

Modeling and Interpretation of Beamforming Gain and Diversity Gain for Underwater Acoustic Communications

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Abstract- Output signal-to-noise ratios for a multichannel decision feedback equalizer were measured from experimental BPSK data as a function of the receiver spacing, number of receivers used, and array aperture and reported in a paper in OCEANS 2005 by the author. This paper discusses the theory and reports numerical modeling results to compare with the data.

I. INTRODUCTION

Noise and inter-symbol interference (ISI) are two primary causes of acoustic communication bit errors. Spatially separated multiple receivers can be used to (1) enhance the signal-to-noise ratio (SNR) as in array beamforming (or equivalently suppression of the noise relative to the signal) and (2) suppress the ISI (or equivalently enhance the signal-to-interference ratio, SIR) using diversity combining algorithms. The spatial processing gain can be illustrated using the passive-phase-conjugation (or passive time-reversal) algorithm. The received data can be expressed in terms of the transmitted symbols as follows:

$$v_k = \sum_{n=0}^L h_n I_{k-n} + \eta_k \quad (1)$$

where k is sample index, $\{h_n\}$ is a discrete representation of the band limited impulse response function of length $L+1$, and $\{v_k\}$, $\{\eta_k\}$ are discrete received signal and noise sequences. Convolution Eq. (1) with the complex conjugate of the channel impulse response function at time t_0 , and summing over the receiver channels, with index j , one obtains after some manipulations in summation indices [1],

$$\begin{aligned} y_k &\equiv \sum_{l=0}^L h_l^* v_{k-l} \\ &= x_0 I_k + \sum_{n=1}^L x_n I_{k-n} + \sum_{n=1}^L x_n^* I_{k+n} + \sum_{l=0}^L h_l^* \eta_{k-l} \end{aligned} \quad (2)$$

where $x_n = \sum_{j=1}^M \sum_{l=1}^{L-n} h_{j,l}^*(t_0) h_{j,l+n}(t)$ is the discrete representation of the auto-correlation of the channel impulse response function, referred to as the Q function [2]. (* denotes complex conjugation.) The first term on the right hand side of Eq. (2) shows that the signal is enhanced using multiple channels. For a time-invariant communication channel $h_{j,l}(t) = h_{j,l}(t_0)$, the (matched field) signal gain for x_0 is $\sim 10 \log M$ for M channels assuming $h_j \approx h_1$. The second and third term show the origin of ISI due to non-zero x_n for $n = 1, \dots, L$. The x_n for $n \neq 0$ are the sidelobes of the Q function [2]. Previous work has shown that the ISI is reduced using multiple channels [2]. The reason is that the sidelobes appear at different positions and are reduced by approximately $1/M$ when averaged over M channels.

For a time-varying channel, $h_j(t) \neq h_j(t_0)$. The matched-field signal gain suffers a loss as a function of time. Figure 1 shows the measured channel impulse responses functions for intra-packet correlations and the decrease of signal gain as a function of time for x_0 . From the physics point of view, signal gain depends on the signal coherence. The loss of signal temporal coherence is responsible for the decrease of the matched field signal gain as a function of time. From the signal processing point of view, the signal gain degradation is caused by the tracking error, namely, that the fixed tap coefficients $h_j(t_0)$ fail to track the temporal variation of the channel. Note that, on the other hand, ISI suppression depend on the incoherent or independent components of the signal. In fact, x_n for $n \neq 0$ becomes smaller as the signal becomes more independent (or less coherent) between the receivers.

This paper models the diversity gain arising from the use of diversity combining algorithms applied to multiple receivers. The calculation is more complicated than the passive phase conjugated case discussed above. Classical

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diversity is defined for receivers receiving independent information. In reality, the signals on spatially separated receivers are never totally independent. The corresponding diversity (for partially correlated signals), as measured by the ability of the receivers to suppress ISI is estimated from the data. The equivalent signal gain for the multichannel decision feedback equalizer is deduced and compared with the array gain of a conventional beamformer applied to the same data.

II. OUTPUT SNR FOR A MULTI-CHANNEL DECISION FEEDBACK EQUALIZER

The output (symbol) SNR for communications is defined as the signal to noise-plus-interference ratio

$$\gamma \equiv \frac{S_{out}}{N_{out} + ISI_{out}}. \quad (3)$$

It is related to the least-mean-square-error (MSE) by

$$\gamma = \frac{1 - J_{min}}{J_{min}}, \quad (4)$$

where J_{min} , for a decision feedback equalizer (DFE), is given below [3,4], assuming that the channel is known:

$$J_{min} = \exp \left\{ \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \left[\frac{N_0}{\bar{Q}(\omega) + N_0} \right] d\omega \right\}, \quad (5)$$

where $\bar{Q}$ is the normalized spectrum $\bar{Q}(\omega) = Q(\omega) / q_0$,

where $Q(\omega) = \sum_{j=1}^M Q_j(\omega)$, $Q_j(\omega) \equiv H_j^*(\omega) H_j(\omega)$,

with j being the channel index, $H(\omega)$ is the Fourier transform of $h(t)$, and

$$q_0 = \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} Q(\omega) d\omega = \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \sum_{j=1}^M Q_j(\omega) d\omega \equiv M \bar{q}_0$$

N_0 is the normalized noise $N_0 = \frac{\sigma_n^2}{q_0}$, where σ_n^2 is the

input noise level (assuming the same noise level for all receivers). We note that $\bar{q}_0$ is the mean input signal level,

$$\bar{q}_0 = \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{1}{M} \sum_{j=1}^M |H_j^{data}(\omega)|^2 d\omega, \quad (6)$$

and N_0 is the inverse of average input SNR divided by M . q_0 , N_0 and $Q(\omega)$ are dependent on the number of channels. The corresponding values for M channels will be denoted as $q_{0,M}$, $N_{0,M}$, etc. In the absence of ISI, $\bar{Q}(\omega) = 1$ one finds $J_{min} = N_0 / (1 + N_0)$. The output symbol SNR is $\gamma =$

$1/N_0$. One finds a spatial processing gain of $10 \log M$. The model assumes a frozen ocean and an ideal solution for the equalizer tap coefficients. Hence one obtains a theoretical processing gain equal to the theoretical array gain for conventional beamforming. In the real world, as remarked above, the signal, particularly its phase, fluctuates rapidly with time, resulting in limited temporal and spatial coherence. Hence the signal-to-noise $1/N_0$ yield less than theoretical $10 \log M$ gain.

In the presence of ISI, one can express Eq. (5) as

$$J_{min} = N_{out} (1 + I_{out}), \quad (7)$$

where

$$N_{out} = \exp \left\{ \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \frac{N_0}{1 + N_0} d\omega \right\} = \frac{N_0}{1 + N_0} \quad (8)$$

for additive Gaussian white noise, and

$$I_{out} = \exp \left\{ \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \left[\frac{1 + N_0}{Q(\omega) + N_0} \right] d\omega \right\} - 1. \quad (9)$$

Note $\mathcal{E}(\omega) = 1 - Q(\omega) \neq 0$ is the cause of ISI.

We note for $J_{min} < 1$, Eq. (4) can be written as

$$\gamma \approx \frac{1}{N_{out} (1 + I_{out})} \quad (10)$$

Comparing Eq. (10) with Eq. (3), one finds $ISI_{out} = N_{out} I_{out}$. Equation (10) can be expressed as

$$OSNR(dB) = 10 \log(\gamma) \\ \approx -10 \log(N_{out}) - 10 \log(1 + I_{out}), \quad (11)$$

where N_{out} is in essence the inverse of output symbol energy over the noise and $1 + I_{out}$ represents the degradation due to ISI.

Equation (11) can be stated as follows: the output (symbol) SNR can be improved by suppressing the noise relative to the symbol energy (N_{out}) and the interference relative to the symbol energy (I_{out}). For the interference, note that $I_{out} = 0$, when $Q(\omega) = 1$. In other words, ISI comes from spectral $Q(\omega)$ often has due to multipath interference or signal fading. The spectral nulls in $Q(\omega)$ are minimized by averaging over independent channels (which possess nulls at different frequencies). This is the reason that spatial diversity, originally conceived to combat temporal fading, can be used to mitigate ISI. In a random ocean, the acoustic signals encounter (repeated) scattering from the ocean inhomogeneities and/or time-varying surfaces (of scales smaller than the spatial coherence length). The spatially uncorrelated (independent)

scattering components of the acoustic signals help to minimize ISI.

Equation (11) can be translated into an expression for the spatial processing gain defined as the output SNR for M channels over that of a single channel. One finds

$$SPG_M(\text{dB}) = NSG_M + ISG_M, \quad (15)$$

where

$$\begin{aligned} NSG_M &= 10 \log \frac{N_{out}(1)}{N_{out}(M)} \\ &\approx 10 \log M + 10 \log \left(\frac{\bar{q}_{0,M}}{\bar{q}_{0,1}} \right) + O\left(\frac{\sigma_n^2}{\bar{q}_0}\right) \end{aligned} \quad (16)$$

and

$$ISG_M = 10 \log \left(\frac{1 + I_{out}(1)}{1 + I_{out}(M)} \right), \quad (17)$$

where $N_{out}(M)$ and $I_{out}(M)$ are given in Eqs. (8) and (9) using N_0 and Q for M channels. The second expression in Eq. (16) assumes a high input SNR. Equation (15) states that spatial processing gain (SPG) is the sum of noise suppression gain (NSG) and ISI suppression gain (ISG) expressed in dB.

III. EXPERIMENTAL DATA

In a previous paper [5], we reported the spatial processing gain as a function of number of receivers using (1) conventional beamforming and (2) a multichannel DFE jointly with a phase-locked loop [1,2]. The data were linear frequency modulated (LFM) and binary-phase-shift-keying (BPSK) signals from the ASCOT01 experiment which took place off the coast of New England [6]. Both signals were centered at 1200 Hz with a 400 Hz bandwidth. The receiver array consisted of 33 elements uniformly spaced at $d = 0.5$ m covering a depth span of 50-66 m. The source was deployed 4 m above the bottom. The water depth was ~ 100 m. The source-receiver range was approximately 10 km. The received signals had a high input SNR (> 15 dB) [5].

We processed the data using subarrays of different number of receivers, with different receiver separations: $D = nd$, $n = 1 \dots 10$, where $d = 0.5$ m. Signal gain was measured by beamforming the various sub-arrays using the LFM signal. To obtain array gain, we assume, for simplicity, uncorrelated noise and hence a noise gain of $10 \log M$, where M is the number of receivers of the subarray. The AG data (see [1,2]) can be fitted with $AG = A \log(1+L/D)$, where L is the array aperture, and A is a coefficient which decreases linearly with increasing element spacing: A lies between 8 and 6 for $D = 1d$ to $10d$.

The output symbol SNR for the multichannel DFE [7] was measured using the BPSK signals for the subarrays with different spacing [5]. The results were plotted as a function of the array aperture as shown in Fig. 2. The data

can be fitted by the following formula: output SNR = $10 \log(L/p) + G$, where $G \approx 8$ for $D = 1d, 2d, 3d, 4d$; $G \approx 5.5$ for $D = 5d, 6d$; and $G \approx 2.5$ for $D = 7d, 8d, 9d, 10d$. The data were also processed as a function of receiver spacing for a fixed number of receivers ($M = 4$) as shown in Fig. 3.

The above result suggests that for a given array aperture, optimal performance is obtained by spacing the element at $D \leq \rho$ (Fig. 2), where ρ is the spatial decorrelation (or coherence) length of the signal $\rho \approx 4d$. To keep the number of elements (and computational complexity) to a minimum, one should set the element spacing at $D \approx \rho$. Note that increasing the number of receivers using smaller separations does not necessarily improve the output SNR (Fig. 2). The result for a fixed number of receivers (Fig. 3) also suggests that the optimal receiver spacing is approximately the signal de-correlation length.

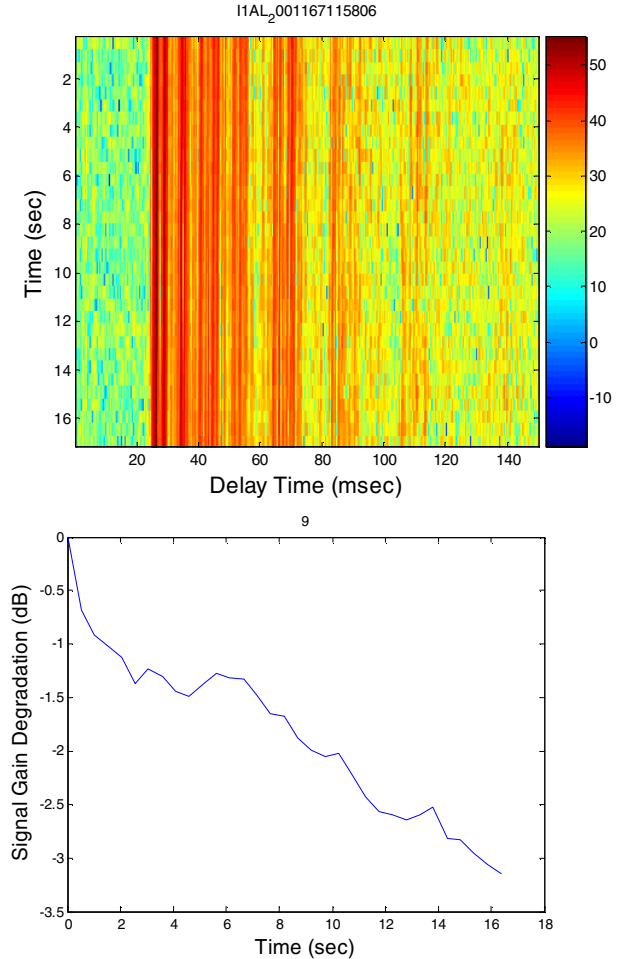


Fig. 1 Channel Impulse response as a function of delay time and geotime (upper figure). Signal gain degradation as a function of geotime (lower figure).

To compare and interpret the experimental results, we calculate numerically the SPG using Eqs. (15-17). The

input is the channel transfer function, which was estimated using the LFM signals. We note that Eqs. (15-17) assume a time-invariant channel which is known to the receiver. In other words, the tap coefficients are well-matched to the channel. In the real world, there is a mismatch between the tap coefficients and the channel, and hence the measured SPG will be less than the theoretical SPG.

The SPG, as given in Eq. (15) is the sum of noise suppression gain (NSG) and ISI suppression gain (ISG). The ISG is modeled using Eq. (17). The results for the various subarrays are shown in Fig. 4. One finds that the ISG is practically only a function of the array aperture and is nearly independent of the element spacing. For the various array configurations mentioned above with different element spacing, the ISG can be fitted with the following formula: $ISG = 10\log(1+L/\rho) - 1$. This result suggests that populating the array with more elements does not always improve the ISG because the additional receivers are NOT receiving new (independent) information. This is not necessarily true for the SPG (or the output SNR).

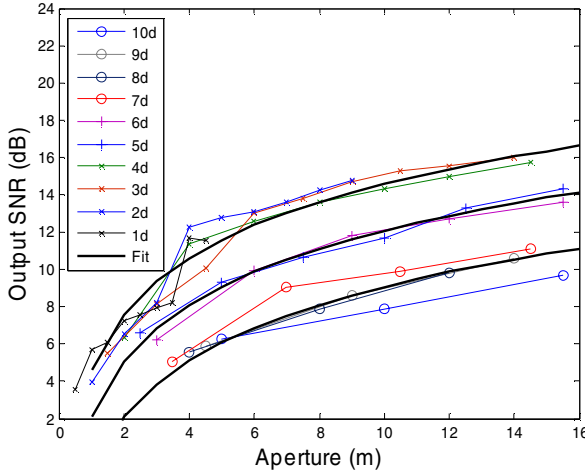


Fig. 2 Output SNR as a function of the array aperture with different receiver separations. Thick black curves are empirical fit to the data (see text).

One finds that the NSG based on the theory is nearly equal to the theoretical maximum of $10\log M = 10\log(1+L/D)$. This is expected based on Eq. (16). Combining ISG with NSG, one obtains a theoretical SPD as: $SPD_{th} = 10\log(1+L/D) + 10\log(1+L/\rho) - 1$. In contrast, the output SNR data in Sec. II produces: $SPD_{Data} = 10\log(L/\rho) + G - OSNR(1)$, where the last term is the output SNR for one channel.

The theoretical SPD result suggests that for a fixed array aperture L , optimal SPD is obtained by spacing element at half wavelength, $D = d$ since SPD_{th} increases with decreasing D (NSG is maximum at $D = d$). In contrast, as noted above, the data suggest that SPD for $D = 1d, 2d, 3d$, and $4d$ are approximately the same. The reason is that the NSG in the real world is different and smaller

than the theoretical value, $10\log M$. Note that in the real world, the signals are time-varying. Note further that the enhancement of the signal over noise (the NSG) requires coherent processing of the signal. The tap coefficients, estimated based on the least mean square error criterion, could not match the temporal variation of the channel. Consequently the NSG of the equalizer is less than the theoretical value.

The NSG in a time-varying channel is difficult to model analytically. Given the measured output SNR from data, one can estimate the NSG by subtracting the ISG from SPD_{Data} . Note that ISI involves the independent (incoherent) component of the signal and can be reliably estimated from the spectrum of the signal. Using data from Fig. 2 and 4, we obtain: $NSG_{data} = -10\log(1+\rho/L) + Q$, $Q = G + C - OSNR(1)$.

Using the formula for ISG and NSG_{data} , one can calculate the SPD for an array of four receivers and then the output SNR for four channels. The modeled output SNR for four receivers is shown in Fig. 3 as the dotted line. It is in good agreement with the data reported above.

For conventional beamforming the measure of performance is the array gain. The measured array gain for the ASCOT01 data was given above. For the multichannel combining algorithm, the measure of performance is diversity gain. What is diversity gain? There are two definitions. Diversity gain has been used loosely in the literature to refer to the improvement in the symbol SNR through the use of a diversity-combining algorithm applied to widely spaced receivers. This was the definition used in [5] and is referred to above as the spatial processing gain. Classical diversity requires that the signals on the different receivers fade independently and are thus uncorrelated between the receivers. For this case, the signal is like noise and cannot be enhanced over noise, or equivalently no NSG is obtained. The performance improvement comes primarily from the suppression of ISI. Thus classical diversity gain is more in line with the ISG. This is the second definition of diversity gain. Note, however that classical diversity does not exist in the real world, since the signals are never totally uncorrelated. For partially correlated signal, the ISG is the “available” diversity gain. The formula for it is given above.

IV. SUMMARY

In this paper, we modeled the spatial processing gain for a multichannel DFE in terms of the signal to noise enhancement (NSG) and signal to interference enhancement (ISG) and compare the results with the measurements reported in a previous paper. The available diversity gain for partially correlated signals is derived. The spatial processing gain for a multichannel DFE are fitted in terms of the signal aperture and signal coherence length and compared with the array gain of conventional beamforming. Details can be found in [8].

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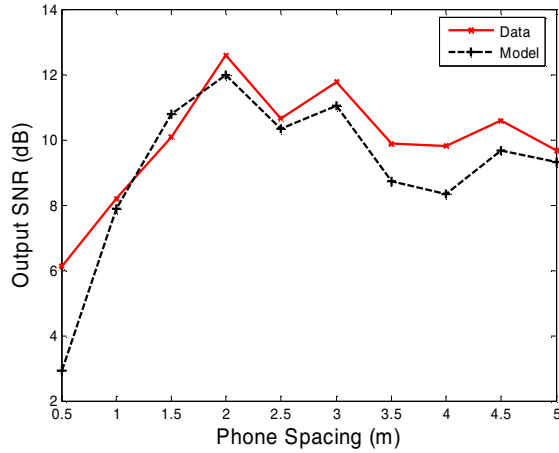


Fig. 3 Output SNR as a function of receiver spacing for an array of four receivers. Solid curve: data. Dashed curve: model.

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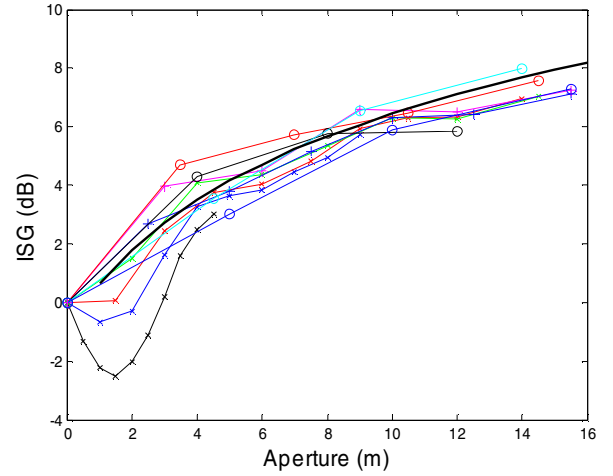


Fig. 4 Modeled ISG as a function of the array aperture with different receiver separations. Thick curve is an empirical fit to the ISG as discussed in text. Same legend as in Fig. 2.