

Adaptive Track Detection for Multi-Static Active Sonar Systems

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Abstract—Multi-static active sonar systems detect contacts of interest by transmitting coherent waveforms and detecting the echoes on one or more receiving sensors. When a target of interest is in a region where its echoes are detectable by more than one receiver it can, in general, be declared sooner by combining the measurements from all sensors. The track detection schemes used in active sonar systems are often based on the Wald Sequential Probability Ratio Test (SPRT) [1] and take as input the amplitudes of the target echoes associated to the track and where the statistical models for the amplitude of a target echo usually depend on a signal-to-noise ratio (SNR) parameter.

Two popular multi-static track detection schemes accumulate a separate SPRT for each target at each sensor. Tracks are declared using one of two rules: if any of the separate SPRTs for a target exceeds the declare threshold then the target is declared (i.e., the OR detector), if the sum of the separate SPRTs goes over the declare threshold then the target is declared (i.e., the SUM detector). The main problem with both methods is that the track detection problem is composite; the distribution for the target-present case depends on the SNR parameter, which is *a priori* unknown and different source/receiver combinations will typically observe different values of SNR on the same target. In practice, a fixed design value (e.g., 10 dB) is often chosen so that each sensor will separately achieve the desired probability of detection for SNR values greater than or equal to the design value. However, when combining measurements from two or more sensors, this approach can be suboptimal when only one sensor is observing a value of SNR at or above the design value and the SNR for the other sensors are lower than the design value. Under such conditions, the OR detector will not achieve any significant increase in the probability of target detection over the single high-SNR sensor and will have a higher probability of false alarm. In the same conditions, the SUM detector will have a lower probability of detection than the separate high-SNR sensor. In effect, the OR and SUM detection schemes will only increase overall system probability of detection when the SNR values for more than one sensor are at or above the design value. Thus, achieving a gain in overall probability of detection requires a scheme that can recognize the conditions under which a group of sensors will observe dissimilar target SNR values and adapt the relevant parameters in the distributions used to compute a single SPRT statistic.

The Multi-Static Adaptive Track Detector (MSATD) is an SPRT based track detection scheme that uses estimates of target aspect derived from track state estimates and a model of bi-static target strength to adapt the parameters in the distribution for

target echo amplitude. Essentially, the SUM detector is modified to use different values for SNR parameter at each sensor. The SNR parameters are determined using a model of bistatic target strength and estimates of the target aspect observed by each sensor computed from the current track state estimate. The theoretical improvement in system track detection performance (i.e., probability of detection and latency) afforded by the proposed method is also presented; theoretically exact expressions for probability of detection and latency are evaluated numerically for all three track detection schemes for a system of one source and two receivers.]

1. INTRODUCTION

Multi-Static active sonar tracking systems are usually built on one of two architectures: distributed or central. In the distributed architecture target tracks are first formed locally at each sensor and a Wald type Sequential Probability Ratio Test (SPRT) is often applied to the normalized amplitudes of the echoes associated to each track used to detect contacts of interest, see figure 1.

Detected target tracks are then reported to a central location where they are compared to detected tracks from the other sensors. Groups of tracks that pass an association test are used to compute a fused track state estimate and detection test statistic. If the fused detection test statistic exceeds a prescribed threshold the fused track is declared to an operator. One popular scheme, the so called OR detector, detects fused tracks by comparing the individual values of the SPRTs from each track in the group. If any of the separate SPRTs exceeds the declare threshold then the track is declared. Another method, the SUM detector, adds the values of the separate SPRTs in each group of tracks and compares it to the declare threshold. In the central architecture the measurements from each sensor are sent to the central location and used to generate a single table of tracks and each track is declared on the basis of a single SPRT, see figure 2. There are several variations of the centralized architecture and the one depicted in figure 2 performs a series of sequential and synchronized updates (one for each sensor) to the tracks at each update cycle.

The main problem with distributed target tracking and detection is that the distribution for the amplitudes of the echoes under the target present (i.e., H_1) hypothesis is

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composite; it depends on an SNR parameter which is a priori unknown.

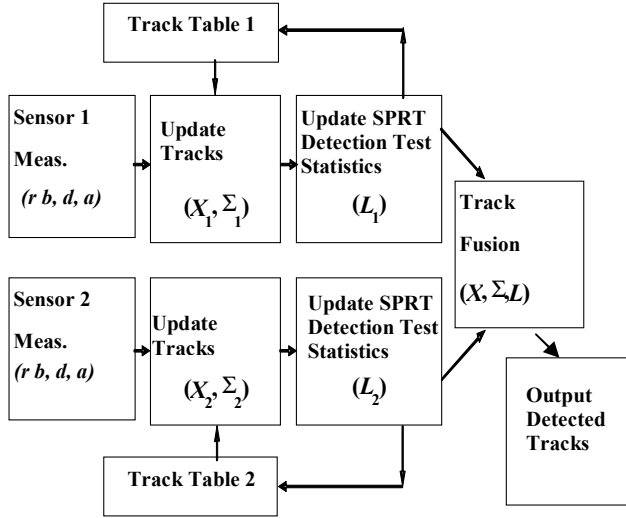


Figure1. Distributed Multi-Sensor Tracking Architecture

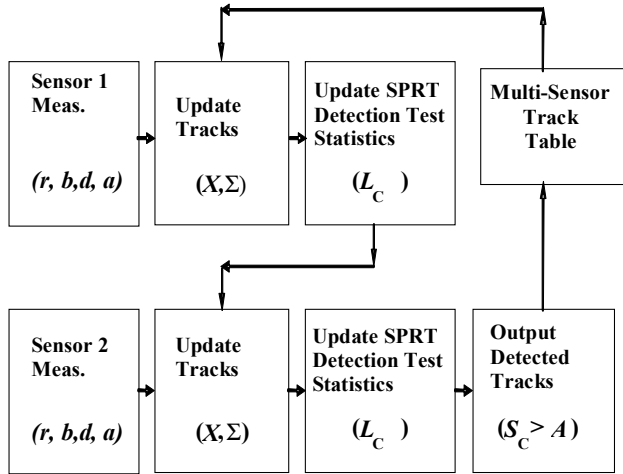


Figure 2. Centralized Multi-Sensor Tracking and Detection

The observed target SNR at each sensor is directly affected by the interference level, environmental propagation conditions and target aspect. Thus each sensor will in general observe a target at a different SNR and the performance of an SPRT using a typical fixed design value (e.g., 10dB) for the SNR parameter will vary significantly. Targets presenting at a lower than design SNR will be detected slower and with lower probability. The tracks of the targets that are missed in this way will never be reported to the central location and used to improve the state estimate and speed the detection of genuine targets of interest. Even when two or more sensors detect a track on a target of interest distributed systems will be

suboptimal when only one sensor is observing a value of SNR at or above the design value and the SNR for the other sensors are lower than the design value. Under such conditions, the OR detector will not achieve any significant increase in the probability of target detection over the single high-SNR sensor and will have a higher probability of false alarm than the SUM detector. Under the same conditions, the SUM detector will have a lower probability of detection than the separate high-SNR sensor because the lower average individual SPRT values from the low SNR sensors will reduce the average combined SPRT value. In effect, the OR and SUM detection schemes will only increase overall system probability of detection when the SNR values for more than one sensor are at or above the design value.

2. PURPOSE

The study reported here investigates the effect of target aspect on target SNR and track detection performance in distributed multi-static active sonar systems. The case where a significant amount of variation in observed SNR due to target aspect is of particular interest. In order to focus on the effect of target aspect the tracking conditions are assumed to be ideal in all other respects: no clutter, zero data registration errors, perfect normalization, perfect data association by all trackers, and a benign environment with identical propagation loss at all sensors. A method to use estimates of target aspect derived from track state estimates and a model of bi-static target strength to adapt the parameters in the distribution for target echo amplitude is presented. Essentially, the SUM detector is employed in a centralized architecture and modified to use different values of the SNR parameter for the data received from each sensor. The theoretical improvement in system track detection performance (i.e., probability of detection and latency) afforded by the proposed method is also presented; theoretically exact expressions for probability of detection and latency are evaluated numerically for all three track detection schemes for a system of one source and two receivers.

3. SIGNAL MODEL AND SEQUENTIAL DETECTION

The development in this section assumes that the measurements from all sensors is perfectly normalized; amplitudes are unit mean Rayleigh distributed in the target absent case (H_0) and Rayleigh distributed with SNR S (in linear scale) under the target present case (H_1). For the case of two sensors the formulas for the relevant distributions are;

$$\begin{aligned} f_1(a_1 | H_0) &= \frac{\pi a_1}{2} e^{-\pi a_1^2 / 4} \\ f_2(a_2 | H_0) &= \frac{\pi a_2}{2} e^{-\pi a_2^2 / 4} \end{aligned} \quad (1)$$

and

$$f_1(a_1|H_1, S_1) = \frac{\pi a_1}{2(1+S_1)} e^{-\pi a_1^2 / 4(1+S_1)} \quad (2)$$

$$f_2(a_2|H_2, S_2) = \frac{\pi a_2}{2(1+S_2)} e^{-\pi a_2^2 / 4(1+S_2)}$$

where S_1 and S_2 are signal to noise ratio parameters. The algorithm described here will be derived for the case of two sensors but can be easily generalized to more than two sensors. For each sensor the separate SPRTs are given by

$$\begin{aligned} L_1(k) &= L_1(k-1) + \ln(f_1(a_1(k)|H_1, S_{D1})) - \dots \\ &\quad \ln(f_1(a_1(k)|H_0)) \\ &= L_1(k-1) - \ln(1+S_{D1}) + \frac{\pi}{4} \left(\frac{S_{D1}}{1+S_{D1}} \right) a_1^2 \\ L_2(k) &= L_2(k-1) + \ln(f_2(a_2(k)|H_1, S_{D2})) - \dots \\ &\quad \ln(f_2(a_2(k)|H_0)) \\ &= L_2(k-1) - \ln(1+S_{D2}) + \frac{\pi}{4} \left(\frac{S_{D2}}{1+S_{D2}} \right) a_2^2 \end{aligned} \quad (3)$$

where S_{D1} and S_{D2} are the design signal to noise ratio values. The decision rule for the OR detector then is

$$\text{If } L_{OR}(k) \equiv \max\{L_1(k), L_2(k)\} > A_{OR}, \quad (4)$$

then declare the track

and the decision rule for the SUM detector has the form

$$\text{If } L_{SUM}(k) \equiv L_1(k) + L_2(k) > A_{SUM}, \quad (5)$$

then declare the track

The rules for dropping a track are similar but with the inequalities reversed and different thresholds.

The centralized Multi-Static Adaptive track Detector SPRT is

$$\begin{aligned} L_{MSATD}(k) &= L_{MSATD} + \dots \\ &\quad \ln(f_1(a_1(k)|H_1, \tilde{S}_1)) - \ln(f_1(a_1(k)|H_0)) \\ &\quad \ln(f_2(a_2(k)|H_1, \tilde{S}_1)) - \ln(f_2(a_2(k)|H_0)) \\ &= L_{MSATD}(k-1) + \dots \\ &\quad \frac{\pi}{4} \left(\frac{\tilde{S}_1}{1+\tilde{S}_1} \right) a_1(k)^2 - \ln(1+\tilde{S}_1) + \dots \\ &\quad \frac{\pi}{4} \left(\frac{\tilde{S}_2}{1+\tilde{S}_2} \right) a_2(k)^2 - \ln(1+\tilde{S}_2) \end{aligned} \quad (6)$$

with decision rule

$$\text{If } L_{MSATD}(k) \Rightarrow A_{MSATD}, \quad (7)$$

then declare the track

The centralized detector has essentially the same form as the SUM detector but where the SNR parameters $\tilde{S}_1$ and $\tilde{S}_2$ are not fixed parameters but instead are modified at each update using an aspect dependent model of target strength and estimates of target aspect computed from the track state estimates and the locations of the sensors.

4. TARGET STRENGTH MODEL AND PARAMETER ADAPTATION METHOD

In distributed multi-static active sonar systems target aspect is a significant source of the variability in target SNR observed by different source/receiver combinations. When a target can be detected by more than one sensor of a centralized system it makes intuitive sense that it can, in general, be detected sooner if all of the measurements are used in one detection test statistic. Figure 1 is a plot of mono-static target strength as a function of aspect from [3]. It shows that there is an 18dB difference in target strength between broadside and forward end fire aspects. Bistatic target strength values of target strength can be obtained by using the Bistatic Theorem; the bistatic target strength is the monostatic target strength at the bisector of the incident and reflected directions

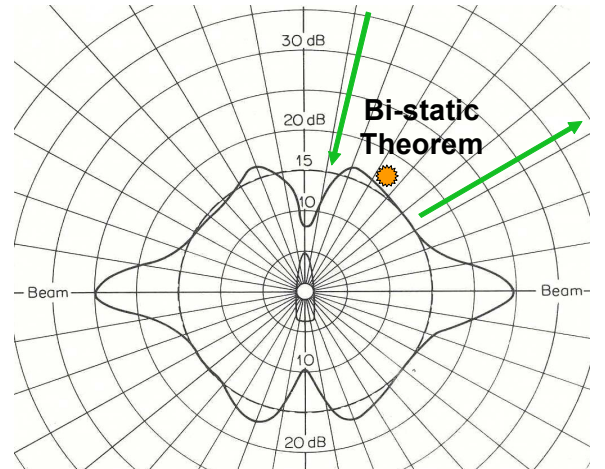


Figure 1. Mono-static target strength vs. aspect.

The variability of the observed target SNR can cause detection schemes that use fixed models of target echo amplitude to perform less than optimally. For example, suppose that two identical but separated sensors can detect a target's echo with comparable interference levels and propagation path lengths but significantly different target strengths due to different target aspects. If the same fixed SNR value is used in equation (2) for both sensors then the increment to the SPRT from at least one sensor will be incorrect. For the sensor observing the target at the lower SNR a missed target detection will cause a greater reduction of the SPRT than it should and a detection may not provide as much of a positive contribution to the SPRT as it should. This, in turn, may delay or prevent the declaration of a target of interest.

Mono-static systems that attempt to use measurements from multiple band separated active waveforms in the same tracking algorithm must contend with the same issue when there are different levels of interference in each frequency band. In [2] the author presents a method to adapt the SNR parameters in equation (2) to differing levels of interference for mono-static systems. This paper applies that method to multi-static systems to adapt to differing target aspects. For simplicity it is assumed that there is one source and two identical receivers each employing the same waveform detector (e.g., matched filter) and that the differences in interference level and propagation path length observed by each receiver are negligible. It is also assumed that an estimate of course (and hence target aspect) is available from the current track state estimates. The method can be extended to more general multi-static situations but that is the subject of future work. Under these circumstances the difference in observed SNR by each receiver will be due to differing target aspect. The method also assumes that the maximum possible target SNR, $S_{\max}$, is known. Although this assumption may seem unreasonable the mono-static version of this method is based on a similar assumption and in [2] it is shown that the track detection performance gain is robust to errors in $S_{\max}$.

Let D_1 and D_2 be the reduction in target strength (in linear scale) from the maximum possible (e.g., broadside aspect) for each receiver as determined by a bi-static target strength model (e.g., Figure 1) using the estimated target aspects. The values for SNR to be used in equation (3) for each receiver are then given by

$$S_1 = D_1 S_{\max} \text{ and } S_2 = D_2 S_{\max} \quad (8)$$

When the value for $S_{\max}$ is correct equation (8) will provide the correct SNR values for equation (2) and ensure that the SPRT update computed using equation (6) is also correct. When the value for $S_{\max}$ is incorrect the SNR values given by equation (8) will also be incorrect but will at least have the correct ratio. In practice this property alone has proven sufficient to provide significant tracking and detection performance gain.

5. THEORETICAL PERFORMANCE PREDICTIONS

In this section the theoretical performance of the three track declaration methods will be compared for the system of one source and two receivers shown in figure 2. The system geometry is deliberately chosen to be symmetric to eliminate the effects of dissimilar propagation path lengths and interference level. In this section the theoretical performance of the three track declaration methods will be compared for the system of one source and two receivers shown in figure 2. The system geometry is deliberately chosen to be symmetric to eliminate the effects of dissimilar propagation path lengths and interference level

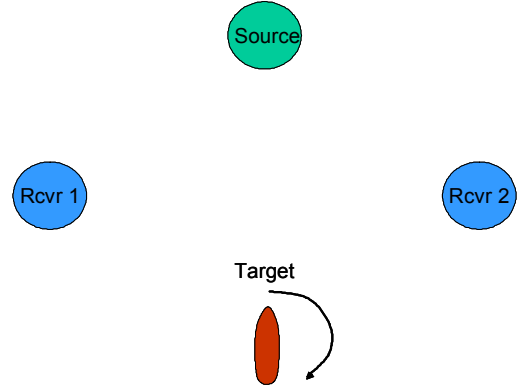


Figure2. Multi-Static System Geometry

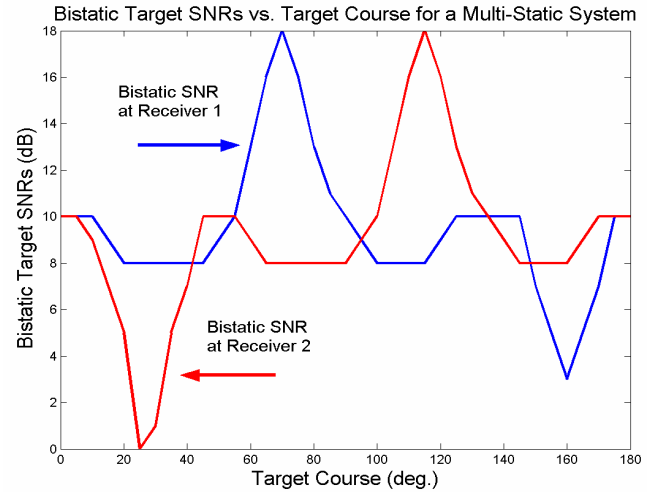


Figure 3. Bistatic Target SNR vs. Target Course for the Multi-Static System, $S_{\max} = 18\text{dB}$.

Figure 3 is a plot of the expected variation in target SNR as a function of aspect for the geometry in figure 2 and the target strength model in figure 1. Note that there is no value of target aspect for which both sensors observe the target at the maximum SNR. Moreover, when one sensor is observing the target at the lowest possible SNR the other sensor is only observing the sensor at approximately 8dB.

The true values of probability of detection (P_d), probability of false alarm (P_{fa}), and average number of scans to decision ($E[N]$) over a wide range of observed SNR values are required to fairly compare the detection and latency performance of the different system architectures. Exact expressions for P_d , P_{fa} , and $E[N]$ can be computed from the probability mass functions for crossing the detect and drop threshold. Let

$$z_i(k) = \frac{\pi}{4} \left(\frac{S_{Di}}{1 + S_{di}} \right) a_i(k)^2 - \ln(1 + S_{Di}) \quad (9)$$

be the update to the SPRT $L_i(k)$ for sensor $i=1$ or 2 at time k . It follows from equations (1) and (2) that the probability density functions for $z_i(k)$ are

$$f_0(z_i(k)) \equiv f(z_i(k)|H_0) = \frac{(1+S_{Di})}{S_{Di}} e^{-\frac{(z_i(k)+\ln(1+S_{Di}))(1+S_{Di})}{S_{Di}}} \quad (10)$$

and

$$f_1(z_i(k)) \equiv f(z_i(k)|H_1, S_i) = \frac{(1+S_{Di})}{S_{Di}(1+S_i)} e^{-\frac{(z_i(k)+\ln(1+S_{Di}))(1+S_{Di})}{S_{Di}(1+S_i)}} \quad (11)$$

The independence of the updates $z_i(k)$ implies that the sequence of SPRT values $L_i(k)$ constitutes a Markov process. In [4] an analytic expression for the probability mass function for time to decision is derived under the assumption that the probabilities that the SPRT $L_i(k)$ will exceed the detection or the drop thresholds at time k for the first time are constant. Although that additional assumption does lead to an analytic solution, it is not realistic and, given modern computing capabilities, not even necessary. It follows from equations (10) and (11) that the probabilities that the SPRT $L_i(k)$ will exceed the detection threshold A at time k for the first time, $P_{Ai}(k|H_0)$ and $P_{Ai}(k|H_1)$, are given by

$$\begin{aligned} P_{Ai}(k|H_0, S_{Di}) &\equiv \Pr[B < L_i(j) < A \\ &\text{for } j < k \text{ and } L_i(k) \geq A|H_0] \\ &= \int_B^A f_0(L_i(1)) \int_B^A f_0(L_i(2)-L_i(1)) \cdots \\ &\int_B^A f_0(L_i(k-2)-L_i(k-1)) \cdots \\ &\int_A^\infty f_0(L_i(k-1)-L_i(k)) dL_i(1) \cdots dL_i(1) \end{aligned} \quad (12)$$

Similarly, the probabilities that the SPRT $L_i(k)$ will cross the drop threshold B at time k for the first time are given by

$$\begin{aligned} P_{Bi}(k|H_0, S_{Di}) &\equiv \Pr[B < L_i(j) < A \\ &\text{for } j < k \text{ and } L_i(k) \leq B|H_0] \\ &= \int_B^A f_0(L_i(1)) \int_B^A f_0(L_i(2)-L_i(1)) \cdots \\ &\int_B^A f_0(L_i(k-2)-L_i(k-1)) \cdots \\ &\int_{-\infty}^B f_0(L_i(k-1)-L_i(k)) dL_i(1) \cdots dL_i(1) \end{aligned} \quad (13)$$

In general, the nested integrals given in equations (12) and

(13) cannot be evaluated analytically, but they can easily be computed numerically by repeated convolution of suitably sampled versions of the integrand functions. Once the probabilities defined in equations (12) and (13) have been obtained, the true probabilities of detection and false alarm for an individual SPRT are given by

$$P_D(S_i) = \sum_{k=1}^{\infty} P_{Ai}(k|H_1, S_{Di}, S_i) \quad (14)$$

and

$$P_{fa} = \sum_{k=1}^{\infty} P_{Ai}(k|H_0, S_{Di}). \quad (15)$$

The true value for the average number of scans to decision is

$$\begin{aligned} E[N|H_1, S_{Di}, S_i] &= \sum_{k=1}^{\infty} k [P_{Ai}(k|H_1, S_{Di}, S_i) \\ &+ P_{Bi}(k|H_1, S_{Di}, S_i)]. \end{aligned} \quad (16)$$

The expressions for P_d , P_{fa} and $E[N]$ for the OR, SUM and MSATD decision rules are computed similarly and given in [2].

Figure 4 is a plot of the probability of detection for all three track detection systems as a function of target course. Thresholds for all three detectors were chosen that achieved a maximum probability of false alarm of 1.0×10^{-6} and a probability of detection of 0.99 when $S_1=18$ dB and $S_2=8$ dB. Essentially, the detectors were designed to achieve the design value for probability of detection (i.e., 0.99) when the observed SNR values are at the maximum possible for the system geometry in figure 2. The theoretical probability of detection was then computed for the three detectors at all possible values of target course in five degree increments and the results plotted in figure 4. The OR and SUM detectors use fixed values for the design SNR values (i.e., $S_{D1}=10$ dB and $S_{D2}=10$ dB) and hence have significantly reduced probability of detection when the observed SNR values are less than the values used to determine the thresholds (i.e., $S_1=18$ dB and $S_2=8$ dB). The SUM detector suffers the greatest loss of probability of detection when the target course is 25 degrees (i.e., when $S_1=8$ dB and $S_2=0$ dB) and the OR detector is only slightly better. The MSATD detector, however, is able to maintain the design probability of detection at all values of target course.

Achieving this level of improvement requires exact knowledge of the target course. In practice only an estimate of target course containing some amount of error will be available. The dotted green line in figure 4 is the probability of detection for the MSATD using target course values with 25 degrees of error. With the erroneous target course values the MSATD probability of detection performance is slightly reduced at the less favorable values of target course but still substantially better than the OR and SUM detectors. These

results show that the MSATD can achieve significant improvement in probability of target detection over the OR and SUM detectors and the gain is robust to errors in the target course estimate.

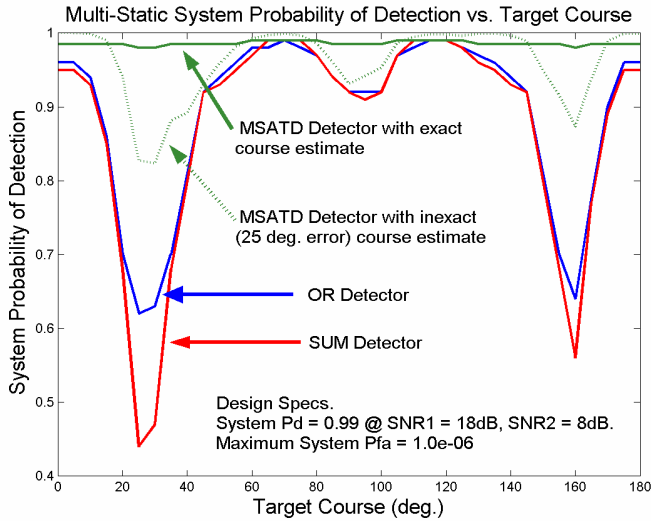


Figure 4. Multi-Static System Probability of Detection vs. Target Course.

System latency (i.e., the expected time to decision) is another important performance metric for sequential detection methods. The theoretical system latency as a function of target course for all three detectors is shown in figure 5. In general, the latency of a sequential detector is inversely proportional to the detection performance; better the detection performance requires longer latency. Accordingly, the SUM detector, which consistently has the poorest detection performance also has the lowest latency. The OR detector is next best in both detection and latency performance and the MSATD has the highest latency. However, the difference in latency between the MSATD and SUM detector is at most one scan which for most applications is a reasonable trade off to achieve significantly better detection performance. The latency of the MSATD with erroneous target course estimates is only negligibly different from the MSATD latency with exactly correct course values.

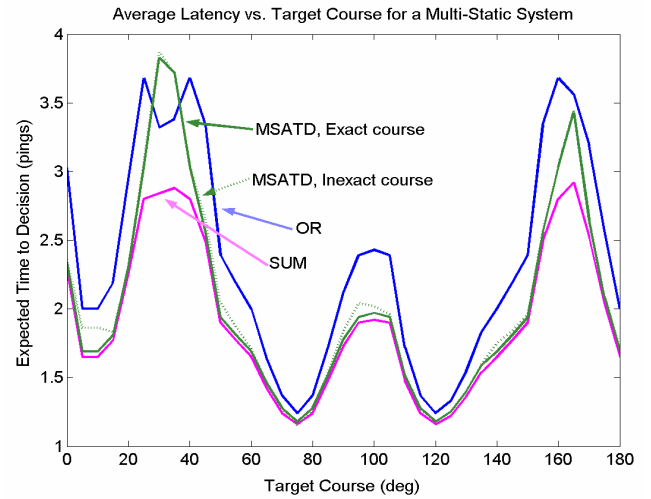


Figure 5. Multi-Static System Latency vs. Target Course.

6. CONCLUSIONS

The results in the preceding section clearly show that adapting the distribution for target echo amplitude to the observed target aspect improves track detection performance in multi-static active sonar systems. In the near future the author plans to incorporate more sophisticated target strength models and to generalize this method to adapt to differing levels of interference, propagation path length, and environmental conditions.

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