D Canada – Atlantic; June 2008.

Background

In the early 1990s the DRDC Atlantic (formerly Defence Research Establishment Atlantic) instigated a Sonobuoy Suspension Design Study involving DREA scientists and engineers, sonobuoy and hydrodynamics contractors, and university engineering researchers. As a result, an improved mathematical model of sonobuoy suspensions was generated, called Sonobuoy Dynamic Simulator (SDS). These suspensions are necessary to allow the surface buoy to follow the large vertical motions of the sea surface while leaving the sensitive acoustic sensor at the lower end relatively undisturbed, thereby suppressing pseudo-acoustic noise at very low frequencies (below 20 Hz). One of the steps in validating SDS was to collect controlled experimental data on the hydrodynamic isolation properties of bungee cord-damper disk suspensions. It turned out that the data collected could be fit by a very simple model of the suspension valid over a wide scale of dimensions.

Previous Publication

This document is a reprint of a paper presented in 1990 at a workshop held under the auspices of The Technical Cooperation Program (TTCP) but which should have enjoyed wider dissemination. The original citation is: The Technical Cooperation Program, Subgroup G (Undersea Warfare) Technical Panel GTP-10, 25th Sonobuoy Working Party, Warminster, PA, USA, 3-5 December, 1990. The first author (DMFC) was a DREA Defence Scientist; the second author (DBAK) was an engineer at Sparton of Canada (London, Ontario, Canada) who performed the experimental work under an R&D contract.

Measurements and Theory

Sonobouy suspensions at 1/3 and 1/6 scale were constructed and tested in an experimental tank with a drive of variable amplitude and frequency applied to the top. For a given combination of amplitude and frequency, the apparatus was run until all transient response had dissipated, then the output amplitude was measured. Similar data was requested and retrieved from other experimenters. Above the bungee cord-damper disk resonance frequency, all transfer function data were observed to roll off with increasing frequency at about 30 dB/decade, rather than the 40 dB/decade expected from the elementary mechanics of the simple harmonic oscillator.

Detailed investigation of hydrodynamic added mass and damping of disks in sinusoidal motion revealed that these coefficients may not be constant, but may in fact be amplitude-dependent. This complicated the model, but an iterative solution was possible. Full application of these details produced a universal transfer function curve that simultaneously fit all data. In fact, well above the suspension resonance, the analysis showed there would be a theoretical rolloff of 26.7 dB/decade, tantalizingly close to the observed 30 dB/decade

Significance

This model could prove useful for practical engineering of sonobuoy suspensions not needing sophisticated modeling and simulation.

Sommaire

Mouvements verticaux sinusoïdaux du mécanisme de suspension de la bouée acoustique : données expérimentales et modèle théorique

David M.F. Chapman et Dusko B. A. Kezele; RDDC Atlantique TM 2008-121; R & D pour la défense Canada — Atlantique; juin 2008.

Introduction

Au début des années 1990, RDDC Atlantique (anciennement le Centre de recherches pour la défense Atlantique) a lancé une étude sur la conception du mécanisme de suspension de la bouée acoustique (Sonobuoy Suspension Design Study) en collaboration avec des scientifiques et des ingénieurs de RDDC Atlantique, des entrepreneurs spécialistes des bouées acoustiques et de l'hydrodynamique, ainsi que des chercheurs universitaires en ingénierie. L'étude a ainsi permis de créer un modèle mathématique amélioré du mécanisme de suspension de la bouée acoustique appelé Sonobuoy Dynamic Simulator (SDS). Le mécanisme de suspension est indispensable pour permettre à la bouée, qui flotte à la surface de l'eau, de suivre les grands mouvements verticaux de l'océan tout en laissant le capteur acoustique sensible plus ou moins stationnaire au fond de l'eau, ce qui élimine les bruits pseudo-acoustiques de très basse fréquence (moins de 20 Hz). Parmi les étapes à suivre pour valider le modèle SDS, il a fallu recueillir des données expérimentales de contrôle concernant les propriétés d'isolation hydrodynamique du mécanisme de suspension extenseur/disque d'amortissement. On a découvert qu'un modèle très simple du mécanisme de suspension, applicable à une grande variété de dimensions, donnait des résultats correspondant aux données expérimentales recueillies.

Publication antérieure

L'actuel document est une réimpression d'une présentation faite en 1990 à un atelier organisé par le Programme de coopération technique (TTCP) et qui n'avait pas été très répandue. Le renvoi d'origine était *The Technical Cooperation Program, Subgroup G (Undersea Warfare) Technical Panel GTP-10, 25th Sonobuoy Working Party, Warminster, PA, USA, 3-5 December, 1990.* L'auteur principal (DMFC) était un scientifique de RDDC – Atlantique, et le second (DBAK), un ingénieur chez Sparton of Canada (à London (Ontario), Canada) qui a mené les expériences en vertu d'un contrat du RDDC.

Mesures et théorie

On a créé des modèles d'un tiers et d'un sixième de la grandeur des mécanismes de suspension de la bouée acoustique, on les a mis à l'essai dans un réservoir et on a appliqué diverses valeurs d'amplitude et de fréquence à la bouée à la surface. Pour chacune des combinaisons amplitude/fréquence, on a laissé le mécanisme fonctionner jusqu'à ce que la réponse transitoire se dissipe entièrement avant de mesurer l'amplitude de sortie. Des données semblables ont été demandées et recueillies auprès d'autres chercheurs. Au-delà de la fréquence de résonance du mécanisme extenseur-disque d'amortissement, toutes les données de la fonction de transfert avaient une pente de diminution dont la fréquence augmentait progressivement à environ 30 dB par décade plutôt qu'à 40 dB par décade, ce qui était pourtant prévu pour la mécanique rudimentaire de l'oscillateur harmonique simple.

À la suite d’enquêtes approfondies sur la masse ajoutée et l’amortissement hydrodynamiques des disques en mouvement sinusoïdal, on a découvert que ces coefficients n’étaient peut-être pas constants et qu’ils dépendaient vraisemblablement de l’amplitude, ce qui a complexifié le modèle. Nous avons toutefois trouvé une solution itérative. L’entière intégration de ces détails nous a permis de produire une courbe universelle de la fonction de transfert qui correspond à toutes les données. En effet, l’analyse suggère une pente de diminution théorique bien au-delà de la fréquence de résonance escomptée, soit de 26,7 dB par décade, ce qui se rapproche de très près de la pente mesurée de 30 dB par décade.

Portée

Pour l’ingénierie appliquée du mécanisme de suspension des bouées acoustiques qui n’a pas besoin de modèles et de simulations perfectionnés, le présent modèle pourrait s’avérer utile.

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Acknowledgements

The development of the mathematical model was motivated to a large extent by earlier work by NADC scientists; the authors thank James McEachern and Joseph McCandless of NADC for providing copies of NADC reports on this topic. Thanks also to Dave Sparks and Dave Bruder of Sparton Electronics (US) for allowing the use of their full-scale suspension data.

Sinusoidal vertical motion of a sonobuoy suspension: experimental data and a theoretical model

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ABSTRACT

To isolate its acoustic sensor from the large vertical motions of the buoy at the ocean surface, a typical sonobuoy employs a low-pass mechanical filter comprising a bungee cord (spring) and a damper disk (hydrodynamic mass and damping). If the system behaved as a driven simple harmonic oscillator, the transfer function of vertical motion would roll-off at 40 dB/decade above resonance; however, experimental bungee/damper systems exhibit a roll-off of approximately 30 dB/decade. Experiments have shown that the inertia coefficient C_I and the drag coefficient C_D of a circular disk in sinusoidal motion are not constant, but are functions of the dimensionless ratio of the amplitude of motion to the disk diameter: A/d. Incorporating this concept, the simple harmonic oscillator model has been enhanced to produce an iterative model for the transfer function of the bungee/damper system that agrees well with the data. Well above resonance, the model simplifies to a closed-form expression giving a 27 dB/decade roll-off.

1. INTRODUCTION

The mechanical suspension system of a sonobuoy – consisting of an elastic bungee cord terminated by a damper disk, in the simplest arrangement – is designed to decouple the large vertical motions of the buoy at the sea surface from the acoustic sensor at the lower end. For a pressure sensor (i.e. a hydrophone), micro-motion of the sensor in the ocean's vertical pressure gradient translates into a pseudo-acoustic signal that represents part of the system self-noise at low frequencies. The simplest mathematical model views the suspension system as a low-pass mechanical filter consisting of a mass (the hydrodynamic inertial mass of the damper) at the end of a spring with a linear force-extension relation (the bungee cord). The resonance frequency of this simple harmonic oscillator is chosen to fall below the significant range of ocean wave frequencies so that motion at these frequencies is highly attenuated, being above the system resonance.

The simple harmonic oscillator model would have the vertical motion transfer function roll-off above resonance at the rate 40 dB per frequency decade, yet experiments indicate a gentler roll-off of about 30 dB/decade. Clearly a better mathematical model of the suspension system is needed.

This paper describes sinusoidal vertical motion experiments on simple sonobuoy suspension systems and presents a model that has been developed to explain the observed frequency dependence of the transfer function. It would seem that both the inertial mass and drag coefficient of a damper disk are not constant quantities for sinusoidal motion, but depend upon the amplitude of the oscillation. The model presented below – which is an enhancement of the simple harmonic oscillator model – not only fits our own vertical motion data but also fits data from entirely independent investigations. The model is straightforward, but requires numerical iteration; a simple closed-form model, based on the asymptotic behaviour of the iterative model, predicts a roll-off of 27 dB/decade, very close to the 30 dB/decade observed in the experiments.

2. THE VERTICAL MOTION EXPERIMENTS

Transfer function measurements were made on the vertical motion of a bungee/damper suspension system at the Sparton of Canada (SOC) facilities in London, Ontario. As illustrated in Fig. 1, a wave motion simulator was used to drive the suspension system in sinusoidal motion at a specified amplitude and frequency. A nylon drive cable transfered the input motion to the top end of the suspension through a system of pulleys. The bungee cord was a standard SOC sonobuoy component: natural rubber surgical tubing (SOC Dwg. No. 110-3687). The damper was a standard SOC full-sized (12" diameter) circular polyethylene disk tensioned by a spring-metal hoop. A calibrated Sparton pressure sensor was installed below the damper disk to sense the pressure signal associated with the vertical component of the output motion in the water tank. The data were collected, digitized, and saved on floppy disk with an HP 3562A signal analyser. The

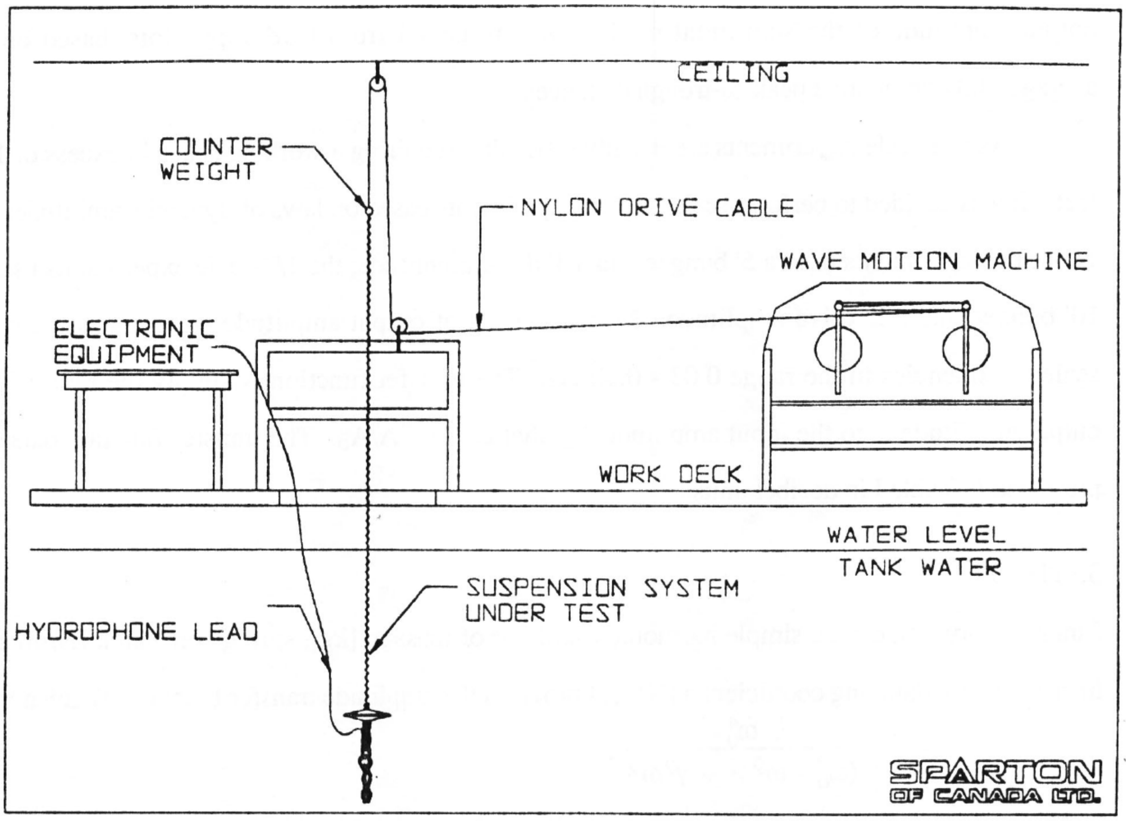

Figure 1 Schematic diagram of the experimental apparatus.

TABLE I: Vertical Motion Transfer Function Data

1/6-Scale System [a]		1/3-Scale System [b]	
Frequency f [Hz]	Transfer Function 20 log |T| [dB]	Frequency f [Hz]	Transfer Function 20 log |T| [dB]
.029	-0.05	.028	-0.56
		.052	-6.91
.075	-2.83	.075	-11.75
.123	-10.27	.123	-18.34
.174	-14.87	.169	-22.41
		.225	-25.83
.285	-21.69	.283	-28.76
.345	-24.39	.339	-32.20
.399	-26.37	.398	-35.71
.456	-28.40	.454	-35.61
.504	-29.92		

a) 5' bungee, 1' input amplitude, 12" disk
b) 10' bungee, 2' input amplitude, 12" disk

output amplitude of the sinusoidal motion was measured from hard-copy plots, based on an average of 3 consecutive peak-to-trough distances.

As full-scale experiments are usually difficult – requiring a working space in excess of 100 feet – it was decided to perform scaled-down experiments based on laws of dynamic similitude: the 1/6-scale experiments used a 5' bungee and a 1' drive amplitude; the 1/3-scale experiments used a 10' bungee and a 2' drive amplitude. Measurements of output amplitude were made at eleven scaled frequencies in the range 0.03 - 0.50 Hz. The transfer function is simply the ratio of the output amplitude A to the input amplitude A_0, that is, $|T| = A/A_0$. The transfer function data are presented in Table I in decibel units.

3. THEORY

Linear theory of a driven simple harmonic oscillator of mass m [kg], spring constant k [N/m] and linear-velocity damping coefficient a [N/m/s] provides the amplitude transfer function |T| given by

$$|T|^2 = \frac{\omega_0^4}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}, \tag{1}$$

in which $\omega_0 = \sqrt{k/m}$ is the resonance frequency in rad/s, ω is the drive frequency, and $\gamma = a/m$ is the loss factor. (The system quality factor is $Q = \omega_0/\gamma$.) Above resonance,

$$|T|^2 \approx \omega_0^4/\omega^4, \qquad (\omega \gg \omega_0) \tag{2}$$

which represents a roll-off of 40 dB/decade. This simple model is inadequate for several reasons: (a) there is evidence[1] that the hydrodynamic mass of a disk in sinusoidal motion is not constant, but depends upon the amplitude of motion; (b) the dominant damping force is hydrodynamic drag, which is proportional to the square of the velocity, not linear with velocity; and (c) the same study[1] that found that the inertia coefficient is non-constant for sinusoidal motion also found that the drag coefficient is amplitude-dependent for sinusoidal motion.

The bungee itself may have a non-linear force-extension relation and may introduce its own dissipation terms. For this analysis, we assume that the bungee is operating within a linear range and that the bungee damping is negligible compared with hydrodynamic damping. Our analysis concentrates on the hydrodynamic forces at the damper disk.

3.1. Inertia and Drag Coefficients for Uniformly-Accelerated Motion

If a thin circular disk of diameter d is accelerated uniformly in a direction perpendicular to the disk plane in a fluid of density ρ, the inertial mass of the disk is primarily the hydrodynamic added mass

$$m_h = C_I \rho d^3, \tag{3a}$$

where C_I is the inertia coefficient (about 1/3 for uniformly-accelerated flow). In what follows, we neglect the inertia due to the terminal weight used to extend the suspension system to its operating point.

It is well known that the drag force F_D for thin disks is not linear in the flow speed v, but quadratic, of the form $F_D = C_D \rho\pi(d^2/8)\, v^2$, where C_D is the drag coefficient (about 1.12 for uniform flow). For nearly-sinusoidal motion, the effect of a non-linear damping force can be approximated by an effective linear damping coefficient a_{eff}, calculated by considering the energy lost over one cycle[2]. The effective damping factor depends upon the amplitude of motion A and the drive frequency ω: for the v^2 drag force,

$$a_{eff} = (8/3\pi)\, C_D\, \rho\pi(d^2/8)\, A\, \omega. \tag{3b}$$

3.2. Inertia and Drag Coefficients for Sinusoidal Motion

Experiments at the US Naval Air Development Center (NADC)[1] have shown that the inertia coefficient C_I and drag coefficient C_D for a disk in sinusoidal motion are not always constant, but depend on the ratio of motion amplitude to disk diameter, A/d. The departure from the constant values $C_I = 1/3$ and $C_D = 1.12$ can be summarized by the relations

$$C_I = 1.2\, (A/d)^{1/2} \qquad (A/d > 0.08) \tag{4a}$$

and

$$C_D = 2.2\, (A/d)^{-1/2}. \qquad (A/d < 3.9) \tag{4b}$$

The precise physical reasons for this phenomenon are currently under separate investigation by SOC. Their initial exploratory findings suggest that the phenomenon may be related to the fact that an oscillating body periodically reverses the direction of its motion, passes through previously-disturbed fluid, and sheds vortex rings at each reversal. Also, there is evidence that the coefficients C_I and C_D may be frequency-dependent as well as amplitude-dependent. The thrust of this paper is to examine the effect of including these non-constant coefficients in the analysis of the vertical motion transfer function of a sonobuoy suspension, so we will not dwell upon the hydrodynamics.

If the dynamic inertia coefficient from Eq.(4a) and the dynamic drag coefficient from Eq.(4b) are substituted into Eq.(3a) and Eq.(3b), bearing in mind the appropriate limitations, the effective mass and effective damping coefficient of the damper disk in sinusoidal motion are

$$m_{eff} = 1.2\ \rho\ d^{5/2} A^{1/2} \tag{5a}$$

and

$$a_{eff} = (1/3)\ 2.2\ \rho\omega\ d^{5/2} A^{1/2}. \tag{5b}$$

3.3. The Iterative Model

Having established that both the effective mass and the effective damping factor of the damper disk possibly depend upon the amplitude of oscillation, we must now include these facts in the computation of the transfer function. If the damper disk were directly coupled to the input drive of amplitude A_o, the effective mass would be $M = 1.2\ \rho\ d^{5/2} A_o^{1/2}$; however, since the amplitude A of the disk motion is related to the driving amplitude by $A = |T| A_o$, we find that the effective mass is indirectly dependent upon frequency; $m_{eff} = M\ |T(\omega)|^{1/2}$. (We will call the quantity M the *reference mass*.) This means the resonance frequency ω_o in Eq.(1) needs to be replaced by the effective quantity

$$\omega_{eff} = \sqrt{k/m_{eff}} = \Omega_o |T(\omega)|^{-1/4} \tag{6a}$$

in which $\Omega_o = \sqrt{k/M}$. (We will call the quantity $F_o = \Omega_o/2\pi$ the *scaling frequency*.) Since most bungees have a non-linear stress-strain relation, the value used for k should be the dynamic spring constant at the appropriate operating point.

Following a similar line of reasoning, the loss factor γ in Eq.(1) needs to be replaced by the effective quantity

$$\gamma_{eff} = a_{eff}/m_{eff} = \alpha\ \omega, \tag{6b}$$

in which α is the number (1/3)(2.2/1.2) = 0.61. Strictly speaking, these relations apply only in the range $0.08 < (A_o/d)\,|T| < 3.9$.

With these substitutions, Eq.(1) is no longer a simple closed-form expression for |T|, as this quantity appears on both sides. If we assume that the new transfer function takes the same general form of the previous transfer function, we can substitute Eqs.(6a) and (6b) into Eq.(1). After simplifying, we find that

$$|T|^2 = \frac{1}{(1 - x^2\,|T|^{1/2})^2 + \alpha^2\,x^4\,|T|}\,, \tag{7}$$

in which $x = \omega/\Omega_o = f/F_o$ is the normalized drive frequency. To find the value of |T| for a given value of x, we could write Eq.(7) in the form of a polynomial in $|T|^{1/2}$ and apply a root-finding algorithm. Alternatively, we can use Eq.(7) in an iterative manner, guessing a value for |T|, substituting on the right-hand side, solving for the left-hand side and repeating the procedure until the answer converges. This is the method we employ in this paper, making all calculations on a programmable pocket calculator. Note that all systems are described by a universal curve if we normalize the forcing frequency by the scaling frequency F_o, which is a combination of the parameters k, d, and A_o. The only other model parameter is α, which is a fixed number.

3.4. The Simple Model

If we define resonance to be the vanishing of the first term in the denominator of Eq.(7), then analysis shows that the resonance frequency f_r is

$$f_r = \alpha^{1/4}\,F_o \approx 0.88\,F_o\,. \tag{8}$$

Well above this frequency, |T| has the approximate form

$$|T| \approx (1+\alpha^2)^{-1/3}\,x^{-4/3} \qquad (x \gg \alpha^{1/4}) \tag{9a}$$

or

$$|T| \approx 0.90\,(F_o/f)^{4/3}, \qquad (f \gg 0.88\,F_o) \tag{9b}$$

which represents a roll-off of 26.7 dB/decade.

4. COMPARISON OF EXPERIMENT AND THEORY

4.1. Sparton of Canada Data

To test this model, let us compare the SOC data with model results using the appropriate system characteristics k, d, and A_o. At DREA we measured the elasticity of the SOC bungee cord: a 35.3 cm length had a dynamic spring constant of 11.2 N/m. For the 5' length of the 1/6-scale suspension, this translates into a spring constant of k = 2.59 N/m; for the 10' length of the 1/3-scale suspension, k = 1.30 N/m. According to our model, a 12" damper disk moving harmonically with amplitude 1' has a reference mass M = 34.0 kg; for the 2' amplitude, M = 48.0 kg. (Compare these with the conventional effective mass of 9.4 kg calculated from Eq.(3a) assuming uniform motion.) These masses, combined with the corresponding k values, provide the scaling frequencies F_o = 0.0439 Hz for the 1/6-scale suspension and F_o = 0.0261 Hz for the 1/3-scale suspension. In Fig. 2 we have plotted the SOC experimental data for the 1/6-scale suspension from Table I, along with two theoretical curves: the "iterative model" of Eq.(7) and the "simple model" of Eq.(9a). The data and the corresponding theoretical curves for the 1/3-scale suspension are shown in Fig. 3.

The agreement between data and theory is quite good, especially since there are no adjustable parameters in the model. The simple model, which has the 26.7 dB/decade roll-off, is a good fit at the higher frequencies but generally over-estimates the attenuation at the lower frequencies. The iterative model behaves asymptotically like the simple model at high frequencies. At the midrange and lower frequencies, both the experimental data and the iterative model show a roll-off closer to 30 dB/decade. At the higher frequencies, both the simple model and the iterative model slightly underestimate the attenuation.

4.2. Other Data Sets

Data from similar NADC [3] experiments confirm the frequency dependence of the transfer function roll-off: their data show values of about 29 dB/decade. The experiments were conducted with 30' of bungee, a 12" damper disk, and input amplitudes of 1.5' and 3'. The drive frequency varied

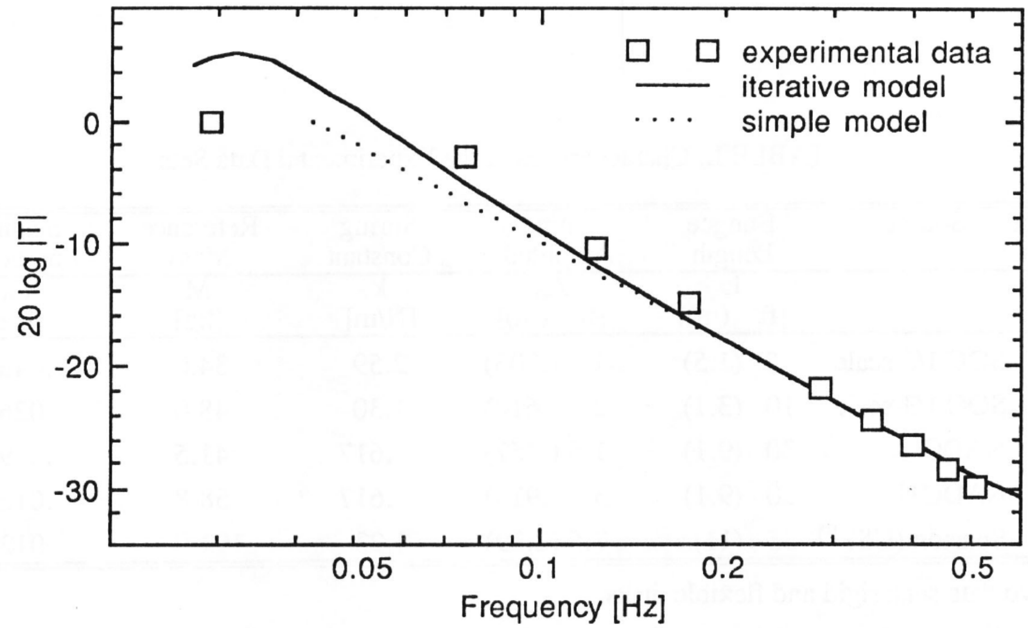

Figure 2 *Transfer function data and model calculations for the SOC 1/6-scale suspension: 1' input amplitude, 5' bungee, and 12" damper disk.*

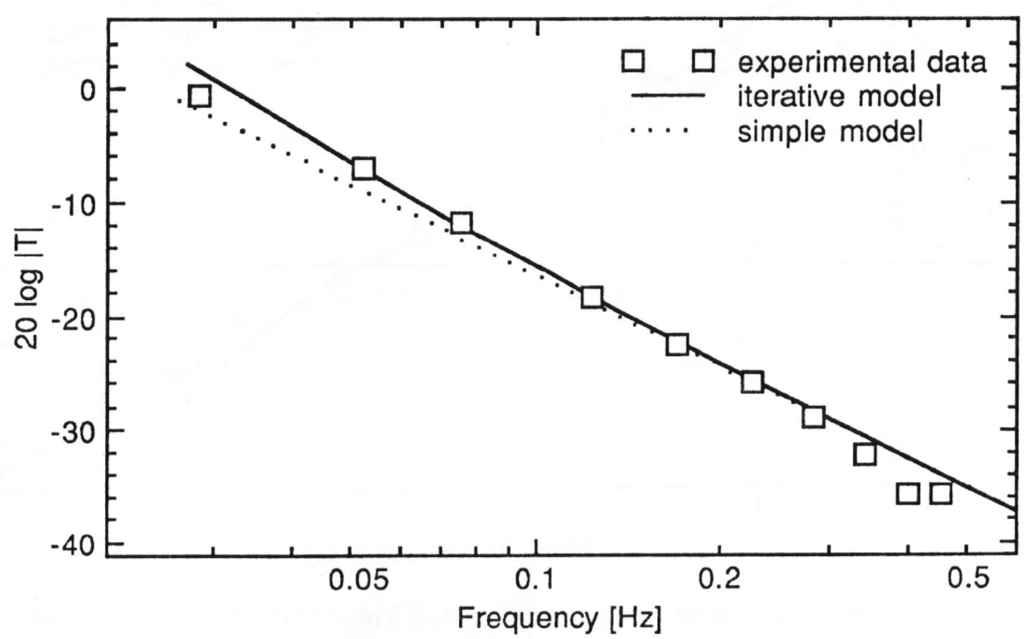

Figure 3 *Transfer function data and model calculations for the SOC 1/3-scale suspension: 2' input amplitude, 10' bungee, and 12" damper disk.*

TABLE II: Characteristics of the Experimental Data Sets

Data Set	Source	Bungee Length L [ft (m)]	Input Amplitude A_0 [ft (m)]	Spring Constant k [N/m]	Reference Mass M [kg]	Scaling Frequency F_0 [Hz]
1	SOC 1/6-scale	5 (1.5)	1 (.305)	2.59	34.0	.0439
2	SOC 1/3-scale	10 (3.1)	2 (.610)	1.30	48.0	.0261
3	NADC	30 (9.1)	1.5 (.457)	.617	41.5	.0194
4	NADC [a)]	30 (9.1)	3 (.914)	.617	58.8	.0163
5	Sparton (US) [b)]	35 (11)	7.5 (2.29)	1.23	191.0	.0128

a) Two data sets: rigid and flexible disks.

b) 12" diameter damper disks used throughout except for Data Set 5, which used a 16" disk.

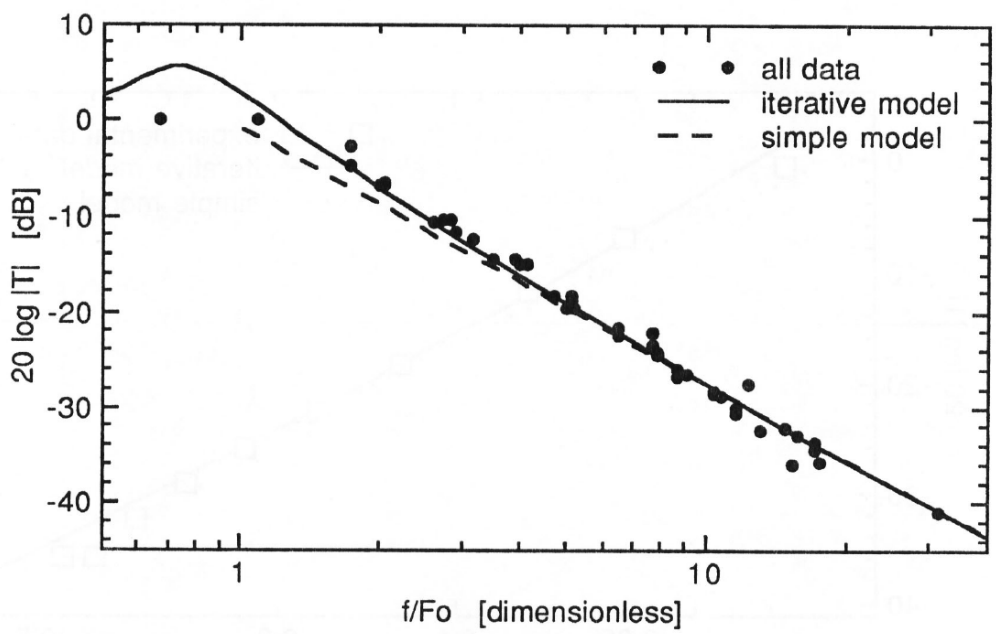

Figure 4 Transfer function data for the 5 data sets of Table II plotted as a function of normalized frequency, along with the universal model curve.

between 0.033 Hz and 0.286 Hz. Another data set supplied by Sparton (US)[4] also supports the present model. Table II summarizes the characteristics of all the data sets examined. Note that data sets 3, 4, and 5 represent full-scale suspension tests. To emphasize the universal nature of the model, the data from all 5 experiments from the 3 different sources have been plotted together in Fig. 4 as a function of normalized frequency f/F_o.

4.3. Discussion

Although the agreement between experiment and theory is encouraging, there are a couple of deviations that deserve comment. At the high frequency end, the experiments show higher attenuation than the model; this might be remedied if the terminal weight were added to the hydrodynamic mass. At the low frequency end, the model predicts an amplification of motion, where the experiments show little or no gain; there must be some physical mechanism in the experiment that limits the system motion but is absent from the present model. SOC, however, have been successful in modelling this region by using a first-order Fourier series approximation to the non-linear damping term[5]. The resulting solution is closed-form and closely models the response of a sonobuoy suspension near the system resonance frequency. On the other hand, this model diverges from experimental data by predicting a 40 dB/decade roll-off above resonance.

A brief word on limits of applicability: expressions (3a) and (3b) for the inertia and drag coefficients are both satisfied in the range $0.08 < (A_o/d)\,|T| < 3.9$. For the SOC 1/6-scale data set, this means that $20\log|T|$ must lie in the range -22 dB to +12 dB; for the SOC 1/3-scale data set, $20\log|T|$ should lie in the range -28 dB to +6 dB. Most of the data presented satisfy these conditions.

It is worth noting that the amplitude-dependent inertia and drag coefficients described here have been included in a sophisticated buoy-cable-body numerical simulation code CABUOY [6] and that this has been successfully used [3] to model the NADC suspension experiments. In the process, a CABUOY bug needed to be fixed; it is not clear whether existing versions of CABUOY have been modified or not.

The model presented here seems appropriate for interpreting experiments of the type described, that is, for strictly vertical sinusoidal motion. However, if there were shear flow in addition to vertical motion, as in the real ocean environment, the applicability of the model would be brought into question. In the same vein, it is not obvious how to apply the theory if the motion involves multiple frequencies.

One final point worth noting: although we have not presented the data here, suspension experiments with a terminal mass formed by a volume of water entrained in a light plastic bag display a transfer function roll-off of 40 dB/decade, as would be expected, since the entrained water mass remains constant.

5. POSSIBLE FUTURE WORK

A more sophisticated version of this model could include several additional parameters: inertial mass of the damper or terminal weight; linear-velocity damping of the bungee; and non-linear force-extension relation for the bungee. For the moment, there is little indication that these parameters are required to support the data. Experiments with dual spring/damper suspensions have been conducted; it would be useful to extend the model to that case. The values of C_I and C_D need to be defined over a broader range of amplitudes and a suspected dependence on frequency should be investigated. Experiments and theory involving multiple-frequency input drives would prove most interesting.

6. CONCLUSIONS

Sinusoidal vertical motion experiments on simple bungee/damper sonobuoy suspensions show a transfer function frequency dependence of about 30 dB/decade above resonance; this observation is confirmed by several independent experiments. The data have been successfully modelled by enhancing the simple harmonic oscillator model to admit inertia and drag coefficients that are not constant, but depend upon the amplitude of motion. The transfer function data from several different suspensions have been shown to lie on a universal model curve when the drive frequency is normalized by a scaling frequency F_0, a parameter that is a combination of the dynamic spring constant, the diameter of the damper disk, and the input amplitude of the motion. The enhanced model – although straightforward – requires numerical iteration; asymptotic analysis of the iterative model provides a simpler model that shows a 27 dB/decade roll-off, close to the trend of the higher-frequency experimental data.

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List of symbols/abbreviations/acronyms/initialisms

DRDC Atlantic	Defence Research & Development Canada - Atlantic
DREA	Defence Research Establishment Atlantic (former name for DRDC Atlantic)
NADC	Naval Air Development Center (US)
SOC	Sparton of Canada

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To isolate its acoustic sensor from the large vertical motions of the buoy at the ocean surface, a typical sonobuoy employs a low-pass mechanical filter comprising a bungee cord (spring) and a damper disk (hydrodynamic added mass and damping). If the system were to behave as a driven simple harmonic oscillator, the transfer function of vertical motion would roll off at 40 dB/decade above resonance; however, experimental systems of this sort exhibit a rolloff of approximately 30 dB/decade. Experiments have shown that the inertia coefficient C_I and the drag coefficient C_D of a circular disk in sinusoidal motion are in fact not constants, but depend on the dimensionless ratio of amplitude of motion to disk diameter, A/d. Incorporating this concept, the simple harmonic oscillator model has been enhanced to produce an iterative model for the transfer function of the bungee/disk system that agrees well with experimental data. Well above resonance, the model simplifies to an expression giving a 27 dB/decade rolloff.

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